Networked Control and Autonomy

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Slides available at http://www.ee.kth.se/~kallej

Outline

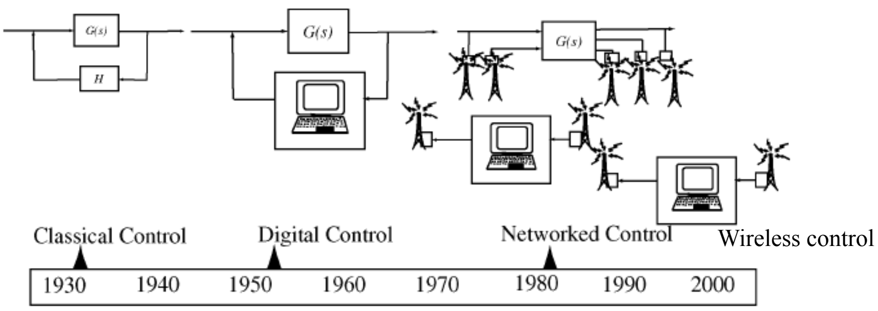
Lecture 1: Motivating applications and challenges

Lecture 2: Wireless control systems

Lecture 2 Outline

- What's new with wireless networked control?
- State-based scheduling for control
- Exploiting wireless protocols for control
- Event-based control
- Conclusions

A history of control

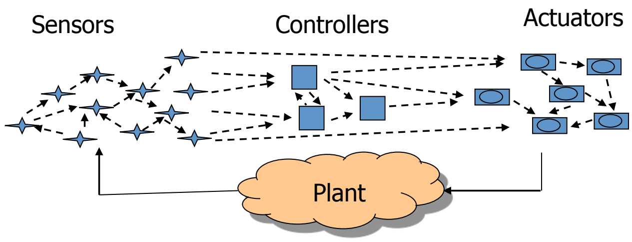


From dedicated communication links and networks for control systems
To open and ubiquitous wireless networks for control applications

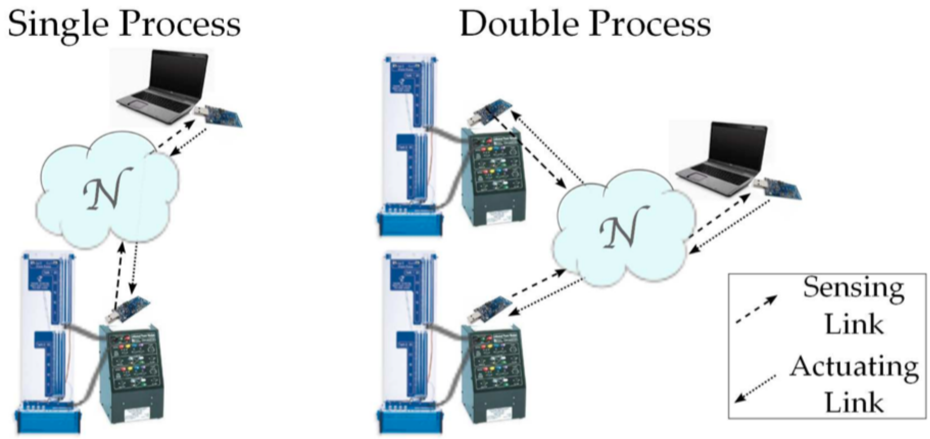
Adopted from [Baillieul & Antsaklis, 2007]

Wireless control system

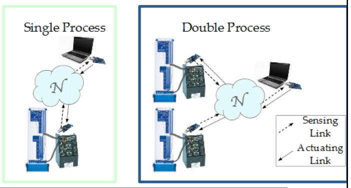
How share common network resources while maintaining guaranteed closed-loop performance?

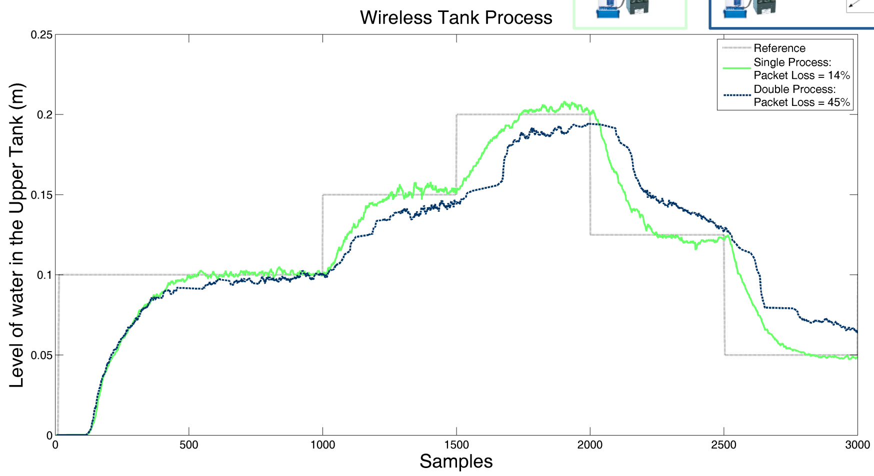


What is the effect on control performance of a shared wireless network?



Packet loss influence on control performance





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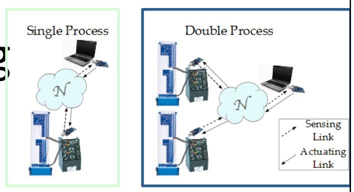
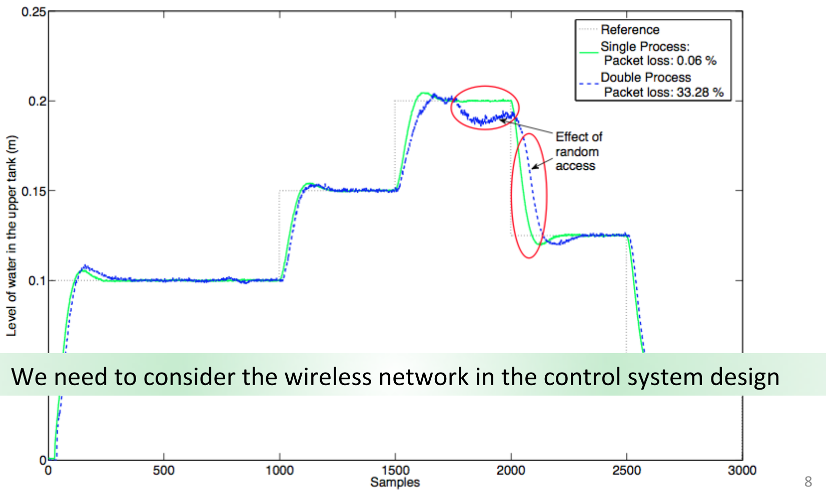
Partial improvement using CSMA/CA medium access





We need to consider the wireless network in the control system design

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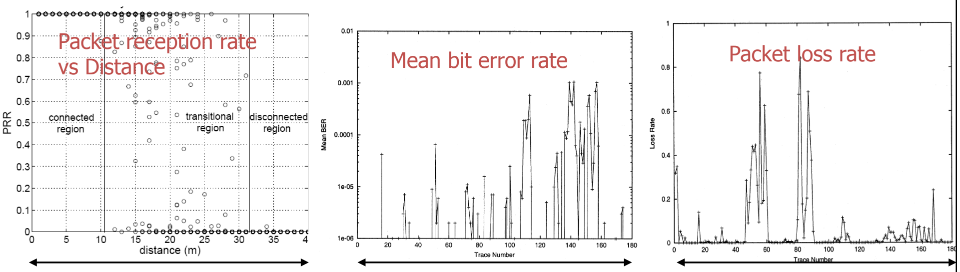
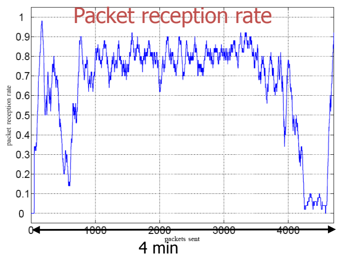
Uncertainty on several communication layers



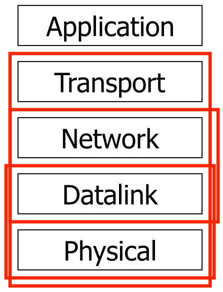
40 m
Zuniga & Krishnamachari, *SECON*, 2004

4 h　4 h
Willig et al., *IEEE Trans. Ind. Electron.*, 49, 6, 2002



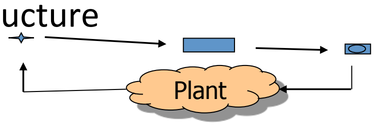
4 min
Park et al., KTH, 2007

Large variations in
- Connectivity
- Bit and packet delivery
- End-to-end delivery



What's new with control over wireless networks?

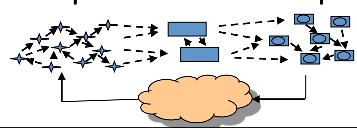
- **Traditional control** systems design is based on assumption of **perfect information** being circulated in the system
- Information flow are dedicated to specific control loop
- Devices need to be fixed to infrastructure



Wireless control systems have

- **Non-ideal communication** between system devices;
  leads to interference, congestion, delay, loss, outages etc.

+ **Information** can be **shared** between components and loops
+ Enhanced **mobility and flexibility**



Lecture 2 Outline

- What's new with wireless networked control?
- State-based scheduling for control
- Exploiting wireless protocols for control
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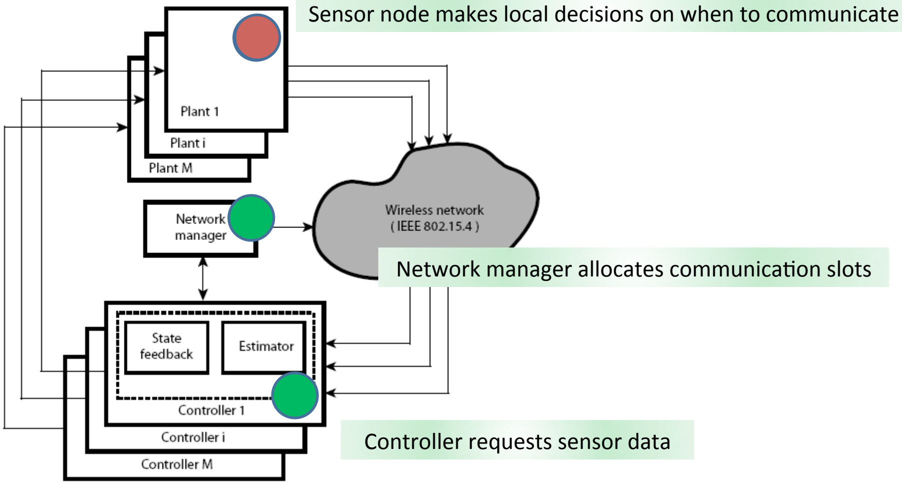
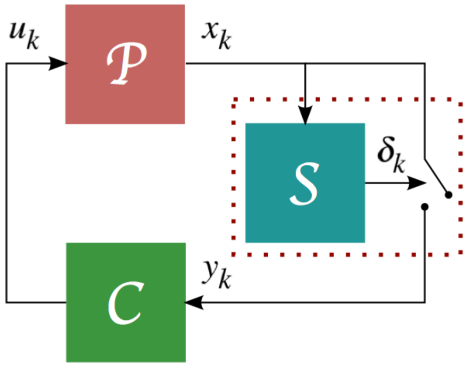
Where taking medium access decisions?



Is there a separation between scheduling-estimation-control?



Stochastic control formulation

Plant:

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

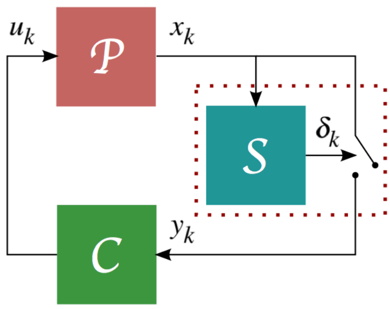
$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$

$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$

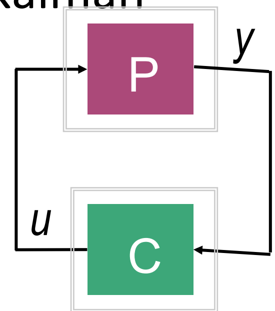
$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$



Cost criterion:

$$J(f,g) = \mathrm{E}[x_N^T Q_0 x_N + \sum_{s=0}^{N-1} (x_s^T Q_1 x_s + u_s^T Q_2 u_s)]$$

Control without scheduling = Classical LQG control of Kalman



The controller minimizing

$$J = \mathbb{E}\left[x_N^T Q_0 x_N + \sum_{s=0}^{N-1}(x_s^T Q_1 x_s + u_s^T Q_2 u_s)\right]$$

is given by

$$u_k = -L_k \hat{x}_{k|k}\ ,$$
$$L_k = (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A$$

where

$$S_k = Q_1 + A^T S_{k+1} A - A^T S_{k+1} B (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A$$

$\hat{x}_{k|k} = \mathbb{E}[x_k | \{y\}_0^k u_0^{k-1}]$ is the minimum mean-square error (MMSE) estimate

Kalman, 1960

Event-based scheduler

Plant:

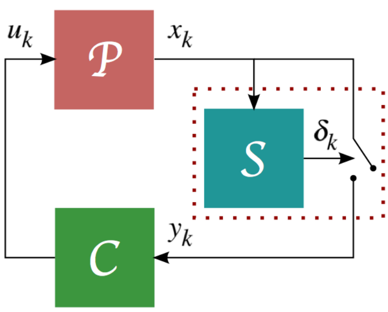
$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$
$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$
$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$



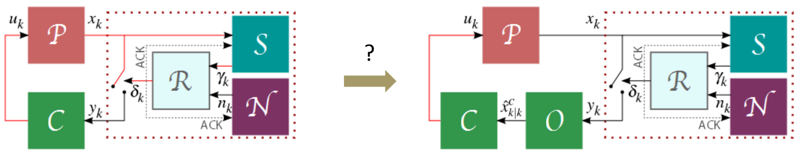
The separation principle does not hold for the optimal closed-loop system, so the design of the (event-based) scheduler, estimator, and controller is coupled

Feldbaum, 1965; Åström, 1970; Bar-Shalom and Tse, 1974

Ramesh et al., 2011

Conditions for Separation

Corollary: The optimal controller for the system $\{\mathcal{P}, \mathcal{S}(f), \mathcal{C}(g)\}$, with respect to the cost J is certainty equivalent if and only if the scheduling decisions are not a function of the applied controls.



Nice architecture achieved at the cost of optimality

Ramesh et al., 2011

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Event-based control architecture

- Plant $\mathcal{P}$:

$$x_{k+1} = ax_k + bu_k + w_k$$

- State-based Scheduler $\mathcal{S}$:

$$\gamma_k = \begin{cases} 1, & |x_k - \hat{x}^s_{k|\tau_{k-1}}|^2 > \epsilon_d, \\ 0, & \text{otherwise.} \end{cases}$$

$$\hat{x}^s_{k|\tau_{k-1}} = a\hat{x}^c_{k-1|k-1} + bu_{k-1}$$

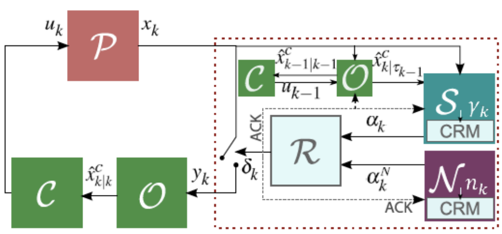
- CRM: $\mathbb{P}(\alpha_k{=}1|\gamma_k{=}1) = \mathbb{P}(\alpha^N_k{=}1|n_k{=}1) = p_\alpha$

$$\delta_k = \alpha_k(1 - \alpha^N_k)$$

- Observer $\mathcal{O}$: $y^{(j)}_k = \delta^{(j)}_k x^{(j)}_k$

$$\hat{x}^c_{k|k} = \bar{\delta}_k(a\hat{x}^c_{k-1|k-1} + bu_{k-1}) + \delta_k x_k$$

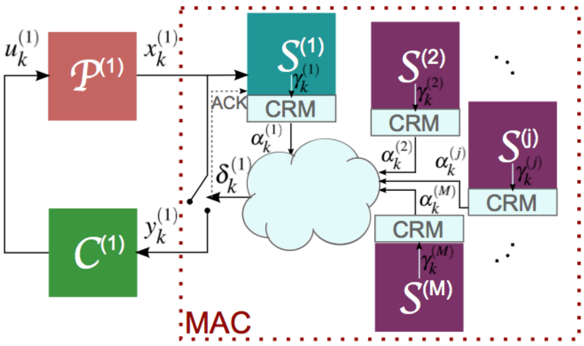
- Controller $\mathcal{C}$: $u_k = -L\hat{x}^c_{k|k}$



Ramesh et al., CDC, 2012

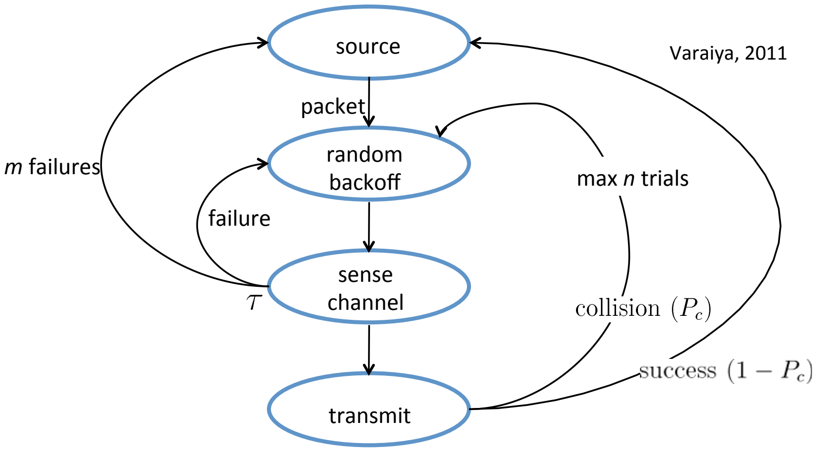
How to integrate contention resolution mechanisms?

- Hard problem because of correlation between transmissions (and the plant states)
- Closed-loop analysis can still be done for classes of event-based schedulers and MAC's



Ramesh et al., CDC 2011

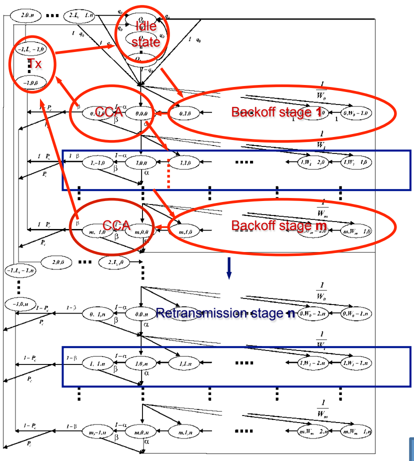
Contention resolution through CSMA/CA



- Every transmitting device executes this protocol
- For analysis, assume carrier sense events are independent [Bianchi, 2000]

CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance

Detailed model of CSMA/CA in IEEE 802.15.4



- Markov state (s,c,r)
  - s: backoff stage
  - c: state of backoff counter
  - r: state of retransmission counter
- Model parameters
  - q_0: traffic condition (q_0=0 saturated)
  - m_0, m, m_b, n: MAC parameters
- Computed characteristics
  - α: busy channel probability during CCA1
  - β: busy channel probability during CCA2
  - P_c: collision probability

- Detailed model for numerial evaluations
- Reduced-order models for control design
- Validated in simulation and experiment

Park, Di Marco, Soldati, Fischione, J, 2009

Cf., Bianchi, 2000; Pollin et al., 2006

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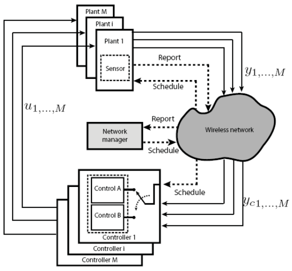
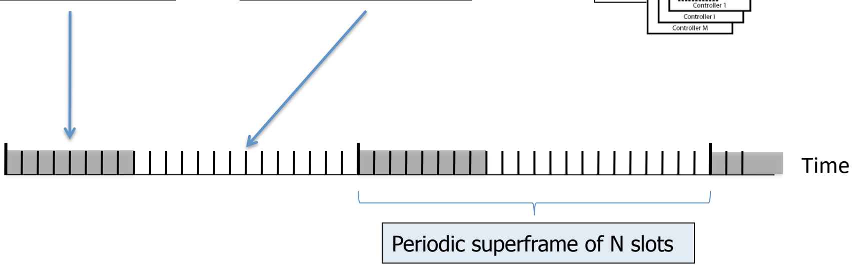
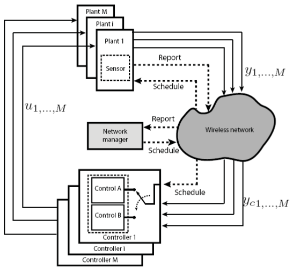
Slotted medium access



Many medium access protocols have slotted contention-free and contention access periods

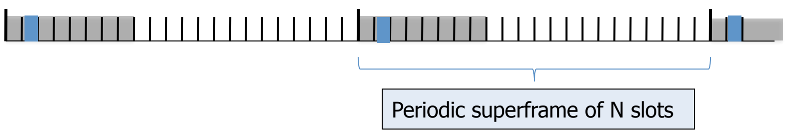


Hybrid MAC protocols

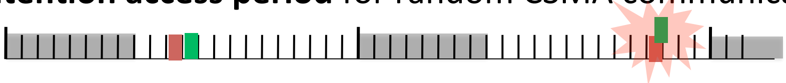


Exploit the mix of CFP's and CAP's for networked control

Contention-free period for TDMA scheduled communication



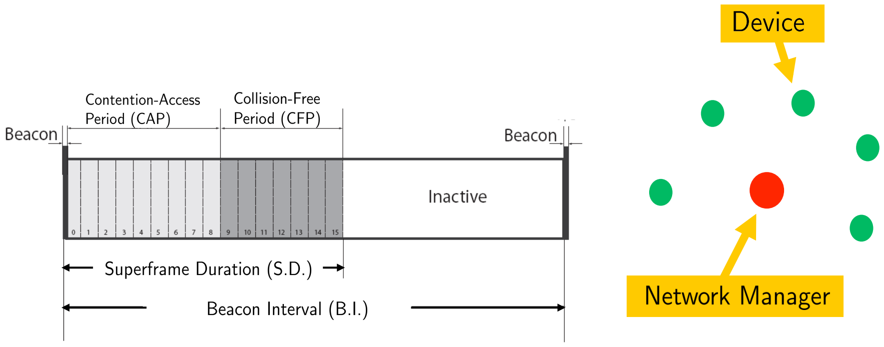
Contention access period for random CSMA communication



Araujo et al., 2010, Tiberi et al., 2010

TDMA = Time division multiple access, CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance

IEEE 802.15.4 MAC



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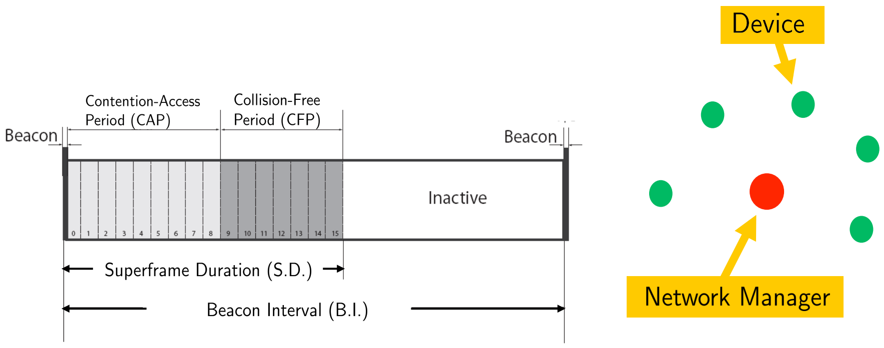
IEEE 802.15.4 MAC



- 16 slots for CAP and CFP
- Maximum 7 slots for CFP

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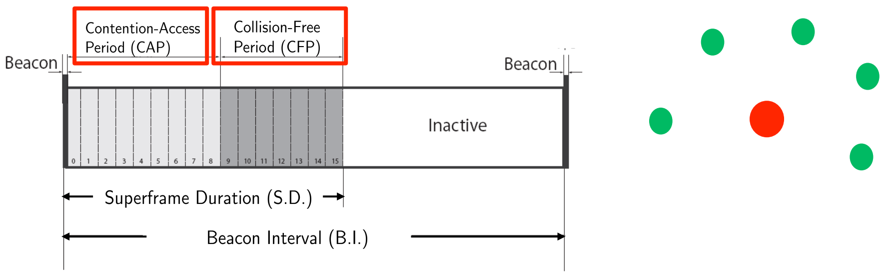
IEEE 802.15.4 MAC



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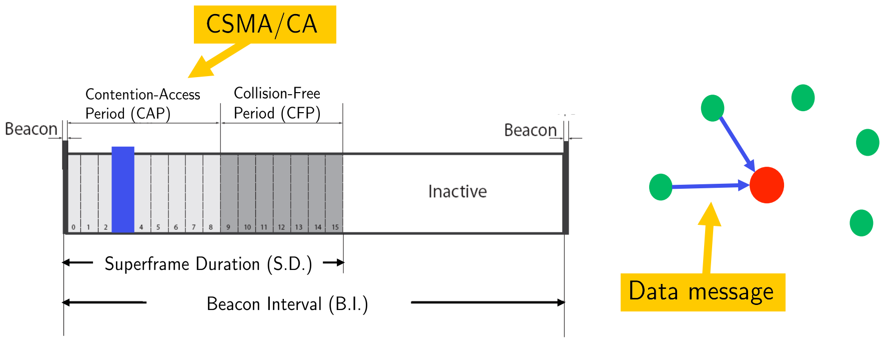
IEEE 802.15.4 MAC



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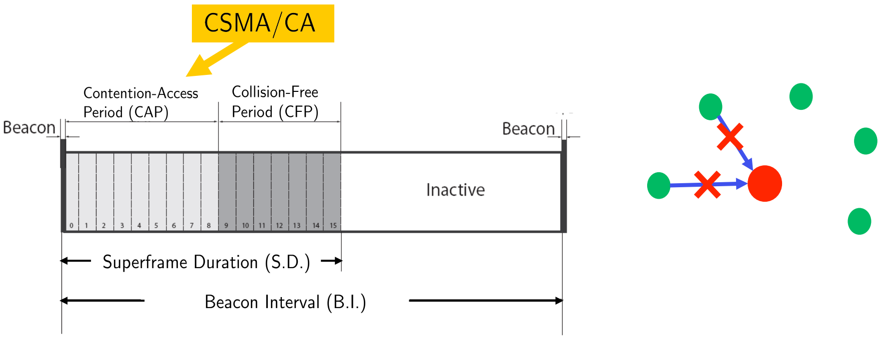
IEEE 802.15.4 MAC



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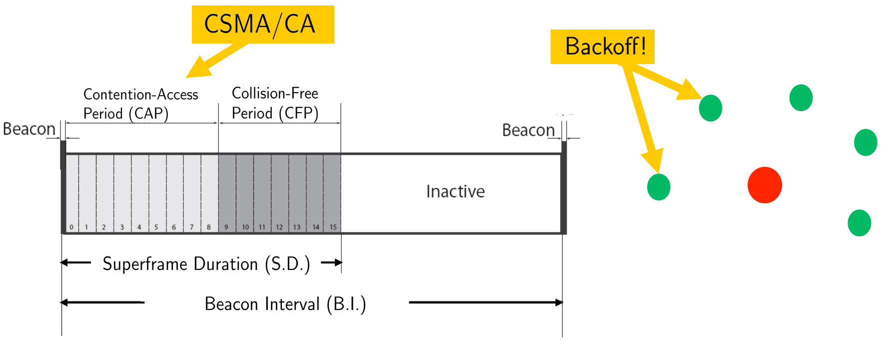
IEEE 802.15.4 MAC



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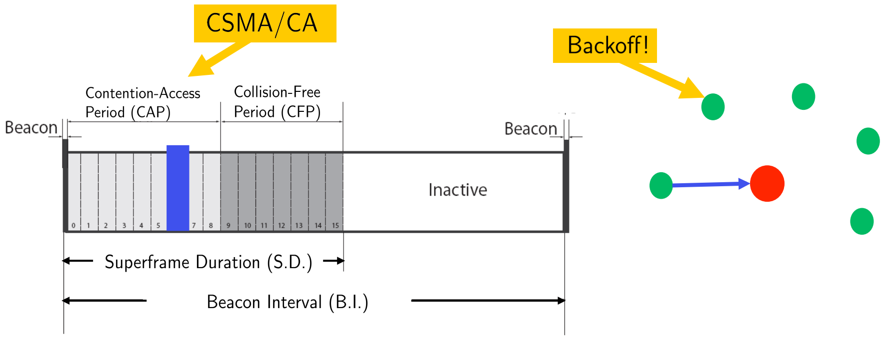
IEEE 802.15.4 MAC



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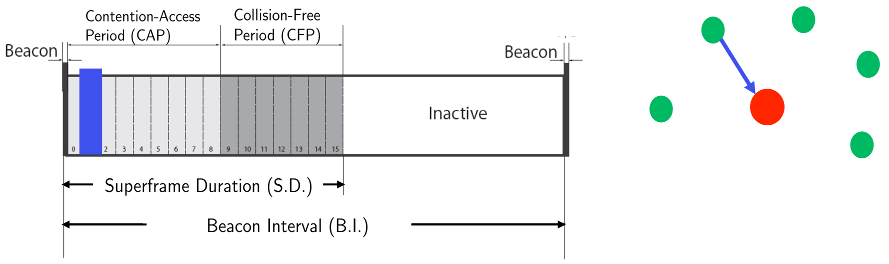
IEEE 802.15.4 MAC



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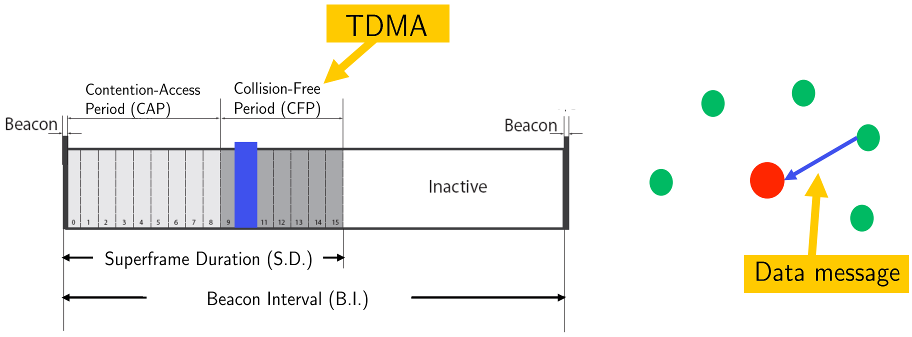
IEEE 802.15.4 MAC



- CFP slot allocation as First-Come First-Served

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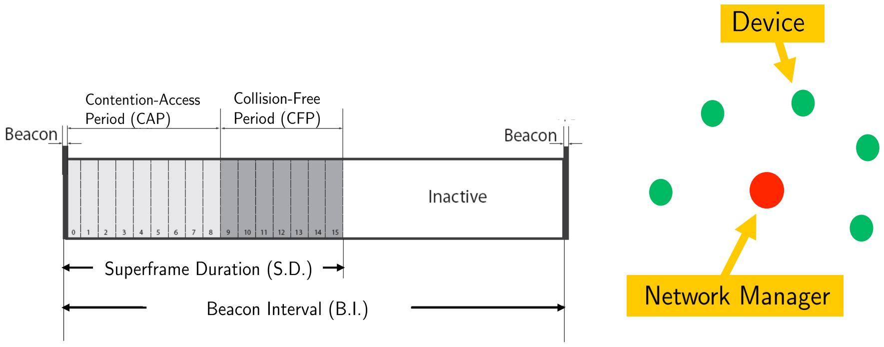
IEEE 802.15.4 MAC



- 16 slots for CAP and CFP
- Maximum 7 slots for CFP
- CFP slot allocation as First-Come First-Served

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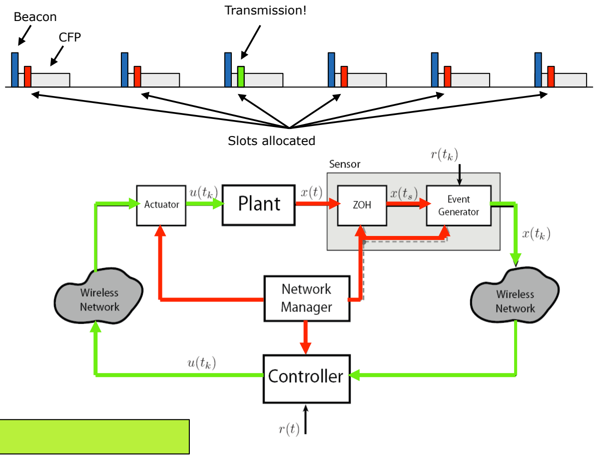
Event-based sensor communication

1. Fixed scheduling of sensing/actuation slots
2. Check triggering condition at every allocated slot
   - One-step ahead triggering condition
3. If triggering condition is true, transmit measurement and perform actuation



- **Robust to disturbances**

- **Unnecessary bandwidth utilization**
- **Energy spent on checking triggering condition**

Araujo, 2011 35

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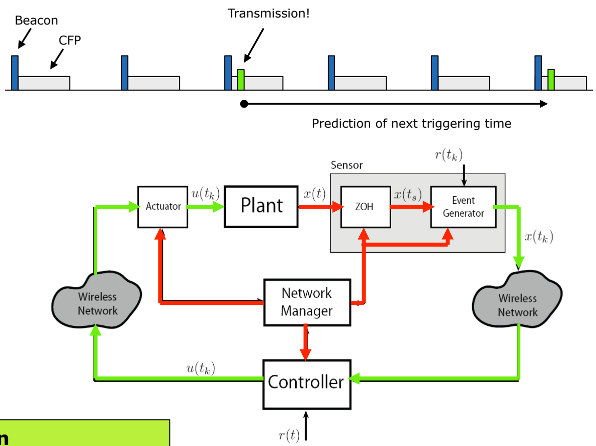
Predictive sensor communication

1. Scheduling of sensing/ actuation slots when required, at beacon times
2. If triggering condition is predicted to be true, transmit measurement and perform control action
3. At every transmission, predict and schedule the next triggering time
   - Set node to sleep until next transmission



- **Efficient bandwidth utilization**
- **Low energy consumption**

- **Less robust to disturbances**

Araujo, 2011 36

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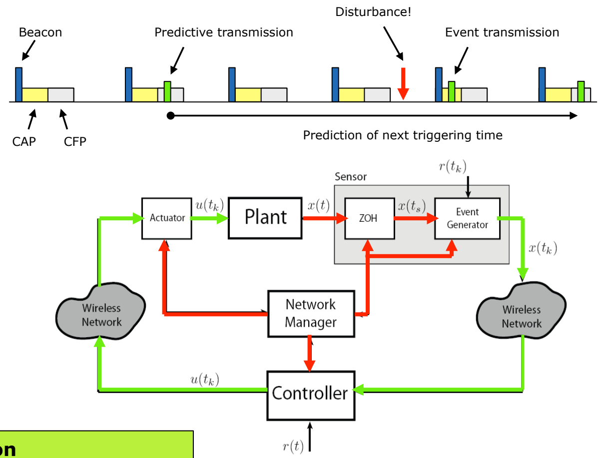
Hybrid sensor communication

1. Scheduling of slots as predictive scheme
2. Sensor node checks triggering condition continuously (or during CAP)
3. If triggering condition is true, transmit measurement and perform control action



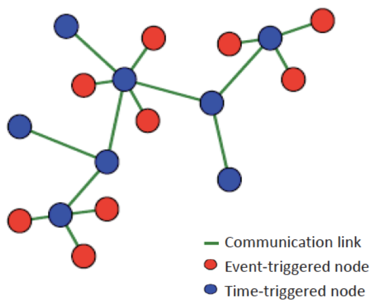
- **Efficient bandwidth utilization**
- **Robust to disturbances**

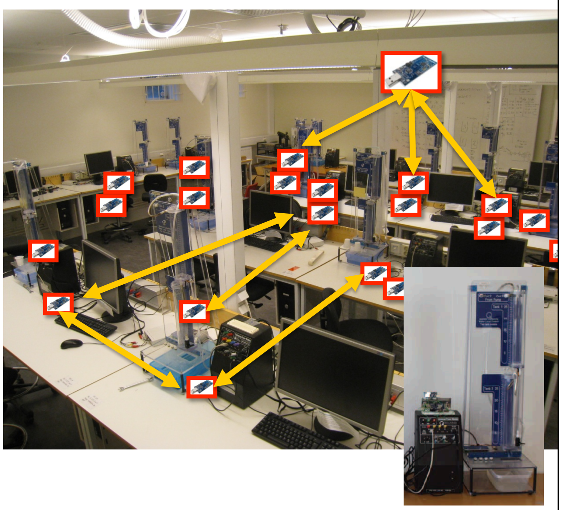
- **Energy spent on checking triggering condition**

Araujo, 2011 37

Multi-hop networks

- Routing decisions
- Time delays
- Hidden terminal problem





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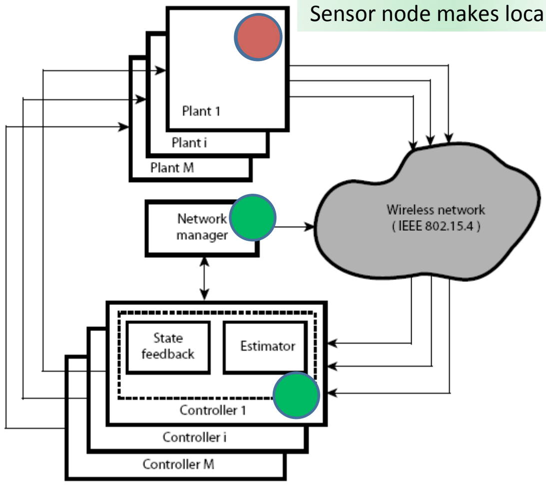
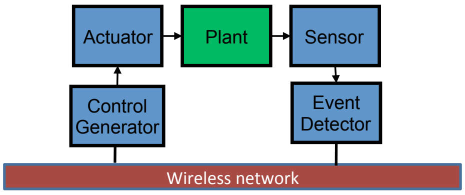
Event-based control over wireless network

Sensor node makes local decisions on when to communicate



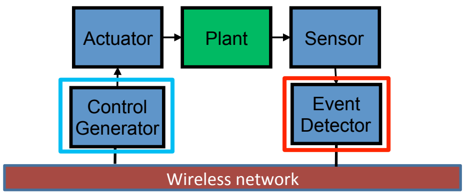
Event-based control loop



Åström, *2007,* Rabi and J., *WICON, 2008*

When to transmit?

- Event detector mechanism on sensor side
  - E.g., threshold crossing



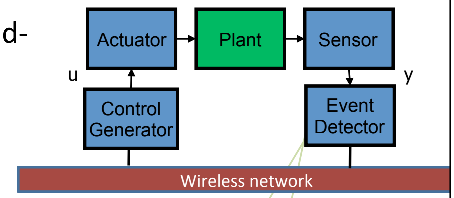
How to control?

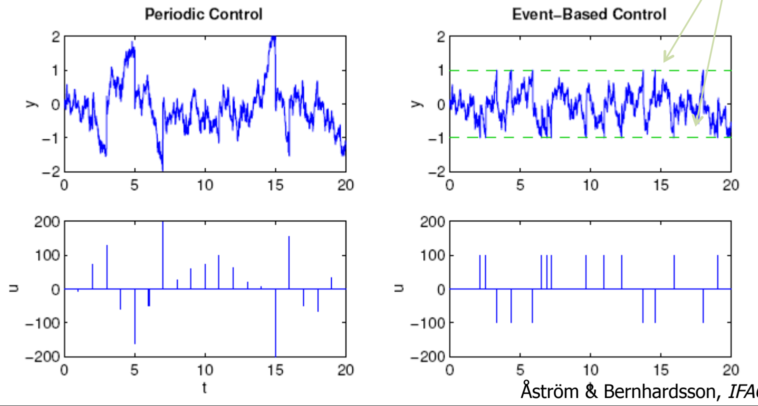
- Execute control law at actuator side
  - E.g., piecewise constant controls, impulse control

Rabi et al., *2008*

Example: Fixed threshold with impulse control

- Event-detector implemented as fixed-level threshold at sensor
- Event-based impulse control better than periodic impulse control



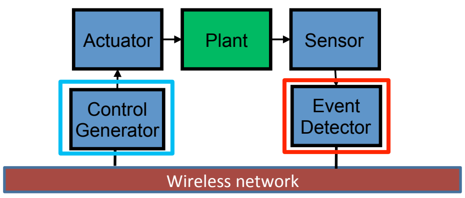


Åström & Bernhardsson, *IFAC*, 1999

Control generators and event detectors

1. Impulse
2. Zero order hold
3. Higher order hold

1. Fixed threshold
2. Time-varying
3. Adaptive



Plant model

Plant $dx = udt + dv,$

Stochastic differential equation, interpreted as

$$x(s+\tau) - x(\tau) = \int_{\tau}^{s+\tau} u(t)dt + \int_{\tau}^{s+\tau} dv(t)$$

with one ordinary (Lebesgue) integral and one stochastic (Ito) integral.

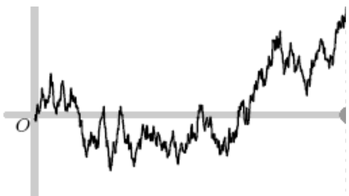
v is a Wiener process (or Brownian motion)

See Øksendal (2003) for an introduction to stochastic differential equations

Wiener process

A Wiener process v(t) fulfills

1. v(0)=0
2. v(t) is almost surely continuous
3. v(t) has independent increments with v(t)-v(s) ~ N(0,t-s) for t>s≥0



Remark The variance of a Wiener process is growing like

$$E(v(t+s) - v(t))^2 = |s|$$

Plant model

Plant $dx = udt + dv,$

Stochastic differential equation, interpreted as

$$x(s+\tau) - x(\tau) = \int_{\tau}^{s+\tau} u(t)dt + \int_{\tau}^{s+\tau} dv(t)$$

with one ordinary (Lebesgue) integral and one stochastic (Ito) integral.

When $s > 0$ is a small, the change of $x(\tau)$ is
normally distributed with mean $su(\tau)$ and variance s.

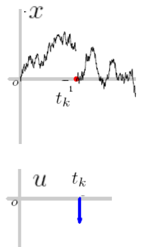
Plant model and control cost

Plant $dx = udt + dv,$

v is a Wiener process: $E(v(t+s) - v(t))^2 = |s|$

Cost function $V = \frac{1}{T} E \int_0^T x^2(t)dt.$

Periodic impulse control



Impulse applied at events t_k

$$u(t) = -x(t_k)\delta(t - t_k),$$

Periodic reset of state every event.

State grows linearly as

$$E(v(t+s) - v(t))^2 = |s|$$

between sample instances, because $dx = udt + dv,$

Average variance over sampling period h is $\frac{1}{2}h$ so the cost is

$$V_{PIH} = \frac{1}{2}h.$$

Åström, 2007

Periodic ZoH control

Traditional sampled-data control theory gives that $V = \frac{1}{h}\int_0^h Ex^2(t)\,dt$ is minimized for the sampled system

$$x(t+h) = x(t) + hu(t) + e(t),$$

with

$$u = -Lx = \frac{1}{h}\frac{3+\sqrt{3}}{2+\sqrt{3}}x$$

derived from

$$S = \Phi^T S\Phi + Q_1 - L^T RL, \quad L = R^{-1}(\Gamma^T S\Phi + Q_{12}^T), \quad R = Q_2 + \Gamma^T S\Gamma,$$

The minimum gives the cost

$$V_{PZOH} = \frac{3+\sqrt{3}}{6}h$$

Åström, 2007

Event-based impulse control with fixed threshold

Suppose an event is generated whenever

$$|x(t_k)| = a$$

generating impulse control

$$u(t) = -x(t_k)\delta(t - t_k),$$

One can show that the average time between two events is

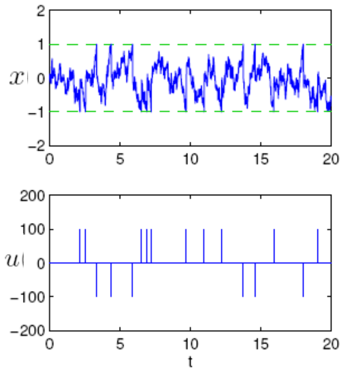
$$h_E := E(T_{\pm d}) = E(x^2_{T_{\pm d}}) = a^2$$

and that the pdf of x is triangular:

$$f(x) = (a - |x|)/a^2$$

The cost is

$$V_{EIH} = \frac{a^2}{6} = \frac{h_E}{6}$$

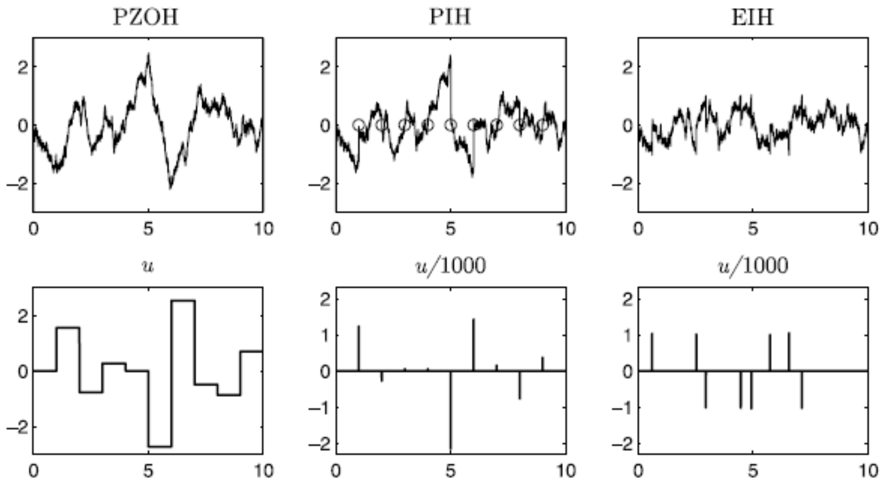


Åström, 2007

Pdf $f(x) = (a - |x|)/a^2$ is the solution to the forward Kolmogorov forward equation (or Fokker–Planck equation)

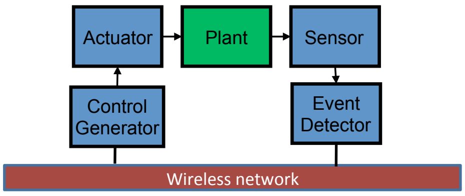
$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(x) - \frac{1}{2}\frac{\partial f}{\partial x}(d)\delta_x + \frac{1}{2}\frac{\partial f}{\partial x}(-d)\delta_x, \qquad f(-a) = f(a) = 0,$$

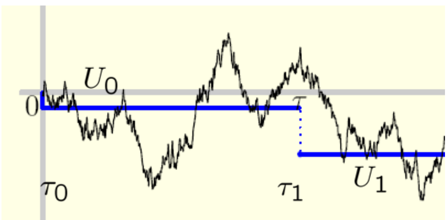
Comparison



Åström, 2007

Event-based ZoH control with adaptive sampling



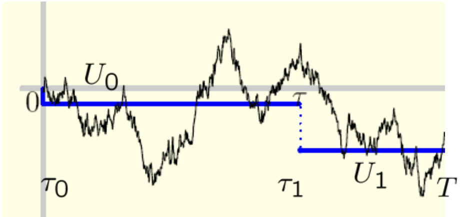


How choose $\{U_i\}$ and $\{\tau_i\}$ to minimize $V = \frac{1}{T} E \int_0^T x^2(t) dt.$

Rabi et al., *2008*

Optimal control with one sampling event

$$dx_t = u_t dt + dB_t$$



$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$

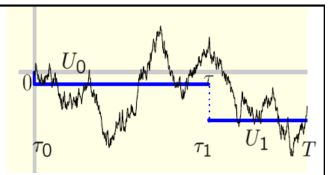
$$= \min_{U_0,U_1,\tau} \left[\mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

A joint optimal control and optimal stopping problem

Rabi et al., *2008*

$$dx_t = u_t dt + dB_t$$

$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$



If τ chosen deterministically (not depending on x_t) and $x_0 = 0$:

$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{T/2}}{T} \qquad \tau^* = T/2$$

If τ is event-driven (depending on x_t) and $x_0 = 0$:

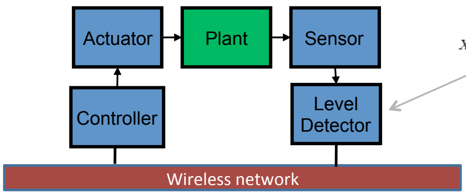
$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{\tau^*}}{2(T-\tau^*)}$$

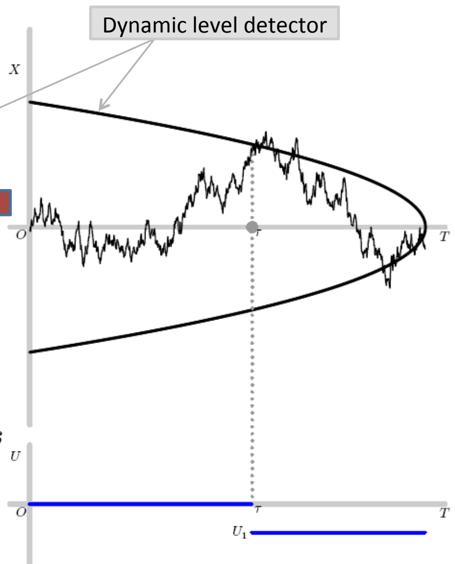
$$\tau^* = \inf\{t : \ x_t^2 \geq \sqrt{3}(T-t)\}$$

Envelope defines optimal level detector

Rabi et al., *2008*

Optimal level detector





$$dx_t = u_t dt + dB_t$$

$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$

Rabi et al., *2008*

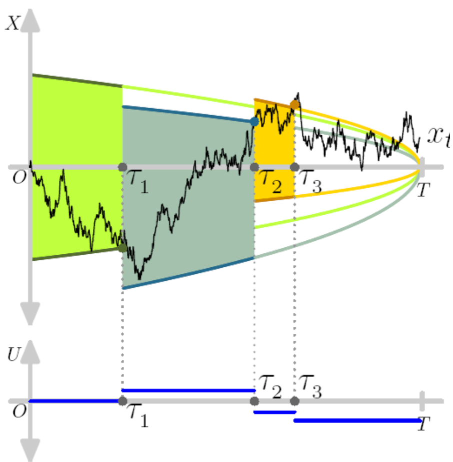
Multiple samples

Extension to N>1 samples

$$J_N\left(x_0, \mathcal{U}, \{\tau\}_{i=1}^N\right) = \mathbb{E}\left[\int_0^T x_s^2 ds \middle| x_0\right]$$

through nested single sample problems

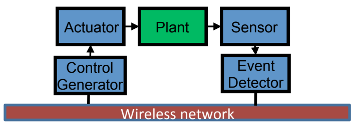
Extension to variable budget sampling, allowing number of samples to depend on x.



Event-based impulse control over wireless network with communication losses

Plant $dx_t = dW_t + u_t dt, \ x(0) = x_0,$

Sampling events $\mathcal{T} = \{\tau_0, \tau_1, \tau_2, \ldots\},$



Impulse control $u_t = \sum_{n=0}^{\infty} x_{\tau_n} \delta(\tau_n)$

Average sampling rate $R_{\tau} = \limsup_{M \to \infty} \frac{1}{M} \mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\tau_n \le M\}} \delta(s - \tau_n)\, ds\right]$

Average cost $J = \limsup_{M \to \infty} \frac{1}{M} \mathbb{E}\left[\int_0^M x_s^2 ds\right]$

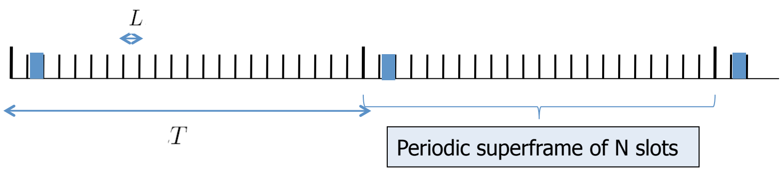
Periodic impulse control

Sampling events $\tau_n = nT$ for $n \geq 0$

Slot length L gives $T = NL$

Average sampling rate $R_{\text{Periodic}} = \frac{1}{T}$

Average cost $J_{\text{Periodic}} = \frac{T}{2}$

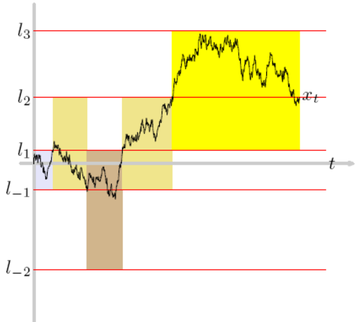


Level-triggered event-based control

Ordered set of levels $\mathcal{L} = \{\ldots, l_{-2}, l_{-1}, l_0, l_1, l_2, \ldots\}$ $l_0 = 0$

Multiple levels needed because we allow packet loss

Lebesgue sampling $\tau = \inf\left\{\tau \middle| \tau > \tau_i, x_\tau \in \mathcal{L}, x_\tau \notin x_{\tau_i}\right\}$



Level-triggered control

For Brownian motion, equidistant sampling is optimal

$$\mathcal{L}^* = \{k\Delta | k \in \mathbb{Z}\}$$

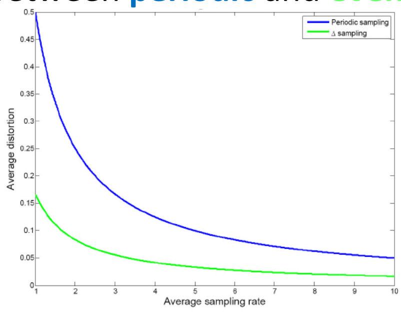
First exit time

$$\tau_\Delta = \inf\{\tau | \tau \geq 0, x_\tau \notin (\xi - \Delta, \xi + \Delta), x_0 = \xi\}$$

Average sampling rate $R_\Delta = \frac{1}{\mathbb{E}[\tau_\Delta]} = \frac{1}{\Delta^2},$

Average cost $J_\Delta = \frac{\mathbb{E}\left[\int_0^{\tau_\Delta} x_s^2 ds\right]}{\mathbb{E}[\tau_\Delta]} = \frac{\Delta^2}{6}.$

Comparison between **periodic** and **event-based** control



$T = \Delta^2$ gives equal average sampling rate for periodic control and event-based control

Event-based impulse control is 3 times better than periodic impulse control

What about the influence of communication losses?
When is event-based sampling better and vice versa?

Influence of communication losses

Times when packets are successfully received $\rho_i \in \{\tau_0 = 0, \tau_1, \tau_2, \ldots\}$,

$\{\rho_0 = 0, \rho_1, \rho_2, \ldots\}$. $\rho_i \geq \tau_i$,

Average rate of packet reception

$$R_\rho = \limsup_{M \to \infty} \frac{1}{M} \mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\rho_n \leq M\}} \delta(s - \rho_n)\, ds \right] = p \cdot R_\tau$$

Define the times between successful packet receptions $\rho_{(p,\Delta)}$

Average cost $J_p = \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T x_s^2 ds \right] = \frac{\mathbb{E}\left[\int_0^{\rho_{(p,\Delta)}} x_s^2 ds \right]}{\mathbb{E}\left[\rho_{(p,\Delta)} \right]}$

Periodic control under packet losses

Sampling with fixed period T with loss probability p gives cost

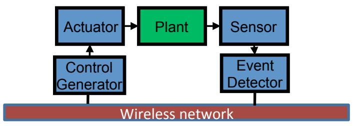
$$J = \frac{T(1+p)}{2(1-p)}$$

Compared with event-based control by setting

$$T = \Delta^2$$

so that the average use of the communication channel is equal

Even-based control under packet losses



Proposition

If packet losses are IID with prob p, then equidistant event-based (Lebesque) sampling gives

$$J_p = \frac{\Delta^2\,(5p+1)}{6\,(1-p)}$$

Remark

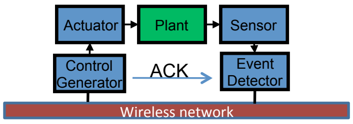
- Event-based control with losses always better than periodic with losses.
- Event-based control with losses outperformed by periodic control without losses if

$$\frac{(1+5p)}{3(1-p)} \geq 1$$

so if $p \geq 0.25$ then periodic sampling do better than event-based sampling.

Rabi and J., *2009*

Sensor data ACK's



If controller perfectly acknowledges packets to sensor, event detector can adjust its sampling strategy

Let $\Delta(l) = \sqrt{l+1}\Delta_0$

where $l \geq 0$ number of samples lost since last successfully transmitted packet

Gives $\mathbb{E}\left[\tau_{i+1}^{\uparrow} - \tau_i^{\uparrow}\right]$ independent of i.

Better performance than fixed $\Delta(l)$ for same sampling rate:

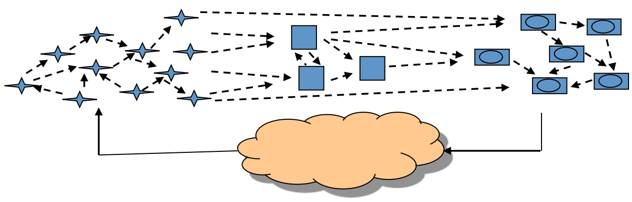
$$J_p^{\uparrow} = \frac{\Delta^2(1+p)}{6(1-p)} \leq \frac{\Delta^2(1+5p)}{6(1-p)} = J_p.$$

Lecture 2 Outline

- What's new with wireless networked control?
- State-based scheduling for control
- Exploiting wireless protocols for control
- Event-based control
- Conclusions

Conclusions

- **Wireless control and networking** are enabling technologies in many emerging industrial applications
- Fundamental challenges related to
  - **time-driven**, synchronous, sampled data control theory, vs
  - **event-driven**, asynchronous, ad hoc wireless networking
- New principles for control in large-scale wireless systems



Take-home message

Lecture 1: Motivating applications and challenges

- Networked control systems have societal importance
- Many new applications with challenging problems

Lecture 2: Wireless control systems

- Everything will be wireless, including control systems
- Interesting research challenges on the intersection between sensor networks, wireless communication, and control theory

http://www.ee.kth.se/~kallej