Fr.l #19 (forts. på Laplace för system).

Rep:

$$\begin{cases} x' = 4x - 2y + 2u_1(t) \\ y' = 3x - y + u_1(t) \end{cases}$$

$x(0) = 0, \quad y(0) = 1/2.$

Laplace:

$$sX(s) = 4X(s) - 2Y(s) + \frac{2}{s}e^{-s}$$

$$sY(s) - \frac{1}{2} = 3X(s) - Y(s) + \frac{1}{s}e^{-s}$$

$\Rightarrow$

$$(*) \begin{cases} (s-4)X(s) + 2Y(s) = \frac{2}{s}e^{-s} \\ -3X(s) + (s+1)Y(s) = \frac{1}{2} + \frac{1}{s}e^{-s} \end{cases}$$

Kramer: $X(s) = \dfrac{-1 + 2e^{-s}}{(s-1)(s-2)}$

$$\begin{vmatrix} s-4 & 2 \\ -3 & s+1 \end{vmatrix}$$

$\Rightarrow x(t) = \mathcal{L}^{-1}(X(s)) = e^{t} - e^{2t} + 2u_1(t)\left(e^{2(t-1)} - e^{t-1}\right)$

$Y = ?$ $\quad Y = \begin{vmatrix} a & e \\ c & f \end{vmatrix} \Big/ \begin{vmatrix} a & b \\ c & d \end{vmatrix}$ $= (s-1)(s-2)$

$\begin{vmatrix} a & e \end{vmatrix} = \begin{vmatrix} s-4 & 2 \end{vmatrix}$ (se tidigare)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} \cdot & \cdot \\ -3 & (s+1) \end{vmatrix} = \dots = (s-1)(s-2)$$

$$\begin{vmatrix} a & e \\ c & f \end{vmatrix} = \begin{vmatrix} s-4 & \frac{2}{s}e^{-s} \\ -3 & \frac{1}{2}+\frac{1}{s}e^{-s} \end{vmatrix}$$

$$= (s-4)\left(\frac{1}{2}+\frac{1}{s}\cdot e^{-s}\right) + \frac{6}{s}e^{-s} =$$

$$\frac{s}{2} + e^{-s} - 2 - \frac{4}{s}e^{-s} + \frac{6}{s}\cdot e^{-s} =$$

$$1+\frac{2}{s} = \frac{s+2}{s}$$

$$\frac{s}{2} + e^{-s} - 2 + \frac{2}{s}e^{-s}$$

$$Y(s) = \det / \det = \frac{s/2 + e^{-s} - 2 + \frac{2}{s}e^{-s}}{(s-1)(s-2)}$$

$$= \frac{s/2 - 2}{(s-1)(s-2)} + \frac{(2+s)e^{-s}}{s(s-1)(s-2)}$$

Partial bråk #1: A. B.

$$\frac{s/2 - 2}{(s-1)(s-2)} = \frac{3}{2}\frac{1}{(s-1)} - \frac{1}{(s-2)}$$

$$\Rightarrow \mathcal{L}^{-1}\left\{\frac{s/2 - 2}{(s-1)(s-2)}\right\} = \frac{3}{2}\cdot e^{t} - e^{2t}$$

(tabell!)

Part. bråk #2:

$$A=\frac{2}{(-1)\cdot(-2)}=1 \quad B \quad C$$

$$\frac{s+2}{s(s-1)(s-2)} = \frac{1}{s} + \frac{-3}{s-1} + \frac{2}{s-2}$$

(övning: kolla! övning 2: gör själv!)

$$\mathcal{L}^{-1}\left\{\frac{(s+2)\cdot e^{-s}}{s(s-1)(s-2)}\right\} =$$

$$= u_1(t)\left(1 - 3\cdot e^{t-1} + 2\cdot e^{2(t-1)}\right)$$

$$\therefore y(t) = \frac{3}{2}e^{t} - e^{2t} + u_1(t)\left(1 - 3e^{t-1} + 2e^{2(t-1)}\right)$$

hopp i t=1 =0 vid t=1

$$\mathcal{L}^{-1}\left\{\left(\frac{1}{s} + \frac{-3}{s-1} + \frac{2}{s-2}\right)\right\} = 1 - 3e^{t} + 2e^{2t}$$

e^{-s}

$t \to t-1$

(och mult. med $u_1(t)$)

Kan skriva allt som:

=0 om t=1

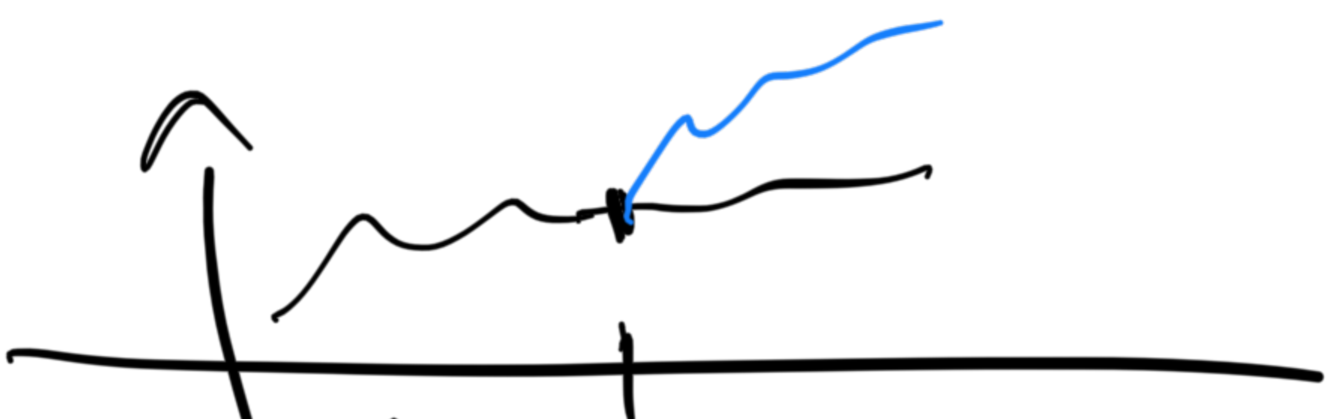
$$x(t) = \begin{cases} e^{t} - e^{2t}, & 0 \le t < 1 \\ e^{t} - e^{2t} + 2\left(e^{2(t-1)} - e^{t-1}\right), & t \ge 1 \end{cases}$$

$$y(t) = \begin{cases} \frac{3}{2}e^{t} - e^{2t}, & 0 \le t < 1 \end{cases}$$

$$\begin{cases} \frac{3}{2}e^{t} - e^{2t} + 1 - 3e^{t-1} + 2e^{2(t-1)}, & t \geq 1 \end{cases}$$

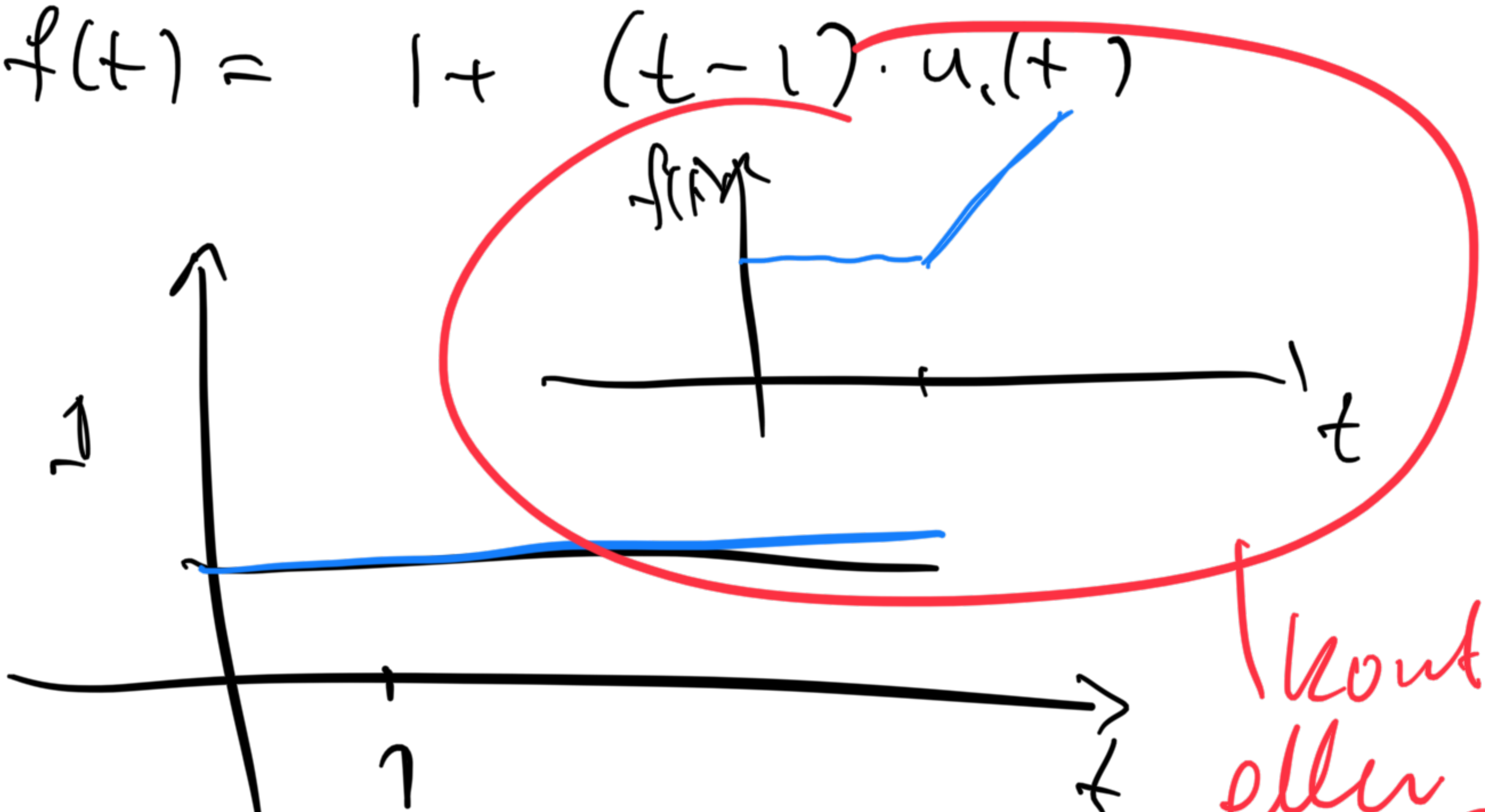
u_1 "påslagen" om $t \geq 1$.

u_1 "avslagen" om $0 \leq t < 1$.

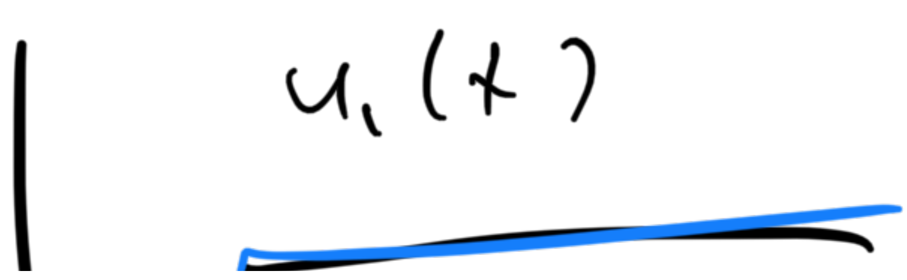


$f(t)$ $\qquad$ $f(t) + u_1(t)\cdot(t-1)$

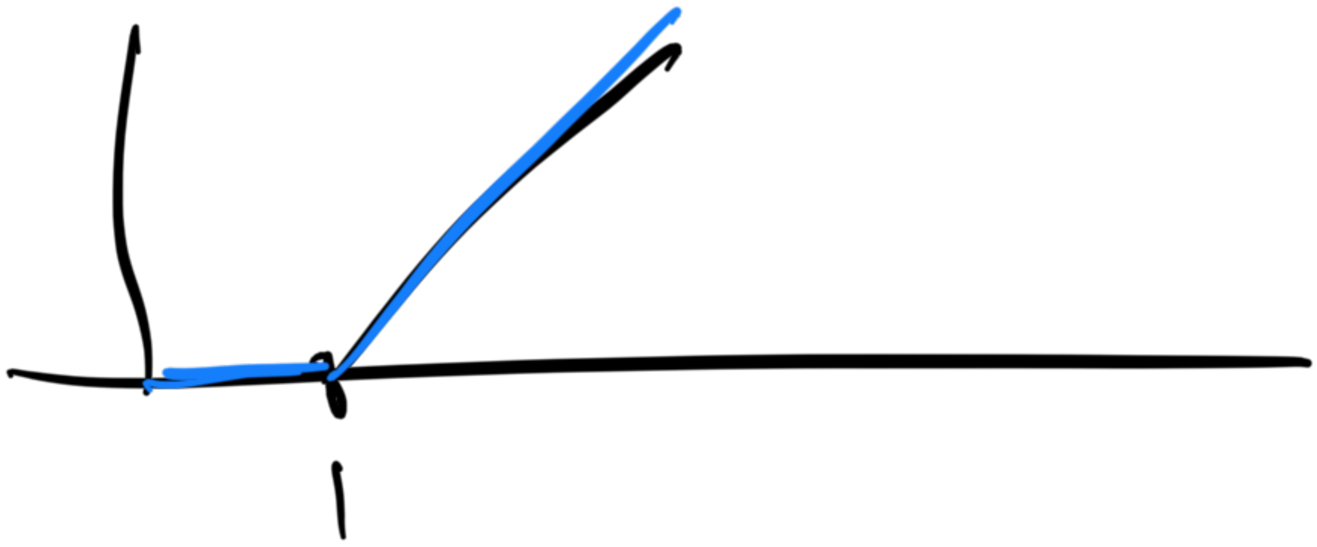
$$f(t) = 1 + (t-1)\cdot u_1(t)$$



kont. eller inte?!



$u_1(t)\cdot(t-1)$



SLUT på nytt material!

Hurra! :)

Efter rast:
jag visar lite filmer.

Onsdg: ~~repert~~ repetition/genomgång av HELA

Kusen.