Event-Based Control of Multi-Agent Systems

Karl H. Johansson
ACCESS Linnaeus Center & School of Electrical Engineering
KTH Royal Institute of Technology, Sweden

IEEE Int'l Conference on Event-based Control, Communication, and Signal Processing
Krakow, 17-19 June 2015

Acknowledgements

Presentation based on joint papers with students

Antonio Adaldo, Georg Kiener, Chithrupa Ramesh, Georg Seyboth

postdocs

Daniel Lehmann, Davide Liuzza, Maben Rabi

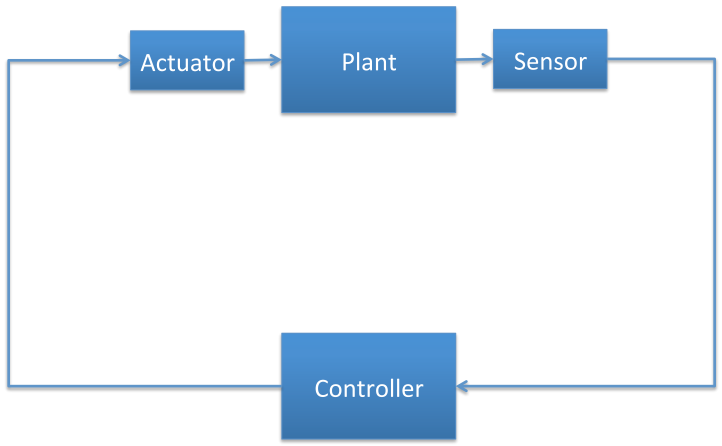
and colleagues

Dimos Dimarogonas, Henrik Sandberg

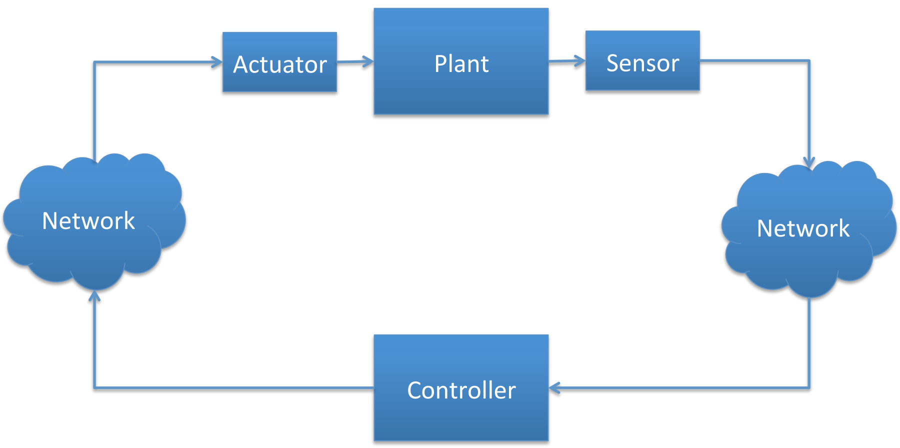
together with collaborations and inspiring discussions with more.

Funding sources:

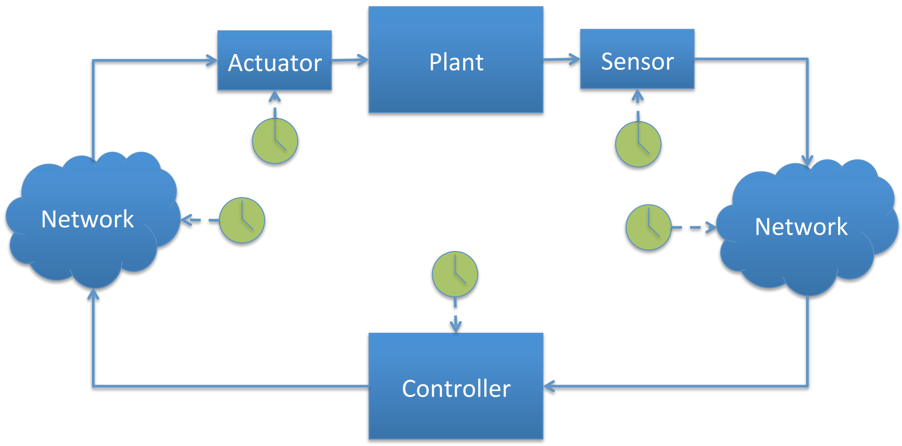
Feedback Control System



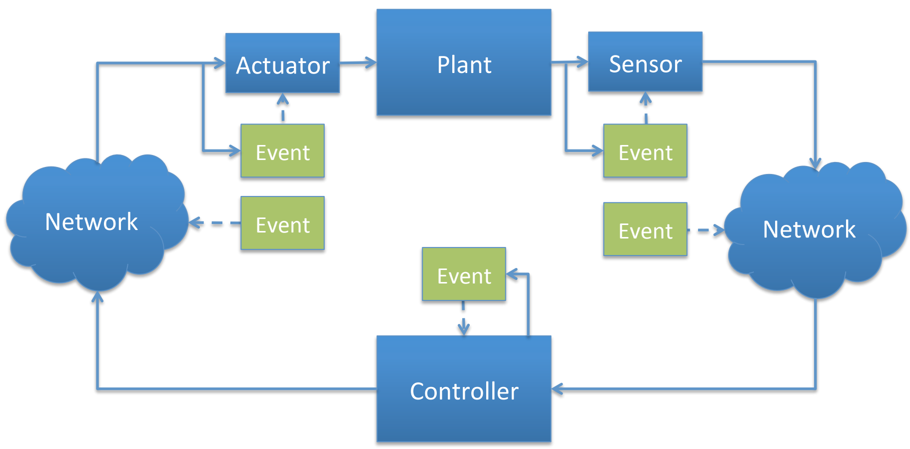
Networked Control System



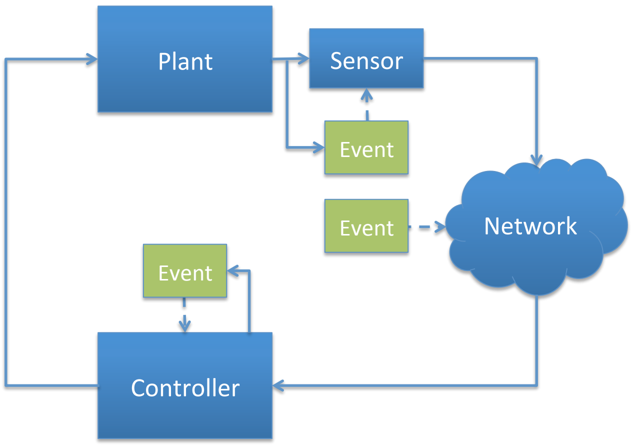
Time-Triggered Control System



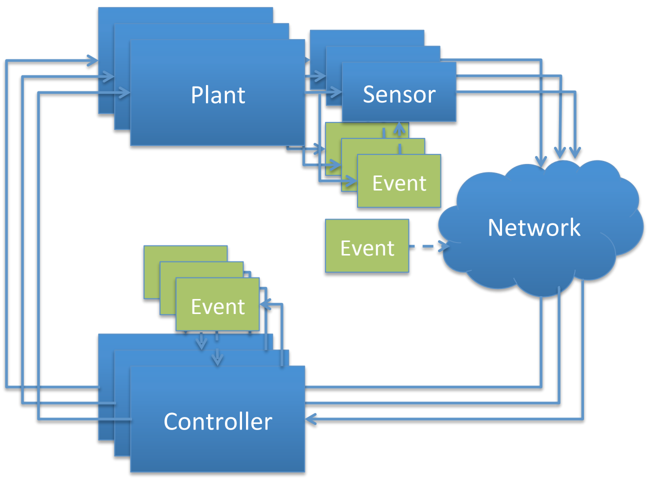
Event-Based Control System



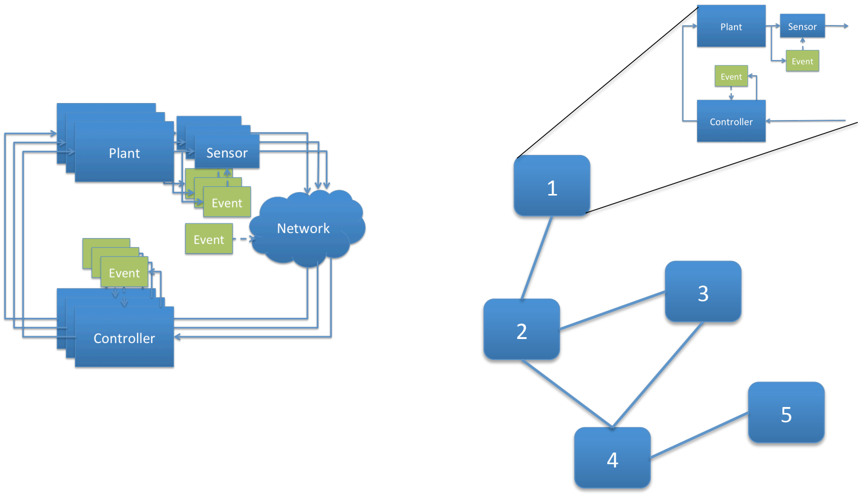
Event-Based Control System



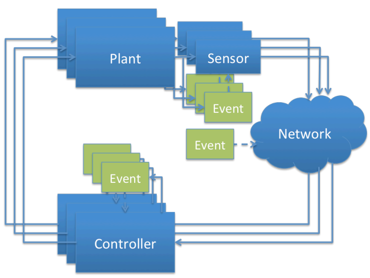
Event-Based Control System



Event-Based Multi-Agent System



Goal: Guarantee Control Performance under Limited Resources



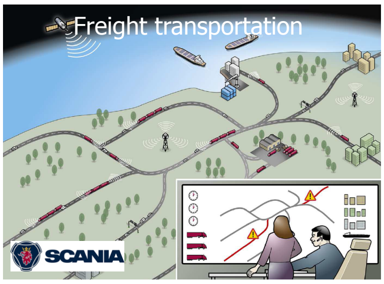
Resources

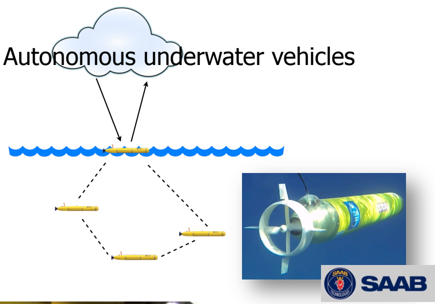
- Sensing
- Sensor communication
- Network
- Actuation
- (Computing)

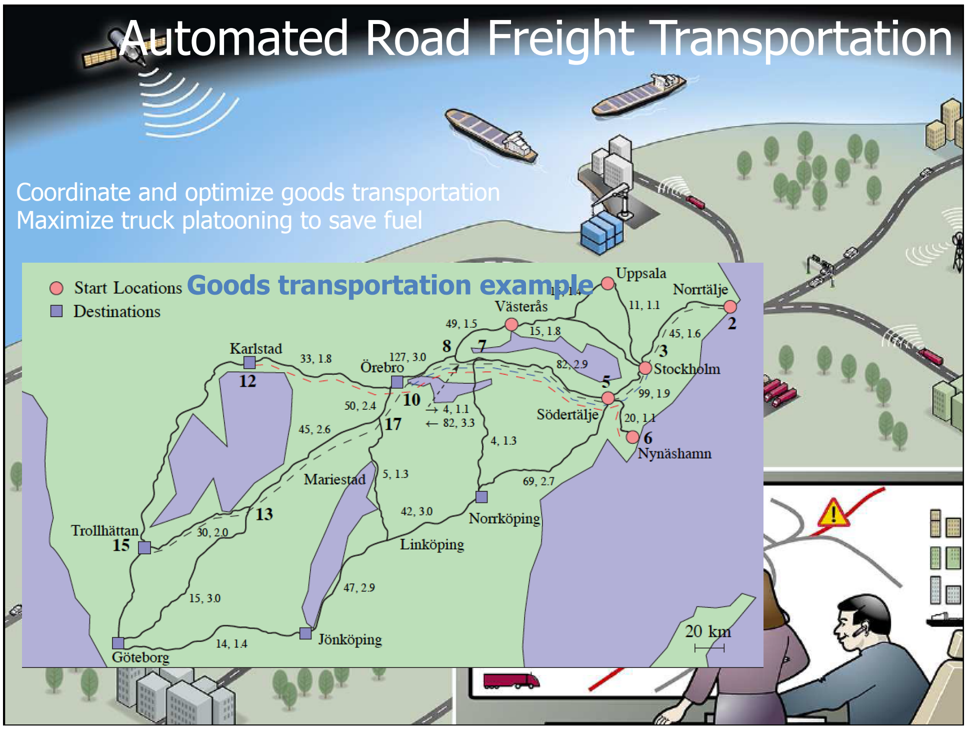
Outline

- Introduction
- Motivating applications
- Optimal event-based control
- Distributed event-based control
- Implementation aspects
- Conclusions

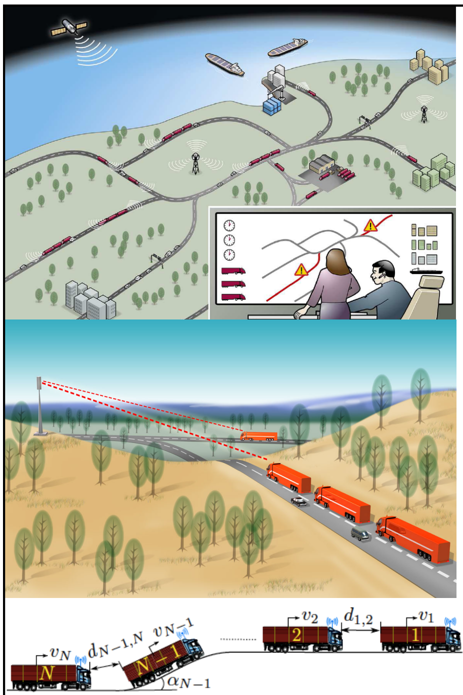
Motivating Applications

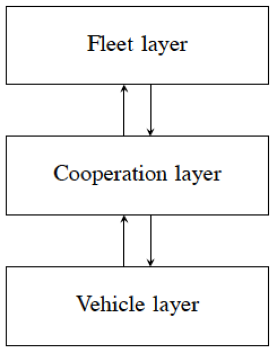




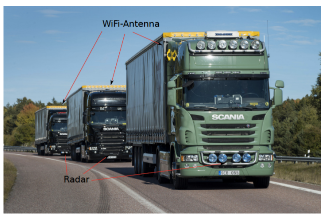


Automated Road Freight Transportation
Coordinate and optimize goods transportation
Maximize truck platooning to save fuel
Goods transportation example
Start Locations
Destinations
Uppsala
Norrtälje
Västerås
Karlstad
Örebro
Stockholm
Södertälje
Nynäshamn
Mariestad
Norrköping
Linköping
Trollhättan
Göteborg
Jönköping
20 km



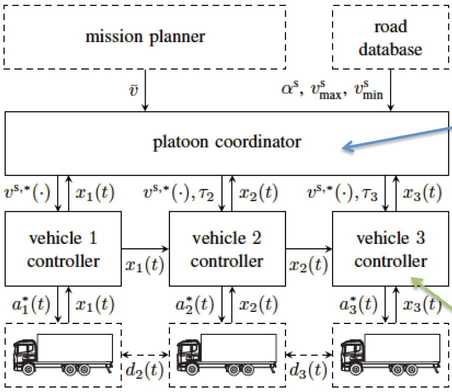


Fleet layer
Cooperation layer
Vehicle layer



WiFi-Antenna
Radar

Event-based Control of Vehicle Platoon



- **Platoon coordinator** optimize velocity based on road grade, vehicle models, and traffic info
- Dynamic program solved every 4 s
- Event-based replanning due to changing operating conditions

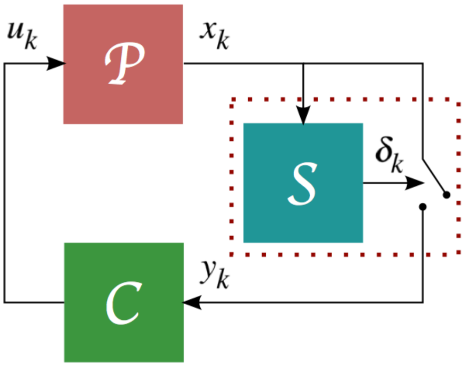
- **Vehicle controller** tracks platoon velocity
- Receding horizon controller with dynamic safety constraints and update rate 50 ms
- Event-based disturbance compensation

Turri et al, 2015

Outline

- Introduction
- Motivating applications
- Optimal event-based control
- Distributed event-based control
- Implementation aspects
- Conclusions

Optimal Event-Generation and Control



Stochastic Control Formulation

Plant:

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

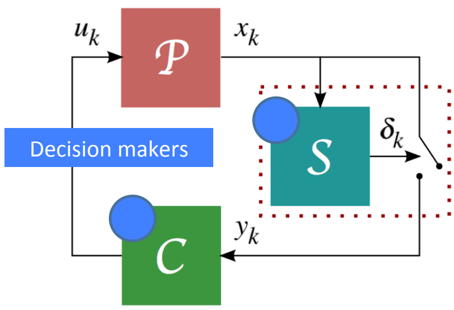
$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$

$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$

$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$



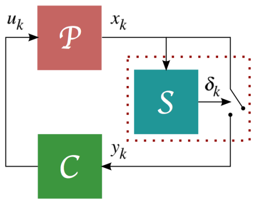
Cost criterion:

$$J(f,g) = \mathrm{E}[x_N^T Q_0 x_N + \sum_{s=0}^{N-1} (x_s^T Q_1 x_s + u_s^T Q_2 u_s)]$$

- Non-classical information pattern
- Hard to find optimal solutions in general
- Special cases lead to tractable problems

Cf., Witsenhausen, Hu & Chu, Varaiya & Walrand , Borkar, Mitter & Tatikonda, Rotkowitz etc

Example



Plant

$$x_{k+1} = x_k + u_k + w_k, \quad x_0 = 2, Ew_k^2 = 0.7^2$$

Certainty equivalent controller

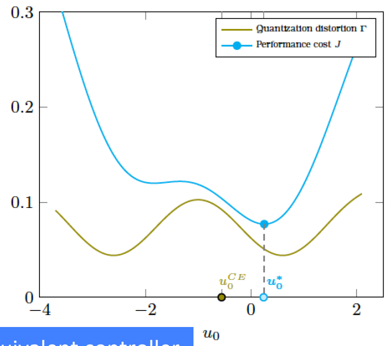
$$u_k^{\mathrm{CE}} = -K_k^{\mathrm{CE}}\left(E[x_k|\{y_k\}_0^k, \{u_k\}_0^{k-1}] + E[w_k|\{y_k\}_0^k, \{u_k\}_0^{k-1}]\right)$$

Event-generator encodes state as

$$\xi(x_k) = \begin{cases} 1, & \text{if } x_k \in (\infty, -\theta) \\ 2, & \text{if } x_k \in (-\theta, \theta) \\ 3, & \text{if } x_k \in (\theta, \infty) \end{cases}$$

Cost for time-horizon $N = 1$

$$J(u_0) = \sigma_w^2 + qu_0^2 + \left(p + \frac{qa^2}{q+1}\right)\mathbb{E}\left[x_1^2 | x_0, u_\right.$$

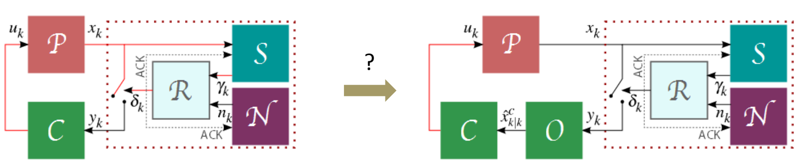


Optimal performance is not obtained by a certainty equivalent controller

Rabi et al, 2015

Condition for Certainty Equivalence

Corollary: The optimal controller for the system {𝒫,𝒮(f),𝒞(g)}, with respect to the cost J is certainty equivalent if the scheduling decisions are not a function of the applied controls.

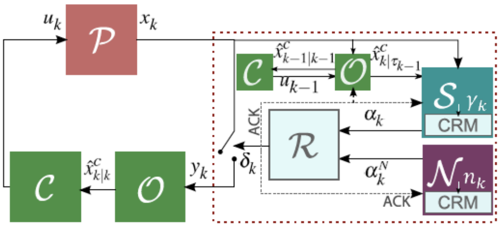


Certainty equivalence achieved at the cost of optimality

Bar-Shalom & Tse, 1974; Ramesh et al., 2011

20

Architecture with Certainty Equivalent Controller



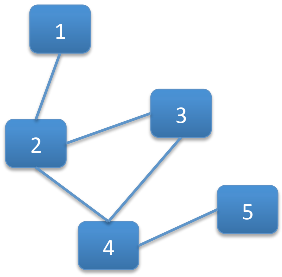
Ramesh et al., 2012, 2013

Outline

- Introduction
- Motivating applications
- Optimal event-based control
- Distributed event-based control
- Implementation aspects
- Conclusions

Distributed Event-Based Control

- How to implement event-based control over a distributed system?
- Local control and communication, but global objective



Approach: Consider a prototype distributed control problem and study it under event-based communication and control

Average Consensus Problem

Multi-agent system model

- Group of N agents

$$\dot{x}_i(t) = u_i(t)$$

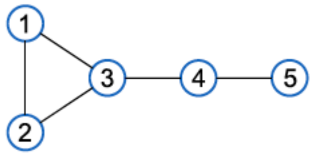
- Communication graph G
  A: undirected, connected

Adjacency matrix A with $a_{ij} = 1$ if agents i and j adjacent, otherwise $a_{ij} = 0$

Degree matrix D is the diagonal matrix with elements equal to the cardinality of the neighbor sets N_i

Objective: Average consensus

$$x_i(t) \overset{t\to\infty}{\longrightarrow} a = \frac{1}{N}\sum_{i=1}^{N} x_i(0)$$



Consensus protocol

$$u_i(t) = -\sum_{j\in N_i} (x_i(t) - x_j(t))$$

Closed-loop dynamics

$$\dot{x}(t) = -Lx(t)$$

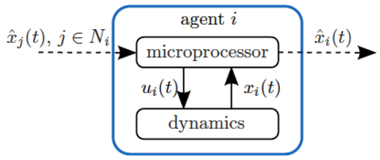
with Laplacian matrix $L = D - A$

Event-based implementation?

Olfati-Saber & Murray, 2004

Event-Based Average Consensus

Event-based scheduling of measurement broadcasts:



Event-based broadcasting

$$\hat{x}_i(t) = x_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i[$$
$$0 \le t_0^i \le t_1^i \le t_2^i \le \cdots$$

- Consensus protocol

$$u_i(t) = -\sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t))$$

- Measurement errors

$$e_i(t) = \hat{x}_i(t) - x_i(t)$$

- Closed-loop

$$\dot{x}(t) = -L\hat{x}(t) = -L(x(t) + e(t))$$

- Disagreement

$$\delta(t) = x(t) - a\mathbf{1}, \qquad \mathbf{1}^T \delta(t) \equiv 0$$

Seyboth et al, 2013

Trigger Function for Event-Based Control

Trigger mechanism: Define *trigger functions* $f_i(\cdot)$ and trigger when

$$f_i\left(t, x_i(t), \hat{x}_i(t), \bigcup_{j \in N_i} \hat{x}_j(t)\right) > 0$$

Defines sequence of events: $t_{k+1}^i = \inf\{t :\ t > t_k^i, f_i(t) > 0\}$

Extends [Tabuada, 2007] single-agent trigger function to multi-agent systems

Find f_i such that
- $|x_i(t) - x_j(t)| \to 0,\ t \to \infty$
- no Zeno (no accumulation point in time)
- few inter-agent communications

Cf., Dimarogonas et al., De Persis et al., Donkers et al., Mazo & Tabuada, Wang & Lemmon, Garcia & Antsaklis, Guinaldo et al.

Seyboth et al, 2013

Event-Based Control with Constant Thresholds

$$\dot{x}(t) = u(t), \qquad u(t) = -L\hat{x}(t) \qquad (1)$$

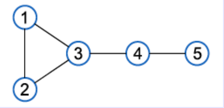
Theorem (constant thresholds)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(e_i(t)) = |e_i(t)| - c_0,$$

with $c_0 > 0$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and for $t \to \infty$,

$$\|\delta(t)\| \leq \frac{\lambda_N(L)}{\lambda_2(L)}\sqrt{N}c_0.$$



Proof ideas:

- Analytical solution of disagreement dynamics yields

$$\|\delta(t)\| \leq e^{-\lambda_2(L)t}\|\delta(0)\| + \lambda_N(L)\int_0^t e^{-\lambda_2(L)(t-s)}\|e(s)\|ds$$

- Compute lower bound τ on the inter-event intervals

Seyboth et al, 2013

Event-Based Control with Exponentially Decreasing Thresholds

$$\dot{x}(t) = u(t), \qquad u(t) = -L\hat{x}(t) \qquad (1)$$

Theorem (exponentially decreasing thresholds)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(t, e_i(t)) = |e_i(t)| - c_1 e^{-\alpha t},$$

with $c_1 > 0$ and $0 < \alpha < \lambda_2(L)$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and as $t \to \infty$,

$$\|\delta(t)\| \to 0.$$

Remarks

- Asymptotic convergence: $|x_i(t) - x_j(t)| \to 0$, $t \to \infty$
- $\lambda_2(L)$ is the rate of convergence for continuous-time consensus, so threshold need to decrease slower

Seyboth et al, 2013

Event-Based Control with Exponentially Decreasing Thresholds and Offset

$$\dot{x}(t) = u(t), \qquad u(t) = -L\hat{x}(t) \qquad (1)$$

Theorem (exponentially decreasing thresholds with offset)

Consider system (1) with undirected connected graph G. Suppose that

$$f_i(t, e_i(t)) = |e_i(t)| - (c_0 + c_1 e^{-\alpha t}),$$

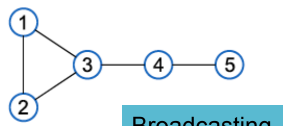
with $c_0, c_1 \geq 0$, at least one positive, and $0 < \alpha < \lambda_2(L)$. Then, for all $x_0 \in \mathbb{R}^N$, the system does not exhibit Zeno behavior and for $t \to \infty$,

$$\|\delta(t)\| \leq \frac{\lambda_N(L)}{\lambda_2(L)} \sqrt{N} c_0.$$

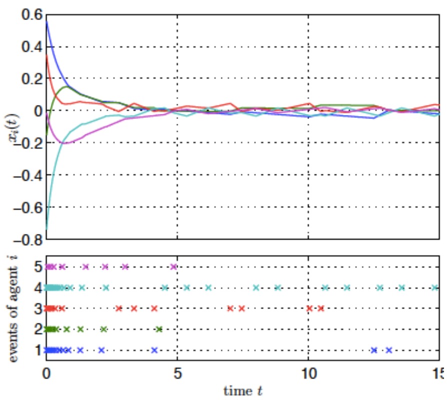
Seyboth et al, 2013

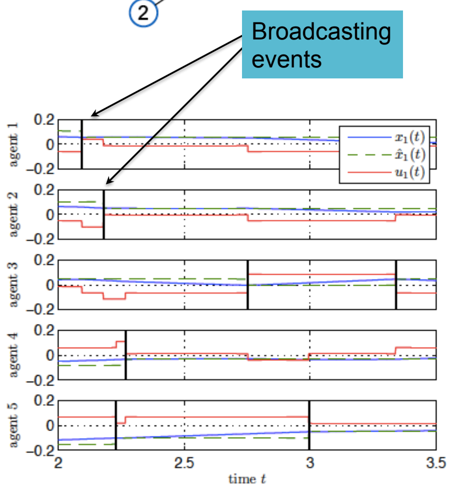
Example

$$\dot{x}(t) = -L\hat{x}(t)$$
$$|e_i(t)| \leq c_0 + c_1 e^{-\alpha t}$$



$$|e_i(t)| \leq 0.050$$



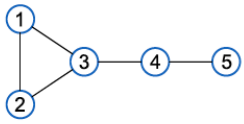


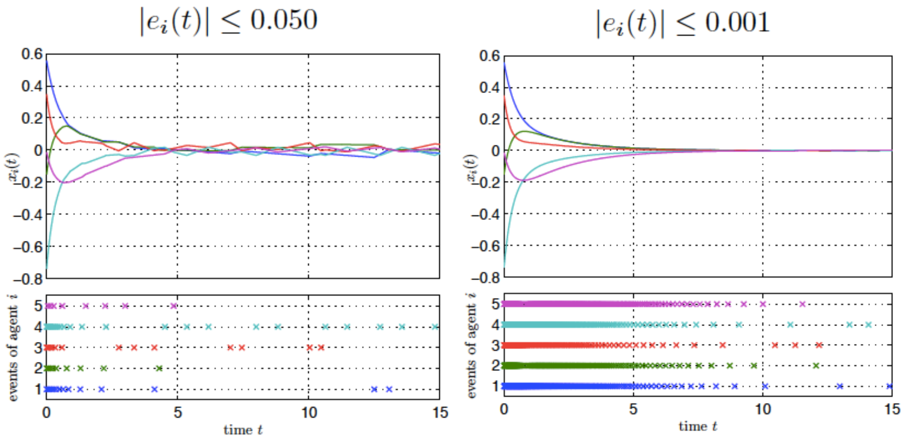
Seyboth et al, 2013

Example: Threshold Tuning

$\dot{x}(t) = -L\hat{x}(t)$

$|e_i(t)| \leq c_0 + c_1 e^{-\alpha t}$



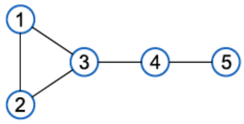


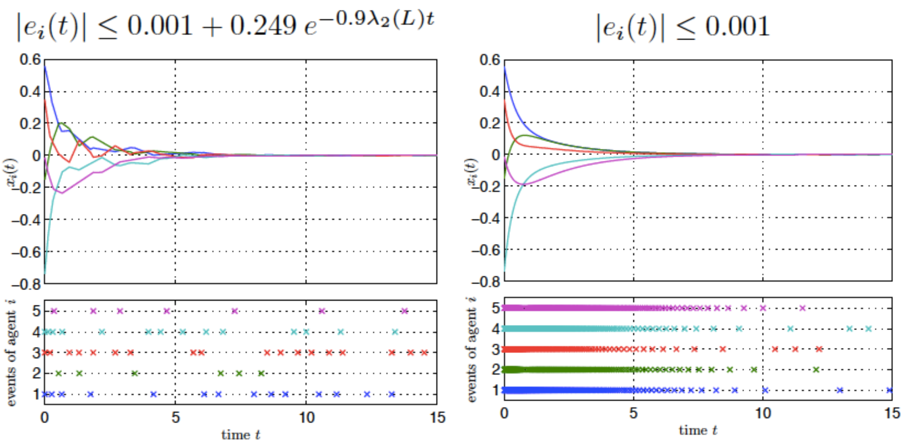
Seyboth et al, 2013

Example: Threshold Tuning

$\dot{x}(t) = -L\hat{x}(t)$

$|e_i(t)| \leq c_0 + c_1 e^{-\alpha t}$

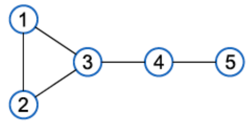




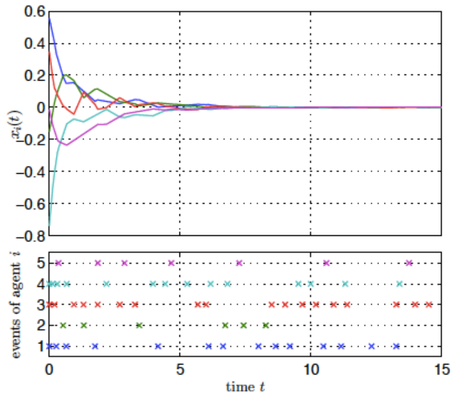
Seyboth et al, 2013

Example: Threshold Tuning

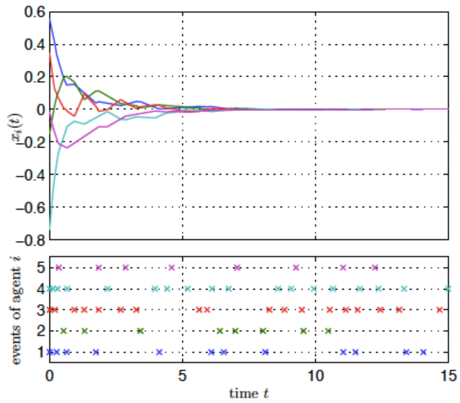
$$\dot{x}(t) = -L\hat{x}(t)$$
$$|e_i(t)| \leq c_0 + c_1 e^{-\alpha t}$$



$$|e_i(t)| \leq 0.001 + 0.249\, e^{-0.9\lambda_2(L)t}$$

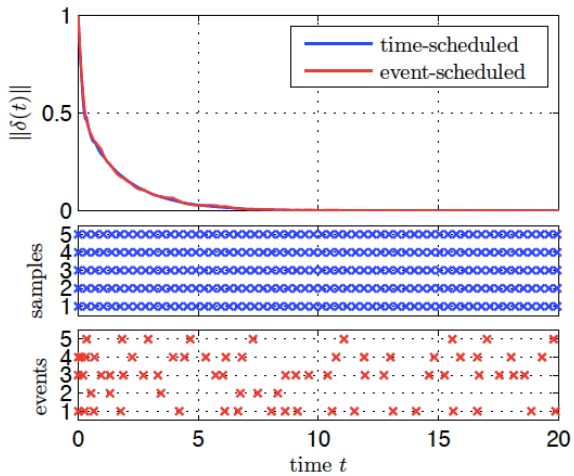


$$|e_i(t)| \leq 0.250\, e^{-0.9\lambda_2(L)t}$$

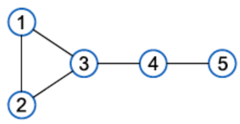


Seyboth et al, 2013

Example: Event- vs Time-Triggered Sampling



Graph:



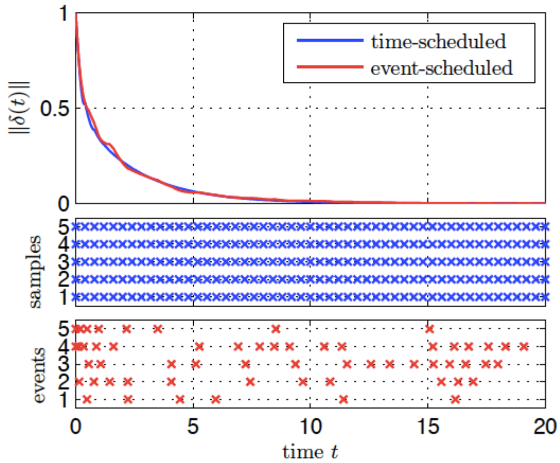
Sampling periods:

- Time-scheduling:
  $\tau_s = 0.350$
  $\tau_{max} = 0.480$
- Event-scheduling:
  $\tau_{mean} = 1.389$

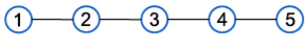
τ_{max} : largest stabilizing sampling period, see *G. Xie et al., ACC2009*

Seyboth et al, 2013

Example: Event- vs time-triggered sampling



Graph:

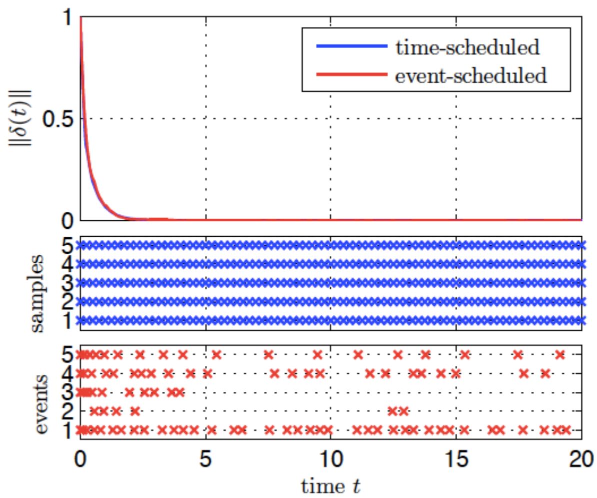


Sampling periods:

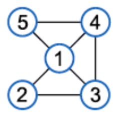
- Time-scheduling:
  $\tau_s = 0.400$
  $\tau_{max} = 0.553$
- Event-scheduling:
  $\tau_{mean} = 1.724$

Seyboth et al, 2013

Example: Event- vs time-triggered sampling



Graph:

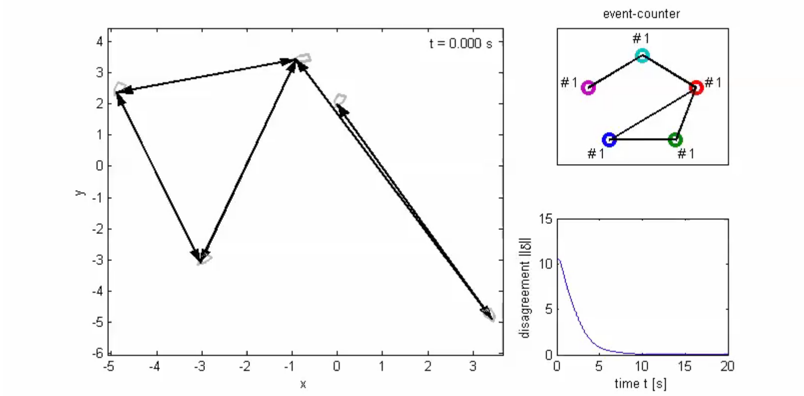


Sampling periods:

- Time-scheduling:
  $\tau_s = 0.250$
  $\tau_{max} = 0.400$
- Event-scheduling:
  $\tau_{mean} = 1.053$

Seyboth et al, 2013

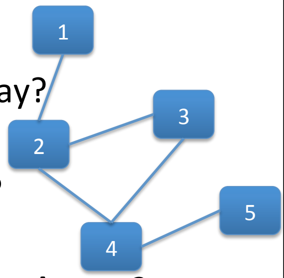
Event-Based Formation Control



- **Non-holonomic mobile robots under feedback linearization**
- **Event-based communication based on threshold for double-integrator network**

Seyboth et al, 2013

Extensions



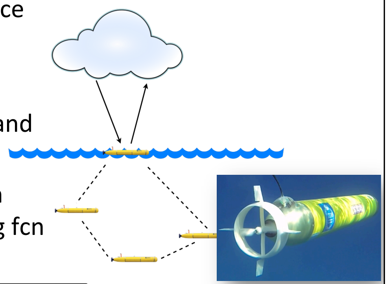
- How to **estimate λ_2(L)** in a distributed way?
  - Aragues et al., 2014
- How to handle **general** agent **dynamics**?
  - Guinaldo et al. 2013
- How to handle network **delays** and packet **losses**?
  - Guinaldo et al., 2014
- **Pinning** (leader-follower) control and switching networks
  - Adaldo et al., 2015
- Event-triggered **pulse width modulation**
  - Meng et al., 2015
- Event-triggered **cloud access**
  - Adaldo et al., 2015

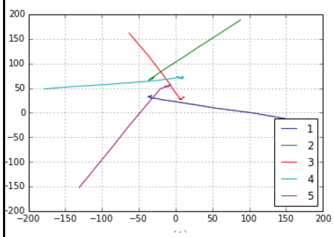
Event-triggered Cloud Access

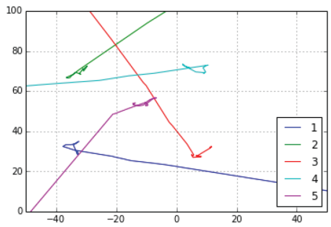
- Agent dynamics with unknown drift disturbance

$$\dot{x}_i(t) = u_i(t) + \omega_i(t), \quad i = 1, \ldots, N,$$

- Agents exchange state, control, disturbance, and timing data through a shared data base
- Schedule next data base access time based on dynamic estimates and event-based triggering fcn







Data base access times

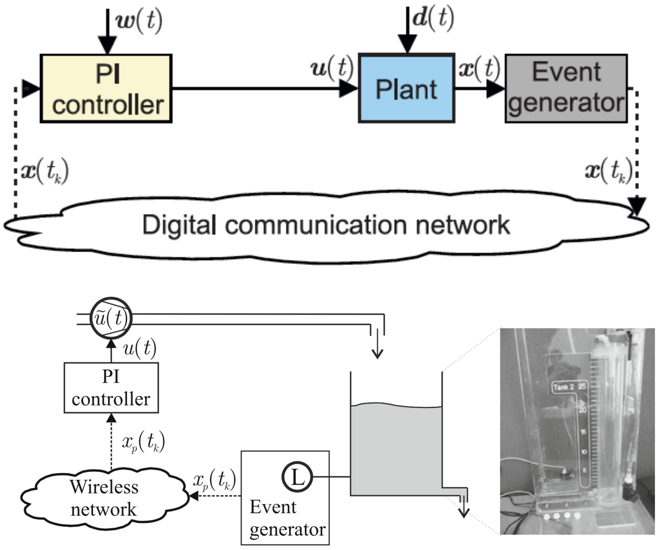
k	$t_{1,k}$	$t_{2,k}$	$t_{3,k}$	$t_{4,k}$	$t_{5,k}$
0	0.00	0.00	0.00	0.00	0.00
1	5.01	6.21	7.41	8.51	10.11
2	12.72	14.72	16.72	18.81	21.31
3	23.32	25.82	28.02	30.41	32.61
4	34.92	37.23	39.63	41.92	44.22
5	46.53	48.84			

Adaldo et al., 2015

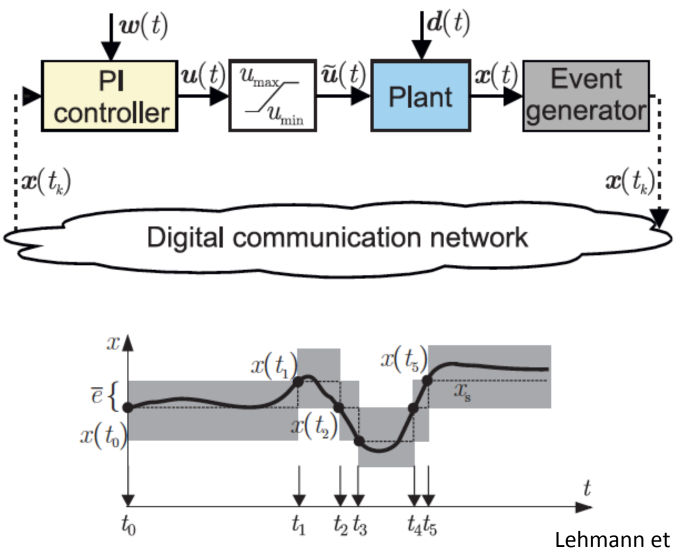
Outline

- Introduction
- Motivating applications
- Optimal event-based control
- Distributed event-based control
- Implementation aspects
- Conclusions

Event-Based Wireless PI Control

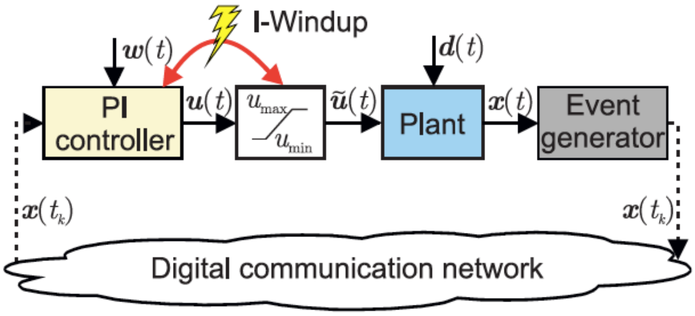


Event-Based PI Control with Saturation



Lehmann et al., 2012

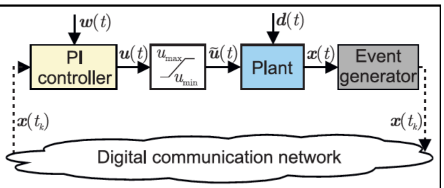
Event-Based PI Control with Saturation



- Industrial applications are generally affected by actuator limitations.

  1. Does actuator saturation affect event-triggered PI control?
  2. Under what conditions can we guarantee stability?
  3. How to overcome potential effects of actuator saturation?

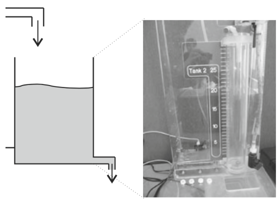
Example



- Plant:

$$\dot{x}(t) = 0.1x(t) + \tilde{u}(t) + 0.1d(t), \quad x(0) = 0$$
$$y(t) = x(t)$$

- Exogenous signals:



$$w(t) = \bar{w} = 1.5$$
$$d(t) = \bar{d} = 0.1$$

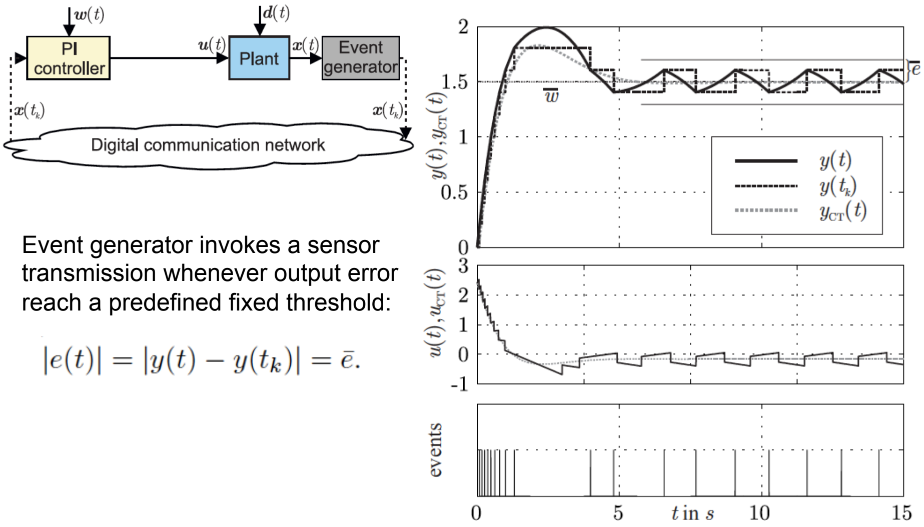
- Actuator saturation:

$$\tilde{u}(t) = \begin{cases} 0.4, & \text{for } u(t) > 0.4; \\ u(t), & \text{for } -0.4 \le u(t) \le 0.4; \\ -0.4, & \text{for } u(t) < -0.4; \end{cases}$$

- PI controller

$$\dot{x}_I(t) = y(t) - w(t), \quad x_I(0) = 0$$
$$u(t) = -x_I(t) - 1.6(y(t) - w(t))$$

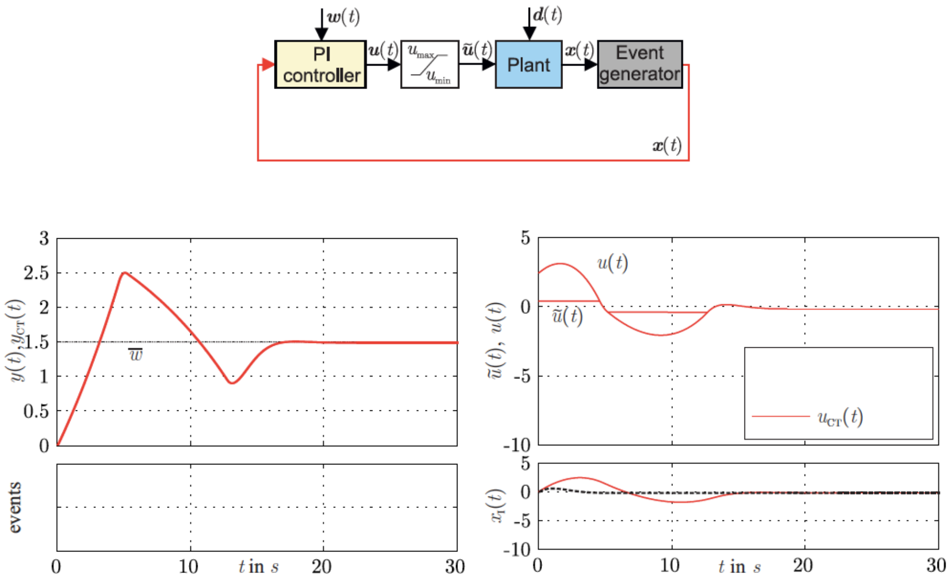
Example: Without Saturation



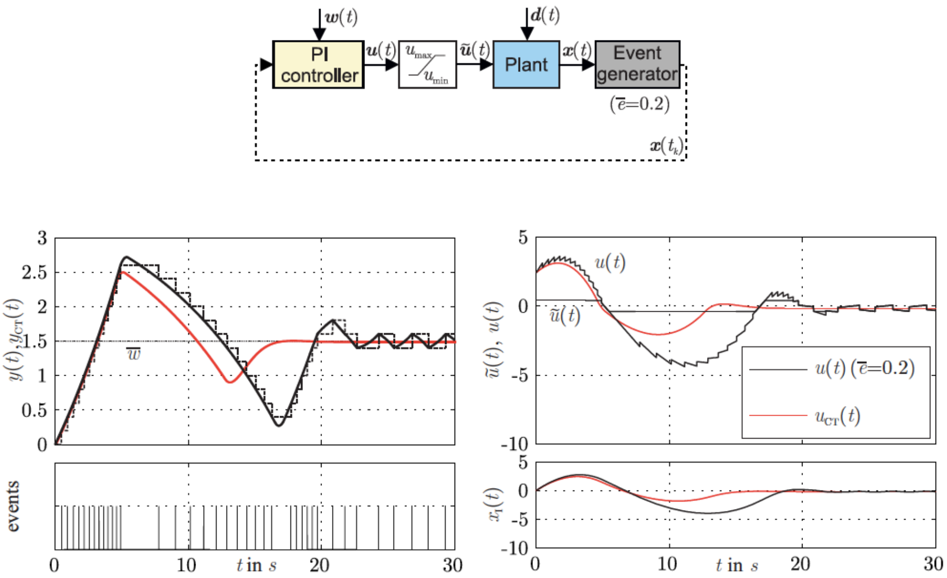
Event generator invokes a sensor transmission whenever output error reach a predefined fixed threshold:

$$|e(t)| = |y(t) - y(t_k)| = \bar{e}.$$

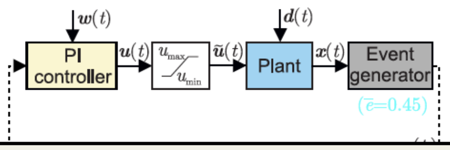
Motivating Example



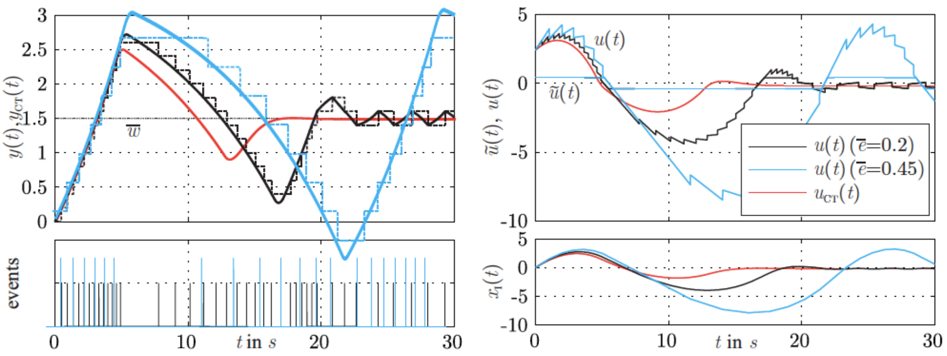
Motivating Example



Motivating Example



Need to take saturation and wind-up into account when designing event-based control systems



Mathematical Model

- Plant:

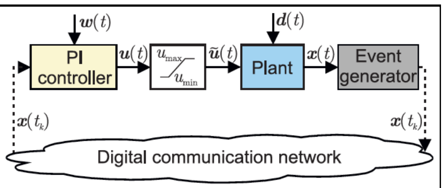
$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\tilde{\boldsymbol{u}}(t) + \boldsymbol{E}\boldsymbol{d}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \tilde{\boldsymbol{u}}(t) &= \text{sat}(\boldsymbol{u}(t)) \\ \text{sat}(u_i(t)) &= \begin{cases} u_0, & \text{for } u_i(t) > u_0 \\ u_i(t), & \text{for } -u_0 \le u(t) \le u_0 \quad \forall i \in \{1,2,..,m\} \\ -u_0, & \text{for } u_i(t) < -u_0 \end{cases} \end{aligned}$$

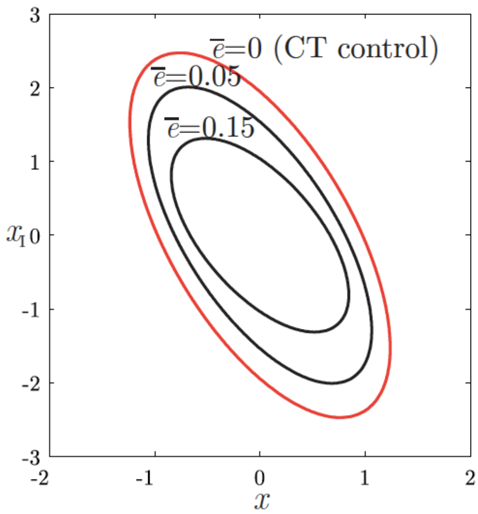
- Event generator: $\|\boldsymbol{x}(t) - \boldsymbol{x}(t_k)\| = \bar{e}$
- PI controller:

$$\begin{aligned} \dot{\boldsymbol{x}}_{\text{I}}(t) &= \boldsymbol{x}(t) - \boldsymbol{e}(t) - \boldsymbol{w}(t), \quad \boldsymbol{x}_{\text{I}}(0) = \boldsymbol{x}_0 \\ \boldsymbol{u}(t) &= \boldsymbol{K}_{\text{I}}\boldsymbol{x}_{\text{I}}(t) + \boldsymbol{K}_{\text{P}}(\boldsymbol{x}(t) - \boldsymbol{e}(t) - \boldsymbol{w}(t)) \end{aligned}$$

- **State error:** $e(t) = \boldsymbol{x}(t) - \boldsymbol{x}(t_k)$
- For the sake of simplicity: $\boldsymbol{w}(t) = \boldsymbol{d}(t) = \boldsymbol{0}$

Stability Regions

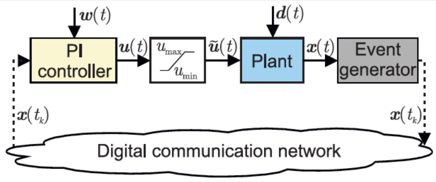


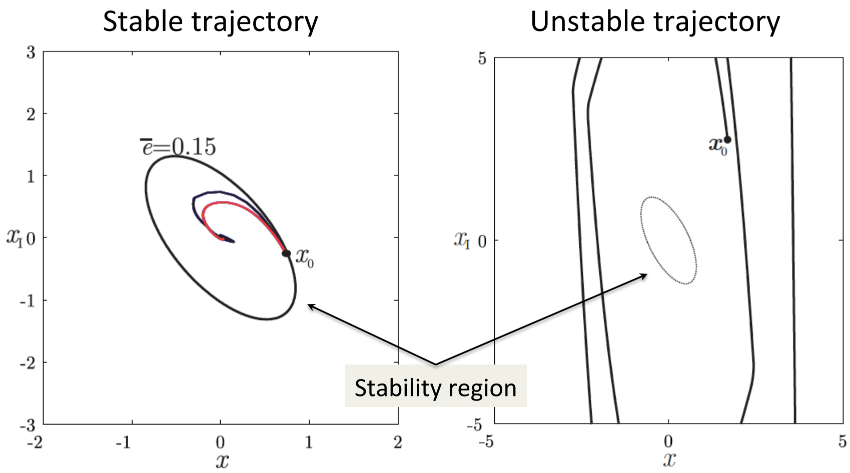


LMI condition to estimate region of stability

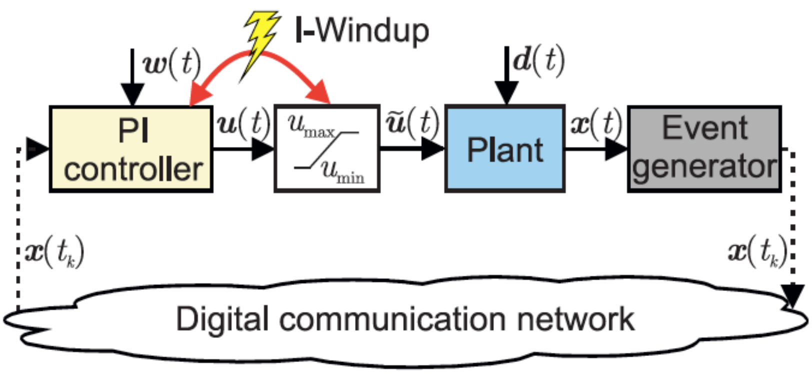
Lehmann et al., 2012

Simulations

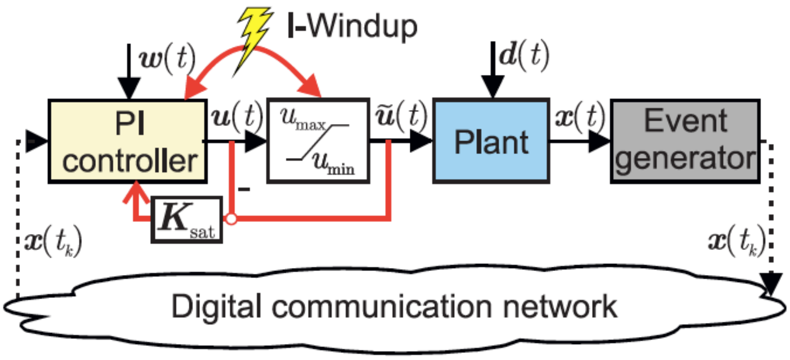




Anti-Windup for Event-Based Control

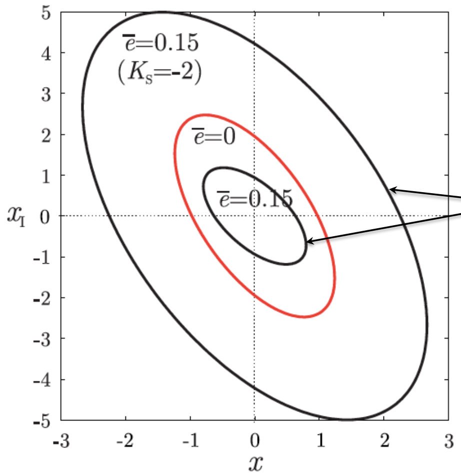


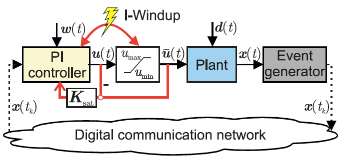
Anti-Windup for Event-Based Control



Cf., anti-windup for conventional control systems [Åström & Hägglund, 1995]

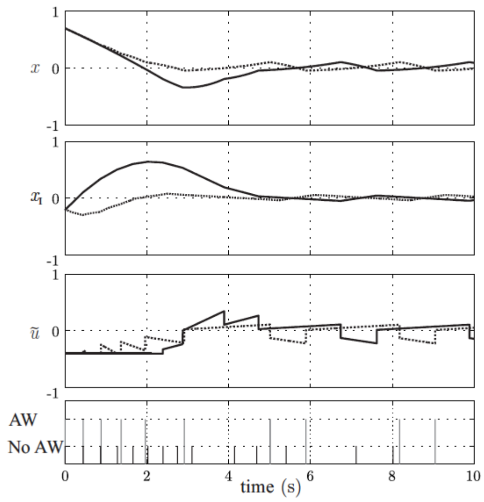
Stability Regions with Anti-Windup



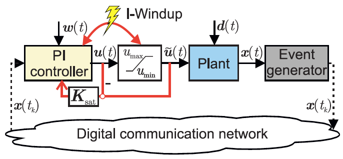


Anti-windup increases the region of stability

System Evolution with Anti-Windup

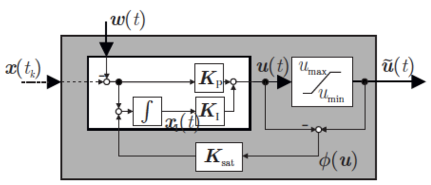


(solid line: no anti windup, dotted line: anti windup)

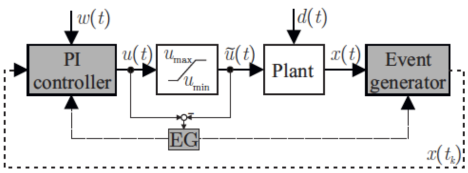


Anti-windup improves the system response

Event-Based Communication for Anti-Windup



Anti-windup event generated when actuator saturates (ETAW)



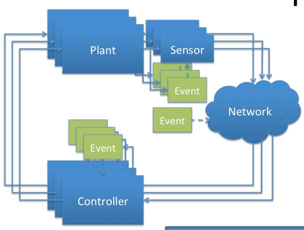
$$|u(t) - u(t_k)| = \bar{e}_c \quad \text{if} \quad \tilde{u}(t) - u(t) \neq 0$$

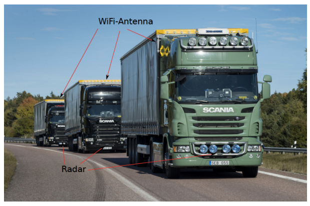
Outline

- Introduction
- Motivating applications
- Optimal event-based control
- Distributed event-based control
- Implementation aspects
- Conclusions

Conclusions

- **Event-based control** of multi-agent systems
- Hard to **jointly optimize** event condition and control law
- Certain **architectures** lead to strong results
- **Applications** in goods transportation, mobile robotics, and wireless automation
- Event-based **revisions** of classical control architectures: event-based anti-windup, feedforward, cascade control





http://people.kth.se/~kallej