KTH Electrical Engineering

# On Optimal Input Design in System Identification for Model Predictive Control

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# Outline

1. Theory
2. Identification Algorithm
3. MPC Example
4. Conclusions
5. Future Work

# Introduction

Framework for experiment design in system identification for control, specifically MPC.

- Objective:

  Find an input signal that minimizes the cost related to the system identification experiment.

- Constraint:

  A specified control performance is guaranteed when using the estimated model in the control design.

# Notation

The model structure is parametrized by $\theta$.

- True system is given by $\theta_0$.
- Estimated model is given by $\hat{\theta}$.

# Application Set

- Application cost: $V_{app}(\theta, \theta_0)$, for example

$$V_{app}(\theta, \theta_0) = \frac{1}{T} \sum_{t=1}^{T} \|y_t(\theta_0) - y_t(\theta)\|_2^2.$$

- Application specification:

$$V_{app}(\theta, \theta_0) \leq \frac{1}{\gamma}, \ \gamma > 0.$$

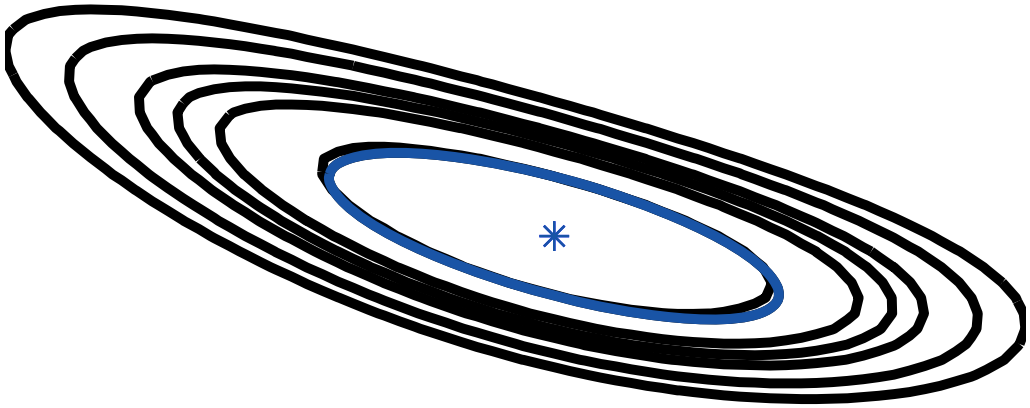
- Acceptable parameter set:

$$\Theta_{app}(\gamma) = \left\{ \theta \mid V_{app}(\theta, \theta_0) \leq \frac{1}{\gamma} \right\}.$$

# Application Set (cont.)

Ellipsoidal approximation:

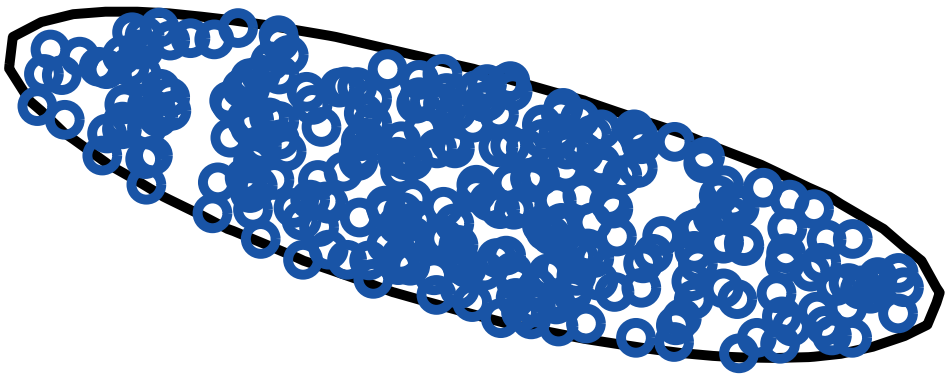
$$\Theta_{app}(\gamma) \approx \mathscr{E}_{app}(\gamma) = \left\{ \theta \mid (\theta - \theta_0)^{\mathrm{T}} V''_{app}(\theta_0, \theta_0)(\theta - \theta_0) \leq \frac{2}{\gamma} \right\}.$$



# Application Set (cont.)

Scenario approach:

$$\Theta_{app}(\gamma) \approx \left\{ \theta_i,\ i = 1 \ldots M < \infty | V_{app}(\theta_i, \theta_0) \leq \frac{1}{\gamma} \right\}.$$



More on scenario approach:

G. C. Calafiore and M. C. Campi, 2006.

# System Identification Set

Asymptotic quality property:

$$\hat{\theta} \in \mathscr{E}_{SI}(\alpha) = \left\{ \theta \mid (\theta - \theta_0)^{\mathrm{T}} \mathbf{I}_F (\theta - \theta_0) \leq \frac{\chi^2_\alpha(n)}{N} \right\}.$$

(Key result from prediction error/maximum likelihood system identification.)

# Optimal Input Signal Design

- Estimated parameters:

$$\hat{\theta} \in \mathscr{E}_{SI}(\alpha) = \left\{ \theta \mid (\theta - \theta_0)^{\mathrm{T}} \mathbf{I}_F (\theta - \theta_0) \leq \frac{\chi^2_\alpha(n)}{N} \right\}.$$

- Acceptable parameters in application:

$$\hat{\theta} \in \Theta_{app}(\gamma) = \left\{ \theta \mid V_{app}(\theta, \theta_0) \leq \frac{1}{\gamma} \right\}.$$

- Experiment cost:

$$f_{cost}(\Phi_u).$$

# Optimal Input Signal Design (cont.)

## Optimization Problem

$$
\begin{aligned}
\underset{\Phi_u}{\text{minimize}} \quad & f_{cost}(\Phi_u) \\
\text{subject to} \quad & \mathscr{E}_{SI}(\alpha) \subseteq \Theta_{app}(\gamma) \\
& 0 \leq \Phi_u(\omega), \quad \forall\, \omega
\end{aligned}
$$

# Optimal Input Signal Design (cont.)

## Optimization Problem

$$\begin{aligned} \underset{\Phi_u}{\text{minimize}} \quad & f_{cost}(\Phi_u) \\ \text{subject to} \quad & \mathscr{E}_{SI}(\alpha) \subseteq \Theta_{app}(\gamma) \\ & 0 \leq \Phi_u(\omega), \quad \forall\, \omega \end{aligned}$$

Can be approximated as a convex problem!

Using:

- ellipsoidal approximation $\Rightarrow$ LMI,
- scenario approach $\Rightarrow$ scalar linear inequalities,
- finite dimensional parametrization $\Rightarrow$ LMI.

# Identification Algorithm

Issues:

- $\theta_0$ is unknown.
- Evaluation of $V_{app}(\theta, \theta_0)$.

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- $\theta_0$ is unknown.
- Evaluation of $V_{app}(\theta, \theta_0)$.

Solutions:

- Use estimates instead of $\theta_0$.
- Evaluate $V_{app}(\theta, \theta_0)$ in simulation.

# Identification Algorithm (cont)

Proposed algorithm:

1. Find an initial estimate of $\theta_0$.
2. Evaluate $V_{app}(\theta, \theta_0)$ in simulation.
3. Design the optimal input signal.
4. Find a new estimate of $\theta_0$.

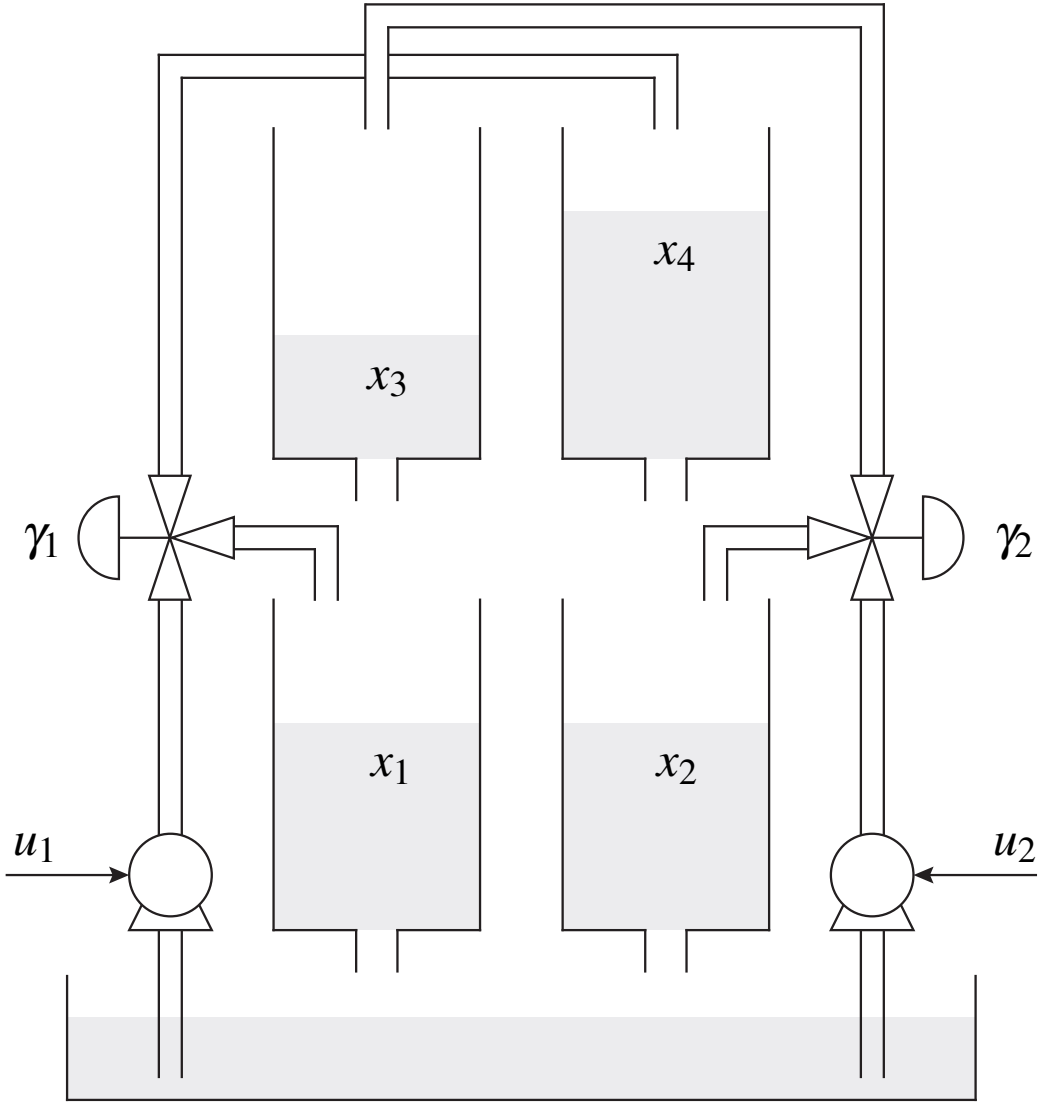
Discussion on iterative approach:

L. Gerencsér, H. Hjalmarsson, J. Mårtensson, 2009.

# MPC Example

Control objective:

Reference tracking of the lower tank levels using MPC.



# MPC Example (cont)

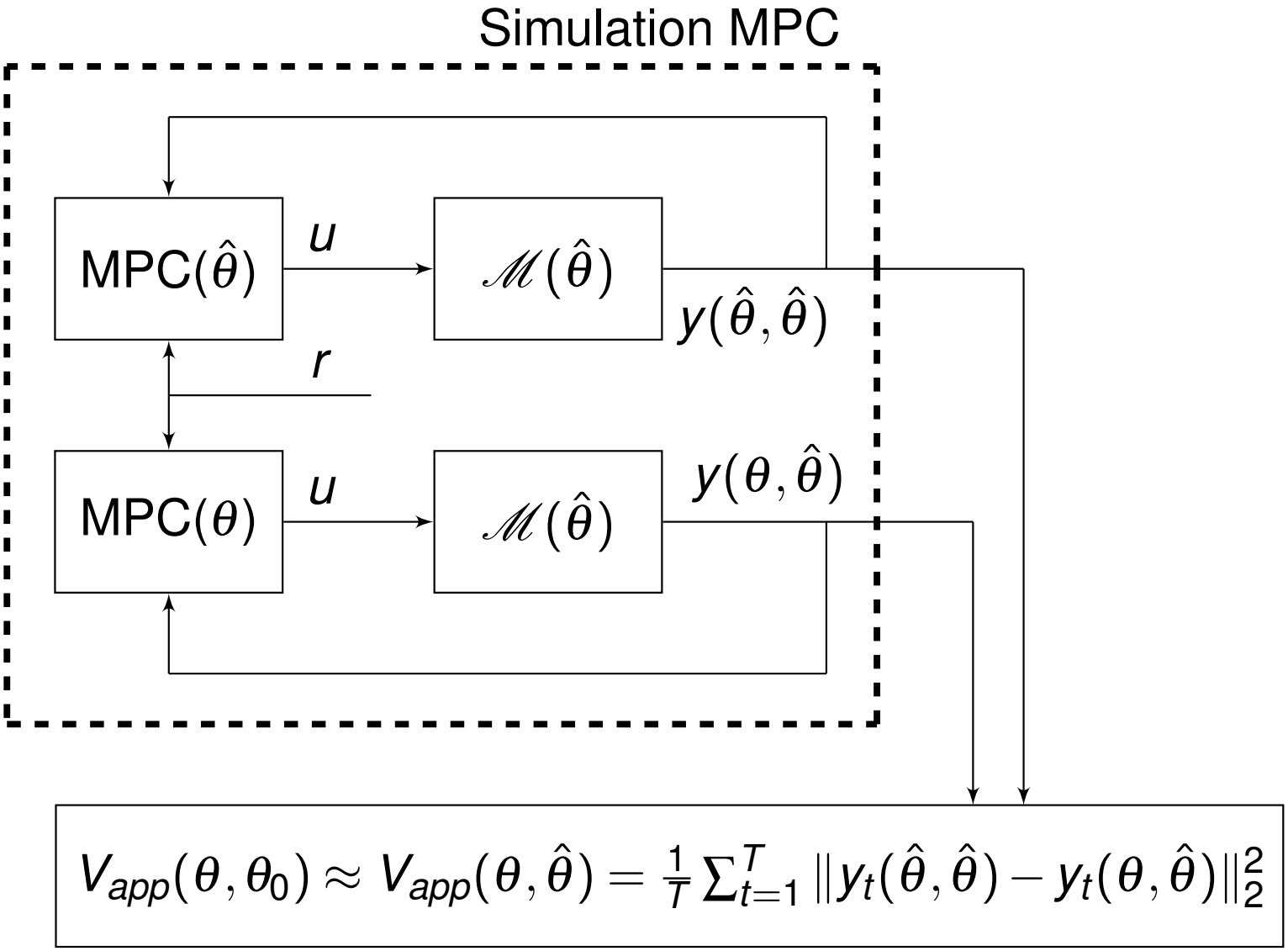
- Application cost:

$$V_{app}(\theta, \theta_0) = \frac{1}{T} \sum_{t=1}^{T} \|y_t(\theta_0) - y_t(\theta)\|_2^2.$$

- Experiment cost: Input power,

$$f_{cost}(\Phi_u) = \text{trace}\left( \frac{1}{2\pi} \int_{-\pi}^{\pi} \phi_u(\omega) d\omega \right).$$

# MPC Example (cont)




$$V_{app}(\theta,\theta_0) \approx V_{app}(\theta,\hat{\theta}) = \frac{1}{T}\sum_{t=1}^{T} \|y_t(\hat{\theta},\hat{\theta}) - y_t(\theta,\hat{\theta})\|_2^2$$

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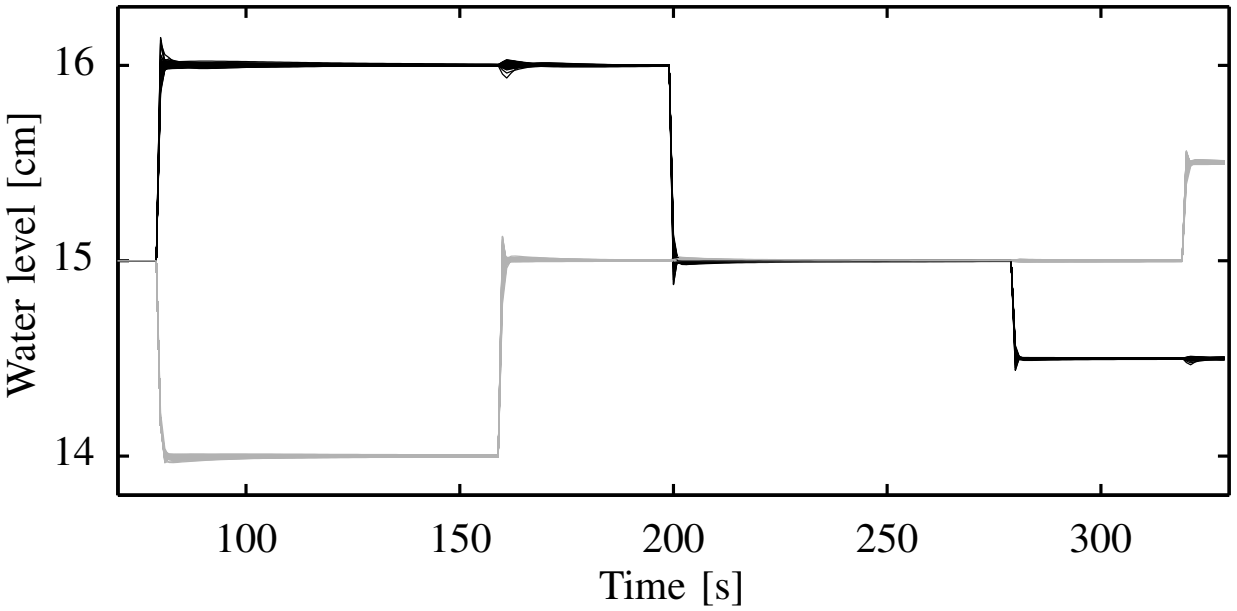
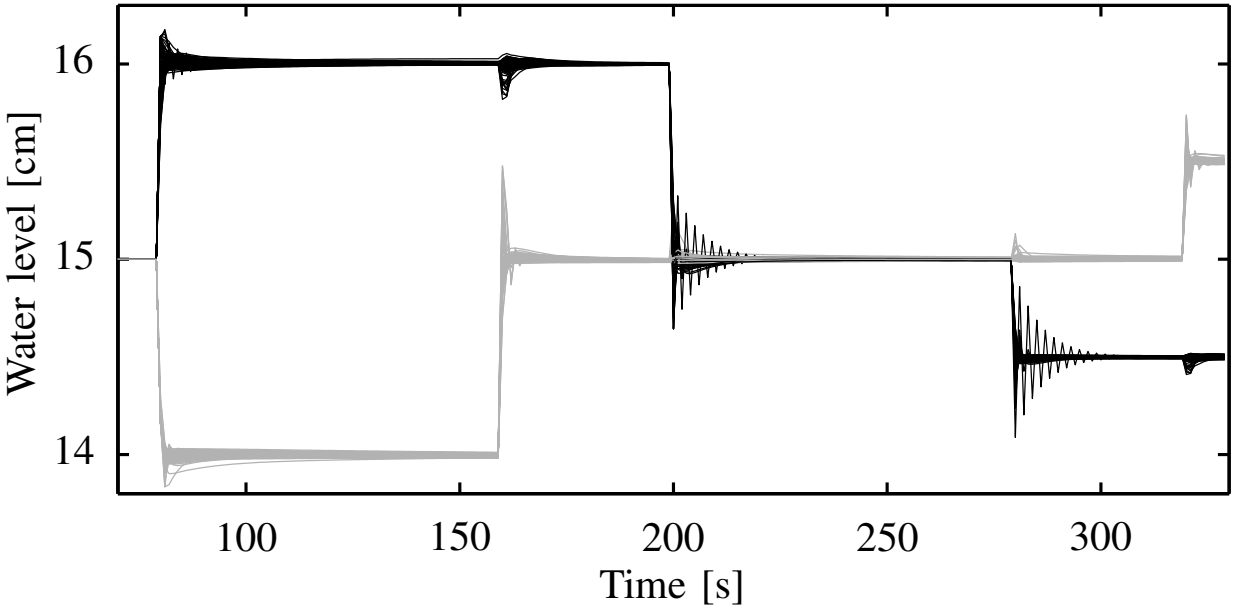
- Optimal:

  91 % success.




- White:

  15 % success.




White: $8 \times N$ gives same success rate as optimal.

# Conclusions

- Identification algorithm for MPC.
- Increased control performance.
- Linear framework applicable on nonlinear systems.

# Future Work

- How to choose $V_{app}$.
- Realistic MPC applications.
- Closed-loop identification.
- Toolbox for optimal input design, MOOSE.

# MOOSE, www.ee.kth.se/moose

MOOSE is a model based optimal input design toolbox developed for Matlab.

It features

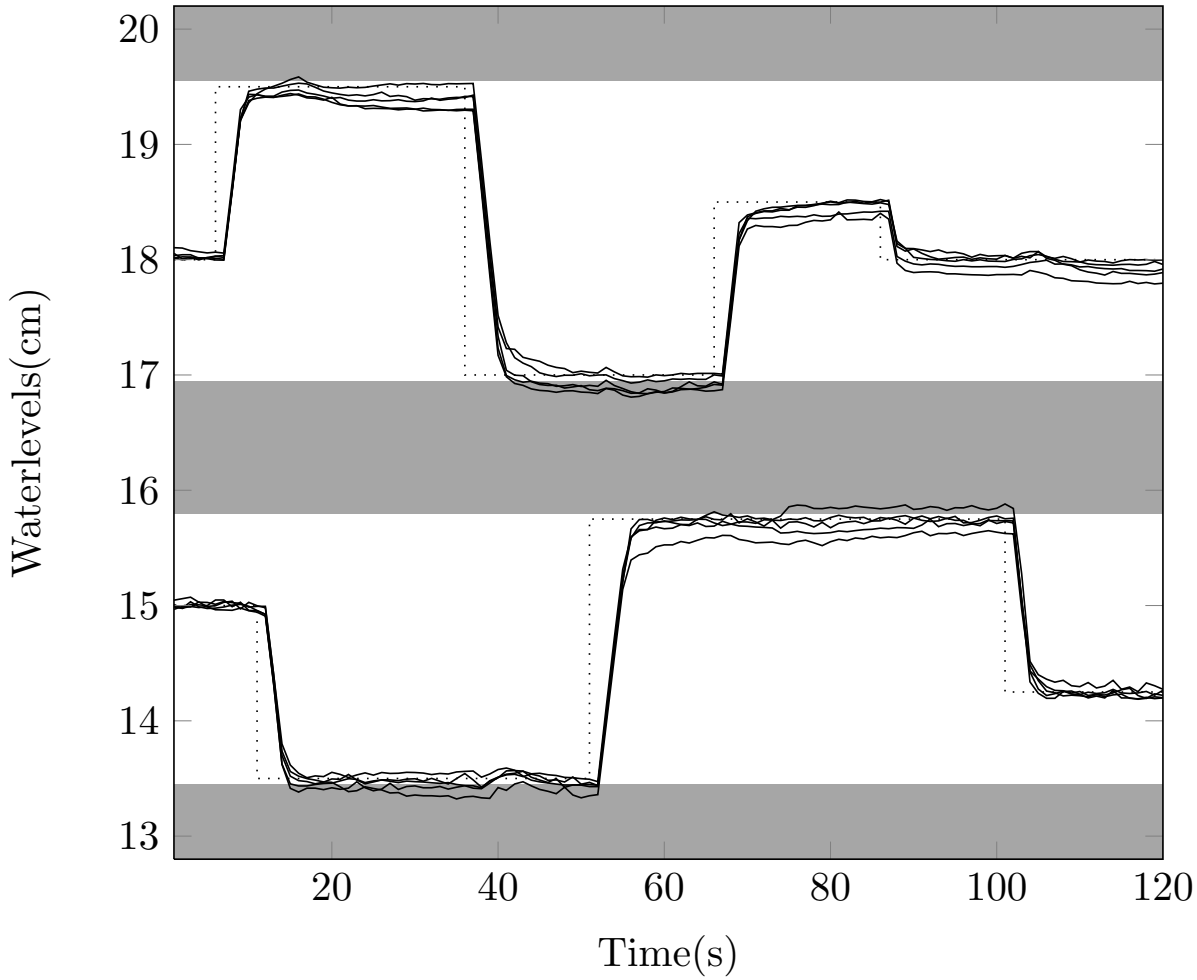
- optimal input design,
- easy-to-use text interface,
- compatibility with Matlab Control System Toolbox.

# Thank you!

# Additional slides

# Real Water Process

Optimal:

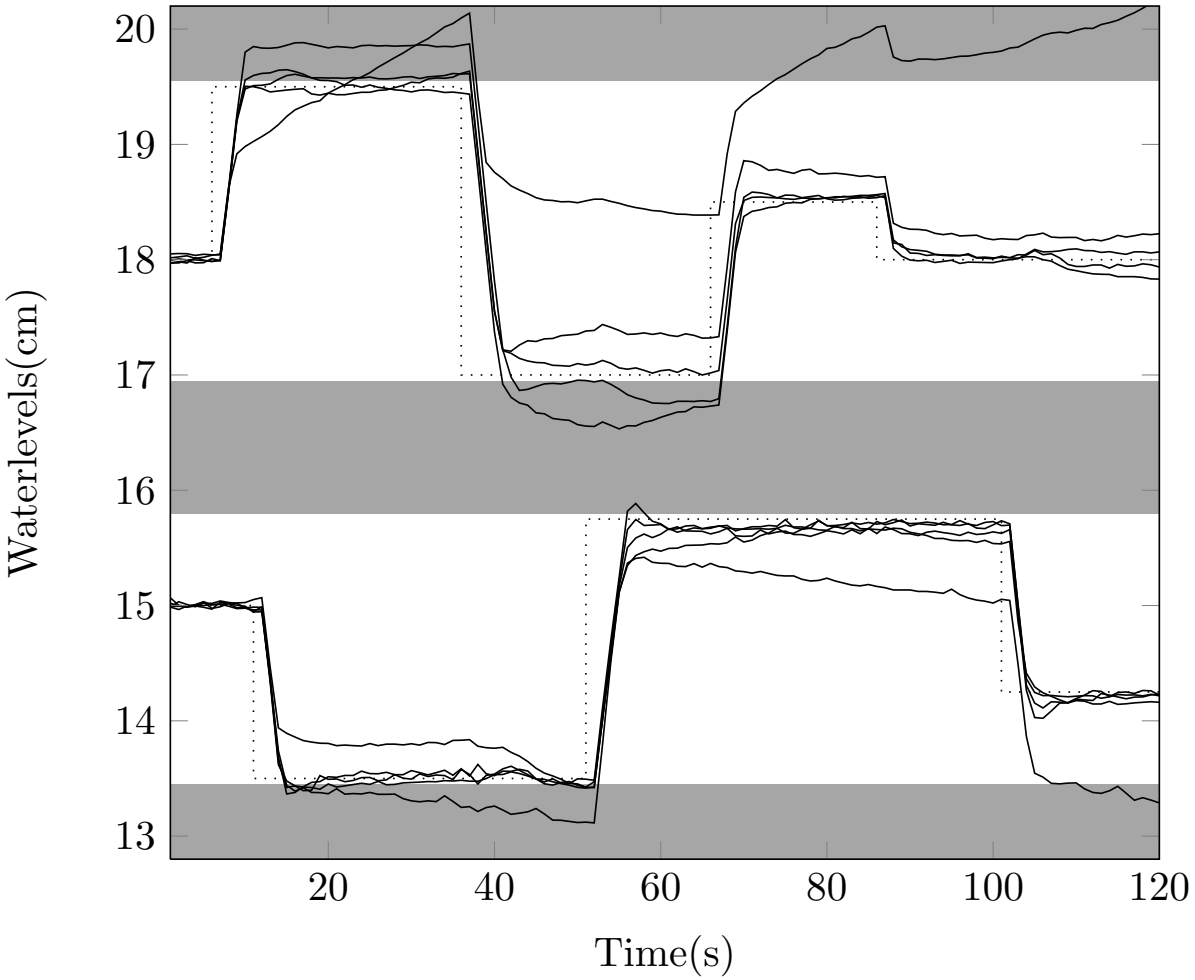



For details:

C. A. Larsson, Licentiate Thesis, 2011.

# Real Water Process (cont)

White:

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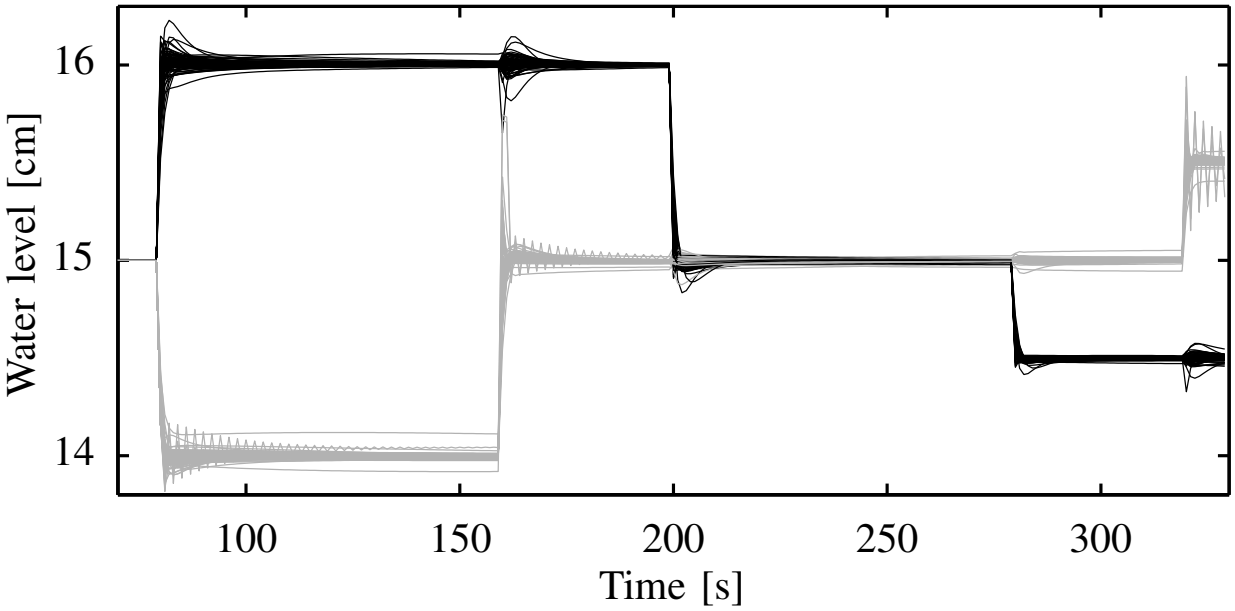
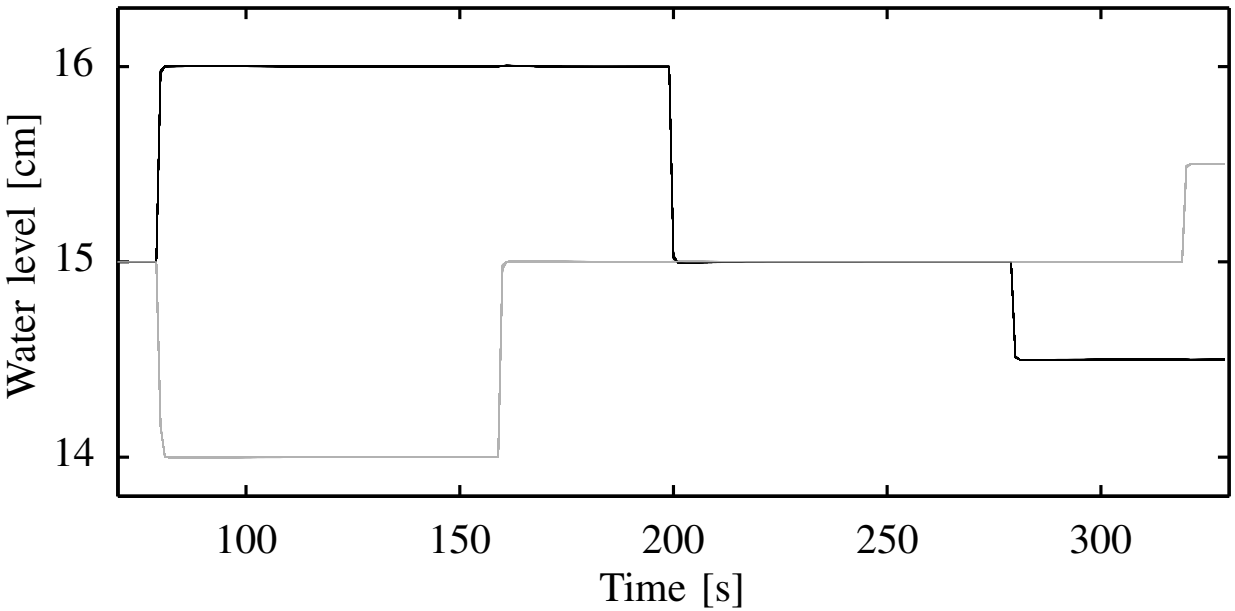
- High power:

  0 % success.




- Low power:

  85 % success.



# Optimal Input Signal Design (cont.)

Geometric interpretation

- Cost function:

  minimize input energy $\Leftrightarrow$ maximize $\mathscr{E}_{SI}$.

- Region constraint:

  $\mathscr{E}_{SI}(\alpha) \subseteq \Theta_{app}(\gamma) \rightarrow$
  $\mathscr{E}_{SI}(\alpha) \subseteq \mathscr{E}_{app}(\gamma)$.

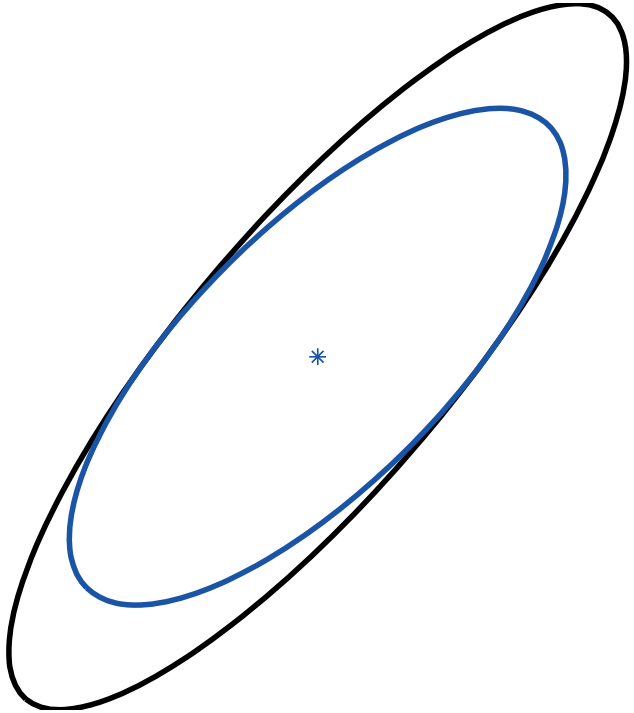


Figure: $\mathscr{E}_{SI}$ (blue) and $\mathscr{E}_{app}$ (black).