Wireless Sensor Network Scheduling for Remote Estimation under Energy Constraints*

Takuya Iwaki[1], Yuchi Wu[2], Junfeng Wu[3], Henrik Sandberg[1] and Karl Henrik Johansson[1]

Abstract— **This paper studies a design problem of how a group of wireless sensors are selected and scheduled to transmit data efficiently over a multi-hop network subject to energy-saving consideration, when they are observing multiple independent discrete-time linear systems. Each time instant, some sensors are selected to transmit their measurements to a remote estimator. We formulate an optimization problem, minimizing a linear combination of the averaged estimation error and the averaged transmission energy consumption to obtain suitable network scheduling and estimation algorithms. Necessary conditions for optimality are derived and these conditions help trim the feasible solution space so that the optimal solution can be computed efficiently. A numerical example is provided to demonstrate the theoretical results.**

I. INTRODUCTION

Recent development of wireless sensor technology enables control and estimation of process states over wireless sensor networks, which is of significant interest for process and automation industries [1], [2]. Wireless sensor networks provide advantages through enhanced and massive sensing, flexible deployment and operation, and more efficient maintenance. However, since wireless sensors have usually no inexhaustible or reliable energy sources, energy limitation of wireless sensors affect system performance. In this context, energy-aware protocols, real-time algorithms as well as empirical studies for optimizing the performance of wireless sensor networks have been discussed in [3], [4], and [5]. In addition, as the number of wireless sensors over an area increases, data packets in the network may be lost due to interference or network congestion. This leads to poor estimation and control performance of the overall networked systems.

To tackle these problems, sensor scheduling approaches have been investigated by several research groups [6], [7], [8], [9], and [10]. For example, collision-free TDMA-like sensor scheduling scheme was proposed in [6]. This scheme considered networks where the sensors are connected directly to a shared network, and the gateway transmits one sensor measurement from a single sensor to a remote estimator at each time. Later, a more general version of sensor scheduling allowing more than one sensors to be scheduled was proposed in [7]. Other significant works on sensor scheduling features event-based scheme [11] and scheduling problem among multiple sensors observing indentical process [12], [13].

In this paper, we consider the problem of how to select and schedule a set of sensors to transmit sensor data efficiently over a multi-hop network subject to energy constraints when the sensors observe independent discrete-time linear systems. The problem set up is motivated by an industrial case study at a Swedish paper plant [14]. At each time step, some sensors are scheduled to transmit to the estimator. By not allowing all sensors to send data every time, the energy consumption of the sensors can be reduced. Different from [6], the measurements are not directly sent to the estimator but through some intermediate nodes and a gateway.

For the medium access and the communication protocol, we consider a periodic superframe structure common for many existing wireless sensor network protocols [15]. A superframe between every sampling interval is divided into timeslots. We assume only one point-to-point link is activated at a time. Then, by activating links in a certain order, the measurement data of selected sensor nodes can be efficiently conveyed to the estimator. The link activation is jointly determined with the sensor selection by considering data aggregation techniques [16], [17], [18], constrained by the energy consumption of the sensor nodes.

The contributions of this work are as follows:

1) We study the offline optimal sensor network scheduling for remote estimation under sensor energy constraints. We find necessary conditions for optimality.
2) By exploiting the necessary conditions, we can find a periodic optimal sensor network schedule.

The remainder of the paper is organized as follows. Section 2 describes the notation and preliminaries. System description including process, communication, energy consumption models together with remote estimation is given in Section 3. Optimal sensor network schedules are discussed in Section 4. A numerical example is provided in Section 5. Section 6 presents the conclusion.

II. PRELIMINARIES

Notations: $\mathbb{N}$ and $\mathbb{R}$ are the sets of nonnegative integers and real numbers, respectively. The set of n by n positive semi-definite (positive definite) matrices (that are restricted to be Hermitian) over the field $\mathbb{R}$ is denoted as $\mathbb{S}^n_+$ ($\mathbb{S}^n_{++}$). For simplicity, we write $X \geq Y$ ($X > Y$), where $X, Y \in \mathbb{S}^n_+$,

*This work was supported in part by the VINNOVA PiiA project "Advancing System Integration in Process Industry," the Knut and Alice Wallenberg Foundation, the Swedish Strategic Research Foundation, and the Swedish Research Council.

[1]The authors are with the ACCESS Linnaeus Centre, Electrical Engineering, KTH Royal Institute of Technology, SE-100 44 Stockholm, Sweden. Emails: {takya, hsan, kallej}@kth.se

[2]Yuchi Wu is with the Department of Electronic and Computer Engineering, The Hong Kong University of Science and Technology, Hong Kong. Email: ywubj@ust.hk

[3]Junfeng Wu is with the College of Control Science and Engineering, Zhejiang University, Hangzhou, P. R. China. Email: jfwu@zju.edu.cn

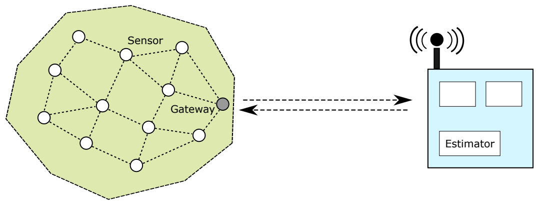

Fig. 1. Wireless sensor network with a remote estimator

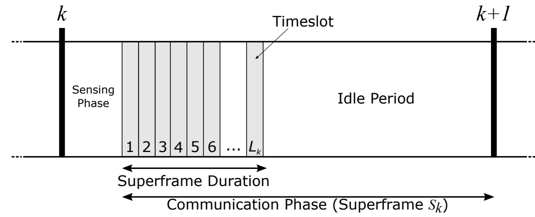

Fig. 2. A superframe and timeslots

if $X - Y \in \mathbb{S}^n_+$ ($X - Y \in \mathbb{S}^n_{++}$) and $X \geq 0$ ($X > 0$) if $X \in \mathbb{S}^n_+$ ($X \in \mathbb{S}^n_{++}$). For a matrix A, we use $\lambda_{\max}(A)$ to denote an eigenvalue of A that has the largest magnitude.

A directed graph is an ordered pair $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is a set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of ordered pairs of nodes. An ordered pair of nodes $(j, i) \in \mathcal{E}$, called a directed edge, means there is a link from node j to node i. For an edge $e \in \mathcal{E}$, denote the the node that e departs from as $v_{\text{out}}(e)$ and the one that e flows into as $v_{\text{in}}(e)$. Let $\mathcal{N}_i^{\text{in}}$ and $\mathcal{N}_i^{\text{out}}$ denote the in- and out-neighbors of node i, respectively, i.e.,

$$\mathcal{N}_i^{\text{in}} = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\},$$
$$\mathcal{N}_i^{\text{out}} = \{j \in \mathcal{V} \mid (i, j) \in \mathcal{E}\}.$$

In a directed graph $\mathcal{G}$, a directed path from node i_1 to node i_l is a sequence of nodes $(i_1, \ldots, i_l)$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \ldots, l-1$. A in-tree with a root $r \in \mathcal{V}$ is a directed subgraph of $\mathcal{G}$ such that every node i, where $i \in \mathcal{V}/\{r\}$, has exactly one directed path from itself to node r. A spanning in-tree is an in-tree that contains all the nodes of $\mathcal{G}$. If a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is an in-tree, we define a partial order $\succeq$ over $\mathcal{E}$. For any $e, \tilde{e} \in \mathcal{E}$, we say $e \succeq \tilde{e}$ if there exist a direct path from $v_{\text{out}}(e)$ to $v_{\text{in}}(\tilde{e})$. It is straightforward to see that $\succeq$ is reflexive, antisymmetric, and transitive, i.e., the following properties for $e, \tilde{e}, \bar{e} \in \mathcal{E}$:

(i). Reflexivity: $e \succeq e$.
(ii). Antisymmetry: if $e \succeq \tilde{e}$ and $\tilde{e} \succeq e$, then $e = \tilde{e}$.
(ii). Transitivity: if $e \succeq \tilde{e}$ and $\tilde{e} \succeq \bar{e}$, then $e \succeq \bar{e}$.

This justifies that $\succeq$ defines a partial order over $\mathcal{E}$.

III. System Description

A set of sensors, denoted by $\mathcal{V}_s = \{1, 2, \ldots, N\}$, are distributed in an area, monitoring N independent discrete-time linear time-invariant (DLTI) processes. The sensors are interconnected via a wireless network and they pass measurements through the network to the remote estimator via a gateway (Fig. 1). We denote the gateway as node 0 and denote the whole node set including the gateway as $\mathcal{V} = \mathcal{V}_s \cup \{0\}$. The estimator generates state estimates based on the received information. We will elaborate the main components of the system in the following part.

A. Process Model

We consider N DLTI processes, the ith of which is described as follows:

$$x_{k+1}^{(i)} = A_i x_k^{(i)} + w_k^{(i)}, \quad k = 0, 1, \ldots, \quad i \in \mathcal{V}_s \tag{1}$$

where $x_k^{(i)} \in \mathbb{R}^n$ is the ith process's state vector at time k, $w_k^{(i)} \in \mathbb{R}^n$ is zero-mean independent and identically distributed (i.i.d.) noises, described by the probability density function (pdf) $\mu_{w^{(i)}}$ with $\mathbb{E}[w_k^{(i)} (w_k^{(i)})^{\mathrm{T}}] = W_i$ ($W_i > 0$). The initial state $x_0^{(i)}$, independent of $w_k^{(i)}, k \in \mathbb{N}$, is described by pdf $\mu_{x_0^{(i)}}$, with mean $\mathbb{E}[x_0^{(i)}]$ and covariance $\Sigma_0^{(i)}$. Without loss of generality, we assume $\mathbb{E}[x_0^{(i)}] = 0$. Notice that the state $x_k^{(i)}$ can be observed directly by sensor i. The system parameters are all known to the sensors as well as the remote estimator. To make the problem of interest nontrivial, we assume the plants are unstable, i.e., $|\lambda_{\max}(A_i)| > 1, \ \forall i$.

B. Communication Model

The sensors communicate to the estimator through intermediate sensors and a gateway which configure underlying communication network. The communication network is described by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{E}$ is the set of all communication links.

We assume that all the sensors are perfectly time-synchronized. Then the time horizon are partitioned into strips of identical sampling time intervals. Each time interval is further divided into two phases: sensing phase and communication phase, where the former one for a sensor to obtain the precess state $x_k^{(i)}, \ i \in \mathcal{V}_s$ and the later one is a time period for message delivery (Fig. 2). The communication phase between time k and $k+1$, which we also call superframe s_k, is divided into $L(k)$ timeslots k_l, where $l \in \mathcal{L}_k = \{1, 2, \ldots, L(k)\}, \ L(k) \leq L_{\max}, \ \forall k$. Superframe structures that can be categorized into this abstract model are practically used in industrial wireless communication protocols [19], [20], built upon the IEEE 802.15.4 MAC layer [21].

The transmission during a superframe over the network is performed in the following way:

1) At each timeslot, only one link $e \in \mathcal{E}$ can be activated to avoid interference across the network, and the order of link activation $\{e_{k_1}, \ldots, e_{k_{L(k)}}\}$ is pre-scheduled.
2) Let $\mathcal{I}_{k_l}^{(i)}$ be a set of the measurements stored by sensor i at timeslot k_l which consists of the one sampled by itself and the ones received from its neighboring nodes. If $e_{k_l} = (i, j)$, the measurement set $\mathcal{I}_{k_l}^{(i)}$ or part of it is transmitted without failure to sensor j with unicast-based communication protocol. Then $\mathcal{I}_{k_l}^{(i)}$ can

be recursively written by

$$\mathcal{I}_{k_l}^{(i)} = \begin{cases} \mathcal{I}_{k-1}^{(i)} \cup \{x_k^{(i)}\}, \ \text{if } l = 1, \\ \mathcal{I}_{k_{l-1}}^{(i)} \cup \tilde{\mathcal{I}}_{k_l}(e_{k_{l-1}}), \\ \qquad\qquad \text{if } l \geq 2 \text{ and } i = v_{\text{in}}(e_{k_{l-1}}), \\ \mathcal{I}_{k_{l-1}}^{(i)}, \qquad \text{if } l \geq 2 \text{ and } i \neq v_{\text{in}}(e_{k_{l-1}}), \end{cases}$$

with $\mathcal{I}_0^{(i)} = \emptyset$, where $\tilde{\mathcal{I}}_{k_l}(e_{k_l})$ is the measurement set transmitted over the link e_{k_l}. Obviously, $\tilde{\mathcal{I}}_{k_l}(e_{k_l}) \subseteq \mathcal{I}_{k_l}^{(i)}$ with $i = v_{\text{out}}(e_{k_l})$.

3) After $L(k)$th timeslot, the gateway transmits all the measurement $\mathcal{I}_k := \mathcal{I}_{k_{L(k)}}^{(0)}$ to the estimator.

We assume that the maximum number of timeslots $L_{\max}$ is sufficiently large for accommodating all communication links in $\mathcal{G}$. We also make the realistic assumption that communication is much faster than sampling of the processes considered. Hence, the time delays due to communication allocation within a superframe can be ignored. The problem we are interested in is to find the optimal sensor network schedule $(e_{k_l}, \tilde{\mathcal{I}}_{k_l})_{l=1}^{L(k)}$ for all superframe $s_k,\ k = 1, 2, \ldots$, which is defined and discussed later.

C. Energy Consumption

The sensors consume certain amount of energy when they receive data from and transmit data to the other sensors. Here we introduce the energy consumption model used in the LEACH protocol [16], [17]. The energy consumption for receiving a packet which contains p-bits information is,

$$E_r(p) = E_{\text{elec}} p, \tag{2}$$

where the energy coefficient E_{elec} is determined by the electronics, coding etc. The energy consumption for sending p-bits information is,

$$E_s(p, d) = E_{\text{elec}} p + E_{\text{amp}} d^2 p, \tag{3}$$

where E_{amp} is the energy coefficient for the amplifier and d is the distance to the receiving sensor or a gateway. When transmitting multiple measurements, each sensor can join them in a single packet in order to reduce the transmission overhead. This technology is called packet aggregation. Assume that a single measurement from any sensor has c bits. Then the bits of information after aggregation is

$$p(q) = c \cdot [1 + (q - 1)(1 - r)], \tag{4}$$

where q is the number of measurements and $r \in [0, 1]$ is the data aggregation rate (DAR) [22]. Notice that it is difficult to aggregate collected data from different sensors perfectly, but some part of the data such as header can be removed when aggregating; in this case we let $0 < r < 1$. Let $q_{k_l} := |\tilde{\mathcal{I}}_{k_l}(e_{k_l})|$ be the number of measurements transmitted to the downstream sensor over link e_{k_l} and d_{k_l} be the distance of e_{k_l} at timeslot k_l. Then the total energy consumption for sensor i in a superframe s_k is

$$E_k^{(i)} = \sum_{l \in \mathcal{T}_{\text{r},k}^{(i)}} E_r(p(q_{k_l})) + \sum_{l \in \mathcal{T}_{\text{s},k}^{(i)}} E_s(p(q_{k_l}), d_{k_l})), \tag{5}$$

where $\mathcal{T}_{\text{r},k}^{(i)} = \{l \in \mathcal{L}_k | v_{\text{in}}(e_{k_l}) = i\}$ and $\mathcal{T}_{\text{s},k}^{(i)} = \{l \in \mathcal{L}_k | v_{\text{out}}(e_{k_l}) = i\}$.

D. Remote Estimation

Let $\tau_i(k) = \max_{t_i}\{t_i : x_{t_i}^{(i)} \in \mathcal{I}_k\}$ be the last superframe that node 0 receives measurement from the ith process. The optimal remote state estimate for the ith process, denoted by $\hat{x}_k^{(i)}$, is computed as

$$\hat{x}_k^{(i)} = \begin{cases} x_k^{(i)}, & \text{if } \tau_i(k) = k; \\ A_i^{k-\tau_i(k)} \hat{x}_{\tau_i(k)}^{(i)}, & \text{if } \tau_i(k) \leq k - 1. \end{cases} \tag{6}$$

The error covariance of $x_k^{(i)}$ is denoted as

$$P_k^{(i)} = \mathbb{E}[(x_k^{(i)} - \hat{x}_k^{(i)})(x_k^{(i)} - \hat{x}_k^{(i)})^\top | \mathcal{I}_k]. \tag{7}$$

It can be recursively updated as follows:

$$P_k^{(i)} = \begin{cases} W_i, & \text{if } \tau_i(k) = k; \\ h_i^{k-\tau_i(k)}(W_i), & \text{if } \tau_i(k) \leq k - 1, \end{cases} \tag{8}$$

where $h_i : \mathbb{S}_n^+ \to \mathbb{S}_n^+$ is an operator defined as $h_i(X) = A_i X A_i^\top + W_i$.

IV. OPTIMAL SENSOR NETWORK SCHEDULE

A. Problem of Interest

Let $\theta_k = (e_{k_l}, \tilde{\mathcal{I}}_{k_l})_{l=1}^{L(k)}$ and $\Theta := (\theta_1, \ldots, \theta_i, \ldots)$ be a feasible schedule. The problem of interest is to find an optimal schedule that minimizes the trace of the error covariance subject to an average sensor energy constraints. That is,

$$\min_{\Theta} \qquad \limsup_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} \sum_{i=1}^{N} \operatorname{tr}(P_k^{(i)}(\Theta)), \tag{9a}$$

$$\text{s.t.} \qquad \limsup_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} E_k^{(i)}(\Theta) \leq \alpha_i,\ i \in \mathcal{V}_s, \tag{9b}$$

where $\alpha_i > 0$ is the average sensor energy constraints for node i.

To solve problem (9), we use a Lagrangian technique similar to [23], [24], to derive the unconstraint problem:

$$\min_{\Theta} \quad \limsup_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} \sum_{i=1}^{N} \operatorname{tr}(P_k^{(i)}(\Theta)) + \beta_i E_k^{(i)}(\Theta) \tag{10}$$

where $\beta_i > 0$ is the Lagrange multiplier. Problem (10) corresponds to jointly optimize a weighted average of estimation error and sensor energy consumption. Minimization of (10) with given values of β_i corresponds to an optimal schedule of (9) with respect to given energy constraints α_i. We consider the problem (10) in the rest of the work.

B. Optimality of Sensor Network Scheduling

In this part, we study properties of sensor network schedules and state estimator. We will give necessary conditions for stable estimation and optimality of sensor network scheduling. Here, stable estimation under a given schedule Θ refer to uniform boundedness of $P_k^{(i)}(\Theta)$ for all processes, i.e., $\sup_{k\in\mathbb{N}} \mathrm{tr}(P_k^{(i)}(\Theta)) < \infty$ for all $i \in \mathcal{V}_s$.

Lemma 1: If graph $\mathcal{G}$ has a spanning in-tree with a unique root node 0, then there exists at least one schedule Θ such that the estimation is stable.

Proof: Let $\mathcal{G}_{\text{st}} = (\mathcal{V}, \mathcal{E}_{\text{st}})$ be a spanning in-tree contained in $\mathcal{G}$ with node 0 being the unique root. Then each node $i \in \mathcal{V}_s$ has an unique directed path going from node i to 0. Next we shall complete the proof by constructing a simple schedule Θ to ensure the remote estimation is stable.

Denote the unique path from node j to 0 by $(j, i_1, \ldots, i_{m_j}, 0)$. Set $\theta_k = (e_{k_l}, \tilde{\mathcal{I}}_{k_l})_{l=1}^{L(k)}$ with

$$e_{k_l} = \begin{cases} (j, i_1), & \text{if } l = 1, \\ (i_{m-1}, i_m), & \text{if } l = 2, \ldots, m_j, \\ (i_{m_j}, 0), & \text{if } l = m_j + 1, \end{cases}$$

and $\tilde{\mathcal{I}}_{k_l} = \{x_k^{(j)}\}$ for all $l \in \mathcal{L}_k$. Then we repeat $\theta_1, \ldots, \theta_N$ every N period, i.e., $\theta_{j+tN} = \theta_j$ for $t \in \mathbb{N}$ and $j \in \{1, \ldots, N\}$. By constructing Θ is this way, it yields

$$\lim_{T\to\infty} \frac{1}{T} \sum_{k=0}^{T-1} \mathrm{tr}(P_k^{(i)}(\Theta)) = \frac{1}{N} \sum_{l=0}^{T-1} \mathrm{tr}(h_i^l(W_i)) < \infty.$$

This completes the proof. ■

To guarantee the existence of a stable estimator, the following assumption is made.

Assumption 2: The graph $\mathcal{G}$ contains a spanning in-tree with a unique root being node 0.

Remark 3: By Lemma 1, as long as $\mathcal{G}$ contains a spanning in-tree, we can find a schedule Θ for any arbitrary small $\alpha_i > 0$ leading to stable estimate. Hence, the optimization problem (10) is always feasible.

If we consider all the communication links within a single superframe jointly and analysis the resulting graph by treating these links as a whole, it is helpful to introduce the notation of joint graph. Let us define the joint graph for a superframe s_k under an optimal sensor network schedule Θ^* in the following way. We denote by $\mathcal{E}_k^*$ the sequence of the communication links in s_k according to time order, i.e., $\mathcal{E}_k^* := (e_{k_1}, \ldots, e_{k_{L(k)}})$. Then we call $\mathcal{G}^*(s_k) = \{\mathcal{V}, \mathcal{E}_k^*\}$ the joint graph of s_k under an optimal schedule Θ^*.

The following property holds for $\mathcal{E}_k^*$ and $\mathcal{G}^*(s_k)$ respectively.

Lemma 4: Assume that the problem (10) has an optimal solution. Then, for all $k \in \mathbb{N}$, $\mathcal{G}^*(s_k)$ is an in-tree with node 0 being the unique root.

Proof: The roadmap of the proof has two steps: *1)* We first show that $\mathcal{G}^*(s_k)$ is a disjoint union of in-trees. *2)* We show that $\mathcal{G}^*(s_k)$ has a unique root which is node 0.

From energy saving point of view, for each θ_k^* in Θ^*, $\mathcal{G}^*(s_k)$ is a disjoint union of in-trees, i.e., any two nodes are connected by at most one path. If not, without loss of generality, assume that there are two pathes, denoted as $e = (i, i_1, \ldots, i_l, j)$ and $e' = (i, j_1, \ldots, j_{l'}, j)$, going from nodes i to j and the lengths of e and e' are $l(e)$ and $l(e')$ respectively with $l(e) \le l(e')$. Node i has q ($q \ge 1$) measurements to be transmitted to node j, among which q_j number of measurements are transmitted through node j_2 and q_i number of measurements through node i_2. Evidently, $q_i + q_j = q$. The number of bits node i transmits to node i_2 is

$$p(q_i) = c(1-r)q_i + cr$$

and the number of bits node i transmits to node j_2 is

$$p(q_j) = c(1-r)q_j + cr.$$

We consider another scheduling decision $\tilde{\theta}_k$, where θ_k^* and $\tilde{\theta}_k$ are the same except for that the q measurements of node i are all transmitted to node j through node i_2. We compare θ_k^* with $\tilde{\theta}_k$ In this case, the number of bits node i transmits to the downstream node i_2 is

$$p(q) = c(1-r)q + cr.$$

Since $p(q_i) + p(q_j) > p(q)$ and $l(e) \le l(e')$, to transmit the p measurements of node i to node j, θ_k^* consumes more energy compared to $\tilde{\theta}_k$. This contradicts the optimality of Θ^*, showing that $\mathcal{G}^*(s_k)$ is a disjoint union of in-trees.

Next we will show that $\mathcal{G}^*(s_k)$ has a unique root which is node 0. Notice that in (6), $\hat{x}_k^{(i)}$ is updated based on the latest information of the ith process that node 0 receives. Therefore, removing the in-trees not having node 0 as the root does not affect the calculation of $\hat{x}_k^{(i)}$, while energy for receiving and sending packets is saved.

The proof is now complete. ■

In virtue of Lemma 4, we have the following lemma.

Lemma 5: Assume that the problem (10) has an optimal solution and denote $\Theta^* := (\theta_1^*, \ldots, \theta_k^*, \ldots)$ and $\theta_k^* = (\tilde{\mathcal{I}}_{k_l}^*, e_{k_l}^*)_{l=1}^{L(k)}$. Then

(i). $\tilde{\mathcal{I}}_{k_l}^*(e_{k_l}^*) = \{x_k^{(i)} : i = v_{\text{out}}(e_{k_j}^*), e_{k_j}^* \succeq e_{k_l}^*\}$ for $e_{k_l}^* \in \mathcal{E}_k^*$;

(ii). $k_i \le k_j$ if $e_{k_i}^* \succeq e_{k_j}^*$ for $e_{k_i}^*, e_{k_j}^* \in \mathcal{E}_k^*$.

Proof: By Lemma 4, $\mathcal{G}^*(s_k)$ is an in-tree with node 0 being the unique root. Also notice that $\hat{x}_k^{(i)}$ in (6) is updated only based on the latest information about the ith process received by node 0. By letting each sensor i in $\mathcal{G}^*(s_k)$ sends $x_k^{(i)}$ following upstream-node-first order, all measurements sampled and sent within s_k can reach node 0 free of delays. Otherwise, if either (i) or (ii) is violated, some measurements received by node 0 will be delayed. In this sense, any Θ violating either (i) or (ii) can never by optimal. ■

By Lemmas 4 and 5, we only need to consider the cases that $\mathcal{G}(s_k)$ are in-trees and the network schedules in each superframe always follow Lemma 5. Thus, the cost defined by (10) can be determined only by the selection of the sensors $\mathcal{I}_k$ and their paths to the estimator, i.e., the topology of in-trees $\mathcal{G}(s_k)$. Therefore, the cost function (10) is redefined by $\Xi := (\xi_1, \ldots, \xi_k, \ldots,)$ instead of Θ where $\xi_k = (\mathcal{I}_k, \mathcal{G}(s_k))$, and the cost function is

$$\min_{\Xi} \quad J(\Xi) := \limsup_{T\to\infty} \frac{1}{T} W(\Xi, T), \tag{11}$$

where

$$W(\Xi, T) = \sum_{k=0}^{T-1}\sum_{i=1}^{N} \mathrm{tr}(P_k^{(i)}(\Xi)) + \beta_i E_k^{(i)}(\xi_k). \tag{12}$$

Note that the energy consumption term in (12) is only determined by current sensor selection and network topology $\xi_k = (\mathcal{I}_k, \mathcal{G}(s_k))$. With this formulation, we can simplify the problem of finding the optimal sensor network schedule by formulating Markov decision Process (MDP). Before that, we present the follwoing lemma, which can restrict state-space of MDP into finite space.

Lemma 6: Let $\tilde{\tau}_i(k) = k - \tau_i(k)$. Assume that the problem (11), (12) has at least an optimal schedule Ξ^*. Then there exists an optimal schedule Ξ^* of which $\tilde{\tau}_i(k)$ is bounded, i.e., there exists $\delta_i := \max_k\{\tilde{\tau}_i(k) : k \in \mathbb{N}\} < \infty$.

Proof: The proof is inspired by that of Theorem 1 in [6]. Main idea is to use the fact that the error covariance term of (12) increases exponentially but it can be reset by sending the latest measurement with some energy consumption. In that sense, any optimal schedule have to ensure that each sensor can only exceeds a certain bound δ_i in "off-duty" period $\tilde{\tau}_i(k)$ for finite times over the entire time horizon; otherwise we can always find a better schedule. Since the cost over finite time steps will vanish while taking average over $T \to \infty$, there is no loss of performance if we directly search for optimal strategy in a state space truncated by the bounds δ_i. ■

C. MDP formulation

The problem (11), (12) can be described as finite-space MDP to derive the optimal schedule with a tuple $(\mathbb{S}, \mathbb{A}, \Pr(\cdot|\cdot,\cdot), R(\cdot,\cdot))$, where

- the state space $\mathbb{S} = \{\tau = [\tau_1, \ldots, \tau_N]^{\mathrm{T}} \in \mathbb{N}^N : \tau_i = 1, \ldots, \delta_i,\ i \in \mathcal{V}_s\}$ represents the time duration between current time and the last instance when the ith sensor transmits the measurement;
- the action space $\mathbb{A} = \{a = (\mathcal{S}, g) : \mathcal{S} \in 2^{\mathcal{V}_s}, g \in \mathcal{G}_{\mathrm{tr}}\}$ where $\mathcal{G}_{\mathrm{tr}}$ is the set of sub-graphs in $\mathcal{G}$ which are in-trees with root node 0;
- the transition probability form state τ to τ' is defined as
$$\Pr(\tau'|\tau, a) = \begin{cases} 1, & \text{if } \tau_i' = 1,\ i \in \mathcal{S}, \\ & \text{ and } \tau_j' = \tau_j + 1,\ j \notin \mathcal{S}, \\ 0, & \text{otherwise;} \end{cases}$$
- the reward function is defined as
$$R(\tau, a) = -\sum_{i=1}^{N} \mathrm{tr}(h_i^{\tau_i - 1}(W_i)) + \beta_i E^{(i)}(a),$$
where $E^{(i)}(a),\ a = (\mathcal{S}, g)$ is the energy cost for ith sensor with selected sensor set $\mathcal{S}$ over the graph g.

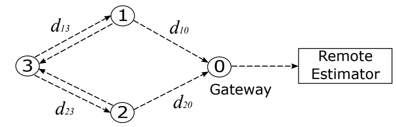

Fig. 3. A small sensor network with $N = 3$

With this set-up, we formulate the MDP problem to find a policy $\{\pi_k\}_{k=1}^{\infty}$ which maximizes the average expected reward

$$g_\pi(\tau_0) = \lim_{T\to\infty} \frac{1}{T} \mathbb{E}_{\tau_0}^{\pi}\left[\sum_{k=0}^{T-1} R(\tau, a)\right], \tag{13}$$

where τ_0 is the initial state.

Lemma 7: For the finite-state MDP (13), there exists a stationary optimal policy π^*.

Proof: It is follows from Theorems 9.1.4 and 9.1.7 in [25]. The proof is omitted due to space limit. ■

Now we are ready to state our main result which shows that the optimal sensor schedule is periodic.

Theorem 8: There exists a periodic policy π^* that is optimal to (13), that is, there exists a sensor network schedule Ξ^* satisfying $\xi_k^* = \xi_{k+d}^*$ for some positive integer d.

Proof: Now, there exists a stationary policy π^*, which is deterministic, since even if there exists more than one optimal actions for some $\tau \in \mathbb{S}$ under π^*, we can take arbitrary one action as an optimal one. Furthermore, as $\mathbb{S}$ is finite, there exists a recurrent state over π^*. Thus, if the system reaches the recurrent state again, the state transition will repeat. In other words, there exists the periodic schedule which is optimal. ■

V. NUMERICAL EXAMPLE

To illustrate our theoretical results in the previous section, we consider a small network case with $N = 3$ and a gateway shown in Fig. 3. To be specific, the system parameters of the three plants are

$$A_1 = \begin{bmatrix} 1.2 & 0 \\ 0 & 1.4 \end{bmatrix},\ A_2 = \begin{bmatrix} 2.0 & 0 \\ 0 & 1.8 \end{bmatrix},\ A_3 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.2 \end{bmatrix}.$$

For communication parameters, we assume that $d_{10} = d_{20} = d_{13} = d_{23} = 1$, $c = 1$ and $r = 0.5$. In cost function, we assume $\beta_1 = 0.1, \beta_2 = 0.2,\ \beta_3 = 0.3$.

The action set consists of every possible subset of sensors selected to transmit accompanied by all possible routes among them as shown in Table I.

Since the convergence of value function is proved, we adopt value iteration algorithm in MDP toolbox [26] to calculate the optimal policy, which will induce a Markov chain with least expected value in cost function. Then according to Theorem 8, given an arbitrary initial state, we can get a periodic network schedule Ξ^*, as shown in Fig. 4.

In the figure, we can see the period length of the optimal schedule is 7. In which only action 3 and 13 (Figure 5) are taken and the network is inactive (action 0) in the rest of time.

Action index	Sensor selection: $\mathcal{S}$	In-tree graph: g
0	ϕ	-
1	1	$1 \to 0$
2	1	$1 \to 3 \to 2 \to 0$
3	2	$2 \to 0$
4	2	$2 \to 3 \to 1 \to 0$
5	3	$3 \to 1 \to 0$
6	3	$3 \to 2 \to 0$
7	1,2	$1 \to 0, 2 \to 0$
8	1,2	$1 \to 3 \to 2 \to 0$
9	1,2	$2 \to 3 \to 1 \to 0$
10	2,3	$3 \to 2 \to 0$
11	2,3	$3 \to 1 \to 0, 2 \to 0$
12	2,3	$2 \to 3 \to 1 \to 0$
13	3,1	$3 \to 1 \to 0$
14	3,1	$3 \to 2 \to 0, 1 \to 0$
15	3,1	$1 \to 3 \to 2 \to 0$
16	1,2,3	$3 \to 1 \to 0, 2 \to 0$
17	1,2,3	$3 \to 2 \to 0, 1 \to 0$
18	1,2,3	$1 \to 3 \to 2 \to 0$
19	1,2,3	$2 \to 3 \to 1 \to 0$

TABLE I

ALL POSSIBLE SENSOR SELECTION AND THEIR ROUTES TO THE GATEWAY

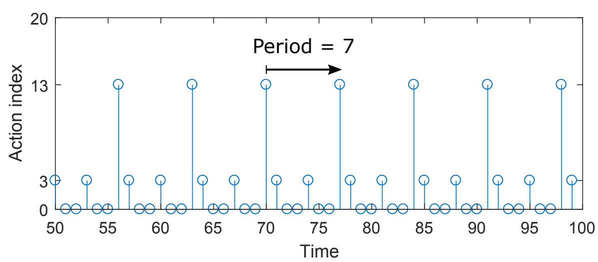

Fig. 4. Optimal periodic schedule

VI. CONCLUSIONS

In this paper, we considered a sensor network scheduling for remote estimation of multiple DLTI systems. We proposed the offline sensor network scheduling under energy consumption constraints by formulating the optimal problem which minimizes the infinite time averaged estimation error covariance. The periodic optimal solution can be found by exploiting necessary conditions for optimality. Possible future works will focus on the cases that communications have channel fading.

REFERENCES

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: a survey," *Computer networks*, vol. 38, no. 4, pp. 393–422, 2002.

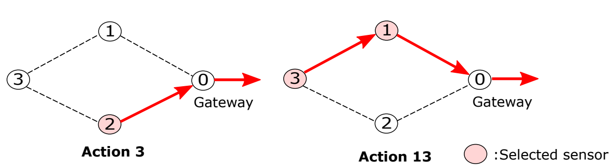

Fig. 5. Network topologies ξ_k in the optimal schedule

[2] N. P. Mahalik, *Sensor Networks and Configuration*. Springer, 2007.

[3] D. Hasenfratz, A. Meier, C. Moser, J.-J. Chen, and L. Thiele, "Analysis, comparison, and optimization of routing protocols for energy harvesting wireless sensor networks," in *Sensor Networks, Ubiquitous, and Trustworthy Computing (SUTC), 2010 IEEE International Conference on*. IEEE, 2010, pp. 19–26.

[4] M. Sha, D. Gunatilaka, C. Wu, and C. Lu, "Empirical study and enhancements of industrial wireless sensor-actuator network protocols," *IEEE Internet of Things Journal*, 2017.

[5] O. Chipara, Z. He, G. Xing, Q. Chen, X. Wang, C. Lu, J. Stankovic, and T. Abdelzaher, "Real-time power-aware routing in sensor networks," in *Quality of Service, 2006. IWQoS 2006. 14th IEEE International Workshop on*. IEEE, 2006, pp. 83–92.

[6] D. Han, J. Wu, H. Zhang, and L. Shi, "Optimal sensor scheduling for multiple linear dynamical systems," *Automatica*, vol. 75, pp. 260–270, 2017.

[7] S. Wu, X. Ren, S. Dey, and L. Shi, "Optimal scheduling of multiple sensors with packet length constraint," *arXiv preprint arXiv:1611.08055*, 2016.

[8] L. Zhao, W. Zhang, J. Hu, A. Abate, and C. J. Tomlin, "On the optimal solutions of the infinite-horizon linear sensor scheduling problem," *IEEE Transactions on Automatic Control*, vol. 59, no. 10, pp. 2825–2830, 2014.

[9] Y. Mo, E. Garone, and B. Sinopoli, "On infinite-horizon sensor scheduling," *Systems & control letters*, vol. 67, pp. 65–70, 2014.

[10] Z. Lin and C. Wang, "Scheduling parallel kalman filters for multiple processes," *Automatica*, vol. 49, no. 1, pp. 9–16, 2013.

[11] J. Wu, Q.-S. Jia, K. H. Johansson, and L. Shi, "Event-based sensor data scheduling: Trade-off between communication rate and estimation quality," *IEEE Transactions on automatic control*, vol. 58, no. 4, pp. 1041–1046, 2013.

[12] Y. Mo, R. Ambrosino, and B. Sinopoli, "Sensor selection strategies for state estimation in energy constrained wireless sensor networks," *Automatica*, vol. 47, no. 7, pp. 1330–1338, 2011.

[13] D. Shi and T. Chen, "Approximate optimal periodic scheduling of multiple sensors with constraints," *Automatica*, vol. 49, no. 4, pp. 993–1000, 2013.

[14] P. Agrawal, A. Ahlén, T. Olofsson, and M. Gidlund, "Characterization of long term channel variations in industrial wireless sensor networks," in *Communications (ICC), 2014 IEEE International Conference on*. IEEE, 2014, pp. 1–6.

[15] J. Araújo, M. Mazo, A. Anta, P. Tabuada, and K. H. Johansson, "System architectures, protocols and algorithms for aperiodic wireless control systems," *IEEE Trans. on Industrial Informatics*, vol. 10, no. 1, pp. 175–184, 2014.

[16] W. R. Heinzelman, A. Chandrakasan, and H. Balakrishnan, "Energy-efficient communication protocol for wireless microsensor networks," in *Proc. of 33rd Hawaii Inter. Conf. on Syst. Sci.* IEEE, 2000, pp. 1–10.

[17] W. B. Heinzelman, A. P. Chandrakasan, and H. Balakrishnan, "An application-specific protocol architecture for wireless microsensor networks," *IEEE Trans. on Wireless Communications*, vol. 1, no. 4, pp. 660–670, 2002.

[18] R. Rajagopalan and P. K. Varshney, "Data aggregation techniques in sensor networks: A survey," 2006.

[19] ZigBee Alliance, "Zigbee specification," 2006.

[20] HART Communication Foundation, "Wirelesshart data sheet," 2007.

[21] IEEE 802.15.4 Standard: Wireless Medium Access Control (MAC) and Pysical Layer (PHY) Specification for Low-Rate Wireless Personal Area Networks (WPANs).

[22] J. Dou, Z. Guo, J. Cao, and G. Zhang, "Data aggregation rate assumption for wireless sensors," in *Proc. of IEEE Inter. Conf. on Radio-Frequency Integration Technology*. IEEE, 2007, pp. 278–281.

[23] A. S. Leong, S. Dey, G. N. Nair, and P. Sharma, "Power allocation for outage minimization in state estimation over fading channels," *IEEE Trans. on Signal Processing*, vol. 59, no. 7, pp. 3382–3397, 2011.

[24] A. S. Leong and S. Dey, "Power allocation for error covariance minimization in kalman filtering over packet dropping links," in *Proc. of IEEE Conf. Dec. Contr.* IEEE, 2012, pp. 3335–3340.

[25] M. L. Puterman, *Markov decision processes: discrete stochastic dynamic programming*. John Wiley & Sons, 2005.

[26] I. Chadès, G. Chapron, M.-J. Cros, F. Garcia, and R. Sabbadin, "Mdptoolbox: a multi-platform toolbox to solve stochastic dynamic programming problems," *Ecography*, vol. 37, no. 9, pp. 916–920, 2014.