Effects of Rayleigh-Lognormal Fading on IEEE 802.15.4 Networks

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***Abstract*—The IEEE 802.15.4 communication protocol is a de-facto standard for wireless applications in industrial and home automation. Although the performance of the medium access control (MAC) of the IEEE 802.15.4 has been thoroughly investigated under the assumption of ideal wireless channel, there is still a lack of understanding of the cross-layer interactions between MAC and physical layer in the presence of realistic wireless channel models that include path loss, multi-path fading and shadowing. In this paper, an analytical model of these dynamics is proposed. The analysis considers simultaneously a composite Rayleigh-lognormal channel fading, interference generated by multiple terminals, the effects induced by hidden terminals, and the MAC reduced carrier sensing capabilities. It is shown that the reliability of the contention-based MAC over fading channels is often far from that derived under ideal channel assumptions. Moreover, it is established to what extent fading may be beneficial for the overall network performance.**

***Index Terms*—IEEE 802.15.4, Fading Channel, Multi-hop.**

I. Introduction

The development of wireless sensor network (WSN) systems relies heavily on the behavior of underlying communication mechanisms. A clear understanding of the achievable performance and cross-layer interactions over realistic environment is missing for the IEEE 802.15.4 standard [1], which is widely used in industrial control, home automation, health care, and smart grid applications.

Ongoing research activities focus on performance characterization of the IEEE 802.15.4 MAC layer in terms of reliability (i.e., successful packet reception probability), packet delay, throughput, and energy consumption [2]–[4]. These analytical studies are almost all based on extensions of the Markov chain model originally proposed by Bianchi for the IEEE 802.11 MAC protocol [5], under ideal channel conditions. In [6], a model of packet losses due to channel fading has been introduced into the homogeneous Markov chain presented in [3]. However, fading is considered only for single packet transmission attempts, the effect of contention and multiple access interference is neglected, and the analysis is neither validated by simulations nor by experiments. In [7], [8], the optimal carrier sensing range is derived to maximize the throughput for IEEE 802.11 networks; however, statistical modeling of wireless fading has not been considered, but a two-ray ground radio propagation model is used in [7] and only the path loss in [8].

Recent studies have investigated the performance of multiple access networks in terms of multiple access interference and capture effect [9], [10]. However, the models of the MAC mechanism are limited to homogeneous single-hop networks (same wireless channel for every node) with uniform random deployment. Moreover in [9], the fading caused by multi-path propagation and shadowing effects have not been considered, while it is known that it may have a crucial impact on the performance of packet access mechanisms [11].

In this paper we propose an analytical model that is able to capture the cross-layer interactions of IEEE 802.15.4 MAC and physical layer over interference-limited wireless channels with composite Rayleigh-lognormal fading in single-hop and multi-hop networks. We describe how to account for statistical fluctuations of the signal-to-(interference plus noise)-ratio (SINR) in the model of the MAC. Based on the new model, we determine the impact of fading conditions on the MAC performance under various settings for traffic, inter-node distances, carrier sensing range, and SINR. Moreover, we discuss system configurations in which a certain severity of the fading may be beneficial for overall network reliability.

To determine the network operating point and the performance indicators in terms of reliability, a moment matching approximation for the linear combination of lognormal random variables based on [12] and [13] is adopted in order to build a model of the MAC mechanism that embeds the physical layer behavior. An extended version of the performance evaluation presented in this paper is reported in [14].

The remainder of the paper is organized as follows. In Section II, we introduce the network model. In Section III, we derive an analytical model of the unslotted IEEE 802.15.4 MAC over fading channels. The accuracy of the model is evaluated by Monte Carlo simulations in Section IV. Section V concludes the paper and prospects our future work.

II. Network Model

We illustrate the network model by considering the two topologies sketched in Fig. 1. Nevertheless, the analytical results that we derive in this paper are applicable to any fixed topology.

The topology in Fig. 1a) refers to a single-hop (star) network, where node i is deployed at distance $r_{i,0}$ from the

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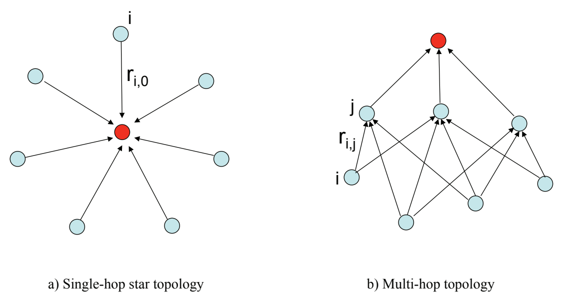

Fig. 1. Example of topologies: single-hop star topology (on the left) and multi-hop topology with multiple end-devices (on the right).

root node at the center, and where nodes forward their packets with single-hop communication to the root node. In Fig. 1b), we illustrate a multi-hop topology with multiple end-devices that generate and forward traffic according to an uplink routing policy to the root node.

Consider node i that is transmitting a packet with transmission power $P_{\mathrm{tx},i}$. We consider an inverse power model of the link gain, and include shadowing and multi-path fading as well. The received power at node j, which is located at a distance $r_{i,j}$, is then expressed as follows

$$P_{\mathrm{rx},i,j} = \frac{c_0 P_{\mathrm{tx},i}}{r_{i,j}^k} f_i \exp(y_i)\,. \tag{1}$$

The constant c_0 represents the power gain at the reference distance 1 m, and it can account for specific propagation environments and parameters, e.g., carrier frequency and antennas. In the operating conditions for IEEE 802.15.4 networks, the inverse of c_0 (i.e., the path loss at the reference distance) is in the range $40-60$ dB [1]. The exponent k is called path loss exponent, and varies according to the propagation environment in the range $2-4$. The factor f_i models the channel fading due to multi-path propagation, which we assume to follow a Rayleigh distribution, i.e., exponential distribution of the power attenuation, with p.d.f.

$$p_{f_i}(z) = z\exp(-z).$$

A random lognormal component models the shadowing effects due to obstacles, with $y_i \sim \mathcal{N}(0,\sigma_i^2)$. The standard deviation σ_i is called spread factor of the shadowing. The model we adopt is accurate for IEEE 802.15.4 in a home or urban environment, in which devices may not be in visibility.

In the rest of the paper, we use the index l to indicate a link, where i is the transmitting node and j is the receiving node. We use double indices (i,j) for variables that depend on a generic pair of nodes in the network.

III. IEEE 802.15.4 MAC and PHY Layer Model

In this section we propose a novel analytical setup to derive the network performance. We extend the unslotted MAC model presented in [4], which was developed under ideal channel conditions, to include the main features of channel impairments and interference. First, we consider a single-hop case, and then we generalize the model to the multi-hop case in Section III-E.

A. Unslotted IEEE 802.15.4 MAC Mechanism

The unslotted IEEE 802.15.4 MAC mechanism is based on a carrier sense multiple access with collision avoidance (CSMA/CA), and binary exponential backoff. Each node can be in one of the following states: (i) idle state, when the node is waiting for a packet to be generated; (ii) backoff state; (iii) clear channel assessment (CCA) state; (iv) transmission state.

Let the link l be in idle state with probability $b_{0,0,0}^{(l)}$. The three variables given by the number of backoffs NB, the backoff exponent BE, and the retransmission attempts RT are initialized: the default initialization is NB=0, BE=m_0, and RT=0. From idle state, the transmitting node wakes up with probability q_i, which represents the packet generation probability in each time unit of duration $S_b =$ *aUnitBackoffPeriod*, and moves to the first backoff state, where the node waits for a random number of complete backoff periods in the range $[0, 2^{BE}-1]$ time units.

When the backoff period counter reaches zero, the node performs the CCA procedure. If the CCA fails due to busy channel, the value of both NB and BE is increased by one. Once BE reaches its maximum value m_b, it remains at the same value until it is reset. If NB exceeds its threshold m, the packet is discarded due to channel access failure. Otherwise the CSMA/CA algorithm repeats the backoff procedure. The tranmsitting node is in CCA state with probability τ_l, and either moves to the next backoff state if the channel is sensed busy with probability α_l, or moves to transmission state with probability $(1-\alpha_l)$. The reception of the corresponding ACK is interpreted as successful packet transmission. The node moves from the transmission state to idle state with probability $(1-\gamma_l)$. As an alternative, with probability γ_l, the packet sent to the receiving node j is lost, and the variable RT is increased by one. As long as RT is less than its threshold n, the MAC layer initializes BE=m_0 and starts again the CSMA/CA mechanism to re-access the channel. Otherwise the packet is discarded due to the retry limit.

B. MAC-Physical Layer Model

Assume packets are generated with Poisson distribution at rate λ_i. The probability of generation of a new packet after an idle unit time is then $q_i = 1-\exp(-\lambda_i/S_b)$. Effects of a limited buffer size can be included in the expression of q_i, by considering the probability that the node queue is not empty after a packet has been successfully sent, after a packet has been discarded due to channel access failure or due to the retry limit (see [4]).

The expression of the idle state probability $b_{0,0,0}^{(l)}$ can be derived from the normalization condition of the Markov chain model in [4]. From the same Markov chain, the CCA probability τ_l is derived as

$$\tau_l = \left(\frac{1-\alpha_i^{m+1}}{1-\alpha_i}\right)\left(\frac{1-\xi_l^{n+1}}{1-\xi_l}\right) b_{0,0,0}^{(l)}\,. \tag{2}$$

where $\xi_l = \gamma_l(1-\alpha_l^{m+1})$. A step-by-step derivation of Eq. (2) is provided in [4].

In the following, we propose a novel analysis to derive the busy channel probability α_l and the packet loss probability γ_l, in the case of fading.

The busy channel probability is

$$\alpha_l = \alpha_{\mathrm{pkt},l} + \alpha_{\mathrm{ack},l} \,, \tag{3}$$

where $\alpha_{\mathrm{pkt},l}$ is the probability that the transmitting node senses the channel and finds it occupied by an ongoing packet transmission, whereas $\alpha_{\mathrm{ack},l}$ is the probability of finding the channel busy due to ACK transmission. Assume that packet and ACK transmissions occupy L and L_{ack} units of time, respectively.

The busy channel probability due to packet transmissions evaluated at the transmitting node i is the combination of three events: (i) at least one other node has accessed the channel within one of the previous L units of time; (ii) at least one of the nodes that accessed the channel found it idle and started a transmission; (iii) the total received power at node i is larger than a threshold a, so that an ongoing transmission is detected by node i.

The combination of all busy channel events yields

$$\alpha_{\mathrm{pkt},l} = L\,\mathcal{H}_l\left(p_i^{\mathrm{det}}\right) \,, \tag{4}$$

where

$$\begin{aligned}\mathcal{H}_l(\chi) = &\sum_{v=0}^{N-1}\sum_{j=1}^{C_{N,v}}\prod_{k=1}^{v+1}\tau_{k_j}\prod_{h=v+2}^{N}(1-\tau_{h_j})\times \\ &\times\sum_{m=0}^{k-1}\sum_{n=1}^{C_{k,i}}\prod_{z=1}^{m+1}(1-\alpha_{z_n})\,\chi\prod_{r=m+2}^{k}\alpha_{r_n}\,,\end{aligned} \tag{5}$$

$C_{k,v} = \binom{k-1}{v+1}$, and $p_i^{\mathrm{det}} = \Pr\left[\sum_{z=1}^{m+1} P_{\mathrm{rx},z_n,i} > a\right]$ is the detection probability. The index k accounts for the events of simultaneous accesses to the channel and the index j enumerates the combinations of events in which a number l of channel accesses are performed in the network simultaneously. Given N nodes in the network, the index k_j refers to the node in the k-th position in the j-th combination of l out of $N-1$ elements (node i is not included). The index z_n accounts for the combinations of events in which one or more nodes find the channel idle simultaneously.

The busy channel probability due to an ACK transmission follows from a similar derivation. An ACK is sent only after a successful packet transmission:

$$\alpha_{\mathrm{ack},l} = L_{\mathrm{ack}}\mathcal{H}_l\left((1-\gamma_{z_n})p_i^{\mathrm{det}}\right) \,, \tag{6}$$

where L_{ack} is the length of the ACK. The detection probability p_i^{det} is evaluated with respect to the set of destination nodes. By summing up Eqs. (4) and (6), we compute α_l in Eq. (3).

We next derive an expression for the packet loss probability γ_l, namely the probability that a transmitted packet in the link $l = (i,j)$ is not correctly detected in reception, by using similar arguments as above. A packet transmission is not detected in reception if there is at least one interfering node that starts the transmission at the same time and the SINR between the received power from the transmitter and the total interfering power plus the noise level N_0 is lower than a threshold b (outage). In the event of no active interferers, which occurs with probability 1-$\mathcal{H}_l(1)$, the packet loss probability is the probability that the signal-to-noise ratio (SNR) between the received power and the noise level is lower than b. Hence,

$$\begin{aligned}\gamma_l =& (1-\mathcal{H}_l(1))\,p_{i,j}^{\mathrm{fad}} + \mathcal{H}_l\left(p_{i,j}^{\mathrm{out}}\right) \\ &+ (2\,L-1)\,\mathcal{H}_l\left(\left(1-p_i^{\mathrm{det}}\right)p_{i,j}^{\mathrm{out}}\right) \,,\end{aligned} \tag{7}$$

where $p_{i,j}^{\mathrm{fad}} = \Pr\left[P_{\mathrm{rx},i,j}/N_0 < b\right]$ is the outage probability due to fading on the useful link (with no interferers), and $p_{i,j}^{\mathrm{out}} = \Pr\left[P_{\mathrm{rx},i,j}/(\sum_{z=1}^{m+1} P_{\mathrm{rx},z_n,j} + N_0) < b\right]$ is the outage probability in the presence of interferers (with composite channel fading on every link).

The expressions of the carrier sensing probability τ_l in Eq. (2), the busy channel probability α_l in Eq. (3), the collision probability in Eq. (7), for $l = 1, \ldots, N$, form a system of non-linear equations that can be solved through numerical methods [15].

C. Model of Aggregate Multi-path Shadowed Signals

In this subsection, we approach the problem of computing the sum of multi-path shadowed signals that appear in the detection probability and in the outage probability.

Consider node i performing a CCA and let us focus our attention on the detection probability in transmission $\Pr\left[\sum_{n=1}^{x} P_{\mathrm{rx},n,i} > a\right]$, where x is the current number of active nodes in transmission. By recalling the power channel model in Eq. (1), let us define the random variable $Y_i = \ln\left(\sum_{n=1}^{x} A_{i,n}\exp(y_n)\right)$ with $A_{i,n} = c_0 P_{\mathrm{tx},n} f_n / r_{n,i}^k$, and $y_n \sim \mathcal{N}(0,\sigma_n^2)$. Since a closed form expression of the probability distribution function of Y_i does not exist, we resort to a useful approximation instead. In order to characterize Y_i, we apply the Moment Matching Approximation (MMA) method, which approximates the statistics of linear combination of lognormal components with a lognormal random variable, such that $Y_i \sim \mathcal{N}(\eta_{Y_i}, \sigma_{Y_i}^2)$.

According to the MMA method, η_{Y_i} and σ_{Y_i} can be obtained by matching the first two moments of $\exp(Y_i)$ with the first two moments of $\sum_{n=1}^{x} A_{i,n}\exp(y_n)$, i.e.,

$$M_1 \triangleq \exp\left(-\eta_{Y_i} + \frac{1}{2}\sigma_{Y_i}\right) = \sum_{n=1}^{x} E\{A_{i,n}\}\exp\left(\eta_{y_n} + \frac{1}{2}\sigma_{y_n}\right), \tag{8}$$

$$\begin{aligned}M_2 \triangleq \exp\left(-2\eta_{Y_i} + 2\sigma_{Y_i}\right) = &\sum_{m=1}^{x}\sum_{n=1}^{x} E\{A_{i,m}A_{i,n}\}\times \\ &\times\exp\left(\eta_{y_m} + \eta_{y_n} + \left(\frac{\sigma_{y_m}^2}{2} + \frac{\sigma_{y_n}^2}{2} + \rho_{y_m,y_n}\sigma_{y_m}\sigma_{y_n}\right)\right).\end{aligned} \tag{9}$$

Solving Eqs. (8) and (9) for η_{Y_i} and σ_{Y_i} yields $\eta_{Y_i} = 0.5\ln(M_2) - 2\ln(M_1)$, and $\sigma_{Y_i}^2 = \ln(M_2) - 2\ln(M_1)$.

It follows that

$$p_i^{\mathrm{det}} = \Pr\left[\exp(Y_i) > a\right] \approx Q\left(\frac{\ln(a) - \eta_{Y_i}}{\sigma_{Y_i}}\right) \,, \tag{10}$$

where $Q(z) = 1/\sqrt{2\pi}\int_z^{\infty}\exp\left(-\nu^2/2\right)d\nu$.

Similar derivations follow for the outage probability in reception $p_{i,j}^{\text{out}} = \Pr\left[P_{\text{rx},i,j}/(\sum_{z=1}^{m+1} P_{\text{rx},z_n,j} + N_0) < b\right]$. Let us now define the random variable

$$\tilde{Y}_{i,j} = -\ln\left(\sum_{n=1}^{x+1} B_{i,j,n}\exp(\tilde{y}_n)\right) ,$$

where $B_{i,j,n} = \begin{cases} \frac{P_{\text{tx},n} r_{i,j}^k}{P_{\text{tx},i} r_{n,j}^k} f_n & \text{for} \quad n = 1,...,x \\ \frac{N_0 r_{i,j}^k}{P_{\text{tx},i}} f_n & \text{for} \quad n = x+1 \end{cases}$

and $\tilde{y} = \begin{cases} y_n - y_i & \text{for} \quad n = 1,...,x \\ -y_i & \text{for} \quad n = x+1 \end{cases}$.

Again, according to the MMA method, we approximate $\tilde{Y}_{i,j} \sim \mathcal{N}(\eta_{\tilde{Y}_{i,j}}, \sigma^2_{\tilde{Y}_{i,j}})$, where $\eta_{\tilde{Y}_{i,j}}$ and $\sigma_{\tilde{Y}_{i,j}}$ can be obtained by matching the first two moments of $\exp(\tilde{Y}_{i,j})$ with the first two moments of $\sum_{n=1}^{N} B_{i,j,n}\exp(\tilde{y}_n)$, as before. Therefore,

$$p_{i,j}^{\text{out}} = \Pr\left[f_i \exp(\tilde{Y}_{i,j}) < b\right] =$$
$$= \int_0^b \int_0^\infty p_f(z|w) \frac{1}{\sqrt{2\pi}\sigma_{\tilde{Y}_{i,j}} w} \exp\left(-\frac{(\ln(w) - \eta_{\tilde{Y}_{i,j}})^2}{2\sigma^2_{\tilde{Y}_{i,j}}}\right) dw\, dz.$$

The analysis above holds for a generic weighted composition of lognormal fading components. In the case of lognormal channel model, where only shadow fading components are considered, (i.e., $f_i = 1$), the outage probability becomes

$$p_{i,j}^{\text{out}} = \Pr\left[\exp(\tilde{Y}_{i,j}) < b\right] \approx 1 - Q\left(\frac{\ln(b) - \eta_{\tilde{Y}_{i,j}}}{\sigma_{\tilde{Y}_{i,j}}}\right) .$$

For a Rayleigh-lognormal channel, the outage probability becomes

$$p_{i,j}^{\text{out}} = 1 - \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_{\tilde{Y}_{i,j}} w} \exp\left(-\frac{(\ln(w) - \eta_{\tilde{Y}_{i,j}})^2}{2\sigma^2_{\tilde{Y}_{i,j}}} - bw\right) dw.$$

The mean and standard deviation of Y_i and $\tilde{Y}_{i,j}$ can be obtained by inserting the moments of f_i in the moments of $A_{i,n}$ and $B_{i,j,n}$. For Gamma distributed components f_i, we obtain $E\{f_i\} = 1$ and $E\{f_i^2\} = \frac{\kappa+1}{\kappa}$.

We remark here that the evaluation of p_i^{det}, and $p_{i,j}^{\text{out}}$ can be carried out off-line, with respect to the solution of the system of nonlinear equations that need to be solved when deriving τ_l, α_l and γ_l. Therefore, the proposed model can be implemented with only a slight increase of complexity with respect to the analytical model of the IEEE 802.15.4 MAC mechanism presented in [4], and the online computation time is not affected significantly.

D. Reliability

As a following modeling step, we investigate the reliability of the IEEE 802.15.4 MAC over fading channels. For each node of the network, we define the reliability considering the probability that packets are discarded at MAC layer. In unslotted CSMA/CA, packets are discarded due to either (i) channel access failure or (ii) retry limits. A channel access failure happens when a packet fails to obtain clear channel within $m+1$ backoff stages. Furthermore, a packet is discarded if the transmission fails due to repeated packet losses after $n+1$ attempts. According to the IEEE 802.15.4 MAC mechanism described in Section III-A, the probability that the packet in the link $l = (i,j)$ is discarded due to channel access failure can be expressed as

$$p_{\text{cf},l} = \frac{\alpha_l^{m+1}(1 - (\gamma_l(1 - \alpha_l^{m+1}))^{n+1})}{1 - \gamma_l(1 - \alpha_l^{m+1})} ,$$

and the probability of a packet discarded due to retry limits is

$$p_{\text{cr},l} = (\gamma_l(1 - \alpha_l^{m+1}))^{n+1} .$$

Therefore, the reliability is

$$R_l = 1 - p_{\text{cf},l} - p_{\text{cr},l} . \tag{11}$$

It is worthwhile mentioning that the last expressions link the reliability at the MAC level with the statistical description of wireless channel environment through Eq. (7) and the analysis of Subsection III-C.

E. Extension to Multi-hop Communications

Here we extend the model to a general network in which information is forwarded through a multi-hop communication to a sink node.

The model equations derived in Section III-B are solved for each link of the network, by considering that a generic node i forwards aggregate traffic Q_i. The total average aggregated traffic of node i is $Q_i = q_i/S_b$ pkt/s, where q_i is the probability of having a packet to transmit in each time unit and S_b is the duration of the basic time unit in IEEE 802.15.4. We define $\bar{\lambda}$ the vector of node traffic generation rates. We assume that the routing matrix is built such that no cycles exists. The effect of routing can be described by the matrix $\mathbf{M}$ in which $M_{i,j} = 1$ if node j is the destination of node i, and $M_{i,j} = 0$ otherwise.

We define the traffic distribution matrix $\mathbf{T}$ by scaling $\mathbf{M}$ by the probability of successful reception in each link as only successfully received packets are forwarded, i.e. $T_{i,j} = M_{i,j}R_l$ where R_l is given by Eq. (11).

Therefore, the vector of traffic generation probabilities is given by [4]

$$Q = \bar{\lambda}\,[\mathbf{I} - \mathbf{T}]^{-1} , \tag{12}$$

where $\mathbf{I}$ is the identity matrix. Eq. (12) gives the relation between the idle packet generation probability q_i, the routing matrix $\mathbf{M}$, and the performance at MAC layer (through the link reliability R_l). To obtain the multi-hop network model, we couple Eq. (12) with the expressions for τ_l, α_l and R_l, as obtained by Eqs. (2), (3), and (11).

IV. Performance Evaluations

In this section, we present numerical results for the new model for various settings, network topology, and operations. We report extensive Monte Carlo simulations to validate the accuracy of the approximations that we have introduced in the model. The settings are based on the default specifications of the IEEE 802.15.4 [1]. We perform simulations both for single-hop and multi-hop topologies.

First, we consider the single-hop star topology in Fig. 1a). We let the number of nodes be $N = 7$, the MAC parameters

$m_0 = 3$, $m = 4$, $m_b = 7$, $n = 0$, $L = 7$, $L_{\rm ack} = 2$ and the physical layer parameters $P_{{\rm tx},i} = 0$ dBm, and $k = 2$. We validate our model and study the performance of the network by varying the traffic rate $\lambda_i = \lambda$, in the range $0.1-10$ pkt/s, the radius $r_{i,0} = r$, in the range $0.1-10$ m, and the spread of the shadow fading $\sigma_i = \sigma$, in the range $0-6$. Moreover, we show results for different values of the carrier sensing threshold $a = -76, 66, 56$ dBm, and outage threshold $b = 6, 10, 14$ dB.

In Fig. 2, we report the average reliability over all links by varying the node traffic rate λ. The results are shown for different values of the spread σ in the absence of multi-path ($f_i = 1$). There is a good matching between the simulations and the analytical model (11). The reliability decreases as the traffic increases. Indeed, an increase of the traffic generates an increase of the contention level at MAC layer. However, we can observe that the impact of shadow fading can be more relevant with respect to variations in the traffic. Therefore, a prediction based only on Markov chain analysis of the MAC without including the channel behavior, as in the previous literature, is typically inaccurate to capture the performance of IEEE 802.15.4 wireless networks, especially at larger shadowing spreads.

In Fig. 3, the average reliability is reported as a function of the radius r for different values of the spread σ. Again, analytical results, obtained through Eq. (11), are in good agreement with those provided by simulations. For the ideal channel case (i.e., $\sigma = 0$) the size of the network does not affect the reliability in the range $r = 0.1-10$ m. For $\sigma = 6$, the performance degrades significantly as the radius increases. An intermediate behavior is obtained for $\sigma = 3$, where the reliability is comparable to the ideal channel case for short links, but it reduces drastically for $r > 1$ m. The effect is the combination of an increase of the outage probability with the radius (due to the path loss component), and hidden terminals that are not detected by the CCA.

Fig. 4 shows the average reliability as a function of the shadowing spread σ. The results are plotted for different values of the carrier sensing threshold a. The reliability decreases when the threshold a become larger. The impact of the variation of the threshold a is maximum for $\sigma = 0$, and the gap reduces when the spread σ increases.

In Fig. 5, we plot the average reliability as a function of the spread σ for different values of the outage threshold b. The threshold b does not affect the performance noticeably for $\sigma = 0$, while the gap in the reliability increases with σ. Note that for a high threshold the reliability tends to increase with σ as long as σ is small or moderate, and it decreases for large spreads. In our setup, a maximum in the reliability is obtained for $\sigma \approx 2$.

In Fig. 6, we report the combined effects of shadow fading and multi-path fading on the reliability. We show the reliability as a function of the spread σ of the shadow fading for log-normal and composite Rayleigh-lognormal fading. The effect of the multi-path is a further degradation of the reliability. Furthermore, the multi-path fading evidences the presence of the maximum at $\sigma \approx 2$ in the plot of reliability.

Finally, we consider the multi-hop topology in Fig. 1b). We use the same MAC and physical layer parameters as in the single-hop case.

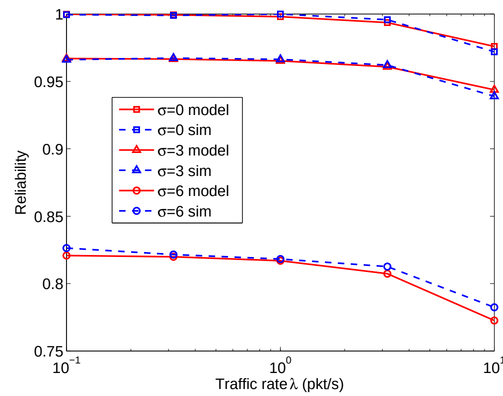

Fig. 2. Reliability vs. traffic rate λ for the star network in Fig. 1a) with $N = 7$ nodes, $r = 1$ m, $a = -76$ dBm, $b = 6$ dB.

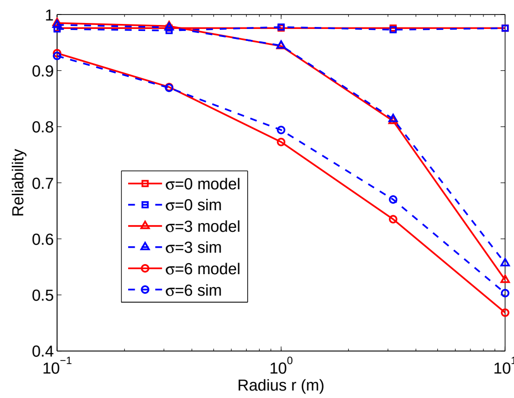

Fig. 3. Reliability vs. radius r for the star network in Fig. 1a) with $N = 7$ nodes, $\lambda = 10$ pkt/s, $a = -76$ dBm, $b = 6$ dB.

Fig. 7 shows the end-to-end reliability by varying the spread σ for different values of b. Differently with respect to the single-hop and linear topologies, a variation of the outage threshold b has a strong impact on the reliability also for small to moderate shadowing spread. In fact, due to the variable distance between each source-destination pair, the fading and the outage probabilities affect the network noticeably. Nonetheless, this effect is well predicted by the analytical model we have developed.

V. Conclusions

In this paper, we proposed an integrated cross-layer model of the MAC and physical layers for unslotted IEEE 802.15.4 networks, by considering explicit effects of multi-path shadow fading channels and presence of interferers. We studied the impact of fading statistics on the MAC performance in terms of reliability by varying traffic rates, inter-nodes distances, carrier sensing range, and SINR threshold. We observed that the severity of the fading and the physical layer thresholds have significant and complex effects on all performance indicator at MAC layer, and the effects are well predicted by the new model. In particular, the fading has a negative impact on the

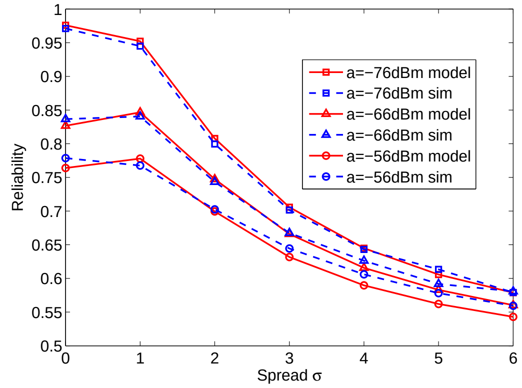

Fig. 4. Reliability vs. σ for the star network in Fig. 1a) with $N = 7$ nodes, $r = 5$ m, $\lambda = 10$ pkt/s, $b = 6$ dB.

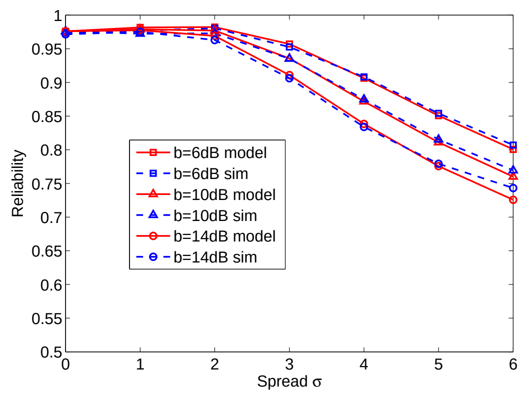

Fig. 5. Reliability vs. σ for the star network in Fig. 1a) with $N = 7$ nodes, $r = 1$ m, $\lambda = 10$ pkt/s, $a = -76$ dB.

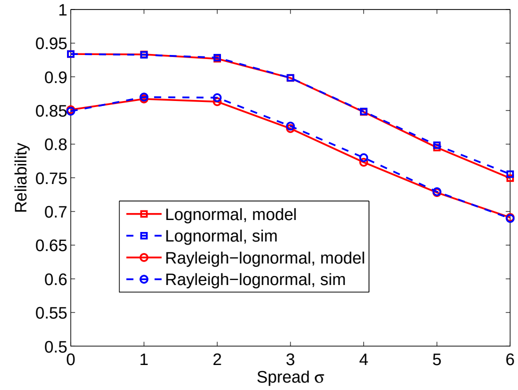

Fig. 6. Reliability vs. σ for the star network in Fig. 1a) with $N = 7$ nodes, $r = 1$ m, $\lambda = 5$ pkt/s, $a = -56$ dB, $b = 6$ dB.

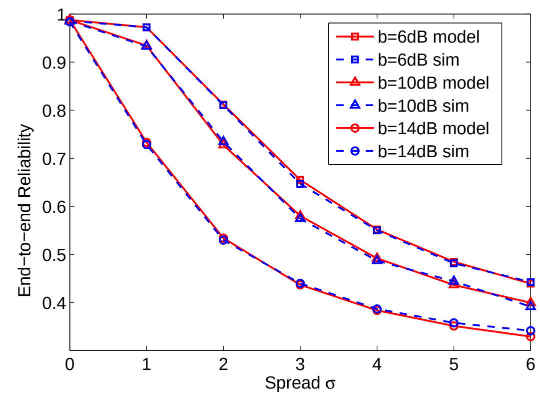

Fig. 7. End-to-end reliability vs. σ for the multi-hop topology in Fig. 1b) with $\lambda = 2$ pkt/s, $a = -76$ dB.

reliability that is more evident as traffic and distance between nodes increase. However, depending on the carrier sensing and SINR thresholds, a fading with small spread can improve the reliability with respect to the ideal case and the performance can be optimized by opportunely tuning the thresholds.

As a future work, a tradeoff between reliability, delay, and power consumption can be exploited by proper tuning of routing, MAC, and physical layer parameters.

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