Number theory session

Location: room 3733, department of mathematics, KTH

Abstracts

Critical zeros of L-functions

Henryk Iwaniec
Rutgers University, USA

This will be a survey talk on the distribution of zeros of the Riemann zeta function and the Dirichlet L-series on the critical line. The most recent result (joint with Brian Conrey and Kannan Soundararajan) asserts that the L-functions have at least 60% zeros on the line on average with respect to the characters. Small and large gaps between zeros will be also discussed.

Small scale equidistribution of eigenfunctions on the torus

Steve Lester
KTH, Sweden

I will describe some recent results on the distribution of the L^2-mass of eigenfunctions of the Laplacian on the torus $\mathbb{T}^d/2\pi\mathbb{R}^d$. A special case of a result of Marklof and Rudnick implies that the L^2-mass of almost all such eigenfunctions equidistributes with respect to Lebesgue measure for $d = 2$. I will discuss results on the scales at which the L^2-mass equidistributes as well as mention some limitations on equidistribution, and relate these questions to arithmetic problems such as representing integers as sums of squares and the distribution of lattice points.

This is joint work with Zeév Rudnick.

Harmonic weak Siegel Maass forms

Martin Westerholt-Raum
Chalmers, Sweden

Harmonic weak Maass forms for $SL(2,\mathbb{R})$ where defined by Bruinier and Funke more than ten years ago. They have been successfully applied to combinatorial problems thanks to their overlap with indefinite theta functions. Their genuine arithmetic, on the other hand, is linked to derivatives of L-series, as demonstrated by Bruinier and Ono. We start by revisiting these connections.

Siegel modular forms are modular forms for the group $Sp(g,\mathbb{R})$. If $g = 1$ they coincide with elliptic modular forms, but in the general case they are more intricate. In joint work with Bringmann and Richter, the speaker studied real analytic Siegel modular forms and connected their Fourier Jacobi coefficients in the case of $g = 2$ to harmonic weak Maass forms for $SL(2,\mathbb{R})$. We showcase this connection, and exhibit the importance of Fourier Jacobi coefficients in the study of Siegel modular forms.

Finally, we discuss the existence of harmonic weak Siegel Maass forms. Using the connection of Eisenstein series and principal series representations, one manages to obtain sufficiently tight control of Dolbeault cohomology to show that every non-holomorphic Saito-Kurokawa lift can be further lifted to a harmonic weak Siegel Maass form. We discuss potential applications to derivatives of L-series.

Counting rational points on cubic curves

Per Salberger
Chalmers, Sweden

We present a new uniform bound for the number of rational points of height at most B on non-singular cubic curves, which improves upon previous bounds of Ellenberg/Venkatesh and Heath-Brown/Testa.

Low-lying zeros of Artin L-functions

Anders Södergren
University of Copenhagen, Denmark

In this talk we discuss the distribution of low-lying zeros of certain families of Artin L-functions attached to geometric parametrizations of number fields. We describe several explicit examples of such families and in each case we verify the Katz-Sarnak heuristics and present the symmetry type of the distribution of low-lying zeros.

This is joint work with Arul Shankar and Nicolas Templier.

The hyperbolic circle problem

Morten Risager
University of Copenhagen, Denmark

We review the hyperbolic circle problem and explain some recent results concerning the error term. We also explain how these results relate to L-functions of certain automorphic forms and conjectures about these.

Bad reduction of curves with CM jacobians

Fabien Pazuki
Chalmers, Sweden

An abelian variety defined over a number field and having complex multiplication (CM) has potentially good reduction everywhere. If a curve of positive genus which is defined over a number field has good reduction at a given finite place, then so does its jacobian variety. However, the converse statement is false already in the genus 2 case, as can be seen in the entry $[I_0 - I_0 - m]$ in Namikawa and Ueno's classification table of fibres in pencils of curves of genus 2. In this joint work with Philipp Habegger, our main result states that this phenomenon prevails for certain families of curves.

We prove the following result: Let F be a real quadratic number field. Up to isomorphisms there are only finitely many curves C of genus 2 defined over $\overline{\mathbb{Q}}$ with good reduction everywhere and such that the jacobian $Jac(C)$ has CM by the maximal order of a quartic, cyclic, totally imaginary number field containing F. Hence such a curve will almost always have stable bad reduction at some prime whereas its jacobian has good reduction everywhere. A remark is that one can exhibit an infinite family of genus 2 curves with CM jacobian such that the endomorphism ring is the ring of algebraic integers in a cyclic extension of $\mathbb{Q}$ of degree 4 that contains $\mathbb{Q}(\sqrt{5})$.

Linear spaces on hypersurfaces with a prescribed discriminant

Julia Brandes
Chalmers, Sweden

For a given form $F \in \mathbb{Z}[x_1, \ldots, x_s]$ we apply the circle method in order to give an asymptotic estimate of the number of m-tuples $\mathbf{x}_1, \ldots, \mathbf{x}_m$ on the hypersurface $F(X) = 0$ having $\det(\mathbf{x}_1, \ldots, \mathbf{x}_m)^t\,(\mathbf{x}_1, \ldots, \mathbf{x}_m)) = b$. As a corollary, we obtain a count of rational linear spaces contained in the hypersurface $F(\mathbf{x}) = 0$ having dimension exactly m, thus addressing a weakness of previous results.

Coding on random lattices

Roope Vehkalahti
University of Turku, Finland

Classical information theory of additive white Gaussian noise channels naturally suggests several coding theoretic problems that can be formulated on lattice theoretic language. For example, performance of a lattice code can be roughly estimated by it's Hermite invariant and behavior of the related theta function.

Modern wireless communication channels assume use of multiple antennas and that the transmitted signal gets faded by scattering environment. On such channels classical lattice codes will not perform well. In this talk we will describe how coding in modern fading channels can be seen as coding on random lattice ensembles and how this perspective transforms classical questions into new ones.

In particular we will show how classical Hermite invariant can be replaced by different homogeneous forms and how class field towers with constant root discriminants and non-commutative algebra can be applied in building capacity approaching lattice codes for fading channels.

If time allows we will discuss how averaging transforms theta functions of lattices into sums of different type and how these sums can be analyzed by employing methods from classical algebraic number theory and ergodic theory on Lie groups.

The talk is based on joint work with Laura Luzzi and Francis Lu.

Algebraicity of automorphic representations

Wushi Goldring
Stockholm University, Sweden

One striking implication of Langlands' conjectures for number fields is that many automorphic representations which are initially defined by analytic and/or representation-theoretic means should have deep algebro-geometric properties, ranging from the algebraicity of Hecke eigenvalues, to the existence of associated Galois representations and ultimately pure motives. An example of a long-standing open problem in this area which admits an elementary formulation is to prove that Maass forms of eigenvalue 1/4 have algebraic Hecke eigenvalues. Two approaches have been used to verify Langlands' predictions: (1) Finding and exploiting a direct link with algebraic geometry and (2) Using Langlands' Functoriality Principle. I will discuss the possibilities and limitations of the two approaches and report on recent work on each approach. The results using the geometric approach are joint work with Jean-Stefan Koskivirta, see arXiv:1507.05032.

Low-lying zeros of quadratic Dirichlet L-functions

James Parks
KTH, Sweden

In this talk we study the 1-level density of low-lying zeros of Dirichlet L-functions attached to real primitive characters of conductor at most X. We obtain an asymptotic expansion of this quantity with lower order terms in descending powers of $\log X$. We show that this is valid under GRH when the support of the Fourier Transform of the implied even test function ϕ is contained in $(-2, 2)$. We also uncover a phase transition when the supremum of the support of $\widehat{\phi}$ reaches 1, where a new lower order term appears.

This is joint work with Daniel Fiorilli and Anders Södergren.

Around transcendence

Tapani Matala-Aho
University of Oulu, Finland

We will discuss on selected tools used in transcendence methods and Diophantine approximations.

Orbits of rotations and beyond

Simon Kristensen
Aarhus University, Denmark

The orbit of a rotation of the unit circle has one of two behaviours as a dynamical system, depending on whether the angle of rotation is a rational or an irrational multiple of 2π. In the rational case, any orbit is periodic; in the irrational case, any orbit is dense and in fact uniformly distributed. A quantitative form of the uniform distribution of the irrational orbits can be studied via the discrepancy of the sequence $\{n\alpha\}$ which in turn depends heavily on the continued fraction expansion of α.

The study of orbits of rotations leads naturally to the notion of twisted Diophantine approximation – a form of inhomogeneous Diophantine approximation, where one fixes the homogeneous parameter and studies the possible approximation properties of the inhomogeneous parameter as it varies.

With Bugeaud, Harrap and Velani, we proved that the natural analogue of badly approximable elements exist in abundance. To be precise, we proved that for any $\alpha \in \mathbb{R}$, the set

$$\left\{x \in [0,1) : \|n\alpha - x\| \geq \frac{K(x)}{n} \text{ for some } K(x) > 0 \text{ for all } n \in \mathbb{N}\right\}$$

has maximal Hausdorff dimension and that this property is stable under intersection with sufficiently nice fractals. In recent work in progress with Tseng, this result is extended to the case when the rotation is replaced by an Interval Exchange Transformation (IET). The added difficulty of the IET setting comes from some highly geometric and dynamical obstacles to a sufficiently nice behaviour of the appropriate analogue of continued fractions.

Diophantine approximation in matrices and Lie groups

Lior Rosenzweig
KTH, Sweden

A classical question in Diophantine approximation is how well can a real number be approximated by rationals. In this talk I will discuss generalization of this question in two direction, namely to approximation on submanifolds of R^n , and more specifically to submanifold of matrices, and approximation in Lie groups. We will also discuss the optimal Diophantine exponent.

This is joint work with M. Aka, E. Breuillard and N. de Saxce

Serre weights and wild ramification

Fred Diamond
King's College London, United Kingdom

Serre's Conjecture (proved by Khare and Wintenberger) asserts that every odd, irreducible representation from $\mathrm{Gal}(K/\mathbb{Q})$ to $GL_2(\mathbb{F}_{p^r})$ arises from a modular form; furthermore Serre specifies the minimal weight and level of a form giving rise to the Galois representation. The recipe for the weight depends on the local behavior of the representation at the prime p, and this dependence becomes much more subtle in generalizations of Serre's Conjecture. In particular, when $\mathbb{Q}$ is replaced by a totally real field, work of Gee and others determines the weights of Hilbert modular forms giving rise to a mod p Galois representation in terms of the existence of crystalline lifts, but the dependence of the weights on wild ramification is not explicit. I'll discuss joint work with Dembele and Roberts towards making it so, or equivalently, describing wild ramification in reductions of two-dimensional crystalline Galois representations.