Energy Efficient Transmissions in MIMO Cognitive Radio Networks

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***Abstract*—In this paper, we study energy-efficient transmissions for multiple-input multiple-output (MIMO) cognitive radio (CR) networks in which the secondary unlicensed users coexist with the primary licensed users. We want to optimize the time allocations and beamforming vectors for the secondary users (SUs), in order to minimize the energy consumption of the SUs while satisfying the SUs' rate requirements and the primary receivers' interference constraints. Compared with the tradition MIMO networks, the challenge here is that the SUs may not always be able to obtain the channel state information (CSI) to the primary receivers. We are interested in two different scenarios. The first is when the SUs have the luxury of knowing the CSI to the primary receivers, and the second is when the SUs do not have such an luxury. The corresponding optimization formulations involve joint time scheduling and beamforming, which are non-convex and are complicated to solve. Fortunately, we show that when the SUs are not able to obtain the CSI, the optimal time allocation and the optimal beamforming vectors can be found very efficiently in polynomial-time through a proper decomposition. When the SUs have perfect knowledge about the CSI, we show that the optimal solutions can still be obtained in polynomial time when the secondary system is *under-utilized*. If the traffic load to the secondary system is heavy, we propose a polynomial-time heuristic to generate a near-optimal solution. The simulation results show that our proposed energy-optimal-transmission algorithms can achieve an energy-saving of 30% to 91%, compared with the simplistic maximum-rate transmission policy, depending on the secondary system's traffic load.**

***Index Terms*—Cognitive radio networks, MIMO, energy-efficiency, scheduling, beamforming.**

I. Introduction

COGNITIVE radio (CR), which allows secondary users (SUs) to opportunistically access the spectrum that is under-utilized by the primary licensed users, is a promising approach to improve the spectrum efficiency [2], [3]. With the aid of multiple-input multiple-output (MIMO) techniques [4], the cognitive spectrum may efficiently work in the underlay mode, where the SUs transmit concurrently with the primary users (PUs) as long as the interferences from the SUs to the PUs are below a tolerable threshold [3].

In this paper, we focus on energy-efficient transmissions for MIMO CR networks. In particular, we consider a CR network where the SUs send up-link traffics to the secondary base station (BS) via time division multiple access (TDMA). There is no interference among the SUs. However, the SUs share the same spectrum with some PUs, and thus the concurrent transmissions of SUs and PUs will cause interference to each other. Unlike most existing results (e.g., [5], [6]) that focus on power minimization of SUs, in this paper we aim to minimize the total *energy consumption* of the SUs while satisfying the SUs' rate requirements and the primary receivers' interference constraints. In particular, the energy consumption of each SU equals its transmit power multiplied by its transmission time. The rate requirement of each SU equals its instantaneous transmission rate multiplied by the fraction of time that this SU is active.

A. Motivation and Contributions

Performing energy efficient transmissions in CR networks is important, as it not only reduces the energy consumption of the SUs but also alleviates the interference to the PUs. The study of energy-efficient transmissions for MIMO CR networks is complicated for the following reasons:

1) Shannon's capacity formula implies that we can reduce the energy consumption for delivering a certain amount of traffic by increasing the transmission time [7]. However, in a TDMA network, the total transmission time is shared by multiple SUs. Increasing transmission time for one SU leads to the reduction of transmission time for others. Therefore, the energy consumptions of SUs trade off against each other.
2) Since the PUs are usually not aware of the existence of the SUs, the SUs are solely responsible for controlling the interference to the primary receivers. Meanwhile, each SU has a QoS requirement, measured as a target rate, to be satisfied. Therefore, the secondary BS needs to allocate the time resource and configure the beamforming patterns for the SUs in a way that could achieve the desirable balance between the SUs' target rates and the interference at the primary receivers.
3) The PUs may not be aware of the existence of the SUs. Therefore, the SUs may not be able to obtain the channel state information (CSI) of primary links. This makes it difficult for SUs to perform proper pre-interference cancellation to avoid the interference to PUs.

The problem formulation involves jointly optimizing the time allocation and beamforming vectors for the SUs, which

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TABLE I
SUMMARY OF THE KEY RESULTS

	Under-utilized secondary system	Heavily-utilized secondary system
Scenario one: no CSI	Polynomial-time **optimal** algorithm	Polynomial-time **optimal** algorithm
Scenario two: perfect CSI	Polynomial-time **optimal** algorithm	Polynomial-time heuristic algorithm

has not been considered in the literature before. We are interested in the following two scenarios: 1) the SUs are not able to obtain the CSI to the primary receivers; 2) the SUs have perfect knowledge of the CSI to the primary receivers. Within this context, this paper has three major contributions listed as follows:

1) In scenario one where the CSI is unavailable to the SUs, we consider the probabilistic interference constraint. We show that the optimal beamforming vectors can be obtained efficiently through a simple matrix eigenvalue-eigenvector computation. Based on the structure of the optimal beamforming vectors, the optimal time allocations can then be found by solving a convex optimization. To summarize, optimal solution of the overall problem can be efficiently computed through looking at two decomposed problems.
2) In scenario two where the SUs have perfect knowledge of the CSI, we consider the deterministic interference constraint. We show that the optimal time allocation and beamforming vectors can still be computed in polynomial-time if the secondary system is *under-utilized*. In particular, we provide a closed-form condition to check whether the under-utilized condition is satisfied.
3) In scenario two with *heavy* traffic load in the secondary system, we propose an iterative algorithm to generate a close-to-optimal solution. Simulation results show that the feasible solution found by the proposed heuristic algorithm is quite close to optimal (with the gap upper-bounded by 12%).

We summarize the key results in this paper in Table I.

B. Related Work

In the literature, most studies on energy-efficient transmissions of MIMO networks are within the traditional MIMO networks [8]–[13]. In [8], the authors considered the transmit power minimization through downlink transmit beamforming, assuming that the perfect CSI is available at the transmitter. In [11], the authors focused on minimizing the total energy consumption of a single MIMO link by considering both the transmission energy and the circuit energy consumption. In [13], the authors studied the energy efficiency of a MIMO-based TDMA cellular system. In particular, the authors proposed a cross-layer approach of joint rate selection and mode switching to save the system energy consumption.

The CR technology brings new challenging issues that do not exist in traditional MIMO networks. In traditional MIMO networks (e.g., ad hoc and cellular networks), all wireless links are responsible for controlling their interferences to each other. However, in a CR network, usually the primary network does not know the existence of the secondary network. Thus, the secondary transmitters are solely responsible for controlling their interferences to the primary receivers. Second, the CSI may not always be available to the secondary system. Therefore, the interference suppression in CR network could only be done with the transmitter-side pre-interference cancellation at the secondary transmitters, and without the CSI to the primary receivers. Existing works on MIMO CR networks mainly focus on maximizing the capacity of the secondary system [14]–[19]. Ref. [14] studied the problem of capacity maximization of a single secondary link subject to interference constraints at the primary receivers. In [15], [16], the authors studied the weighted sum-rate maximization of multiple SUs in a MIMO CR network. However, in [14]–[16], CSI is assumed to be available to the secondary system.

When the CSI is unknown to the secondary system, the deterministic interference constraints at the primary receivers cannot be satisfied and thus are not proper. A robust optimization framework is proposed in [6], [17], which requires the constraints to hold for every possible realization of the channel. Such an approach guarantees the worst-case performance and is thus overly conservative. In practice, many wireless applications can tolerate occasional outages without affecting users' QoS. This motivates us to consider a more realistic interference constraint when the CSI is unavailable, which is to satisfy the interference constraints with high probability [20]. Such practical probabilistic constraints are, however, generally hard to deal with mathematically. In [19], the authors considered the capacity maximization of a single secondary link subject to an interference constraint at primary receivers, under three different levels of the CSI availability, namely complete, partial, or no knowledge of CSI.

Only very recent papers studied the MIMO CR networks with the system objective of reducing the energy/power consumption [5], [6]. In particular, the authors considered the downlink transmissions from one secondary transmitter to multiple secondary receivers in [5], [6]. In [5], [6], [19], there is only one secondary transmitter in the system. Therefore, it is sufficient to only optimize the secondary user's beamforming to achieve power minimization in [5], [6] or capacity maximization in [19]. In this paper, we are interested in the uplink transmissions involving multiple secondary transmitters. In this case, the scheduling and beamforming need to be jointly optimized in order to minimize the total energy consumption of all the SUs.

The remainder of this paper is organized as follows. In Section II, we describe the system model. Section III outlines the general problem formulation. In Section IV, we focus on the Problem decomposition and the methods to find optimal solutions in scenario one. In Section V, we focus on the formulation and the decomposition condition in scenario two. In Section VI, we propose polynomial-time solution methods in scenario two. Section VII presents the simulation results. Section VIII concludes this paper.

II. SYSTEM MODEL

We consider a CR network with K SUs and J PUs. The primary links could be always active, and thus we need

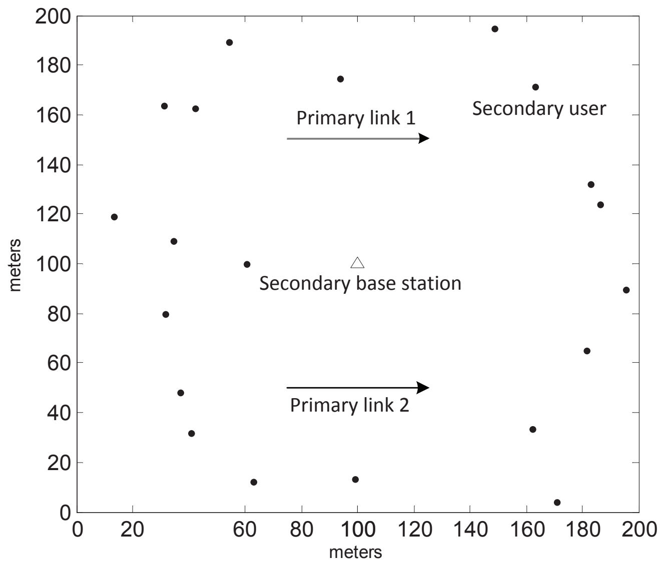

Fig. 1. Network topology with 20 SUs.

to protect them at all times. The network topology for the primary system is general (either a cellular network or an ad hoc network). The secondary system is a cellular network, where the SUs send uplink traffics to the same secondary BS via TDMA. The uplink transmissions are synchronized by the secondary BS so that they are allocated different time for their transmissions and thus do not cause interference to each other. A sample network topology with two primary links and 20 SUs is shown in Fig. 1.

We use S_k to denote the kth SU. Let M_{S_k} denote the number of transmit antennas of S_k and N_{BS} denote the number of receive antennas at the secondary BS. Let $\mathbf{H}_{BS,S_k}$ denote the $N_{BS} \times M_{S_k}$ channel matrix from S_k to the secondary BS. There are J links in the primary network. We use P_j to denote the jth primary link. Let M_{P_j} and N_{P_j} denote the number of transmit antennas and the number of receive antennas of P_j, respectively. Let $\mathbf{H}_{P_i,P_j}$ denote the $N_{P_i} \times M_{P_j}$ channel matrix from the jth primary transmitter to the ith primary receiver. Since the SUs coexist with the PUs, the SU's signal and the PUs' signals may interfere with each other. Let $\mathbf{H}_{P_j,S_k}$ and $\mathbf{H}_{BS,P_j}$ denote the $N_{P_j} \times M_{S_k}$ channel matrix from S_k to the jth primary receiver and the $N_{BS} \times M_{P_j}$ channel matrix from the jth primary transmitter to the secondary BS, respectively. We assume a block fading channel, so that the channel matrices do not change during a TDMA frame, and the channel realizations in different frames are uncorrelated. In particular, we assume Rayleigh fading channels and a rich scattering environment, so that the entries of the channel matrices are independently and identically distributed (i.i.d.) complex Gaussian random variables with a zero mean [21]. The variance of the complex Gaussian variables is half of the path loss from the corresponding transmitter to the corresponding receiver.

Let $\mathbf{u}_{S_k}$ and $\mathbf{v}_{BS_k}$ denote the $M_{S_k} \times 1$ transmit beamforming vector of S_k and the $N_{BS} \times 1$ receive beamforming vector of the secondary BS when S_k is active, respectively. Let $\mathbf{u}_{P_j}$ and $\mathbf{v}_{P_j}$ denote the $M_{P_j} \times 1$ transmit beamforming vector and the $N_{P_j} \times 1$ receive beamforming vector of the primary link P_j, respectively. Without loss of generality, we normalize the receive beamforming vectors such that $\|\mathbf{v}_{BS_k}\|_2^2 = 1$ and $\|\mathbf{v}_{P_j}\|_2^2 = 1$. The CR network considered here is an interference-limited network, and it is better for each SU to transmit one data stream at a time on all its transmit antennas to avoid excessive interference to the other links [22], [23]. Thus we let scalars x_{S_k} and x_{P_j} denote the transmit signals of S_k and P_j, respectively. Without loss of generality, we assume $\mathbb{E}[|x_{S_k}|^2] = 1$ and $\mathbb{E}[|x_{P_j}|^2] = 1$. The received signal of S_k after receive beamforming at the secondary BS is

$$y_{BS_k} = \mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k} x_{S_k} + \sum_{j=1}^{J} \mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j} x_{P_j} + \mathbf{v}_{BS_k}^H \mathbf{n}_{BS}, \quad k = 1, \cdots, K.$$

The vector $\mathbf{n}_{BS}$ is an $N_{BS} \times 1$ circular complex additive Gaussian noise vector with a noise power of $N_0 w$ at the secondary BS, where N_0 is the noise power spectral density and w is the bandwidth used in the secondary system. The received signal-to-interference-plus-noise ratio (SINR) of S_k then becomes

$$\gamma_{BS_k} = \frac{\mathbb{E}\left[\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k} x_{S_k}\right|^2\right]}{\mathbb{E}\left[\sum_{j=1}^{J}\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j} x_{P_j}\right|^2 + \left|\mathbf{v}_{BS_k}^H \mathbf{n}_{BS}\right|^2\right]} = \frac{\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}\right|^2}{\sum_{j=1}^{J}\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j}\right|^2 + N_0 w}, \quad k = 1, \cdots, K. \tag{1}$$

According to the Shannon's capacity formula, the achievable transmission rate of S_k is

$$r_{S_k} = w \log\left(1 + \gamma_{BS_k}\right) = w \log\left(1 + \frac{\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}\right|^2}{\sum_{j=1}^{J}\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j}\right|^2 + N_0 w}\right).$$

The transmit power of S_k is $p_{S_k} = \|\mathbf{u}_{S_k}\|_2^2$, and S_k causes an interference to the jth primary receiver at the level of

$$q_{P_j S_k} = \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k}\right|^2, \quad k = 1, \cdots, K, \quad j = 1, \cdots, J.$$

III. Problem Formulation

Our target is to choose the proper time allocation and the transmit and receive beamforming vectors for each SU, in order to minimize the total energy consumption of the SUs. Each SU has a rate requirement R_{S_k}. The interferences from the SUs to PUs need to be below a certain tolerable threshold. Without loss of generality, the TDMA frame length of the secondary system is normalized to be 1. Each S_k is allocated a time fraction t_{S_k} $(0 \leq t_{S_k} \leq 1)$ to transmit its data. The transmit power of each S_k is limited by a maximum transmit power $p_{S_k,\max}$. This problem can be mathematically formulated as follows:

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \|\mathbf{u}_{S_k}\|_2^2 \\
\text{subject to} \quad & t_{S_k} w \log\left(1 + \frac{\left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}\right|^2}{\sum_{j=1}^{J} \left|\mathbf{v}_{BS_k}{}^H \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j}\right|^2 + N_0 w}\right) \\
& \geq R_{S_k}, \quad \forall k, && \text{(2a)} \\
& \sum_{k=1}^{K} t_{S_k} \leq 1, && \text{(2b)} \\
& \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k}\right|^2 \leq \phi_{P_j}, \quad \forall k, \quad \forall j, && \text{(2c)} \\
& \|\mathbf{u}_{S_k}\|_2^2 \leq p_{S_k,\max}, \quad \forall k, && \text{(2d)} \\
\text{variables} \quad & t_{S_k} \geq 0, \quad \forall k, \\
& \mathbf{u}_{S_k}, \quad \forall k, \\
& \mathbf{v}_{BS_k}, \quad \forall k.
\end{aligned}$$

The objective function in (2) is the total energy consumption of all the SUs. Constraint (2a) guarantees each SU's rate requirement. Constraint (2b) states that the total time allocated to all the SUs is no larger than the TDMA frame length. Constraint (2c) ensures that the interference from each secondary transmitter to each primary receiver is no larger than a tolerable threshold ϕ_{P_j}. Constraint (2d) states that the SUs have limited transmission power. The variables in (2) are the time fraction variables t_{S_k}, the transmit beamforming vectors $\mathbf{u}_{S_k}$, and the receive beamforming vectors $\mathbf{v}_{BS_k}$ of the SUs.

Since we consider a centralized TDMA network for the secondary system, it is reasonable to assume that $\mathbf{H}_{BS,S_k}$ is known to both S_k and the secondary BS. As in a CR network, the secondary system is usually aware of the existence of the primary system, we can further assume that the secondary BS can overhear the transmissions on the primary links. At the secondary BS, the overheard signal of P_j is $\mathbf{H}_{BS,P_j}\mathbf{u}_{P_j}$. Therefore, the secondary BS is able to estimate $\mathbf{H}_{BS,P_j}\mathbf{u}_{P_j}$ for all PUs. However, in a CR network, the secondary system is usually transparent to the primary system. The primary system would not deliberately provide the CSI to the secondary system. Therefore, the secondary system may not know the coefficient vectors $\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}$ in constraint (2c). In this paper, we are interested in both the following two scenarios:

1) The secondary system is not able to obtain either $\mathbf{H}_{P_j,S_k}$ or $\mathbf{v}_{P_j}$.
2) The secondary system has perfect knowledge of the vector $\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}$.

A. Formulation Simplification

Notice that in formulation (2), the receive beamforming variables $\mathbf{v}_{BS_k}$ only appear in constraint (2a). These variables can be eliminated by exploring the optimal receive beamforming, which are the minimum-mean-squared-error (MMSE) receivers [24]:

$$\mathbf{v}_{BS_k}^* = \theta_{S_k} \mathbf{B}_{S_k}^{-1} \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}, \quad k = 1, \cdots, K, \tag{3}$$

where $\mathbf{B}_{S_k} = \sum_{j=1}^{J} \mathbf{H}_{BS,P_j} \mathbf{u}_{P_j} \mathbf{u}_{P_j}^H \mathbf{H}_{BS,P_j}^H + N_0 w \mathbf{I}$, and θ_{S_k} is the normalized factor given by $\theta_{S_k} = \frac{1}{\|\mathbf{B}_{S_k}^{-1} \mathbf{H}_{BS,S_k} \mathbf{u}_{S_k}\|_2}$, which ensures $\|\mathbf{v}_{BS_k}^*\|_2^2 = 1$. The maximum received SINR is then given by

$$\gamma_{BS_k} = \mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k}, \quad k = 1, \cdots, K, \tag{4}$$

where $\mathbf{A}_{S_k} = \mathbf{H}_{BS,S_k}^H \mathbf{B}_{S_k}^{-1} \mathbf{H}_{BS,S_k}$, which is an $M_{S_k} \times M_{S_k}$ Hermitian matrix.

Substituting (4) into constraint (2a), formulation (2) can be simplified to

$$\begin{aligned}
\text{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \|\mathbf{u}_{S_k}\|_2^2 \\
\text{subject to} \quad & t_{S_k} w \log\left(1 + \mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k}\right) \geq R_{S_k}, \forall k, && \text{(5a)} \\
& \sum_{k=1}^{K} t_{S_k} \leq 1, && \text{(5b)} \\
& \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k}\right|^2 \leq \phi_{P_j}, \quad \forall k, \quad \forall j, && \text{(5c)} \\
& \|\mathbf{u}_{S_k}\|_2^2 \leq p_{S_k,\max}, \quad \forall k, && \text{(5d)} \\
\text{variables} \quad & t_{S_k} \geq 0, \quad \forall k, \\
& \mathbf{u}_{S_k}, \quad \forall k.
\end{aligned}$$

The receive beamforming vectors $\mathbf{v}_{BS_k}$ are removed, and the variables in formulation (5) are the time fraction variables t_{S_k} and the transmit beamforming vectors $\mathbf{u}_{S_k}$ of the SUs.

B. Feasibility

The feasible set in (5) may not always be non-empty. For each S_k, its maximum feasible instantaneous transmission rate C_{S_k} depends on its maximum transmit power and the interference constraints at the primary receivers. The link capacity C_{S_k} can be computed by solving the following problem

$$\begin{aligned}
\text{maximize} \quad & \gamma_{BS_k} = \mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k} \\
\text{subject to} \quad & \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k}\right|^2 \leq \phi_{P_j}, \quad \forall j, \\
& \|\mathbf{u}_{S_k}\|_2^2 \leq p_{S_k,\max}, \\
\text{variable} \quad & \mathbf{u}_{S_k}.
\end{aligned} \tag{6}$$

Let $\gamma_{BS_k,\max}$ denote the optimal objective value of problem (6), then the link capacity $C_{S_k} = w \log(1 + \gamma_{BS_k,\max})$. Problem (5) is feasible when the traffic load to the secondary system does not exceed its capacity, i.e.,

$$\sum_{k=1}^{K} \frac{R_{S_k}}{C_{S_k}} \leq 1. \tag{7}$$

Notice that problem (6) is NP-hard in general [19]. Therefore it is difficult to efficiently compute C_{S_k} and check the feasibility of problem (5). In the subsequent sections, we will discuss the feasibility conditions and the methods of solving (5) in scenarios one and two, respectively.

IV. Scenario One: Problem Decomposition and Optimal Solution

In this section, we will show that there is a closed-form feasibility condition in scenario one. Furthermore, the optimal time fractions and transmit beamforming vectors can be obtained efficiently through a proper decomposition.

A. Formulation Recast

In scenario one, the secondary system is not able to obtain either $\mathbf{H}_{P_j,S_k}$ or $\mathbf{v}_{P_j}$. Thus, the left-hand-side of constraint (5c) is random for any given $\mathbf{u}_{S_k}$. The requirement of satisfying constraint (5c) would easily lead to suboptimal or infeasible solutions. Interestingly, many wireless applications (such as video streaming, voice over IP) can tolerate occasional outages without affecting users' QoS. Thus, we consider a more realistic requirement, which is to satisfy the interference constraints with a high probability. In other words, the CR network allows the interference from the secondary transmitters to the primary receivers to exceed the power threshold ϕ_{P_j} with a small outage probability δ_{P_j}. Constraint (5c) is then replaced by

$$\Pr_{\mathbf{H}_{P_j,S_k},\mathbf{v}_{P_j}}\left\{\left|\mathbf{v}_{P_j}^H\mathbf{H}_{P_j,S_k}\mathbf{u}_{S_k}\right|^2\le\phi_{P_j}\right\}\ge 1-\delta_{P_j},\quad\forall k,\forall j,\tag{8}$$

where the probability is taken over both $\mathbf{H}_{P_j,S_k}$ and $\mathbf{v}_{P_j}$.

Since we consider Rayleigh fading channel, the entries of the channel matrix $\mathbf{H}_{P_j,S_k}$ are i.i.d. complex Gaussian random variables with a zero mean and a variance of $\frac{\beta_{P_j,S_k}}{2}$, where β_{P_j,S_k} denotes the path loss from S_k to the jth primary receiver. Furthermore, because $\mathbf{H}_{P_j,S_k}$ and $\mathbf{v}_{P_j}$ are independent of each other, $\left|\mathbf{v}_{P_j}^H\mathbf{H}_{P_j,S_k}\mathbf{u}_{S_k}\right|^2$ follows an exponential distribution with the parameter $\frac{1}{\beta_{P_j,S_k}\|\mathbf{u}_{S_k}\|_2^2}$ [19]. So we have

$$\Pr_{\mathbf{H}_{P_j,S_k},\mathbf{v}_{P_j}}\left\{\left|\mathbf{v}_{P_j}^H\mathbf{H}_{P_j,S_k}\mathbf{u}_{S_k}\right|^2\le\phi_{P_j}\right\}=1-\exp\left(-\frac{\phi_{P_j}}{\beta_{P_j,S_k}\|\mathbf{u}_{S_k}\|_2^2}\right).$$

Therefore, the outage probability constraint (8) is equivalent to

$$\|\mathbf{u}_{S_k}\|_2^2\le\frac{-\phi_{P_j}}{\beta_{P_j,S_k}\log\delta_{P_j}},\quad\forall k,\quad\forall j.\tag{9}$$

Furthermore, after converting the outage probability constraint to (9), we find that (9) can be combined with the maximum transmission power constraint (5d). Let $\lambda_{S_k}=\min\left\{\frac{-\phi_{P_1}}{\beta_{P_1,S_k}\log\delta_{P_1}},\cdots,\frac{-\phi_{P_J}}{\beta_{P_J,S_k}\log\delta_{P_J}},p_{S_k,\max}\right\}$. Constraints (9) and (5d) are equivalent to

$$\|\mathbf{u}_{S_k}\|_2^2\le\lambda_{S_k},\quad\forall k=1,\cdots,K.$$

Therefore, in scenario one, problem (5) can be recast as follows:

$$\begin{aligned}\text{minimize}\quad&\sum_{k=1}^K t_{S_k}\|\mathbf{u}_{S_k}\|_2^2\\ \text{subject to}\quad&t_{S_k}w\log\left(1+\mathbf{u}_{S_k}^H\mathbf{A}_{S_k}\mathbf{u}_{S_k}\right)\ge R_{S_k},\forall k,&\text{(10a)}\\ &\sum_{k=1}^K t_{S_k}\le 1,&\text{(10b)}\\ &\|\mathbf{u}_{S_k}\|_2^2\le\lambda_{S_k},\quad\forall k,&\text{(10c)}\\ \text{variables}\quad&t_{S_k}\ge 0,\quad\forall k,\\ &\mathbf{u}_{S_k},\quad\forall k.\end{aligned}$$

B. Problem Decomposition

In this subsection, we will show that in scenario one, the time fractions t_{S_k} and the transmit beamforming vectors $\mathbf{u}_{S_k}$ can be separately optimized without affecting the overall optimality.

Given any time fraction allocation $(t_{S_1},\cdots,t_{S_K})$, problem (10) reduces to K separate optimization problems among the SUs. For each S_k, the optimization problem is given by

$$\begin{aligned}\text{minimize}\quad&\|\mathbf{u}_{S_k}\|_2^2\\ \text{subject to}\quad&w\log\left(1+\mathbf{u}_{S_k}^H\mathbf{A}_{S_k}\mathbf{u}_{S_k}\right)\ge\frac{R_{S_k}}{t_{S_k}},&\text{(11a)}\\ &\|\mathbf{u}_{S_k}\|_2^2\le\lambda_{S_k},&\text{(11b)}\\ \text{variable}\quad&\mathbf{u}_{S_k}.\end{aligned}$$

Now, we proceed to show that the optimization problem (11) has a closed-form solution by a simple eigenvalue-eigenvector computation. Let $\rho_{S_k,1},\rho_{S_k,2},\cdots,\rho_{S_k,M_{S_k}}$ denote all the eigenvalues of matrix $\mathbf{A}_{S_k}$. Let $\mathbf{z}_{S_k,i}$ ($\|\mathbf{z}_{S_k,i}\|_2^2=1$) denote the normalized eigenvector of $\mathbf{A}_{S_k}$ associated with eigenvalue $\rho_{S_k,i},(1\le i\le M_{S_k})$. Let $\rho_{S_k,\max}$ denote the largest eigenvalue of $\mathbf{A}_{S_k}$ and $\mathbf{z}_{S_k,\max}$ denote the normalized eigenvector of $\mathbf{A}_{S_k}$ associated with $\rho_{S_k,\max}$. The closed-form solution to (11) is given in the following lemma.

Lemma 1. *The necessary and sufficient condition for optimization problem* (11) *to be feasible is*

$$t_{S_k}\ge\frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max}+1\right)},\quad k=1,\cdots,K.\tag{12}$$

When condition (12) *is satisfied, the optimal solution to* (11) *is*

$$\mathbf{u}_{S_k}^*=\sqrt{\frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right)-1}{\rho_{S_k,\max}}}\mathbf{z}_{S_k,\max},\quad k=1,\cdots,K.\tag{13}$$

Proof. Matrix $\mathbf{A}_{S_k}$ is a Hermitian matrix. Therefore $\mathbf{A}_{S_k}$ can be unitarily diagonalized as $\mathbf{A}_{S_k}=\mathbf{Q}_{S_k}\mathbf{\Lambda}_{S_k}\mathbf{Q}_{S_k}^H$, where $\mathbf{Q}_{S_k}$ is a unitary matrix and $\mathbf{\Lambda}_{S_k}$ is a diagonal matrix containing all the eigenvalues of $\mathbf{A}_{S_k}$. So we have

$$\begin{aligned}\mathbf{u}_{S_k}^H\mathbf{A}_{S_k}\mathbf{u}_{S_k}&=\mathbf{u}_{S_k}^H\mathbf{Q}_{S_k}\mathbf{\Lambda}_{S_k}\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\\ &=\left(\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\right)^H\mathbf{\Lambda}_{S_k}\left(\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\right)\le\rho_{S_k,\max}\|\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\|_2^2.\end{aligned}$$

Since matrix $\mathbf{Q}_{S_k}$ is unitary, we further know that $\|\mathbf{Q}_{S_k}^H\mathbf{u}_{S_k}\|_2^2=\|\mathbf{u}_{S_k}\|_2^2$. So we have

$$\mathbf{u}_{S_k}^H\mathbf{A}_{S_k}\mathbf{u}_{S_k}\le\rho_{S_k,\max}\|\mathbf{u}_{S_k}\|_2^2,\tag{14}$$

where the equality is achieved when $\mathbf{u}_{S_k}$ is an eigenvector of $\mathbf{A}_{S_k}$ corresponding to $\rho_{S_k,\max}$.

On the other hand, constraint (11a) is equivalent to

$$\mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k} \geq \exp\left(\frac{R_{S_k}}{wt_{S_k}}\right) - 1. \tag{15}$$

According to (14) and (15), we know that if we only consider constraint (11a) in optimizing Problem (11), the minimum value of the objective function in (11) is $\|\mathbf{u}_{S_k}^*\|_2^2 = \frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right)-1}{\rho_{S_k,\max}}$, and the optimal solution is $\mathbf{u}_{S_k}^* = \sqrt{\frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right)-1}{\rho_{S_k,\max}}}\mathbf{z}_{S_k,\max}$. Since constraint (11b) only states that $\|\mathbf{u}_{S_k}\|_2^2$ should be no greater than λ_{S_k}, therefore (11) is feasible if and only if the minimum value of $\|\mathbf{u}_{S_k}\|_2^2$ satisfies constraint (11b). That is,

$$\frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right) - 1}{\rho_{S_k,\max}} \leq \lambda_{S_k} \Rightarrow t_{S_k} \geq \frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max}+1\right)}.$$

□

According to Lemma 1, the optimal transmit beamforming vectors are explicit functions of the time fraction allocation $(t_{S_1}, \cdots, t_{S_K})$. This enables us to solve optimization problem (10) through a proper decomposition.

Theorem 1. *In scenario one, the necessary and sufficient condition for optimization problem* (10) *to be feasible is*

$$\sum_{k=1}^{K} \frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max}+1\right)} \leq 1. \tag{16}$$

Furthermore, problem (10) *can be solved in polynomial time. In particular, the optimal time fractions are the optimal solutions to the convex optimization*

$$\begin{aligned} \textit{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right) - 1}{\rho_{S_k,\max}} \\ \textit{subject to} \quad & \sum_{k=1}^{K} t_{S_k} \leq 1, \\ & t_{S_k} \geq \frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max}+1\right)}, \quad \forall k, \\ \textit{variables} \quad & t_{S_k} \geq 0, \quad \forall k. \end{aligned} \tag{17}$$

The optimal transmit beamforming vectors are then given by the closed-form solution in (13)*.*

Proof. Substituting (13) into Problem (10), then (10) becomes (17). The condition for the constraint set of (17) to be non-empty is $\sum_{k=1}^{K} \frac{R_{S_k}}{w\log\left(\lambda_{S_k}\rho_{S_k,\max}+1\right)} \leq 1$.

Problem (17) is an optimization problem with the time fraction variables t_{S_k} only. The second order derivative of the objective function in (17) with respect to variable t_{S_k} is

$$\frac{R_{S_k}^2}{w^2\rho_{S_k,\max}t_{S_k}^3} \exp\left(\frac{R_{S_k}}{wt_{S_k}}\right),$$

which is always positive for any nonnegative t_{S_k}. Thus, the objective function in (17) is a convex function. Furthermore, the constraints in (17) are linear. Therefore, (17) is a convex optimization problem, which can be solved in polynomial time with the standard interior-point method [25]. □

Two remarks are in order for Theorem 1:

1) When the secondary system has no knowledge of CSI, the only way for the SUs to meet the probabilistic interference constraints is to control their transmit powers, as shown in constraint (10c). In this case, it is energy-optimal for each SU to scale its transmit beamforming vector with the eigenvector corresponding to the largest eigenvalue of its $\mathbf{A}_{S_k}$. The scaling factor depends on the time resource for each SU. Furthermore, we can find the energy-optimal time allocation by solving convex optimization problem (17).
2) Finding the optimal solutions in scenario one is not difficult. When a new SU sends transmission request to the secondary BS, the secondary BS first checks whether condition (16) is satisfied. If yes, the secondary BS solves the convex optimization problem (17) to obtain the optimal time fractions. Furthermore, the optimal transmit beamforming vectors are then computed by (13). After obtaining the optimal transmit beamforming vectors, the optimal receive beamforming vectors can be computed by (3). If condition (16) is not satisfied, the rate requirements of all the SUs cannot be satisfied. In this case, the secondary BS performs call admission control to block the new SU.

V. Problem Decomposition Condition in Scenario Two

In scenario two, the secondary system has perfect knowledge of the vector $\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}$. The formulation is shown in (5). In this section, we will shown that problem (5) can be decomposed into optimizing the time fractions t_k and optimizing the transmit beamforming vectors $\mathbf{u}_{S_k}$ separately when the secondary system is *under-utilized*.

A. Decomposition Condition

We first consider a relaxation of Problem (5) by removing the interference constraint (5c):

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \|\mathbf{u}_{S_k}\|_2^2 \\ \text{subject to} \quad & t_{S_k} w\log\left(1 + \mathbf{u}_{S_k}^H \mathbf{A}_{S_k} \mathbf{u}_{S_k}\right) \geq R_{S_k}, \forall k, && \text{(18a)} \\ & \sum_{k=1}^{K} t_{S_k} \leq 1, && \text{(18b)} \\ & \|\mathbf{u}_{S_k}\|_2^2 \leq p_{S_k,\max}, \quad \forall k, && \text{(18c)} \\ \text{variables} \quad & t_{S_k} \geq 0, \quad \forall k, \\ & \mathbf{u}_{S_k}, \quad \forall k. \end{aligned}$$

When we do not consider the interference constraints, $\mathbf{u}_{S_k}$ is only upper bounded by $p_{S_k,\max}$, as shown in constraint (18c). Problem (18) is of a similar form to Problem (10). The only difference is that λ_{S_k} in constraint (10c) is replaced by

$p_{S_k,\max}$ in (18c)[1]. Therefore, similar to Theorem 1, we have the following theorem for optimization problem (18).

Theorem 2. *Optimization problem* (18) *is feasible if and only if*

$$\sum_{k=1}^{K} \frac{R_{S_k}}{w \log\left(p_{S_k,\max}\rho_{S_k,\max} + 1\right)} \leq 1. \tag{19}$$

Furthermore, the optimal time fractions are the optimal solutions to the convex optimization

$$\begin{aligned} \textit{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \frac{\exp\left(\frac{R_{S_k}}{w t_{S_k}}\right) - 1}{\rho_{S_k,\max}} \\ \textit{subject to} \quad & \sum_{k=1}^{K} t_{S_k} \leq 1, \\ & t_{S_k} \geq \frac{R_{S_k}}{w \log\left(p_{S_k,\max}\rho_{S_k,\max} + 1\right)}, \quad \forall k, \\ \textit{variables} \quad & t_{S_k} \geq 0, \quad \forall k. \end{aligned} \tag{20}$$

The optimal transmit beamforming vectors are then given by the closed-form solution in (13)*.*

The proof of Theorem 2 is the same as the proof of Theorem 1.

Theorem 2 shows that the relaxed problem (18) can be solved to optimal in polynomial time through a proper decomposition. Moreover, if the optimal solution to (18) satisfies constraint (5c), in this case, there is no gap between the optimal value of (5) and that of (18), and problems (5) and (18) are equivalent. The following theorem shows the condition when this is true. To simplify notations, let $\mu_{S_k} = \min\left\{\frac{\phi_{P_1}}{\left|\mathbf{v}_{P_1}^H \mathbf{H}_{P_1,S_k}\mathbf{z}_{S_k,\max}\right|^2}, \cdots, \frac{\phi_{P_J}}{\left|\mathbf{v}_{P_J}^H \mathbf{H}_{P_J,S_k}\mathbf{z}_{S_k,\max}\right|^2}\right\}$.

Theorem 3. *In scenario two, if the optimal time fractions to the convex optimization* (20) *satisfy*

$$t_{S_k}^* \geq \frac{R_{S_k}}{w \log\left(\mu_{S_k}\rho_{S_k,\max} + 1\right)}, \quad k = 1, \cdots, K, \tag{21}$$

the relaxed problem (18) *is equivalent to the original formulation* (5)*. In this case, Problem* (5) *can be solved to optimal in polynomial time.*

Proof. The optimal transmit beamforming vectors to the relaxed problem (18) are given by (13). Substituting (13) into constraint (5c), we have

$$\begin{aligned} & \frac{\exp\left(\frac{R_{S_k}}{w t_{S_k}}\right) - 1}{\rho_{S_k,\max}} \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}\mathbf{z}_{S_k,\max}\right|^2 \leq \phi_{P_j} \\ \Rightarrow & t_{S_k} \geq \frac{R_{S_k}}{w \log\left(\frac{\rho_{S_k,\max}\phi_{P_j}}{\left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}\mathbf{z}_{S_k,\max}\right|^2} + 1\right)}, \quad j = 1, \cdots, J. \end{aligned}$$

□

Condition (21) states that each SU has enough time for its transmissions. This is more likely to be satisfied when the secondary system is *under-utilized*. In this case, by making use of the time resource and doing the energy efficient optimization, each SU automatically operates at a lower rate and hence does not violate the interference power constraint at the primary receivers. Therefore, only when condition (21) is satisfied, can the decomposition method be applied to find the optimal solutions in scenario two.

B. The Feasible Condition in Scenario Two

In scenario two, condition (21) may not always hold. This happens when the traffic load to the secondary system is heavy but does not exceed its capacity. Here we provide a condition of checking the feasibility of formulation (5) and performing call admission control.

As shown in Section III-B, the link capacity for each S_k can be calculated by solving (6). However, because (6) is NP-hard, we adopt the Semi-Definite Programming (SDP) method in [19] to obtain an approximation of the link capacity, $\hat{C_{S_k}}$ [2]. The call admission control needs to guarantee

$$\sum_{k=1}^{K} \frac{R_{S_k}}{\hat{C_{S_k}}} \leq 1 \tag{22}$$

to ensure Problem (5) is feasible.

VI. The Solution Methods in Scenario Two

In this section we will propose the solution methods in scenario two, i.e., solving Problem (5) for all the possible system parameters. In particular, we propose a polynomial-time *optimal* algorithm when the secondary system is *under-utilized*, and a heuristic algorithm when the secondary system is *heavily-utilized*. The flowchart for scenario two is shown in Fig. 2.

A. Optimal Solution for Under-utilized Secondary System

When a new SU sends transmission request to the secondary BS, the secondary BS first checks condition (19). If it is not satisfied with this additional SU, the relaxed problem (18) is not feasible. Therefore, Problem (5) is not feasible. In this case, the secondary BS performs call admission control and blocks the transmission request of the new SU. If condition (19) is satisfied, the secondary BS then solves convex optimization (20), and checks its optimal solution with condition (21). If condition (21) is satisfied, problem (5) is equivalent to problem (18), and thus can be solved to optimal through a proper decomposition, as shown in Theorem 3. Specifically, the optimal time fractions are the optimal solutions to the convex optimization problem (20), and the optimal transmit beamforming vectors can then be computed by (13).

[1]In Problem (10), the outage probabilistic interference constraints are incorporated in the parameter λ_{S_k}.

[2]When there are no more than two PUs, the SDP method provided in [19] can find the optimal solution to (6). In this case, $\hat{C_{S_k}}$ is exact the link capacity C_{S_k}.

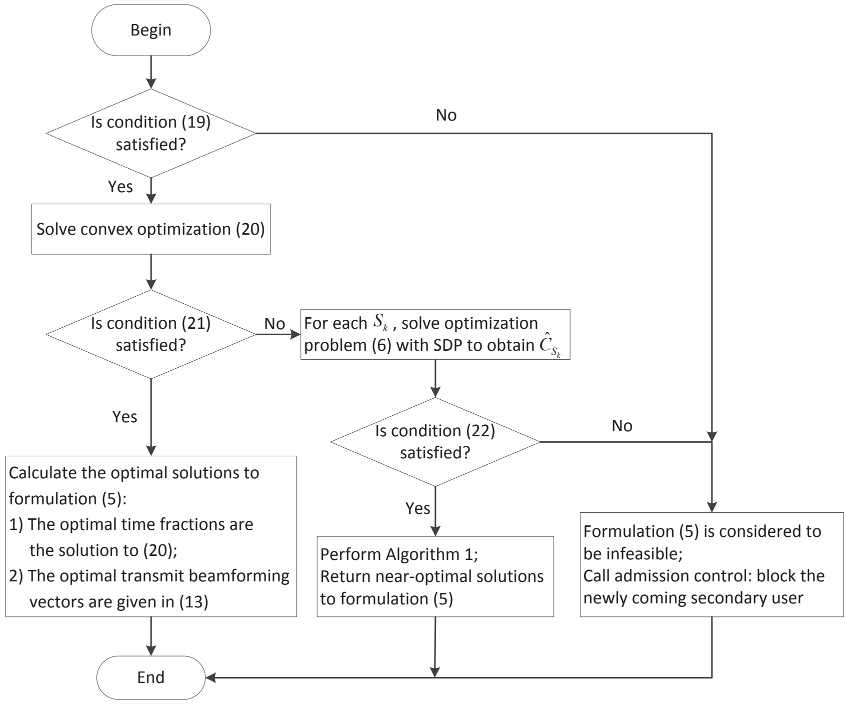

Fig. 2. Flowchart of the algorithms in scenario two.

B. Heuristic Solution for Heavily-utilized Secondary System: Algorithm 1

If condition (21) is not satisfied, the secondary BS then solves problem (6) with the SDP method for each S_k to estimate its link capacity. If condition (22) is satisfied, this indicates that the traffic load to the secondary system is heavy, but does not exceed its capacity. In this case, we propose Algorithm 1 to generate a near-optimal solution to Problem (5). If condition (22) is not satisfied, the secondary BS then performs the call admission control and block the new SU.

In Algorithm 1, we first fix the transmit beamforming vector of each SU, and optimize the time fraction allocation. We then fix the time fractions and optimize the transmit beamforming vector for each SU. We iterate these two modules until termination. Let us first introduce some notations that are used in the following discussions. In general, any transmit beamforming vector $\mathbf{u}_{S_k}$ can be represented by a linear combination of the normalized eigenvectors of matrix $\mathbf{A}_{S_k}$,

$$\mathbf{u}_{S_k} = \sqrt{p_{S_k}} \sum_{i=1}^{M_{S_k}} \sigma_{S_k,i}\mathbf{z}_{S_k,i} = \sqrt{p_{S_k}}\Big(\sigma_{S_k,1}\mathbf{z}_{S_k,1} + \cdots + \sigma_{S_k,i}\mathbf{z}_{S_k,i} + \cdots + \sigma_{S_k,M_{S_k}}\mathbf{z}_{S_k,M_{S_k}}\Big),$$

where $p_{S_k} = \mathbf{u}_{S_k}^H \mathbf{u}_{S_k}$ is the transmit power of S_k, and the coefficient $\sigma_{S_k,i}$ can be calculated by

$$\sigma_{S_k,i} = \frac{1}{\sqrt{p_{S_k}}}\mathbf{z}_{S_k,i}^H \mathbf{u}_{S_k}. \tag{23}$$

The coefficients are normalized such that $\sum_{i=1}^{M_{S_k}} |\sigma_{S_k,i}|^2 = 1$. Each element $|\sigma_{S_k,i}|^2$ denotes the fraction of power allocated to $\mathbf{z}_{S_k,i}$.

1) Initialization: For each S_k, the initial transmit beamforming vector, $\mathbf{u}_{S_k}^0$, is set to be the solution to (6). Then the initial time fractions are $t_{S_k}^0 = \frac{R_{S_k}}{\hat{C}_{S_k}}$. The initial normalized coefficients $\boldsymbol{\sigma}_{S_k}^0 = \left(\sigma_{S_k,1}^0, \sigma_{S_k,2}^0, \cdots, \sigma_{S_k,M_{S_k}}^0\right)$ of each $\mathbf{u}_{S_k}^0$ can be calculated according to (23).

2) Main Body: The main body involves an iteration between two steps. In Step 1, we fix the normalized coefficients $\boldsymbol{\sigma}_{S_k} = \left(\sigma_{S_k,1}, \sigma_{S_k,2}, \cdots, \sigma_{S_k,M_{S_k}}\right)$ of each S_k, and then optimize the time fractions among all the SUs.

Given $\boldsymbol{\sigma}_{S_k} = \left(\sigma_{S_k,1}, \sigma_{S_k,2}, \cdots, \sigma_{S_k,M_{S_k}}\right)$, the received SINR of S_k in constraint (5a) is

$$\gamma_{BS_k} = \left(\sqrt{p_{S_k}} \sum_{i=1}^{M_{S_k}} \sigma_{S_k,i}\mathbf{z}_{S_k,i}\right)^H \mathbf{A}_{S_k} \left(\sqrt{p_{S_k}} \sum_{i=1}^{M_{S_k}} \sigma_{S_k,i}\mathbf{z}_{S_k,i}\right) = p_{S_k} \sum_{i=1}^{M_{S_k}} \rho_{S_k,i} \left|\sigma_{S_k,i}\right|^2.$$

The interference power term that appears in constraint (5c) can then be expressed as

$$q_{P_j S_k} = \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{u}_{S_k}\right|^2 = p_{S_k} \left|\sum_{i=1}^{M_{S_k}} \sigma_{S_k,i} \mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k} \mathbf{z}_{S_k,i}\right|^2.$$

Let η_{S_k} denote $\min \left\{ \frac{\phi_{P_1}}{\left|\sum_{i=1}^{M_{S_k}} \sigma_{S_k,i}\mathbf{v}_{P_1}^H \mathbf{H}_{P_1,S_k}\mathbf{z}_{S_k,i}\right|^2}, \cdots, \right.$

$\left.\frac{\phi_{P_J}}{\left|\sum_{i=1}^{M_{S_k}} \sigma_{S_k,i}\mathbf{v}_{P_J}^H \mathbf{H}_{P_J,S_k}\mathbf{z}_{S_k,i}\right|^2}, p_{S_k,\max}\right\}$. Therefore, given $\boldsymbol{\sigma}_{S_k}$ $(1 \le k \le K)$ of all the secondary users, Problem (5) reduces to

$$\begin{aligned} \text{minimize} \quad & \sum_{k=1}^{K} t_{S_k} \frac{\exp\left(\frac{R_{S_k}}{wt_{S_k}}\right) - 1}{\sum_{i=1}^{M_{S_k}} \rho_{S_k,i} \left|\sigma_{S_k,i}\right|^2} \\ \text{subject to} \quad & \sum_{k=1}^{K} t_{S_k} \le 1, \\ & t_{S_k} \ge \frac{R_{S_k}}{w\log\left(\eta_{S_k}\left(\sum_{i=1}^{M_{S_k}} \rho_{S_k,i}\left|\sigma_{S_k,i}\right|^2\right)+1\right)}, \\ \text{variables} \quad & t_{S_k} \ge 0, \quad \forall k. \end{aligned} \tag{24}$$

Similar to (17) and (20), problem (24) is a convex optimization problem. The optimal time fractions to (24) can be solved in polynomial time. The objective value of (24), denoted by E_1, is the system energy consumption obtained in Step 1.

In Step 2, we fix the active time fraction to be the value obtained in Step 1, and then optimize the transmit beamforming vector $\mathbf{u}_{S_k}$ for each S_k. Given $(t_{S_1}, \cdots, t_{S_K})$, Problem (5) reduces to K separate optimization problems. For each S_k, the optimization problem is given by

$$\begin{aligned} \text{minimize} \quad & \|\mathbf{u}_{S_k}\|_2^2 \\ \text{subject to} \quad & w\log\left(1+\mathbf{u}_{S_k}^H \mathbf{A}_{S_k}\mathbf{u}_{S_k}\right) \ge \frac{R_{S_k}}{t_{S_k}}, \\ & \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}\mathbf{u}_{S_k}\right|^2 \le \phi_{P_j}, \quad \forall j, \\ & \|\mathbf{u}_{S_k}\|_2^2 \le p_{S_k,\max}, \\ \text{variable} \quad & \mathbf{u}_{S_k}. \end{aligned} \tag{25}$$

Problem (25) is a quadratically constrained quadratic programming (QCQP), but it is not a convex optimization as the constraints are not convex. We adopt the SDP relaxation method to solve (25). Let $\mathbf{X}_{S_k}$ denote $\mathbf{u}_{S_k}\mathbf{u}_{S_k}^H$, which is a rank one Hermitian positive semidefinite matrix. We know that for any $\mathbf{A}$, we have $\mathbf{u}_{S_k}^H \mathbf{A}\mathbf{u}_{S_k} = \operatorname{tr}(\mathbf{A}\mathbf{X}_{S_k})$. If we drop the rank constraint $\operatorname{rank}(\mathbf{X}_{S_k}) = 1$, we then obtain the SDP relaxation of problem (25):

$$\begin{aligned} \text{minimize} \quad & \operatorname{tr}(\mathbf{X}_{S_k}) \\ \text{subject to} \quad & \operatorname{tr}(\mathbf{A}_{S_k}\mathbf{X}_{S_k}) \ge e^{\frac{R_{S_k}}{wt_{S_k}}} - 1, \\ & \operatorname{tr}\left(\mathbf{G}_{P_j,S_k}\mathbf{X}_{S_k}\right) \le \phi_{P_j}, \quad \forall j, \\ & \operatorname{tr}(\mathbf{X}_{S_k}) \le p_{S_k,\max}, \\ \text{variable} \quad & \mathbf{X}_{S_k} \succeq \mathbf{0}, \end{aligned} \tag{26}$$

where $\mathbf{G}_{P_j,S_k} = \mathbf{H}_{P_j,S_k}^H \mathbf{v}_{P_j}\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}$, and we use the fact that

$$\begin{aligned} \left|\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}\mathbf{u}_{S_k}\right|^2 &= \mathbf{u}_{S_k}^H \mathbf{H}_{P_j,S_k}^H \mathbf{v}_{P_j}\mathbf{v}_{P_j}^H \mathbf{H}_{P_j,S_k}\mathbf{u}_{S_k} \\ &= \mathbf{u}_{S_k}^H \mathbf{G}_{P_j,S_k}\mathbf{u}_{S_k} = \operatorname{tr}\left(\mathbf{G}_{P_j,S_k}\mathbf{X}_{S_k}\right). \end{aligned}$$

Problem (26) is a SDP, which is a convex optimization problem and can be solved in polynomial time using interior point methods [26]. We first obtain the optimal solution $\mathbf{X}_{S_k}^*$ to (26). The solution $\mathbf{X}_{S_k}^*$ may not be rank one in general. If $\operatorname{rank}\left(\mathbf{X}_{S_k}^*\right) = 1$, then the principal component of $\mathbf{X}_{S_k}^*$ will be the optimal solution to (25). Otherwise we can generate a feasible solution $\mathbf{u}_{S_k}$ to (25) from $\mathbf{X}_{S_k}^*$ using randomization techniques provided in [5]. [3] After obtaining $\mathbf{u}_{S_k}$, the system energy consumption obtained in Step 2 then can be calculated by $E_2 = \sum_{k=1}^{K} t_{S_k}\|\mathbf{u}_{S_k}\|_2^2$. At last, given current $\mathbf{u}_{S_k}$ for each SU, we update its normalized coefficients $\boldsymbol{\sigma}_{S_k} = \left(\sigma_{S_k,1}, \sigma_{S_k,2}, \cdots, \sigma_{S_k,M_{S_k}}\right)$ according to (23), and then start the next iteration.

3) Termination Condition: Notice that in each iteration of Algorithm 1, the system energy consumption is updated both when the time fractions $(t_{S_1}, \cdots, t_{S_K})$ are updated in Step 1 and when the transmit beamforming vectors $(\mathbf{u}_{S_1}, \cdots, \mathbf{u}_{S_K})$ are updated in Step 2. We take the minimum of the two, $E = \min\{E_1, E_2\}$, as the system energy consumption in the current iteration[4]. In each iteration, we compare E with the one obtained in last iteration, and start the next iteration if the reduction ratio is greater than a percentage threshold $\varepsilon \in (0,1)$; otherwise, the algorithm terminates. As E strictly decreases as Algorithm 1 continues, and it is lower-bounded, the algorithm is guaranteed to terminate in a finite number of iterations[5].

VII. Simulation Results

We carry out simulations to evaluate the performance of the proposed algorithms. We simulate a CR network with two primary links[6]. The network topology is shown in Fig. 1. The length of each primary link is 50 meters. The secondary BS is placed at the center of the square area of $200m \times 200m$. The SUs are uniformly distributed in the square, with a minimum distance of 25 meters from the two primary receivers.

The wireless channel is Rayleigh fading channel with the path loss exponent of 4. All the transmitters and receivers are equipped with 4 antennas. The maximum output power is 27.5 dBm. The PUs both transmit at the maximum power. The noise power density is -174 dBm/Hz. The bandwidth and the frame length of the secondary system are 1 MHz and 1 second, respectively. Each SU has a rate requirement of 200 kbps. The tolerable interference power at each primary receiver ϕ_{P_j} is chosen such that $\frac{\phi_{P_j}}{N_0 w}$ is 20 dB. The outage probability δ_{P_j} is set to be 1%. Each point in the curves is an average of 1000 simulation runs with independent SUs' locations.

[3]Since the constraint sets in (25) have both $\ge$ and $\le$ constraints, the chosen candidate vector in the randomization method needs to be scaled in both directions (up or down) in order to satisfy both types of constraints.

[4]Notice that E_1 and E_2 may not always be lower than the previous iteration.

[5]Notice that there is no optimality guarantee (local or global) of the solution found by Algorithm 1. The maximum number of iterations is upper bounded by $\log_{(1-\varepsilon)}\left(\frac{E_{\min}}{E_{in}}\right)$, where E_{in} is the initial system energy consumption in the first iteration and $E_{\min}$ is the minimum system energy consumption.

[6]The reason why we choose two primary links is that the link capacity of each SU can be calculated exactly. In this case, there is no gap in performing the call admission control.

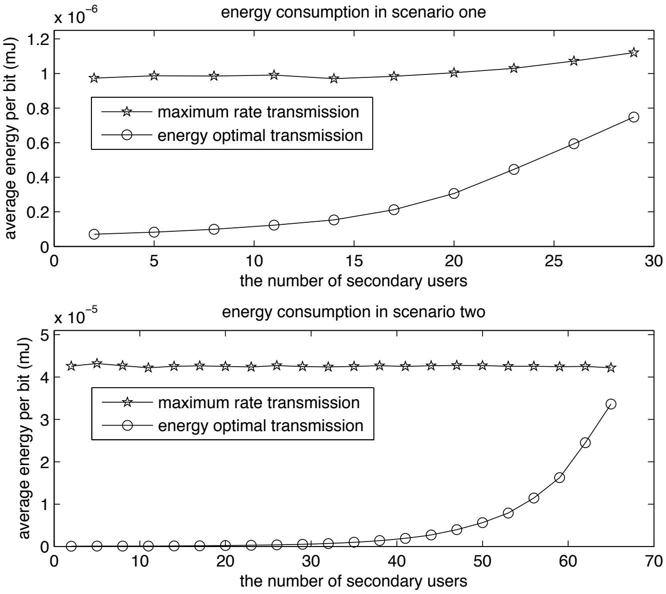

Fig. 3. Average energy consumption per bit of the secondary system vs. the number of SUs.

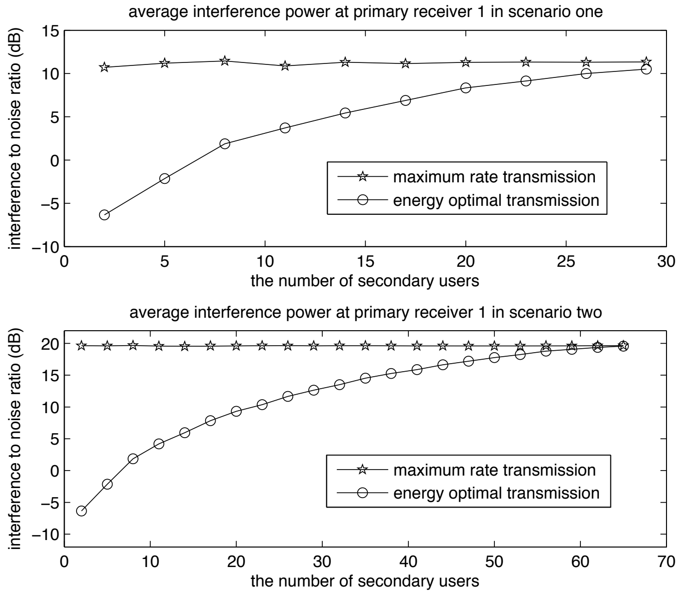

Fig. 4. Average interference power at primary receiver one vs. the number of SUs.

A. Energy Consumption Improvement

We compare the energy consumption of the following two transmission policies:

1) Maximum-rate transmission: each SU transmits at its maximum rate while satisfying the maximum transmit power constraint and the interference constraint to the primary receivers.
2) Energy-optimal transmission: the energy-optimal time scheduling and transmit beamforming proposed in this paper.

Figure 3 shows the energy consumption per bit of the secondary system as a function of the number of SUs. We find that the energy-optimal transmission policy significantly saves the energy consumption. In the maximum-rate transmission policy, the averaged energy consumption per bit does not vary much with the secondary system's traffic load. In the energy-optimal transmission policy, there is a tradeoff between the energy consumption and the system traffic load. The secondary BS reduces the energy consumption by fully utilizing the time resource. Therefore, the energy consumption per bit is adaptive to the system traffic load. When the traffic load is low (i.e., each SU has more available time resource), the energy saving becomes more significant. Compared with the maximum-rate transmission policy, the energy reduction ranges from 30% to 91% and from 20% to 99% in scenario one and scenario two, respectively.

B. Interference Power Level at the Primary Receivers

We now evaluate the interference power from the secondary transmitters to the primary receivers. The maximum tolerable $\frac{\phi_{P_j}}{N_0 w}$ is set to be 20 dB. Since the primary links are placed symmetrically to each other, the interference power level at both the primary receivers are similar. Figure 4 shows the averaged interference power at primary receiver one as a function of the number of SUs. The interference power is measured as the the ratio of the interference power to noise power (in dB scale).

As we can see from Fig. 4, in the maximum-rate transmission policy, the SUs generate almost constant interference power at the primary receiver for different secondary system's traffic loads. The interference power to noise ratio is around 10 dB and 20 dB for scenario one and scenario two, respectively. In the energy-optimal transmission policy, there is a tradeoff between the interference power level and the secondary system's traffic load. The interference power is reduced significantly when the traffic load is low. As shown in Fig. 4, when the number of SUs is no more than 6, the interference power to noise ratio is below 0 dB. In this case, the interference power generated by the transmissions of SUs is no larger than the background noise.

C. Optimality of Algorithm 1

We now investigate the solution optimality of Algorithm 1. As discussed in the previous sections, the exact optimal values in scenario two are quite difficult to obtain when the traffic load is heavy. However, during simulations we find that when the number of SUs, K, ranges from 14 to 22, the secondary system is at the boundary between under-utilized and heavily-utilized due to the randomness of simulation parameters. That is, for each K within this range, among all the 1000 simulated secondary networks, a certain amount of networks are under-utilized and thus optimal solutions can be found, and a certain amount of networks are heavily-utilized and thus we can only find the approximation solutions by the heuristic Algorithm 1. Table II shows the numbers of under-utilized and heavily-utilized secondary networks for each K value. Figure 5 shows the averaged energy consumption of both the two types of secondary networks. For each given K, the averaged value of

TABLE II
THE NUMBER OF UNDER-UTILIZED AND HEAVILY-UTILIZED SECONDARY NETWORKS

The number of SUs	14	16	18	20	22
The number of under-utilized networks	838	686	545	350	204
The number of heavily-utilized networks	162	314	455	650	796

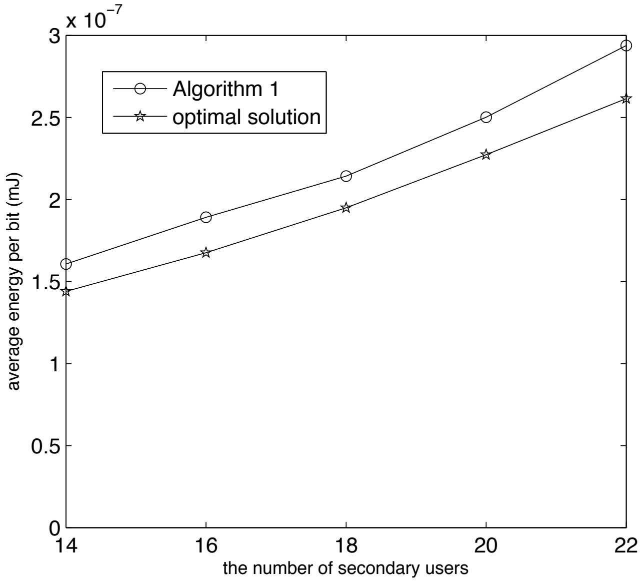

Fig. 5. The optimality of Algorithm 1.

the optimal energy consumption of the under-utilized networks could server as a lower bound of the optimal values for the heavily-utilized networks. As we can see from Fig. 5, the heuristic Algorithm 1 performs very close to the optimal. Compared with the lower bound of the optimal values, the gap is within 12%.

D. The averaged number of iterations of Algorithm 1

When the number of SUs exceeds 32, all the simulated networks are heavily-utilized secondary networks. For each network realization, the number of iterations of Algorithm 1 is a random number. Figure 6 shows that the averaged number of iterations for Algorithm 1 to terminate. We find that for all the simulated networks with different numbers of SUs, the averaged number of iterations is below 6. This indicates that Algorithm 1 terminates very fast.

VIII. CONCLUSION

In this paper, we considered the energy-optimal time allocation and beamforming in MIMO CR networks. We showed that in scenario one with no CSI to the primary receivers, the energy-optimal beamforming vectors can be found efficiently by a simple matrix eigenvalue-eigenvector computation. Based on its nice closed-form structure, we further showed that the overall optimization problem can be solved in polynomial-time with a proper decomposition. In scenario two with perfect CSI to primary receivers, such nice property is still valid and

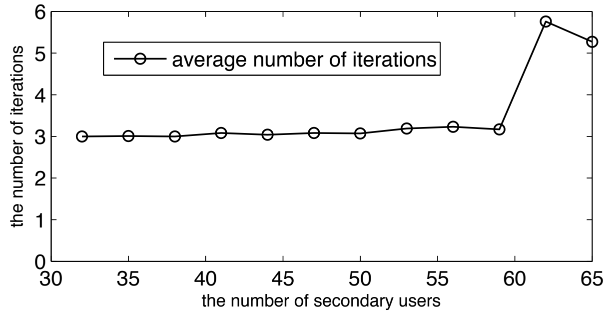

Fig. 6. The average number of iterations of Algorithm 1.

the decomposition method can still be applied to find the optimal solutions if the secondary system is *under-utilized*. For heavily-utilized secondary system in scenario two, we proposed a heuristic algorithm to construct a near-optimal solution. The simulation results showed that performing energy-efficient transmissions in MIMO CR networks not only reduces the energy consumptions of the SUs, but also alleviates the interference generated to the primary receivers. The energy saving benefits and interference power level reduction become more significant when the traffic load of the secondary system is low.

Note that for MIMO CR networks, the secondary users can also coexist with each other with spatial multiplexing. Furthermore, each secondary user can transmit multiple data streams simultaneously. In this case, the secondary users not only interfere with the primary users, but also interfere with each other. There is a trade-off between the interference cancellation and the spatial multiplexing gain. The energy efficient transmission for such MIMO CR networks is an interesting, yet challenging topic for future study.

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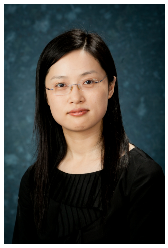

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