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Cooperation Patterns between Fleet Owners for Transport Assignments

Farhad Farokhi, Kuo-Yun Liang, and Karl H. Johansson

***Abstract*— We study cooperation patterns between the heavy-duty vehicle fleet owners to reduce their costs, improve their fuel efficiency, and decrease their emissions. We consider a distributed cooperation pattern in which the fleet owners can communicate directly with each other to form alliances. A centralized cooperation pattern is studied in which the fleet owners pay to subscribe to a third-party service provider that pairs their vehicles for cooperation. The effects of various pricing strategies on the behaviour of fleet owners and their inclusiveness are analyzed. It is shown that the fleet size has an essential role.**

I. Introduction

Heavy-duty vehicles account for a quarter of road transport emissions and about 5% of the total European Union's greenhouse gas emissions [1]. Based on historical data that the carbon emissions from heavy-duty vehicles transport have grown by some 36% between 1990 and 2010, we may extrapolate that a "no policy change" scenario is clearly incompatible with European Union's objective of reducing the greenhouse gas emissions from transport by around 60% of 1990 levels by 2050 [1]. Heavy-duty vehicle platooning is one way to reduce the green house emissions. This is motivated by a study in [2] pointing that we can achieve up to 7.7% reduction in fuel consumption (depending on the distance between the trucks among other factors) at a speed of 70 km/h for two identical trucks. Other studies claim that the reduction can be increased up to 21% [3]. Cooperative driving is also beneficial for fleet owners to reduce their operational costs and to improve their transport efficiency. For instance, consider a scenario in which a fleet owner is contracted to transport a divisible good that takes half a truck between two cities. Further, imagine that another fleet owner is requested to transport the same (or a similar) good of the size of one and half trucks on a close route. In an uncooperative world, the fleet owners need to use three trucks for transporting the goods; however, if they cooperate, this number can be reduced to two. Therefore, both fleet owners can save money and fuel (and reduce their total carbon footprint).

To be able to capitalize on the potential of cooperative driving in reducing the costs and emissions, we need to have a big pool of heavy-duty vehicles that travel on the same (or similar) route and at the same time. It is rarely the case for a single fleet owner to own so many vehicles to satisfy this criterion. One way to solve this problem is to create an "online dating" service for the heavy-duty vehicles. The fleet owners can privately provide their routes and timetables so that the service provider can pair their heavy-duty vehicles for cooperation. For participating in this service, the fleet owners may need to pay a subscription fee. In addition, the fleet owners may need to invest in devices to facilitate cooperation between vehicles (e.g., communication units, automated driving devices, etc.). In this paper, we study the cooperation pattern of the fleet owners induced by these costs as well as the benefits of cooperation.

Firstly, we consider the time-management aspect. In this case, the fleet owners have made the decision to form platoons (to reduce the fuel costs, carbon taxes, etc). However, to do so, each vehicle need to wait until a vehicle that moves in the same direction joins it. By cooperating with other fleet owners, they can reduce their waiting time. We consider both the distributed and the centralized cooperation patterns. In the distributed cooperation pattern, the fleet owners communicate directly with each other to form alliances. In the centralized case, we assume that there exists a third-party service provider that can pair the vehicles if their fleet owners subscribe to it. Subsequently, we consider the fuel-saving aspect of the problem in the centralized cooperation pattern. In this case, the trucks can meet each other on the road and form platoons in an ad-hoc fashion if the fleet owners are cooperating with each other. Therefore, contrary to the earlier case, the vehicles do not wait for their partners. In both time-management and fuel-saving problems, we propose a game-theoretic approach for analysing the decision making of the fleet owners. We find equilibria of the game in each case and study their properties.

There are several studies on technology adoption that have some resemblance to this problem. The general idea, in the economics literature, is to see how people make decisions to adopt a new technology (e.g., smart phones, gaming consoles, genetically modified crops, etc.) and why, sometimes, the decisions seem counter-intuitive (e.g., a better gaming console does not catch up). For instance, a study in [4] uses a static game to analyze a firm's decision to adopt a new technology under uncertain forecasts profitability. It suggests that the firms that have a high initial cost adopt the (potentially cost reducing but uncertain) technology, however, low-cost firms do not take the risk. The model considers the case where the firms are competing in a market characterized by Cournot-Nash quantity-setting behavior. The diffusion of a new technology in a two-firm setup using a dynamic game

F. Farokhi is with the Department of Electrical and Electronic Engineering, University of Melbourne, Parkville, Victoria 3010, Australia. E-mail: ffarokhi@unimelb.edu.au

K.-Y. Liang and K. H. Johansson are with the ACCESS Linnaeus Center, School of Electrical Engineering, KTH Royal Institute of Technology, SE-10044 Stockholm, Sweden. E-mails: {kyliang,kallej}@kth.se

K.-Y. Liang is also with the Research and Developement Department at Scania CV AB, SE-151 87 Södertälje, Sweden E-mail: kuo-yun.liang@scania.com.

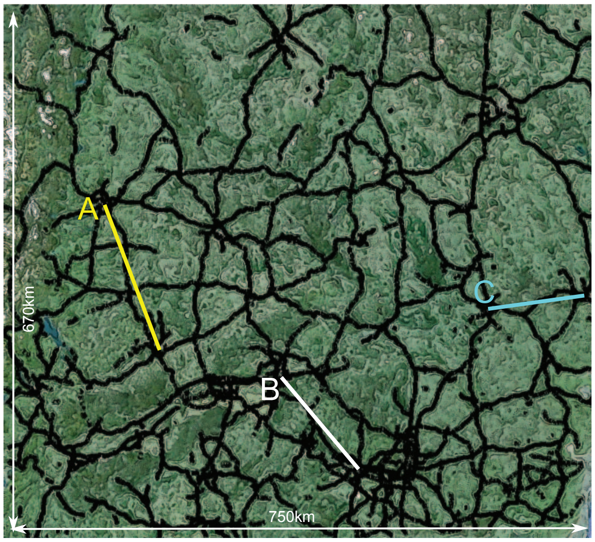

Fig. 1. Three trips where we analyzed trucks traveling the cities. The underlying highway network is based on OpenStreetMap.

setup was considered in [5]. The effect of peer pressure in adopting technologies or buying products was also studied in [6]. Games over networks were studied in [7], where the authors used the concept to understand the influence of the underlying (social) network on adoption of new technologies. Other approaches, such as modelling the technology adoption as contagion propagation over social networks, have also been utilized [8].

The rest of the paper is organized as follows. In Section II, we motivate our work with some real world examples. In Section III, we consider the cooperation pattern from the time-management perspective. We investigate the fuel-saving potentials in Section IV. Finally, we conclude the paper in Section V.

II. Motivation

Before considering cooperation patterns of fleet owners, we first give a real world example to motivate our work. We obtained vehicle probe data from Scania's fleet management system over a region in Europe, depicted in Fig. 1. Over a 24-hour period in May 2013, we obtained data from 7634 trucks; see [9] for a detailed description. The vehicle probe data consisted of vehicle ID, timestamp, latitude, longitude, and heading. We chose six big cities and analyzed how many trips the trucks traveled between two of them, making it a total of three different trips, depicted with A, B, and C in Fig. 1. The distances are 230, 190, and 150 km for trip A, B, and C, respectively. For each truck, we checked if it passed both cities, if so, it counts as a trip. If the truck traveled back and forth, it will be counted as two trips.

For the 24 hours, we got 79, 118, and 58 trips for A, B, and C, respectively. This translates to an average of 2.5–5 trucks per hour. Note that these data are only from one truck manufacturer (Scania) and only from trucks equipped with data communication units, which are a small fraction of the total number of trucks traveling the considered trips. Noting that these trucks do not necessarily belong to the same fleet, there is a need for forming collaborations between the fleets (in centralized or decentralized ways) to capitalize on the platooning opportunities to improve the fuel efficiency across the society. An important key transport corridor is the A15 highway in Netherlands, where the Botlek tunnel has 12 trucks passing by per minute [10].

III. Time Management

We analyse the case where the fleets transport goods on a single road. This abstract model is applicable to the case where the fleets are using parallel highway networks between two major cities. In this section, we study the problem from the time-management perspective. That is, the fleet owners have decided to pair the trucks to form platoons. Therefore, each vehicle needs to wait (or slow down) for another one that uses the same route to form a platoon. Upon cooperating with each other, the fleets can reduce the waiting time of each vehicle, however, this can only be done at a price for forging cooperation.

A. Distributed Cooperation

We consider the case where n fleets are transporting goods over a single road. We assume that fleet i, $1 \leq i \leq n$, dispatch trucks over the road according a Poisson process[1] with rate $\lambda_i \in \mathbb{R}_{>0}$. Hence, the time between dispatching two consecutive trucks is an exponentially distributed random variable with mean $1/\lambda_i$. Note that this is a reasonable assumption due to the Palm-Khintchine theorem, that is, the superposition of many low intensity non-Poisson point processes is close to a Poisson process [11].

Each fleet owner can make a decision to approach another fleet owner for vehicle platooning cooperation. Let $\mathbf{a}_i = (a_{i,j})_{j=1}^n \in \{0,1\}^n$ denote actions of fleet owner i. If $a_{i,j} = 1$ fleet owner i asks fleet owner j for cooperation to increase its platooning opportunities. By definition, we assume that $a_{i,i} = 1$ for all i. An alliance between fleet owners i and j is formed if $a_{i,j} = a_{j,i} = 1$. We call this model distributed cooperation as the fleet owners approach each other individually to forge alliances. Let us denote the set of all the possible actions for fleet i by $\mathcal{A}_i = \{\mathbf{a}_i \in \{0,1\}^n : a_{i,i} = 1\}$.

When forming platoons, the cost of fleet i is

$$U_i(\mathbf{a}_i, \mathbf{a}_{-i}) = \lambda_i \left[\frac{1}{\lambda_i + \sum_{j \neq i} a_{i,j} a_{j,i} \lambda_j} + p_i(\mathbf{a}_i, \mathbf{a}_{-i}) \right],$$

where $1/(\lambda_i + \sum_{j \neq i} a_{i,j} a_{j,i} \lambda_j)$ is the average time that a truck owned by fleet i should wait for another truck, from its own fleet and all the other fleets that are cooperating with it, to arrive for forming a platoon. Here $p_i(\mathbf{a}_i, \mathbf{a}_{-i})$ is the price-per-vehicle that fleet i should pay for cooperating with the other fleets.

Remark 3.1: In the cooperative mode, a truck from fleet i needs to wait until a truck from fleets in $\mathcal{J}_i = \{j : a_{i,j} a_{j,i} =$

[1] A Poisson process $(N(t))_{t \in \mathbb{R}_{>0}}$ with rate $\lambda \in \mathbb{R}_{>0}$ is a stochastic process with $N(0) = 0$ such that $\mathbb{P}\{N(t_2) - N(t_1) = k\} = \exp(-\lambda(t_2 - t_1))(\lambda(t_2 - t_1))^k / k!$ $\forall k \in \mathbb{N}_0$ and $t_2 \geq t_1 \geq 0$.

$1\}$ arrives. Assuming that the Poisson processes are independent (e.g., whenever different fleets are not servicing the same sets of companies and individuals), the Poisson process for the arrival of all these vehicles is $\sum_{j\in\mathcal{J}_i} N_j(t) = N_i(t) + \sum_{j\neq i} a_{i,j}a_{j,i}N_j(t)$, where $(N_j(t))_{t\in\mathbb{R}_{\geq 0}}$ is the Poisson process modeling the arrival of vehicles for fleet j. Easily, we may see that this sum is indeed a Poisson process with the rate $\sum_{j\in\mathcal{J}_i}\lambda_j = \lambda_i + \sum_{j\neq i} a_{i,j}a_{j,i}\lambda_j$ and, hence, the average time that a truck from fleet owner i needs to wait for another truck to form a platoon is equal to $1/(\lambda_i + \sum_{j\neq i} a_{i,j}a_{j,i}\lambda_j)$.

With these definitions in hand, we are ready to introduce the equilibrium of the game in this case.

Definition 3.1: (Equilibrium in Distributed Cooperation): A tuple of actions $(\mathbf{a}_i^*)_{i=1}^n \in \prod_{i=1}^n \mathcal{A}_i$ constitutes an equilibrium in distributed cooperation for time-management if

$$\mathbf{a}_i^* \in \arg\min_{\mathbf{a}_i\in\mathcal{A}_i} U_i(\mathbf{a}_i, \mathbf{a}_{-i}^*).$$

Moreover, this equilibrium is symmetric if $a_{i,j} = a_{j,i}$ for all $1 \leq i \neq j \leq n$.

Remark 3.2: We can reduce each equilibrium to a symmetric equilibrium with the same cost (for each player) if the price-per-vehicle follows $p_i(\mathbf{a}_i, \mathbf{a}_{-i}) = f_i((a_{i,j}a_{j,i})_{j\neq i})$ for some mapping $f_i : \{0,1\}^{n-1} \to \mathbb{R}$. This is the case since, with this assumption on the price-per-vehicle, all the terms in the cost function $U_i(\mathbf{a}_i, \mathbf{a}_{-i})$ are a function of $(a_{i,j}a_{j,i})_{j\neq i}$ which is symmetric.

Following this observation, in the remainder of this section, we only search for a symmetric equilibria.

Let us consider two specific pricing schemes and study their properties.

1) Constant Price for Cooperation: In this case, the price-per-vehicle that fleet i needs to pay for forging alliances with other fleets is equal to

$$p_i(\mathbf{a}_i, \mathbf{a}_{-i}) = \sum_{j\neq i} a_{i,j}a_{j,i}c_1,$$

where $c_1 \in \mathbb{R}_{\geq 0}$ is a given constant. This case is, for instance, motivated by a scenario in which the vehicles are already equipped with modules to form platoon and the fleet, at a higher level, should pay each other to form a alliance (e.g., to create a communication infrastructure between their headquarters).

Theorem 3.1: Define

$$\lambda_{\max}^1 = \min\{\lambda_i | 1/\lambda_i - 1/(\lambda_i+\lambda_j) < c_1, \forall j \neq i\},$$
$$\lambda_{\min}^1 = \max\{\lambda_i | 1/\lambda_j - 1/(\lambda_j+\lambda_i) < c_1, \forall j \neq i\}.$$

There exists a symmetric equilibrium in which $a_{i,j} = 0$ for all $j \neq i$ and i such that $\lambda_i \in (0, \lambda_{\min}^1] \cup [\lambda_{\max}^1, +\infty)$.

Proof: The proof follows from that for $\lambda_i > \lambda_{\max}^1$, the largest decrease in the term $1/(\lambda_i + \sum_{j\neq i} a_{i,j}a_{j,i}\lambda_j)$ (which is achieved upon admitting the first external fleet) is less than the cost of acquiring that alliance c_1 for that fleet. Hence, this fleet will never collaborate with any other fleet. Similarly, we can see that $\lambda_i < \lambda_{\min}^1$, no fleet can benefit enough to justify spending c_1. Note that we can transform this equilibrium to a symmetric one using the idea of Remark 3.2. ■

Algorithm 1 Learning algorithm for distributed cooperation.

Initialize $\mathbf{a}_k = e_k$ for all k
for iter $= 1, 2, \ldots$ **do**
  Pick $i \neq j$ at random
  Set $\mathbf{a}_k^{\text{new}} = \mathbf{a}_k$ for $k = i, j$
  if $a_{i,j}a_{j,i} = 1$ **then**
    Set $a_{i,j}^{\text{new}} = a_{j,i}^{\text{new}} = 0$
    if $U_i(\mathbf{a}_i^{\text{new}}, \mathbf{a}_{-i}) > U_i(\mathbf{a}_i, \mathbf{a}_{-i}) \vee U_i(\mathbf{a}_j^{\text{new}}, \mathbf{a}_{-j}) > U_i(\mathbf{a}_j, \mathbf{a}_{-j})$ **then**
      Set $\mathbf{a}_k = \mathbf{a}_k^{\text{new}}$ for $k = i, j$
    end if
  else
    Set $a_{i,j}^{\text{new}} = a_{j,i}^{\text{new}} = 1$
    if $U_i(\mathbf{a}_i^{\text{new}}, \mathbf{a}_{-i}) > U_i(\mathbf{a}_i, \mathbf{a}_{-i}) \wedge U_i(\mathbf{a}_j^{\text{new}}, \mathbf{a}_{-j}) > U_i(\mathbf{a}_j, \mathbf{a}_{-j})$ **then**
      Set $\mathbf{a}_k = \mathbf{a}_k^{\text{new}}$ for $k = i, j$
    end if
  end if
end for

Remark 3.3: Theorem 3.1 shows that, based on the price coefficient $c_1 > 0$, large fleets may not participate in the forming alliances with the other fleets (since they do not gain sufficiently). Moreover, small fleets can also be left alone as they do not contribute much to the welfare of those with which they potentially cooperate.

Remark 3.4: If we use the result of Theorem 3.1, we can narrow down the space of actions over which we search for an equilibrium. Specifically, the computational complexity of finding a symmetric equilibrium using brute-force search reduces to $\mathcal{O}(n^2 + 2^{k^2})$, with $k = \#\{i|\lambda_i \in (\lambda_{\min}^1, \lambda_{\max}^1)\}$, from $\mathcal{O}(2^{n^2})$. This is a significant improvement if $k \ll n$. Instead of using brute-force search, we may use Algorithm 1. Note, however, that its convergence properties are unknown.

Example 3.1: Let us consider $n = 5$ fleet owners with vehicle arrival rates $\lambda_1 = 0.40$, $\lambda_2 = 0.01$, $\lambda_3 = 0.2$, $\lambda_3 = 0.5$, and $\lambda_5 = 0.1$. Figure 2 illustrates the cooperation graph for distributed cooperation with constant cost $c_1 = 1.0$ (left) and $c_1 = 0.5$ (right) at the equilibrium. The radius of each circle (representing the corresponding fleet) is proportional to the rate of vehicle dispatch for that fleet. For $c_1 = 1.0$, the smallest fleet is left alone, as other fleet owners need to pay for cooperating with it while it cannot contribute much to their welfare. In this case, the large fleets tend not to cooperate with anyone as the price that they need to pay is not worth it (i.e., others cannot contribute to their waiting time much in comparison to the price that they need to pay). Upon reducing the price ($c_1 = 0.5$), larger fleets get involved; however, very small fleet owners are still left alone.

2) Proportional Price for Cooperation: As we saw, small fleets (that need the cooperation the most) are left alone since they are expensive allies without much to offer. This can be fixed by asking smaller fleets to pay more for cooperation (since they are the ones who actually benefit the most from forging an alliance). In such case, the price that fleet i needs

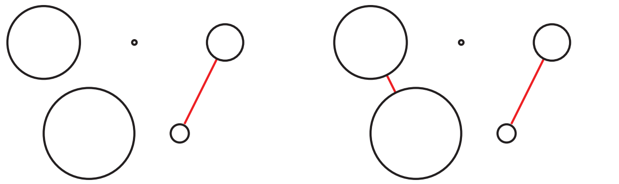

Fig. 2. The cooperation graph for distributed cooperation with constant cost $c_1 = 1.0$ (left) and $c_1 = 0.5$ (right) at the equilibrium for distributed cooperation. The radius of each node (which is representing a fleet) is proportional to the rate of vehicle dispatch λ_i for that fleet. Moreover, there is an edge between nodes $1 \leq i \neq j \leq n$ if $a_{i,j}a_{j,i} = 1$ at the depicted equilibrium.

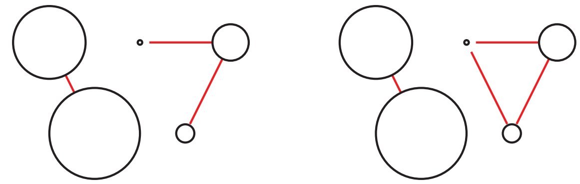

Fig. 3. The cooperation graph for distributed cooperation with proportional cost and $c_2 = 1.0$ (left) and $c_2 = 0.5$ (right) at the Nash equilibrium. The radius of each node (which is representing a fleet) is proportional to the rate of vehicle dispatch λ_i for that fleet. Moreover, there is an edge between nodes $1 \leq i \neq j \leq n$ if $a_{i,j}a_{j,i} = 1$ at the depicted equilibrium.

to pay is equal to

$$p_i(\mathbf{a}_i, \mathbf{a}_{-i}) = \sum_{j \neq i} a_{i,j} a_{j,i} \psi(\lambda_i, \lambda_j) c_2,$$

where $\psi : \mathbb{R}_{>0} \times \mathbb{R}_{>0} \to [0,1]$ is a mapping that determines the share of the cost that each fleet owners needs to pay and obeys the property that $\psi(\lambda_i, \lambda_j) + \psi(\lambda_j, \lambda_i) = 1$ for all $1 \leq i \neq j \leq n$. For instance, we can select

$$\psi(\lambda_i, \lambda_j) = \frac{\lambda_j}{\lambda_i + \lambda_j}, \ \forall \lambda_i, \lambda_j \in \mathbb{R}_{>0},$$

which denote the proportional cost. Further, let $c_2 \in \mathbb{R}_{\geq 0}$ be a given constant.

Theorem 3.2: Define

$$\begin{aligned} \lambda^2_{\text{max}} &= \min\{\lambda_i | 1/\lambda_i - 1/(\lambda_i + \lambda_j) \\ &\qquad\qquad < c_2 \lambda_j/(\lambda_i + \lambda_j), \forall j \neq i\}, \\ \lambda^2_{\text{min}} &= \max\{\lambda_i | 1/\lambda_j - 1/(\lambda_j + \lambda_i) \\ &\qquad\qquad < c_2 \lambda_i/(\lambda_i + \lambda_j), \forall j \neq i\}. \end{aligned}$$

There exists a symmetric equilibrium in which $\mathbf{a}_{i,j} = 0$ for all $j \neq i$ and i such that $\lambda_i \in (0, \lambda^2_{\text{min}}] \cup [\lambda^2_{\text{max}}, +\infty)$.

Proof: The proof follows from the same line of reasoning as in Theorem 3.1. ∎

Now, we show that this actually results in improved cooperation for both small and large fleets. The following result follows immediately from Theorems 3.1 and 3.2.

Proposition 3.1: If $c_1 = c_2$, then $\lambda^2_{\text{max}} \geq \lambda^1_{\text{max}}$ and $\lambda^2_{\text{min}} \leq \lambda^1_{\text{min}}$.

Therefore, the number of excluded fleets reduces, which improves the chance of cooperation for all.

Example 3.2: Let us consider the same setup as in Example 3.1. Figure 3 shows the cooperation graph for distributed cooperation with proportional cost and $c_2 = 1.0$ (left) and $c_2 = 0.5$ (right) at the equilibrium. In this case, very small fleets cooperate with medium size fleets.

B. Centralized Cooperation

Here, we consider the case where a fleet owner can buy a "fleet management product" (e.g., from a truck manufacturer) and, using this product, cooperate with the pool of all the fleets that have purchased this product. Hence, each fleet owner's decision is $a_i \in \{0,1\}$ with $a_i = 1$ denoting the case where the fleet owner buys the product. In the centralized setup, the cost of fleet owner i is equal

$$U_i(a_i, a_{-i}) = \lambda_i \left[\frac{1}{\lambda_i + \sum_{j \neq i} a_i a_j \lambda_j} + p_i(a_i, a_{-i}) \right],$$

where, similarly, $1/(\lambda_i + \sum_{j \neq i} a_i a_j \lambda_j)$ denotes the average waiting time for the trucks of fleet owner i (to find a partner for platooning) and $p_i(a_i, a_{-i})$ is the price-per-vehicle that it should pay for buying the fleet management product.

Definition 3.2: (Equilibrium in Centralized Cooperation): A tuple of actions $(a_i^*)_{i=1}^n \in \{0,1\}^n$ constitutes an equilibrium in centralized cooperation for time management if

$$a_i^* \in \arg\min_{a_i \in \{0,1\}} U_i(a_i, a_{-i}^*).$$

Unfortunately, the centralized cooperation games possess a trivial equilibrium, which does not results in any cooperation. The following theorem captures this equilibrium.

Theorem 3.3: Assume that the price function satisfies $p(1, a_{-i}) \geq p(0, a_{-i})$ for all $a_{-i} \in \{0,1\}^{n-1}$. Then, $(a_i^*)_{i=1}^n$ with $a_i^* = 0$, $\forall i$, constitutes an equilibrium in centralized cooperation.

Proof: The proof follows from

$$\begin{aligned} U_i(0, a_{-i}^*) &= \frac{1}{\lambda_i} + p(0, a_{-i}^*) \\ &= \frac{1}{\lambda_i + \sum_{j \neq i} a_j^* \lambda_j} + p(0, a_{-i}^*) \quad \text{by } a_j^* = 0, \forall j \\ &\leq \frac{1}{\lambda_i + \sum_{j \neq i} a_j^* \lambda_j} + p(1, a_{-i}^*) \\ &= U_i(1, a_{-i}^*). \end{aligned}$$

Therefore, $a_i^* = \{0\} \in \arg\min_{a_i \in \{0,1\}} U_i(a_i, a_{-i}^*)$. ∎

Remark 3.5: This trivial Nash equilibrium is a pattern of behaviour observed in many technology adoption games (e.g., gamers might not switch from console A to console B even if it is much better, because their friends are playing with console A). One way of getting out of the equilibrium is to perturb the player's actions, i.e., the fleet management organization can distribute the product as a gift to one fleet owner, preferably a large fleet, in order to entice the others to join the program.

Let us consider two specific schemes and construct an equilibrium for each case.

1) Constant Price for Cooperation: In this case, the price-per-vehicle offered for the fleet management product

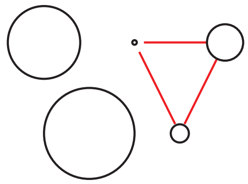

Fig. 4. The cooperation graph associated with a nontrivial Nash equilibrium for constant cost $c_3 = 1.5$. The radius of each node (which is representing a fleet) is proportional to the rate of vehicle dispatch λ_i for that fleet. Moreover, there is an edge between nodes $1 \le i \neq j \le n$ if $a_i a_j = 1$ at the depicted equilibrium.

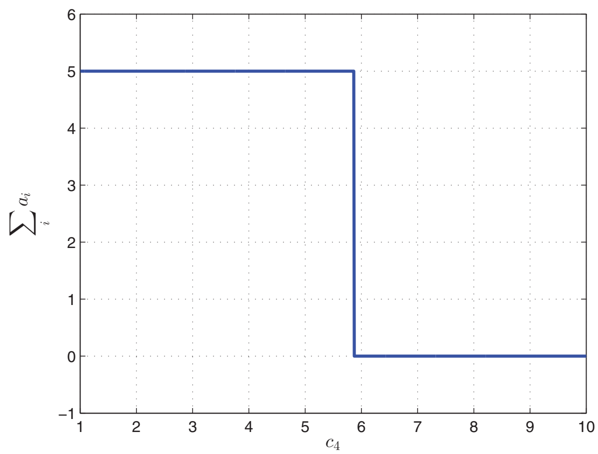

Fig. 5. The number of the fleet owners that buy the product versus the development cost.

is constant and, as a result,

$$p_i(a_i, a_{-i}) = c_3 a_i,$$

where $c_3 > 0$ denotes the cost of acquiring the technology for each vehicle.

Example 3.3: For the same setup as in the previous examples, Figure 4 illustrates the cooperation graph associated with a nontrivial Nash equilibrium for constant cost $c_3 = 1.5$. In this case, small and medium size fleets join the program while larger fleets avoid it.

2) Divisive Price for Cooperation: Here, we assume that the cost of the product is its development cost divided by the number of fleets who wish to buy the product. Therefore,

$$p_i(a_i, a_{-i}) = \frac{c_4 a_i}{\sum_j a_j}$$

where $c_4 > 0$ is the development cost. In this formulation, we adopt the convention that "$\frac{0}{0} = 0$".

Example 3.4: Figure 5 shows the number of the fleet owners that buy the product versus the development cost, for the same setup as before. The fleet owners all buy the product until the price gets too high and, only then, suddenly, all of them stop buying the product.

IV. Fuel Management

In this section, we assume that the vehicles do not wait for forming platooning, but only join other vehicles to create a platoon if they are in their vicinity. Let us generalize the centralized setup so as the costs of the fleets reflect their fuel consumption. The trucks owned by fleet i can cooperate with all the fleets from the set $\mathcal{J}_i(a_i, a_{-i}) = \{j | a_i a_j = 1\} \cup \{i\}$. Therefore, the total rate of dispatch for all the cooperative fleets over the road is $\Lambda_i(a_i, a_{-i}) = \sum_{j \in \mathcal{J}_i} \lambda_j$. We assume that the probability of platooning is proportional to this rate because the more cooperative trucks we have on the road, the higher the probability of forming a platoon with them is. Hence, we assume that mapping $r : \mathbb{R}_{\ge 0} \to [0, 1]$ is given, such that $r(\Lambda_i(a_i, a_{-i}))$ denotes the probability of platooning for each truck owned by fleet i; see Remark 4.1 below for details. Let f denote the fuel consumed by a (single) truck when traveling along the road and η denote the improved fuel efficiency caused by platooning. The cost of fleet i is

$$\begin{aligned} U_i(a_i, a_{-i}) &= \lambda_i [f(1 - r(\Lambda_i(a_i, a_{-i}))) \\ &\quad + (1 - \eta) f r(\Lambda_i(a_i, a_{-i}))) \\ &\quad + p_i(a_i, a_{-i})] \\ &= \lambda_i [f(1 - \eta r(\Lambda_i(a_i, a_{-i}))) + p_i(a_i, a_{-i})], \end{aligned}$$

where $f(1 - r(\Lambda_i(a_i, a_{-i}))) + (1 - \eta) f r(\Lambda_i(a_i, a_{-i})))$ is the expected fuel consumption by each vehicle and $p_i(a_i, a_{-i})$ is the price-per-vehicle for acquiring the fleet management product.

Theorem 4.1: Assume that the price function satisfies $p(1, a_{-i}) \ge p(0, a_{-i})$ for all $a_{-i} \in \{0, 1\}^{n-1}$. Then, $(a_i^*)_{i=1}^n$ with $a_i^* = 0$, $1 \le i \le n$, constitutes an equilibrium in centralized cooperation.

Proof: The proof follows from the same line of reasoning as the proof of Theorem 3.3. ■

For small rates $\Lambda_i(a_i, a_{-i})$, we can linearize the probability of platooning around zero to use the approximate model

$$r(\Lambda_i(a_i, a_{-i})) = \alpha \Lambda_i(a_i, a_{-i}), \tag{1}$$

where $\alpha > 0$ is a constant depending on various parameters, such as the velocity of trucks and their communication range.

Remark 4.1: (Probability of Forming a Platoon): To justify the assumption that the probability of forming platoons is related to the rate of dispatch, we calculate this probability in a simple, yet meaningful, setup. Consider the case in which two trucks can form a platoon if they are in a distance $d > 0$ from each other (e.g., dictated by the range of their communication devices). Moreover, assume that the trucks travel with the velocity $v > 0$. Therefore, two trucks must enter the road within d/v units of time to be able to form a platoon. Now, considering that we assumed that the vehicles which cooperate with fleet i form a Poisson process with rate $\lambda_i + \sum_{j \neq i} a_i a_j \lambda_j$, this probability is equal to

$$\begin{aligned} r(\Lambda_i(a_i, a_{-i})) &= 1 - \exp(-d(\lambda_i + \sum_{j \neq i} a_i a_j \lambda_j)/v) \\ &= 1 - \exp(-d\Lambda_i(a_i, a_{-i})/v). \end{aligned}$$

Following simple algebraic manipulations, we can see that $r(\Lambda_i(a_i, a_{-i}))$ is a decreasing function of $\Lambda_i(a_i, a_{-i})$. Further, if we linearise this probability, we get the linear model in (1) with $\alpha = d/v$.

Let us consider the case where the price-per-vehicle for purchasing the fleet management product is constant, that is

$$p_i(a_i, a_{-i}) = c_5 a_i,$$

where $c_5 > 0$ is a constant. In this case, the cost of fleet i becomes

$$\begin{aligned} U_i(a_i, a_{-i}) &= \lambda_i[f(1 - \eta\alpha\Lambda_i(a_i, a_{-i})) + c_5 a_i] \\ &= \lambda_i\left[f\left(1 - \eta\alpha\lambda_i - \eta\alpha a_i \sum_{j\neq i} a_j\lambda_j\right) + c_5 a_i\right]. \end{aligned} \tag{2}$$

Definition 4.1 (Potential Game): $\Phi : \{0,1\}^n \to \mathbb{R}$ is a potential function for the game if

$$\begin{aligned} &\Phi(a_i, a_{-i}) - \Phi(a'_i, a_{-i}) \\ &\qquad = U_i(a_i, a_{-i}) - U_i(a'_i, a_{-i}), \forall a_i, a'_i \in \{0,1\}. \end{aligned}$$

We say that the game is a potential game if it admits a potential function.

Interestingly, we can prove that the introduced game is a potential game.

Theorem 4.2: Let $\Phi : \{0,1\}^n \to \mathbb{R}$ be a mapping such that

$$\Phi(a) = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j\neq i}\eta\alpha a_i a_j \lambda_i \lambda_j + \sum_{i=1}^{n} c_5 a_i.$$

Then, Φ is a potential function for the introduced fuel management game with costs (2).

Proof: Notice that

$$\begin{aligned} &\Phi(a_i, a_{-i}) - \Phi(a'_i, a_{-i}) \\ &\qquad = -\eta\alpha\lambda_i(a_i - a'_i)\sum_{j\neq i} a_j\lambda_j + c_5(a_i - a'_i) \\ &\qquad = U_i(a_i, a_{-i}) - U_i(a'_i, a_{-i}). \end{aligned}$$

This concludes the proof. ∎

Following this observation, we can use joint strategy fictitious play, as specified in Algorithm 2, to find an equilibrium of the game.

Theorem 4.3: Let the actions of the fleet be generated by the joint strategy fictitious play in Algorithm 2. Then, these actions converge almost surely to an equilibrium of the fuel management game with costs in (2).

Proof: The proof follows from combining Theorem 4.2 with the results of [12]. ∎

Algorithm 2 The joint strategy fictitious play for learning a Nash equilibrium of the fuel management game.

Input: $\beta, \delta \in (0,1)$
Output: An equilibrium a^*
for $i = 1, \ldots, n$ **do**
  Initialize $\hat{U}_i(\xi; t-1) = 0$ for $\xi = 0, 1$
  Initialize $a_i[0] = 0$
end for
for $t = 1, 2, \ldots$ **do**
  for $i = 1, \ldots, n$ **do**
    Calculate $a'_i \in \arg\max_{\xi\in\{0,1\}} \hat{U}_i(\xi; t-1)$
    if $U_i(a'_i, a_{-i}[t-1]) \leq U_i(a_i[t-1], a_{-i}[t-1])$ **then**
      $a_i[t] \leftarrow a_i[t-1]$
    else
      With probability $1-\beta$, $a_i[t] \leftarrow a_i[t-1]$, otherwise $a_i[t] \leftarrow a'_i$
    end if
    Update $\hat{U}_i(\xi; t) = (1-\delta)\hat{U}_i(\xi; t-1) + \delta U_i(\xi, a_{-i}[t])$ for $\xi = 0, 1$
  end for
end for

V. Conclusions

Cooperation patterns between fleet owners were analyzed in this paper. The problem was studied from time-management and fuel-efficiency perspectives. We considered the cases where the fleet owners (*i*) can communicate directly with each other to form alliances or (*ii*) pay to subscribe to a third-party service provider that pairs their vehicles for cooperation. The effects of various pricing strategies on the behaviour of fleet owners and their inclusiveness were quantified. Further, we illustrated these results on numerical examples.

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