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On Optimal Input Design in System Identification for Control

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Outline

1. Theory
2. MPC Example
3. Conclusions
4. Future Work

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Introduction

Framework for experiment design in system identification for control.

- Objective:

  Find a minimum variance input signal to be used in system identification experiment.

- Such that:

  The control application specification is guaranteed when using the estimated model in the control design.

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Notation

The model structure is parametrized by θ.

- True system is given by θ_0.
- Estimated model is given by $\hat{\theta}$.

Application Set

- Application cost: $V_{app}(\theta)$ such that

$$V_{app}(\theta_0) = 0,\ V'_{app}(\theta_0) = 0 \text{ and } V''_{app}(\theta_0) \succeq 0.$$

- Application specification

$$V_{app}(\theta) \leq \frac{1}{2\gamma},\ \gamma > 0.$$

Application Set (cont.)

- Acceptable parameter set

$$\mathscr{E}_{app} = \{\theta \mid V_{app}(\theta) \leq \frac{1}{2\gamma}\}.$$

- Ellipsoidal approximation

$$\mathscr{E}_{app} \approx \{\theta \mid (\theta - \theta_0)^{\mathrm{T}} V''_{app}(\theta_0)(\theta - \theta_0) \leq \frac{1}{\gamma}\}.$$

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Application Set (cont.)

Example 1

- System

$$y(t) = b_1^0 u(t-1) + b_2^0 u(t-2) + d(t), \ \left| b_2^0 / b_1^0 \right| < 1.$$

- P-controller used to reject constant disturbance $d(t)$.
  - Nominal closed loop poles

$$-\beta \text{ and } -\beta b_2/(\beta b_1 - b_2).$$

  - Controller gain

$$K = \frac{\beta^2}{\beta b_1 - b_2}, \quad 0 < \beta \leq 1.$$

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Application Set (cont.)

Example 1 (cont.)

- $V_{app}(\theta)$ is error in static gain of sensitivity function.
- Hessian of application cost

$$V''_{app}(\theta_0) = C \begin{bmatrix} \beta^2 & -\beta \\ -\beta & 1 \end{bmatrix}.$$

- Eigenvalues: $C(\beta^2 + 1)$ and 0.
- Eigenvectors: $[-\beta,\ 1]^T$ and $[1,\ \beta]^T$.

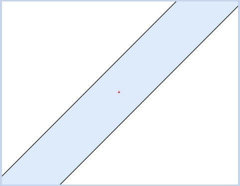

Figure: $\mathscr{E}_{app}$.

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System Identification Set

Asymptotic quality property

$$\hat{\theta} \in \mathscr{E}_{SI} = \{\theta \mid (\theta - \theta_0)^{\mathrm{T}} \mathbf{I}_F (\theta - \theta_0) \leq \eta\}.$$

(Key result from prediction error/maximum likelihood system identification.)

System Identification Set (cont.)

Example 1 (cont.)

The same system as before (first order FIR) yields

$$\mathbf{I}_F = \frac{1}{\lambda_e}\begin{bmatrix} r_0 & r_1 \\ r_1 & r_0 \end{bmatrix}, \; r_\tau = \mathrm{E}\{u(t)u(t-\tau)\}.$$

Optimal Input Signal Design

- Estimated parameters

$$\hat{\theta} \in \mathscr{E}_{SI} = \{\theta \mid (\theta - \theta_0)^{\mathrm{T}} \mathbf{I}_F (\theta - \theta_0) \leq \eta\}.$$

- Acceptable parameters in application

$$\hat{\theta} \in \mathscr{E}_{app} \approx \{\theta \mid (\theta - \theta_0)^{\mathrm{T}} V''_{app}(\theta_0)(\theta - \theta_0) \leq \frac{1}{\gamma}\}.$$

Minimize input power (r_0) subject to $\mathscr{E}_{SI} \subseteq \mathscr{E}_{app}$.
(Hjalmarsson, 2009)

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Optimal Input Signal Design

- Estimated parameters

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Minimize input power (r_0) subject to $\mathscr{E}_{SI} \subseteq \mathscr{E}_{app}$.
(Hjalmarsson, 2009)

Can be formulated as a convex problem!

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Optimal Input Signal Design (cont.)

Optimization Problem

General problem formulation

$$\underset{r_\tau}{\text{minimize}}\ r_0, \text{ subject to } \mathscr{E}_{SI} \subseteq \mathscr{E}_{app}.$$

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Optimal Input Signal Design (cont.)

Optimization Problem

General problem formulation

$$\underset{r_\tau}{\text{minimize}}\ r_0, \ \text{subject to}\ \mathscr{E}_{SI} \subseteq \mathscr{E}_{app}.$$

Convex Optimization Problem (SDP/LMI)

Approximative problem formulation

$$\underset{r_\tau}{\text{minimize}}\ r_0, \ \text{subject to}\ \mathbf{I}_F \succeq \eta\gamma V''_{app}(\theta_0).$$

Optimal Input Signal Design (cont.)

Geometric interpretation

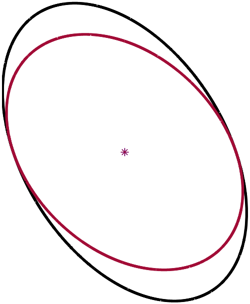
- minimize $r_0 \Leftrightarrow$ maximize $\mathscr{E}_{SI}$.
- $\mathbf{I}_F \succeq \eta\gamma V''_{app}(\theta_0) \leftrightarrow \mathscr{E}_{SI} \subseteq \mathscr{E}_{app}$.

Optimal Input Signal Design (cont.)

Geometric interpretation

- minimize $r_0 \Leftrightarrow$ maximize $\mathscr{E}_{SI}$.
- $\mathbf{I}_F \succeq \eta\gamma V''_{app}(\theta_0) \leftrightarrow \mathscr{E}_{SI} \subseteq \mathscr{E}_{app}$.

Figure: $\mathscr{E}_{SI}$ (red) and $\mathscr{E}_{app}$ (black). $\longrightarrow$



Optimal Input Signal Design (cont.)

Example 1 (cont.)

- Optimization problem solved analytically, since only two parameters.
- Optimal input signal realized by an AR-process

$$u(t) = -\beta u(t-1) + e_u(t).$$

- Experiment design relates to application of model.
  - AR-pole $(-\beta) \Leftrightarrow$ nominal closed loop pole $(-\beta)$.

MPC Example

Example 2a

- System

$$y(t) = b_1^0 u(t-1) + b_2^0 u(t-2) + d(t).$$

- MPC used to reject constant disturbance $d(t)$.
  - MPC cost function

$$V_{MPC}(u(t)) = \sum_{t=1}^{T-1} y^2(t).$$

  - MPC prediction horizon

$$T = 10.$$

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MPC Example (cont.)

Example 2a (cont.)

- $V_{app}(\theta)$ is chosen as error in output signal.
  - Application cost

$$V_{app}(\theta) = \frac{1}{M} \sum_{t=1}^{M} [y(t,\theta) - y(t,\theta_0)]^2.$$

  - Number of measurements

$$M = 10.$$

- The Hessian of $V_{app}(\theta)$ is constructed numerically.

MPC Example (cont.)

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Example 2a (cont.)

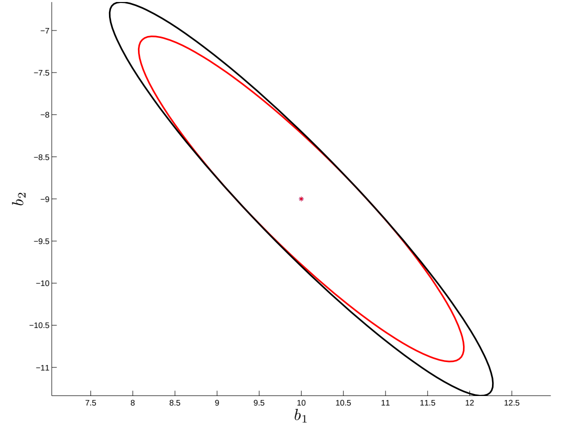

Figure: $\mathscr{E}_{SI}$ (red) and $\mathscr{E}_{app}$ (black).

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MPC Example (cont.)

Example 2b

A cost is added on the input signal in the MPC.

- MPC cost function

$$V_{MPC}(u(t)) = \sum_{t=1}^{T-1} [y^2(t) + u^2(t)].$$

MPC Example (cont.)

Example 2b (cont.)

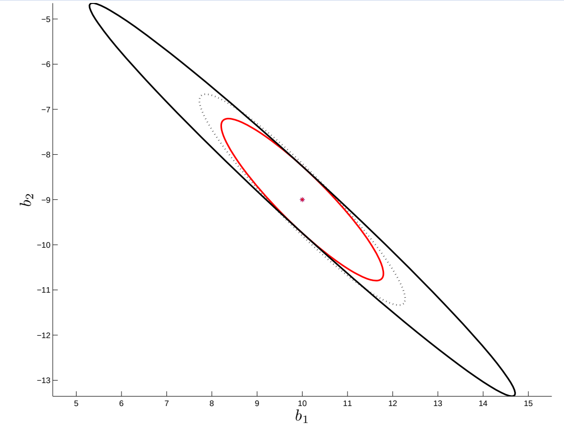

Figure: $\mathscr{E}_{SI}$ (red), $\mathscr{E}_{app}$ (black) and $\mathscr{E}_{app}$ from 2a (dashed).

Conclusions

- Obtained estimated model guarantees (with high probability) that application requirements are met.
- Simple examples give insight in how experiment design relates to intended application of model.

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Future Work

- How to choose V_{app}.
- Realistic MPC applications.
- Toolbox for optimal input design.