On Optimal Input Design in System Identification for Model Predictive Control

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Outline

1. Theory
2. Identification Algorithm
3. MPC Example
4. Conclusions
5. Future Work
Introduction

Framework for experiment design in system identification for control, specifically MPC.

- **Objective:**
  
  Find an input signal that minimizes the cost related to the system identification experiment.

- **Constraint:**
  
  A specified control performance is guaranteed when using the estimated model in the control design.
The model structure is parametrized by $\theta$.

- True system is given by $\theta_0$.
- Estimated model is given by $\hat{\theta}$.
Application Set

- Application cost: \( V_{\text{app}}(\theta, \theta_0) \), for example
  \[
  V_{\text{app}}(\theta, \theta_0) = \frac{1}{T} \sum_{t=1}^{T} \| y_t(\theta_0) - y_t(\theta) \|_2^2.
  \]

- Application specification:
  \[
  V_{\text{app}}(\theta, \theta_0) \leq \frac{1}{\gamma}, \quad \gamma > 0.
  \]

- Acceptable parameter set:
  \[
  \Theta_{\text{app}}(\gamma) = \left\{ \theta \mid V_{\text{app}}(\theta, \theta_0) \leq \frac{1}{\gamma} \right\}.
  \]
Ellipsoidal approximation:

\[ \Theta_{app}(\gamma) \approx \mathcal{E}_{app}(\gamma) = \left\{ \theta \mid (\theta - \theta_0)^T V''_{app}(\theta_0, \theta_0)(\theta - \theta_0) \leq \frac{2}{\gamma} \right\}. \]
Scenario approach:

$$\Theta_{app}(\gamma) \approx \left\{ \theta_i, \ i = 1 \ldots M < \infty \mid V_{app}(\theta_i, \theta_0) \leq \frac{1}{\gamma} \right\}.$$
Asymptotic quality property:

$$\hat{\theta} \in \mathcal{E}_{SI}(\alpha) = \left\{ \theta \mid (\theta - \theta_0)^T I_F (\theta - \theta_0) \leq \frac{\chi^2_{\alpha}(n)}{N} \right\}.$$ (Key result from prediction error/maximum likelihood system identification.)
Optimal Input Signal Design

- Estimated parameters:
  \[
  \hat{\theta} \in \mathcal{E}_{SI}(\alpha) = \left\{ \theta \mid (\theta - \theta_0)^T I_F(\theta - \theta_0) \leq \frac{\chi^2_\alpha(n)}{N} \right\}.
  \]

- Acceptable parameters in application:
  \[
  \hat{\theta} \in \Theta_{app}(\gamma) = \left\{ \theta \mid V_{app}(\theta, \theta_0) \leq \frac{1}{\gamma} \right\}.
  \]

- Experiment cost:
  \[
  f_{cost}(\Phi_u).
  \]
Optimal Input Signal Design (cont.)

Optimization Problem

\[
\begin{align*}
\text{minimize} & \quad f_{\text{cost}}(\Phi_u) \\
\text{subject to} & \quad \mathcal{E}_{SI}(\alpha) \subseteq \Theta_{app}(\gamma) \\
& \quad 0 \leq \Phi_u(\omega), \quad \forall \omega
\end{align*}
\]

Can be approximated as a convex problem! Using:

- ellipsoidal approximation \( \Rightarrow \) LMI,
- scenario approach \( \Rightarrow \) scalar linear inequalities,
- finite dimensional parametrization \( \Rightarrow \) LMI.
### Optimization Problem

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- scenario approach \( \Rightarrow \) scalar linear inequalities,
- finite dimensional parametrization \( \Rightarrow \) LMI.
Identification Algorithm

Issues:

- $\theta_0$ is unknown.
- Evaluation of $V_{app}(\theta, \theta_0)$. 
Identification Algorithm

Issues:

• $\theta_0$ is unknown.
• Evaluation of $V_{app}(\theta, \theta_0)$.

Solutions:

• Use estimates instead of $\theta_0$.
• Evaluate $V_{app}(\theta, \theta_0)$ in simulation.
Proposed algorithm:

1. Find an initial estimate of $\theta_0$.
2. Evaluate $V_{app}(\theta, \theta_0)$ in simulation.
3. Design the optimal input signal.
4. Find a new estimate of $\theta_0$.

Discussion on iterative approach:
MPC Example

Control objective:
Reference tracking of the lower tank levels using MPC.
MPC Example (cont)

- Application cost:

\[ V_{\text{app}}(\theta, \theta_0) = \frac{1}{T} \sum_{t=1}^{T} \| y_t(\theta_0) - y_t(\theta) \|_2^2. \]

- Experiment cost: Input power,

\[ f_{\text{cost}}(\Phi_u) = \text{trace} \left( \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi_u(\omega) d\omega \right). \]
\[ V_{\text{app}}(\theta, \theta_0) \approx V_{\text{app}}(\theta, \hat{\theta}) = \frac{1}{T} \sum_{t=1}^{T} \| y_t(\hat{\theta}, \hat{\theta}) - y_t(\theta, \hat{\theta}) \|^2 \]
• Optimal: 91% success.

• White: 15% success.

White: $8 \times N$ gives same success rate as optimal.
Conclusions

- Identification algorithm for MPC.
- Increased control performance.
- Linear framework applicable on nonlinear systems.
Future Work

• How to choose $V_{app}$.
• Realistic MPC applications.
• Closed-loop identification.
• Toolbox for optimal input design, MOOSE.
MOOSE is a model based optimal input design toolbox developed for Matlab.

It features

- optimal input design,
- easy-to-use text interface,
- compatibility with Matlab Control System Toolbox.
Thank you!
Additional slides
Real Water Process

For details:

Real Water Process (cont)

White:

For details:

- High power: 0 % success.

- Low power: 85 % success.
Optimal Input Signal Design (cont.)

Geometric interpretation

- Cost function:
  minimize input energy ⇔
  maximize $\mathcal{E}_{SI}$.

- Region constraint:
  $\mathcal{E}_{SI}(\alpha) \subseteq \Theta_{app}(\gamma) \rightarrow$
  $\mathcal{E}_{SI}(\alpha) \subseteq \mathcal{E}_{app}(\gamma)$.

Figure: $\mathcal{E}_{SI}$ (blue) and $\mathcal{E}_{app}$ (black).