Network Scheduling and Control Co-Design for Multi-Loop MPC

Kun Liu, Aoyun Ma, Yuanqing Xia, Zhongqi Sun, and Karl Henrik Johansson

***Abstract*—This paper is concerned with model predictive control (MPC) for multi-loop networked control systems over shared bandwidth-limited communication networks. The plants are described by linear discrete-time systems with additive disturbances and input constraints. At each time instant, only a limited number of loops can communicate over the networks, and the scheduling sequence is determined by a network manager. Algorithms are proposed for co-designing the MPC and the scheduling protocol to stabilize all loops with conflict-free transmissions. We give conditions to ensure recursive feasibility of the optimization problems and the stability of all loops. Finally, two numerical examples show that the algorithms work well.**

***Index Terms*—Bandwidth-limited communication networks, input constraints, multi-loop networked control systems, model predictive control, scheduling sequence.**

I. Introduction

In networked control systems (NCS), the information is transmitted through networks [1]–[3]. NCS is usually easy to install and maintain. Consequently, considerable attention has been attracted to the research of NCS in the last two decades. Most of the works mainly consider single-loop NCS [4]–[8]. In fact, multi-loop systems have already been applied in a wide range of areas, such as networked autonomous air vehicles [9] and process industry [10]. In multi-loop NCS, sensors, actuators, and controllers of multiple control loops communicate over shared networks.

In some multi-loop NCS, the shared networks are bandwidth-limited, and only a limited number of loops can communicate over the networks at the same time. In order to stabilize all plants, the controllers and the scheduling protocols need to be designed simultaneously. The authors of [11] use a parameter-dependent Lyapunov function method combined with average-dwell-time technique to achieve simultaneous stability conditions. A scheduling-and-feedback-control co-design procedure is proposed for the simultaneous stabilization. In [12], a novel stochastic scheduling scheme is presented for NCS with shared communication network through dynamic priority assignment. The scheduler decides which transmission requests have the priority by so called p-powered prioritization. The probability to allocate the communication network to a loop increases with a growing p-powered norm of the state error. In [13], the authors study event-triggered data scheduling for stochastic multi-loop NCS communicating over a shared network with communication uncertainties. The scheduler allocates the communication resource to the loop with the biggest state error.

The co-design of model predictive control (MPC) with a delay compensation scheme and scheduling is proposed in [14] for a set of NCS. In [15], the authors present an algorithm for controlling and scheduling multiple linear time-invariant plants on a shared bandwidth-limited communication network using adaptive sampling intervals. The stability and conflict-free transmissions are guaranteed by joint design of a self-triggering rule and model predictive controller. Note that constraints are not considered in [11]–[15], and it is necessary to consider constraints in most actual systems. For instance, actuators are naturally limited, and states are restricted by safety considerations [16], [17]. For multi-loop NCS with input constraints, a scheduling algorithm for MPC is proposed in [18], where the communication scheduling is solved offline at the initial time instant, and it cannot be changed with the actual system states.

In this paper, we propose algorithms for the joint design of MPC and scheduling protocols to stabilize multi-loop NCS with input constraints over shared bandwidth-limited communication networks, and the communication scheduling is determined online at each time instant by weighted errors between actual system states and optimal ones. The main contributions are as follows:

1) We propose an algorithm to guarantee that all loops can be controlled without conflicting transmissions.
2) We provide sufficient conditions to ensure recursive feasibility of the optimization problems and stability of all closed loops.
3) We extend to the case that more than one loop can communicate over the networks at each time instant.

The rest of this paper is organized as follows. Section II introduces the problem. Section III gives the algorithms for co-designing MPC and scheduling, and provides proofs of feasibility and stability. Two numerical examples are presented in Section IV to illustrate the effectiveness of the proposed algorithms. Section V concludes this paper.

Notations: Let $\mathbb{R}$, $\mathbb{N}$, and $\mathbb{R}^n$ denote the set of real numbers, the set of positive integers, and the n-dimensional real space, respectively. $\bar{\lambda}(\cdot)$ and $\underline{\lambda}(\cdot)$ denote the maximum and minimum eigenvalues of a matrix, respectively. Given a symmetric matrix P, $P > 0$ $(P \geq 0)$ means that the matrix P is positive definite (semi-positive definite). For a given column vector x, $\|x\|$ is the Euclidean norm, i.e., $\|x\| \triangleq \sqrt{x^T x}$. P-weighted vector norm is denoted by $\|x\|_P \triangleq \sqrt{x^T P x}$, where $P > 0$. For a matrix A, $\|A\|$ denotes its two-norm, i.e., $\|A\| \triangleq \sup\{\|Ax\| : x \in \mathbb{R}^n \text{ with } \|x\| = 1\}$. $\lfloor \cdot \rfloor$ denotes the largest integer less than or equal to the scalar "$\cdot$," and $\mathrm{mod}(x, y)$ represents the unique nonnegative remainder after the division of x by y. Given two

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sets $Y \subseteq X \subset \mathbb{N}$, the difference between the two sets is defined as $X \setminus Y \triangleq \{x | x \in X, x \notin Y\}$.

II. Problem Formulation

Consider the multi-loop control system in Fig. 1. A controller communicates with sensors and actuators over shared bandwidth-limited communication networks, and the networks are identical. A network manager allocates the communication resources of the networks. In this system, only a limited number of loops can communicate over the networks at each time instant. Our purpose is to stabilize the plants by scheduling the communication sequences and calculating appropriate control inputs.

Plant $\mathcal{P}_l, l \in \mathcal{L} \triangleq \{1, 2, \ldots, L\}$, is described as

$$x_l(t+1) = A_l x_l(t) + B_l u_l(t) + w_l(t),\ t \geq 0, \quad (1)$$

where $x_l(t) \in \mathbb{R}^{n_l}$ is the system state, $u_l(t) \in \mathcal{U}_l \subset \mathbb{R}^{m_l}$ is the control input, and $w_l(t) \in \mathcal{W}_l \subset \mathbb{R}^{n_l}$ is the disturbance. $\mathcal{U}_l$ and $\mathcal{W}_l$ are compact sets containing the origin. The disturbance is bounded by

$$\rho_l \triangleq \sup\{\|w_l(t)\| : w_l(t) \in \mathcal{W}_l\}. \quad (2)$$

Assumption 1: Suppose that system (1) with $w_l(t) = 0$ is stabilizable. Given two matrices $Q_l > 0$ and $R_l > 0$, there exist a state feedback gain K_l, a constant $\varepsilon_l > 0$, and a matrix $P_l > 0$ such that: 1) $\Omega_{P_l}(\varepsilon_l) \triangleq \{x(t) : \|x(t)\|_{P_l} \leq \varepsilon_l\}$ is an invariant set for the system $x_l(t+1) = \Phi_l x_l(t)$, where $\Phi_l = A_l + B_l K_l$; 2) $P_l - Q_l - K_l^T R_l K_l - \Phi_l^T P_l \Phi_l \geq 0$ and $K_l x_l(t) \in \mathcal{U}_l, \forall x_l(t) \in \Omega_{P_l}(\varepsilon_l)$.

For system (1), define the sequence $\{t_k^l,\ k \in \mathbb{N}\}$ as the time instants when the following optimization problem is solved for plant $\mathcal{P}_l$. The vector $u_l(t|t_k^l)$ is the predictive control input of the plant $\mathcal{P}_l$ at time instant t based on the actual state at t_k^l, and the corresponding predictive state is denoted by $x_l(t|t_k^l)$, where $t \in [t_k^l + 1, t_k^l + H_l]$. Note that $x_l(t_k^l|t_k^l) = x_l(t_k^l)$, where $x_l(t_k^l)$ is the actual state of plant $\mathcal{P}_l$ at time instant t_k^l. The optimization problem for plant $\mathcal{P}_l$ at time instant t_k^l is

$$\begin{aligned}
&\min_{U_l(t_k^l)} \quad J_l\left(x_l\left(t_k^l\right), U_l\left(t_k^l\right)\right),\\
&\text{s.t.} \quad x_l\left(t+1|t_k^l\right) = A_l x_l\left(t|t_k^l\right) + B_l u_l\left(t|t_k^l\right),\\
&\qquad\qquad t \in \left[t_k^l, t_k^l + H_l - 1\right],\\
&\qquad \|x_l\left(t|t_k^l\right)\|_{P_l} \leq \frac{H_l \alpha_l \varepsilon_l}{t - t_k^l},\ t \in \left[t_k^l + 1, t_k^l + H_l\right],\\
&\qquad u_l(t|t_k^l) \in \mathcal{U}_l,\ t \in \left[t_k^l, t_k^l + H_l - 1\right],
\end{aligned} \quad (3)$$

where $U_l(t_k^l) = \{u_l(t_k^l|t_k^l), u_l(t_k^l + 1|t_k^l), \ldots, u_l(t_k^l + H_l - 1|t_k^l)\}$ is the predictive control sequence, H_l is the predictive horizon of plant $\mathcal{P}_l$, and $\alpha_l \in (0, 1)$ is called the shrinkage rate [6], [19]. The cost function is defined as

$$J_l\left(x_l\left(t_k^l\right), U_l\left(t_k^l\right)\right) = \sum_{t=t_k^l}^{t_k^l + H_l - 1} \left(\|x_l(t|t_k^l)\|_{Q_l}^2 + \|u_l(t|t_k^l)\|_{R_l}^2\right) + \|x_l(t_k^l + H_l|t_k^l)\|_{P_l}^2, \quad (4)$$

where $Q_l > 0$, $R_l > 0$, and $P_l > 0$ are symmetric matrices satisfying Assumption 1.

The optimal control input and the corresponding optimal state are denoted by $\hat{u}_l(t|t_k^l)$ and $\hat{x}_l(t|t_k^l)$, respectively. The optimal solution of problem (3) is denoted by $\hat{U}_l(t_k^l) = \{\hat{u}_l(t_k^l|t_k^l), \hat{u}_l(t_k^l + 1|t_k^l), \ldots, \hat{u}_l(t_k^l + H_l - 1|t_k^l)\}$, and the corresponding optimal state sequence is $\hat{X}_l(t_k^l) = \{\hat{x}_l(t_k^l + 1|t_k^l), \hat{x}_l(t_k^l + 2|t_k^l), \ldots, \hat{x}_l(t_k^l + H_l|t_k^l)\}$.

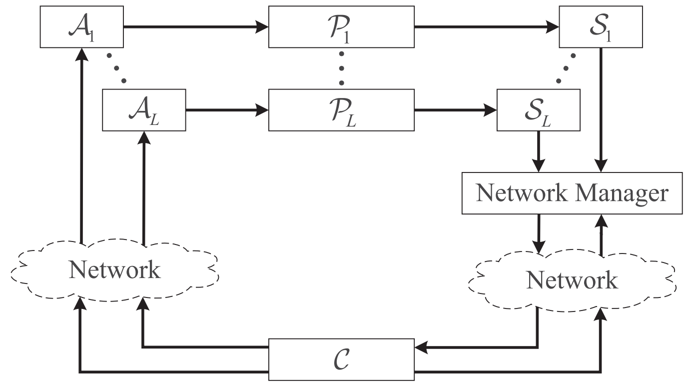

Fig. 1. Multi-loop control system.

Assumption 2: Assume that the optimization problem (3) has a feasible solution at the initial time t_1^l, i.e., $U_l(t_1^l) \neq \varnothing$.

In traditional MPC, the optimal control inputs are calculated at each time instant and the actuator only uses the first control input of the optimal control sequence. However, in our multi-loop MPC with shared communication networks, the actuator uses multiple control inputs. Meanwhile, the actual state may deviate from the optimal state, and the bound of system errors between the actual states and the optimal ones is increasing with the number of inputs used. Hence, we require that no more than M inputs are used, where M is a positive integer satisfying $M \leq H_l$.

III. Network Scheduling and MPC Co-design

At each time instant, the network manager in Fig. 1 determines which states need to be transmitted to the controller based on the states of all plants, then the optimal control inputs will be transmitted to the corresponding actuators.

In this section, we will propose algorithms to determine the scheduling sequences for the overall system and calculate optimal control inputs for each plant. In Section III-A, we consider the case that only one loop communicates over the networks at each time instant. Recursive feasibility of the optimization problems and the stability of the overall system are provided in Sections III-B and III-C, respectively. Then, the case that more than one loop can communicate over the networks at each time instant is extended in Section III-D.

A. Only One Loop Communicates Over the Networks at Each Time Instant

In this section, we give the following algorithm to ensure that all plants can be stabilized without conflicting transmissions. Define $U_l^\star(t_k^l) = \{\hat{u}_l(t_k^l|t_k^l), \hat{u}_l(t_k^l + 1|t_k^l), \ldots, \hat{u}_l(t_k^l + M - 1|t_k^l)\}$ and $X_l^\star(t_k^l) = \{\hat{x}_l(t_k^l + 1|t_k^l), \hat{x}_l(t_k^l + 2|t_k^l), \ldots, \hat{x}_l(t_k^l + M - 1|t_k^l)\}$.

Algorithm 1: From time $t = 0$, the controller executes step 3 in the order of increasing loop index l from $\mathcal{P}_1$, and the input of plant $\mathcal{P}_l$ is zero until time instant t_1^l. The controller transmits $U_l^\star(t_1^l)$ to actuator $\mathcal{A}_l$, and transmits $X_l^\star(t_1^l)$ to the network manager after solving the optimization problem (3). The last time instant of solving optimization problem for plant $\mathcal{P}_l$ before t is denoted by t_{last}^l. At time instant $t = L$,

Step 1: If there exists $l^* \in \mathcal{L}$ such that $t - t_{last}^{l^*} = M$, the network manager transmits $x_{l^*}(t)$ to the controller and updates $t_{last}^{l^*} = t$, then go to step 3; else, go to step 2.

Step 2: Sensors measure the states $x_l(t)$ of all plants, and the network manager determines the index $l^* = \min\{\arg\max_{l \in \mathcal{L}} \|x_l(t) - \hat{x}_l(t|t_{last}^l)\|_{\Psi_l}^2\}$, where Ψ_l is a weight matrix for the state error between the actual state and the optimal one. The state $x_{l^*}(t)$ is transmitted to the controller by the network manager and $t^{l^*}_{last}$ is updated as t, then go to step 3.

Step 3: After solving the optimization problem (3) for plant $\mathcal{P}_{l^*}$, the controller transmits $U^\star_{l^*}(t^{l^*}_{last})$ to actuator $\mathcal{A}_{l^*}$, and transmits $X^\star_{l^*}(t^{l^*}_{last})$ to the network manager.

Step 4: Apply the control input $\hat{u}_l(t|t^l_{last})$ to plant $\mathcal{P}_l$ for $[t, t+1)$, $\forall l \in \mathcal{L}$, and set $t = t+1$, then go to step 1.

Remark 1: Because the controller only sends the first M optimal control inputs to the actuator $\mathcal{A}_l$ at time instant t^l_k, the control inputs are exhausted at $t^l_k + M$ (assuming that the optimization problem (3) is not calculated in the time interval $[t^l_k + 1, t^l_k + M - 1]$). Hence the optimization problem (3) for plant $\mathcal{P}_l$ must be calculated at time $t = t^l_k + M$. Therefore, the controller only needs to transmit $X^\star_l(t^l_k)$ to the network manager. Meanwhile, if $M < L$, some actuators have no control inputs during some control periods. Therefore, we impose that $L \le M \le H_l$ should be satisfied. In addition, if the multi-loop control system is noise-free, the error between actual state $x_l(t)$ and the optimal one $\hat{x}_l(t|t^l_{last})$ is zero. This will cause the information to be transmitted in a fixed order, and the system states will converge to the origin instead of stable in a set containing the origin.

B. Feasibility

According to Algorithm 1, we know that for plant $\mathcal{P}_l$, the time interval $\tau^l_k = t^l_{k+1} - t^l_k$ satisfies $\tau^l_k \in [1, M]$. The optimal control sequence of the optimization problem (3) at time t^l_k is $\hat{U}_l(t^l_k)$, and the control input is $u_l(t) = K_l x_l(t)$ when $t \ge t^l_k + H_l$. In order to prove the recursive feasibility of the optimization problem (3) in Algorithm 1, we define the control input sequence $\tilde{U}_l(t^l_{k+1})$ at time instant t^l_{k+1}. Assume that $\tilde{U}_l(t^l_{k+1})$ is a feasible control input sequence when $t^l_{k+1} = t^l_k + M$, i.e., $\tau^l_k = M$, then the optimization problem (3) has a feasible solution when $\tau^l_k \in [1, M-1]$. Therefore, in order to guarantee that the optimization problem (3) for plant $\mathcal{P}_l$ has a feasible solution at time instant t^l_k, $k \in \mathbb{N}$, we just need to ensure that $\tilde{U}_l(t^l_{k+1})$ is a feasible solution when $\tau^l_k = M$. The control input sequence is $\tilde{U}_l(t^l_{k+1}) = \{\hat{u}_l(t^l_{k+1}|t^l_k), \hat{u}_l(t^l_{k+1}+1|t^l_k), \ldots, \hat{u}_l(t^l_k + H_l - 1|t^l_k), K_l\tilde{x}_l(t^l_k + H_l|t^l_{k+1}), \ldots, K_l\tilde{x}_l(t^l_{k+1} + H_l - 1|t^l_{k+1})\}$ when $L \le M < H_l$ and $\tilde{U}_l(t^l_{k+1}) = \{K_l x_l(t^l_{k+1}), K_l\tilde{x}_l(t^l_{k+1} + 1|t^l_{k+1}), \ldots, K_l\tilde{x}_l(t^l_{k+1} + H_l - 1|t^l_{k+1})\}$ when $M = H_l$.

Theorem 1: For system (1), suppose that Assumptions 1 and 2 hold, $\forall l \in \mathcal{L}$. If the system parameters satisfy the following conditions:

$$\rho_l \le \begin{cases} \min\left\{\frac{(1-\alpha_l)\varepsilon_l}{\|A_l\|^{H_l-M}\bar{\lambda}(\sqrt{P_l})T^l_M}, \frac{M\alpha_l\varepsilon_l}{(H_l-M)N_{A_l}\bar{\lambda}(\sqrt{P_l})T^l_M}\right\}, & L \le M < H_l, \\ \frac{(1-\alpha_l)\varepsilon_l}{\bar{\lambda}(\sqrt{P_l})T^l_M}, & M = H_l, \end{cases}$$

and

$$\|\Phi_l\| \le \begin{cases} \frac{\underline{\lambda}(\sqrt{P_l})}{\bar{\lambda}(\sqrt{P_l})}\frac{H_l-M}{H_l}, & L \le M < H_l, \\ \frac{\underline{\lambda}(\sqrt{P_l})}{\bar{\lambda}(\sqrt{P_l})}\alpha_l, & M = H_l, \end{cases}$$

where ρ_l is defined in (2), and

$$\Phi_l = A_l + B_l K_l,$$
$$N_{A_l} = \max\{\|A_l\|, \|A_l\|^{H_l-M}\},$$
$$T^l_M = \begin{cases} M, & \|A_l\| = 1, \\ \frac{1-\|A_l\|^M}{1-\|A_l\|}, & \|A_l\| \ne 1, \end{cases}$$

then the optimization problem (3) for plant $\mathcal{P}_l$ has a feasible solution at time instant t^l_k, $k \in \mathbb{N}$, under Algorithm 1.

Proof: In order to prove the theorem, we analyse the feasibility conditions in two cases according to the value of M.

Case 1: $L \le M < H_l$

For $t \in [t^l_{k+1} + 1, t^l_k + H_l]$, we have $\tilde{x}_l(t|t^l_{k+1}) = \hat{x}_l(t|t^l_k) + A_l^{t-t^l_{k+1}}(A_l^{M-1}w_l(t^l_k) + A_l^{M-2}w_l(t^l_k+1) + \cdots + w_l(t^l_{k+1}-1))$, hence we have

$$\begin{aligned} \|\tilde{x}_l(t|t^l_{k+1})\|_{P_l} &\le \|\hat{x}_l(t|t^l_k)\|_{P_l} + \|A_l\|^{t-t^l_{k+1}}\bar{\lambda}(\sqrt{P_l})\rho_l \\ &\quad \times (\|A_l\|^{M-1} + \|A_l\|^{M-2} + \cdots + 1) \\ &\le \frac{H_l\alpha_l\varepsilon_l}{t - t^l_k} + N_{A_l}\bar{\lambda}(\sqrt{P_l})T^l_M\rho_l \\ &\le \frac{H_l\alpha_l\varepsilon_l}{t - t^l_k} + \frac{M\alpha_l\varepsilon_l}{H_l - M}. \end{aligned} \tag{5}$$

Define a continuous-time function $f(s) = \frac{1}{s-t^l_{k+1}} - \frac{1}{s-t^l_k}$, $t^l_{k+1} + 1 \le s \le t^l_k + H_l$. Because $\frac{\mathrm{d}}{\mathrm{d}s}f(s) < 0$, we obtain $f(s) \ge f(t^l_k + H_l) = \frac{M}{H_l(H_l-M)}$. Further, we have $\frac{1}{s-t^l_{k+1}} \ge \frac{1}{s-t^l_k} + \frac{M}{H_l(H_l-M)}$. Therefore, $\|\tilde{x}_l(t|t^l_{k+1})\|_{P_l} \le \frac{H_l\alpha_l\varepsilon_l}{t-t^l_{k+1}}$, $t \in [t^l_{k+1} + 1, t^l_k + H_l]$.

Moreover, the following inequality holds

$$\|\tilde{x}_l(t|t^l_{k+1}) - \hat{x}_l(t|t^l_k)\|_{P_l} \le \|A_l\|^{t-t^l_{k+1}}\bar{\lambda}(\sqrt{P_l})T^l_M\rho_l. \tag{6}$$

Substituting (6) into the triangle inequality $\|\tilde{x}_l(t|t^l_{k+1})\|_{P_l} \le \|\tilde{x}_l(t|t^l_{k+1}) - \hat{x}_l(t|t^l_k)\|_{P_l} + \|\hat{x}_l(t|t^l_k)\|_{P_l}$, we have $\|\tilde{x}_l(t^l_k + H_l|t^l_{k+1})\|_{P_l} \le \|A_l\|^{H_l-M}\bar{\lambda}(\sqrt{P_l})T^l_M\rho_l + \alpha_l\varepsilon_l \le \varepsilon_l$.

Therefore, $\tilde{x}_l\left(t^l_k + H_l|t^l_{k+1}\right) \in \Omega_{P_l}(\varepsilon_l)$, which means the state $\tilde{x}_l(t^l_k + H_l|t^l_{k+1})$ goes into $\Omega_{P_l}(\varepsilon_l)$, and the state feedback control can be used. Hence, for $t \in [t^l_k + H_l + 1, t^l_{k+1} + H_l]$, the nominal model is $\tilde{x}_l(t|t^l_{k+1}) = \Phi_l^{t-t^l_k-H_l}\tilde{x}_l(t^l_k + H_l|t^l_{k+1})$. Hence, we have

$$\begin{aligned} \|\tilde{x}_l(t|t^l_{k+1})\|_{P_l} &\le \frac{\bar{\lambda}(\sqrt{P_l})}{\underline{\lambda}(\sqrt{P_l})}\|\Phi_l\|^{t-t^l_k-H_l}\|\tilde{x}_l(t^l_k + H_l|t^l_{k+1})\|_{P_l} \\ &\le \frac{\bar{\lambda}(\sqrt{P_l})}{\underline{\lambda}(\sqrt{P_l})}\|\Phi_l\|^{t-t^l_k-H_l}\frac{H_l\alpha_l\varepsilon_l}{H_l - M}. \end{aligned} \tag{7}$$

Because $\|\Phi_l\|^{t-t^l_k-H_l} \le \|\Phi_l\|$ and $\frac{H_l-M}{t-t^l_{k+1}} \ge \frac{H_l-M}{H_l}$, we have $\|\Phi_l\|^{t-t^l_k-H_l} \le \frac{H_l-M}{t-t^l_{k+1}}\frac{\underline{\lambda}(\sqrt{P_l})}{\bar{\lambda}(\sqrt{P_l})}$. Therefore, $\|\tilde{x}_l(t|t^l_{k+1})\|_{P_l} \le \frac{H_l\alpha_l\varepsilon_l}{t-t^l_{k+1}}$ when $t \in [t^l_k + H_l + 1, t^l_{k+1} + H_l]$.

Moreover, the control inputs in sequence $\tilde{U}_l(t^l_{k+1})$ satisfy the input constraint during the time interval $[t^l_{k+1} + 1, t^l_{k+1} + H_l]$. This indicates that the optimization problem (3) has a feasible solution at time instant t^l_{k+1}.

Case 2: $M = H_l$

For $t = t^l_{k+1} = t^l_k + H_l$, we have $x_l(t^l_{k+1}) = \hat{x}_l(t^l_{k+1}|t^l_k) + A_l^{H_l-1}w_l(t^l_k) + A_l^{H_l-2}w_l(t^l_k+1) + \cdots + w_l(t^l_{k+1}-1)$, and hence we obtain $\|x_l(t^l_{k+1})\|_{P_l} \le \|\hat{x}_l(t^l_{k+1}|t^l_k)\|_{P_l} + \bar{\lambda}(\sqrt{P_l})T^l_M\rho_l \le \varepsilon_l$, i.e., the actual state $x_l(t^l_{k+1}) \in \Omega_{P_l}(\varepsilon_l)$. Thus, the state feedback control can be used and the control inputs in sequence $\tilde{U}_l(t^l_{k+1})$ satisfy the input constraint.

For $t \in [t_{k+1}^l + 1, t_{k+1}^l + H_l]$, we have $\tilde{x}_l(t|t_{k+1}^l) = \Phi_l^{t-t_{k+1}^l} x_l(t_{k+1}^l)$. Therefore, we obtain

$$\begin{aligned} \|\tilde{x}_l(t|t_{k+1}^l)\|_{P_l} &\leq \frac{\bar{\lambda}(\sqrt{P_l})}{\underline{\lambda}(\sqrt{P_l})} \|\Phi_l\|^{t-t_{k+1}^l} \|x_l(t_{k+1}^l)\|_{P_l} \\ &\leq \frac{\bar{\lambda}(\sqrt{P_l})}{\underline{\lambda}(\sqrt{P_l})} \|\Phi_l\| \varepsilon_l \\ &\leq \alpha_l \varepsilon_l. \end{aligned} \tag{8}$$

Inequality (8) implies that the state $\tilde{x}_l(t|t_{k+1}^l)$ satisfies the state constraint in the optimization problem (3) for $t \in [t_{k+1}^l + 1, t_{k+1}^l + H_l]$, i.e., the control input sequence $\tilde{U}_l(t_{k+1}^l)$ is a feasible solution for the optimization problem (3) at time instant t_{k+1}^l.

Therefore, the optimization problem (3) of each plant $\mathcal{P}_l$ has a feasible solution at time instant t_k^l, $k \in \mathbb{N}$, under Algorithm 1. ■

Theorem 1 gives the conditions for recursive feasibility of the optimization problem (3). Recursive feasibility can be satisfied when $\|\Phi_l\|$ and the bound ρ_l satisfy corresponding conditions.

C. Stability

Stability of the closed-loop system is investigated in this section. The bound of system errors between the actual states and the optimal ones is increasing with τ_k^l. Therefore, if the system stability with $\tau_k^l = M$ can be guaranteed, then the system is stable when $\tau_k^l \in [1, M]$. The following theorem gives sufficient conditions to guarantee stability.

Theorem 2: For system (1), suppose that Assumptions 1 and 2 hold, $\forall l \in \mathcal{L}$, and the scheduling sequences of overall system and optimal control inputs for each plant are calculated by Algorithm 1.

1) $L \leq M < H_l$: if the conditions in Theorem 1 are satisfied, and $\eta_l > 0$ with $\eta_l = \frac{M\varepsilon_l^2}{\bar{\lambda}(P_l)} - \frac{\bar{\lambda}(Q_l)}{\underline{\lambda}(Q_l)} \rho_l T_M^l (\frac{2H_l\alpha_l\varepsilon_l}{M\underline{\lambda}(\sqrt{P_l})} + \rho_l T_M^l) - \frac{\bar{\lambda}(\sqrt{P_l})}{\underline{\lambda}(Q_l)} \rho_l (1+\alpha_l)\varepsilon_l \|A_l\|^{H_l - M} T_M^l - \frac{\bar{\lambda}(Q_l)}{\underline{\lambda}(Q_l)\underline{\lambda}(\sqrt{P_l})} \rho_l H_l \alpha_l \varepsilon_l T_M^l T_S^l$, then the system states converge to the set $\mathcal{Y}_l \subset \Omega_{P_l}(\varepsilon_l)$, which is described as

$$\begin{aligned} \mathcal{Y}_l = \Bigg\{ x_l \in \mathbb{R}^{n_l} : \|x_l\|_{P_l}^2 \leq \frac{\bar{\lambda}(P_l)}{M\underline{\lambda}(Q_l)} \Bigg\{ &\frac{2\bar{\lambda}(Q_l)}{\underline{\lambda}(\sqrt{P_l})} \rho_l \varepsilon_l T_M^l T_{H_l} \\ &+ \bar{\lambda}(Q_l) T_M^l \rho_l \left(\frac{2\varepsilon_l}{\underline{\lambda}(\sqrt{P_l})} + T_M^l \rho_l \right) + \bar{\lambda}(Q_l) \left(T_M^l \rho_l\right)^2 T_{H_l^2} \\ &+ 2M T_M^l \rho_l \frac{\bar{\lambda}(Q_l)}{\underline{\lambda}(\sqrt{P_l})} \varepsilon_l + M(T_M^l \rho_l)^2 \bar{\lambda}(Q_l) \\ &+ \bar{\lambda}\left(\sqrt{P_l}\right) \|A_l\|^{H_l - M} T_M^l \rho_l (1+\alpha_l)\varepsilon_l \Bigg\} \Bigg\}, \end{aligned} \tag{9}$$

where

$$T_{H_l} = \begin{cases} H_l - M - 1, & \|A_l\| = 1, \\ \frac{\|A_l\| - \|A_l\|^{H_l - M}}{1 - \|A_l\|}, & \|A_l\| \neq 1, \end{cases}$$

$$T_{H_l^2} = \begin{cases} H_l - M - 1, & \|A_l\| = 1, \\ \frac{\|A_l\|^2 - \|A_l\|^{2(H_l - M)}}{1 - \|A_l\|^2}, & \|A_l\| \neq 1, \end{cases}$$

$$T_S^l = \begin{cases} \sum_{i=1}^{H_l - M - 1} \|A_l\|^i \left(\frac{1}{i} + \frac{1}{M+i}\right), & L \leq M < H_l - 1, \\ 0, & M = H_l - 1. \end{cases}$$

2) $M = H_l$: if the conditions in Theorem 1 are satisfied, then the system states converge to the invariant set $\Omega_{P_l}(\varepsilon_l)$.

Proof: This theorem is provided by investigating two cases.

Case 1: $L \leq M < H_l$

Define the Lyapunov function as $V_l(t_k^l) = J_l(x_l(t_k^l), \hat{U}_l(t_k^l))$, the difference between the values of Lyapunov function at t_k^l and t_{k+1}^l satisfies

$$\begin{aligned} \Delta V_k^l &= V_l(t_{k+1}^l) - V_l(t_k^l) \\ &\leq \sum_{t=t_{k+1}^l}^{t_k^l + H_l - 1} \left(\left\|\tilde{x}_l(t|t_{k+1}^l)\right\|_{Q_l}^2 - \left\|\hat{x}_l(t|t_k^l)\right\|_{Q_l}^2 \right) \\ &+ \sum_{t=t_k^l + H_l}^{t_{k+1}^l + H_l - 1} \left(\left\|\tilde{x}_l(t|t_{k+1}^l)\right\|_{Q_l}^2 + \left\|\tilde{u}_l(t|t_{k+1}^l)\right\|_{R_l}^2 \right) \\ &+ \left\|\tilde{x}_l(t_{k+1}^l + H_l|t_{k+1}^l)\right\|_{P_l}^2 - \left\|\hat{x}_l(t_k^l + H_l|t_k^l)\right\|_{P_l}^2 \\ &- \sum_{t=t_k^l}^{t_{k+1}^l - 1} \left(\left\|\hat{x}_l(t|t_k^l)\right\|_{Q_l}^2 + \left\|\hat{u}_l(t|t_k^l)\right\|_{R_l}^2 \right). \end{aligned} \tag{10}$$

Based on the fact that $\tilde{u}_l(t|t_{k+1}^l) = \hat{u}_l(t|t_k^l)$, $t \in [t_{k+1}^l, t_k^l + H_l - 1]$, we split the right hand of (10) into $\Delta V_k^l(1)$, $\Delta V_k^l(2)$, and $\Delta V_k^l(3)$, where

$$\Delta V_k^l(1) = \sum_{t=t_{k+1}^l}^{t_k^l + H_l - 1} \left(\left\|\tilde{x}_l(t|t_{k+1}^l)\right\|_{Q_l}^2 - \left\|\hat{x}_l(t|t_k^l)\right\|_{Q_l}^2 \right), \tag{11}$$

$$\begin{aligned} \Delta V_k^l(2) = &\sum_{t=t_k^l + H_l}^{t_{k+1}^l + H_l - 1} \left(\left\|\tilde{x}_l(t|t_{k+1}^l)\right\|_{Q_l}^2 + \left\|\tilde{u}_l(t|t_{k+1}^l)\right\|_{R_l}^2 \right) \\ &+ \left\|\tilde{x}_l(t_{k+1}^l + H_l|t_{k+1}^l)\right\|_{P_l}^2 - \left\|\hat{x}_l(t_k^l + H_l|t_k^l)\right\|_{P_l}^2, \end{aligned} \tag{12}$$

$$\Delta V_k^l(3) = -\sum_{t=t_k^l}^{t_{k+1}^l - 1} \left(\left\|\hat{x}_l(t|t_k^l)\right\|_{Q_l}^2 + \left\|\hat{u}_l(t|t_k^l)\right\|_{R_l}^2 \right). \tag{13}$$

In order to prove the theorem, we analyse ΔV_k^l in two cases according to whether $x_l(t_k^l)$ belongs to $\Omega_{P_l}(\varepsilon_l)$ or not.

i) $x_l(t_k^l) \notin \Omega_{P_l}(\varepsilon_l)$. By the triangle inequality and Theorem 1, we have

$$\begin{aligned} &\left\|\tilde{x}_l(t_{k+1}^l|t_{k+1}^l)\right\|_{Q_l}^2 - \left\|\hat{x}_l(t_{k+1}^l|t_k^l)\right\|_{Q_l}^2 \\ &\leq \left\|\tilde{x}_l(t_{k+1}^l|t_{k+1}^l) - \hat{x}_l(t_{k+1}^l|t_k^l)\right\|_{Q_l} \\ &\quad \times \left(\left\|\tilde{x}_l(t_{k+1}^l|t_{k+1}^l)\right\|_{Q_l} + \left\|\hat{x}_l(t_{k+1}^l|t_k^l)\right\|_{Q_l} \right) \\ &\leq \bar{\lambda}\left(\sqrt{Q_l}\right) \rho_l T_M^l \cdot \left(\frac{\bar{\lambda}\left(\sqrt{Q_l}\right)}{\underline{\lambda}\left(\sqrt{P_l}\right)} \frac{2H_l\alpha_l\varepsilon_l}{M} + \bar{\lambda}\left(\sqrt{Q_l}\right) \rho_l T_M^l \right) \\ &\leq \bar{\lambda}(Q_l) \rho_l T_M^l \left(\frac{2H_l\alpha_l\varepsilon_l}{\underline{\lambda}\left(\sqrt{P_l}\right) M} + \rho_l T_M^l \right). \end{aligned} \tag{14}$$

Therefore, we obtain

$$\begin{aligned}\Delta V_k^l(1) = & \sum_{t=t_{k+1}^l}^{t_k^l+H_l-1} \left(\|\tilde{x}_l\left(t|t_{k+1}^l\right)\|_{Q_l}^2 - \|\hat{x}_l\left(t|t_k^l\right)\|_{Q_l}^2\right) \\ & \leq \bar{\lambda}\left(Q_l\right)\rho_l T_M^l\left(\frac{2H_l\alpha_l\varepsilon_l}{\underline{\lambda}\left(\sqrt{P_l}\right)M} + \rho_l T_M^l\right) \\ & + \sum_{t=t_{k+1}^l+1}^{t_k^l+H_l-1}\left(\left\|\tilde{x}_l\left(t|t_{k+1}^l\right)\right\|_{Q_l}^2 - \left\|\hat{x}_l\left(t|t_k^l\right)\right\|_{Q_l}^2\right) \\ & \leq \bar{\lambda}\left(Q_l\right)\rho_l T_M^l\left(\frac{2H_l\alpha_l\varepsilon_l}{\underline{\lambda}\left(\sqrt{P_l}\right)M} + \rho_l T_M^l\right) + \frac{\bar{\lambda}\left(Q_l\right)}{\underline{\lambda}\left(\sqrt{P_l}\right)} \\ & \times \rho_l H_l\alpha_l\varepsilon_l T_M^l \sum_{i=1}^{H_l-M-1}\|A_l\|^i\left(\frac{1}{i} + \frac{1}{M+i}\right) \\ & \leq \bar{\lambda}\left(Q_l\right)\rho_l T_M^l\left(\frac{2H_l\alpha_l\varepsilon_l}{\underline{\lambda}\left(\sqrt{P_l}\right)M} + \rho_l T_M^l\right) \\ & + \frac{\bar{\lambda}\left(Q_l\right)}{\underline{\lambda}\left(\sqrt{P_l}\right)}\rho_l H_l\alpha_l\varepsilon_l T_M^l T_S^l. \end{aligned} \tag{15}$$

According to Theorem 1, we have $\tilde{x}_l\left(t_k^l + H_l|t_{k+1}^l\right) \in \Omega_{P_l}(\varepsilon_l)$. For $\Delta V_k^l(2)$, it holds that

$$\begin{aligned}\Delta V_k^l(2) = & \sum_{t=t_k^l+H_l}^{t_{k+1}^l+H_l-1}\left(\left\|\tilde{x}_l\left(t|t_{k+1}^l\right)\right\|_{Q_l}^2 + \left\|\tilde{u}_l\left(t|t_{k+1}^l\right)\right\|_{R_l}^2\right) \\ & + \left\|\tilde{x}_l\left(t_{k+1}^l + H_l|t_{k+1}^l\right)\right\|_{P_l}^2 - \left\|\hat{x}_l\left(t_k^l + H_l|t_k^l\right)\right\|_{P_l}^2 \\ & \leq \left\|\tilde{x}_l\left(t_k^l + H_l|t_{k+1}^l\right)\right\|_{P_l}^2 - \left\|\hat{x}_l\left(t_k^l + H_l|t_k^l\right)\right\|_{P_l}^2 \\ & \leq \bar{\lambda}\left(\sqrt{P_l}\right)\|A_l\|^{H_l-M}T_M^l\rho_l(1+\alpha_l)\varepsilon_l. \end{aligned} \tag{16}$$

Since $x_l(t_k^l) \notin \Omega_{P_l}(\varepsilon_l)$, we have

$$\begin{aligned}\Delta V_k^l(3) = & -\sum_{t=t_k^l}^{t_{k+1}^l-1}\left(\left\|\hat{x}_l\left(t|t_k^l\right)\right\|_{Q_l}^2 + \left\|\hat{u}_l\left(t|t_k^l\right)\right\|_{R_l}^2\right) \\ \leq & -\sum_{t=t_k^l}^{t_{k+1}^l-1}\left\|\hat{x}_l\left(t|t_k^l\right)\right\|_{Q_l}^2 \\ \leq & -M\frac{\underline{\lambda}(Q_l)}{\bar{\lambda}(P_l)}\varepsilon_l^2. \end{aligned} \tag{17}$$

By summing the results of $\Delta V_k^l(1), \Delta V_k^l(2)$, and $\Delta V_k^l(3)$, we obtain $\Delta V_k^l < -\underline{\lambda}(Q_l)\eta_l$, i.e., $V_l(t_{k+1}^l) < V_l(t_k^l) - \underline{\lambda}(Q_l)\eta_l$, which implies that the system states will go into the invariant set $\Omega_{P_l}(\varepsilon_l)$ in finite time.

ii) $x(t_k^l) \in \Omega_{P_l}(\varepsilon_l)$. Since $\hat{x}_l(t|t_k^l) \in \Omega_{P_l}(\varepsilon_l)$ holds during the time interval $t \in [t_{k+1}^l, t_k^l + H_l - 1]$, we obtain

$$\begin{aligned}\Delta V_k^l(1) = & \sum_{t=t_{k+1}^l}^{t_k^l+H_l-1}\left(\left\|\tilde{x}_l(t|t_{k+1}^l)\right\|_{Q_l}^2 - \left\|\hat{x}_l(t|t_k^l)\right\|_{Q_l}^2\right) \\ & \leq \bar{\lambda}(Q_l)T_M^l\rho_l\left(\frac{2\varepsilon_l}{\underline{\lambda}(\sqrt{P_l})} + T_M^l\rho_l\right) \\ & + \frac{2\bar{\lambda}(Q_l)}{\underline{\lambda}(\sqrt{P_l})}T_M^l\rho_l\varepsilon_l T_{H_l} + \bar{\lambda}(Q_l)\left(T_M^l\rho_l\right)^2 T_{H_l^2}. \end{aligned} \tag{18}$$

$\Delta V_k^l(2)$ in this case is the same as the one in the previous case. For $\Delta V_k^l(3)$, it is obvious that the term $\|\hat{x}_l(t|t_k^l)\|_{Q_l}^2 + \|\hat{u}_l(t|t_k^l)\|_{R_l}^2$ is decreasing with the increase of t when the state feedback control law is applied to the system. Hence, $-\|\hat{x}_l(t|t_k^l)\|_{Q_l}^2 - \|\hat{u}_l(t|t_k^l)\|_{R_l}^2 \leq -\|\hat{x}_l(t_{k+1}^l|t_k^l)\|_{Q_l}^2, \quad t \in [t_k^l, t_{k+1}^l - 1]$. Because $\|x_l(t_{k+1}^l)\|_{Q_l} \leq \|\hat{x}_l(t_{k+1}^l|t_k^l)\|_{Q_l} + \|A_l^{M-1}w_l(t_k^l) + A_l^{M-2}w_l(t_k^l+1) + \cdots + w_l(t_{k+1}^l)\|_{Q_l} \leq \|\hat{x}_l(t_{k+1}^l|t_k^l)\|_{Q_l} + \bar{\lambda}(\sqrt{Q_l})T_M^l\rho_l$, we have

$$\begin{aligned}\Delta V_k^l(3) = & -\sum_{t=t_k^l}^{t_{k+1}^l-1}\left(\left\|\hat{x}_l(t|t_k^l)\right\|_{Q_l}^2 + \left\|\hat{u}_l(t|t_k^l)\right\|_{R_l}^2\right) \\ \leq & -M\left\|\hat{x}_l(t_{k+1}^l|t_k^l)\right\|_{Q_l}^2 \\ \leq & -M\frac{\underline{\lambda}(Q_l)}{\bar{\lambda}(P_l)}\|x_l(t_{k+1}^l\|_{P_l}^2 + M(T_M^l\rho_l)^2\bar{\lambda}(Q_l) \\ & + 2MT_M^l\rho_l\frac{\bar{\lambda}(Q_l)}{\underline{\lambda}(\sqrt{P_l})}\varepsilon_l. \end{aligned} \tag{19}$$

Consequently, we obtain

$$\begin{aligned}\Delta V_k^l \leq \quad & \Delta V_k^l(1) + \Delta V_k^l(2) + \Delta V_k^l(3) \\ \leq \quad & \bar{\lambda}(Q_l)T_M^l\rho_l\left(\frac{2\varepsilon_l}{\underline{\lambda}(\sqrt{P_l})} + T_M^l\rho_l\right) \\ & + \frac{2\bar{\lambda}(Q_l)}{\underline{\lambda}(\sqrt{P_l})}T_M^l\rho_l\varepsilon_l T_{H_l} + \bar{\lambda}(Q_l)\left(T_M^l\rho_l\right)^2 T_{H_l^2} \\ & + 2MT_M^l\rho_l\frac{\bar{\lambda}(Q_l)}{\underline{\lambda}(\sqrt{P_l})}\varepsilon_l + M(T_M^l\rho_l)^2\bar{\lambda}(Q_l) \\ & + \bar{\lambda}\left(\sqrt{P_l}\right)\|A_l\|^{H_l-M}T_M^l\rho_l(1+\alpha_l)\varepsilon_l \\ & - M\frac{\underline{\lambda}(Q_l)}{\bar{\lambda}(P_l)}\left\|x_l(t_{k+1}^l)\right\|_{P_l}^2, \end{aligned} \tag{20}$$

which means the system is stable, and the system states of plant $\mathcal{P}_l$ will converge to the set $\mathcal{Y}_l$.

Case 2: $M = H_l$

According to Theorem 1, we have $x_l(t_{k+1}^l) \in \Omega_{P_l}(\varepsilon_l)$. Therefore, the system states converge to the invariant set $\Omega_{P_l}(\varepsilon_l)$ if the conditions in Theorem 1 are satisfied.

The proof is completed. ■

Remark 2: As the plants $\mathcal{P}_l, l = 1, 2, \ldots, L$, are only coupled through the shared communication networks, recursive feasibility of the optimization problem and the stability of each loop are influenced by each plant's system parameters and the common parameter M. For plant $\mathcal{P}_l$, $\forall l \in \mathcal{L}$, the conditions of Theorem 1 guarantee the recursive feasibility of the optimization problem, and the conditions of Theorem 2 ensure the stability of the closed-loop system.

Remark 3: The linear system (1) is Lipschitz continuous in state x_l, and $\|A_l\|$ is a Lipschitz constant. For nonlinear system with satisfying Lipschitz conditions and Assumptions 1 and 2, we can get similar algorithms and conditions for recursive feasibility and stability.

D. A Limited Number of Loops Communicate Over the Networks at Each Time Instant

The previous section describes the case that only one loop communicates over the networks at each time instant. In this section, we will consider the case that a limited number of loops N can communicate over the networks at each time instant. In order to ensure the communication without conflict, we assume $L \leq MN$.

Algorithm 2: From time $t = 0$, step 3 is executed in the order of increasing loop index l, and the input of plant $\mathcal{P}_l$ is zero when $t < t_1^l$.

The first time of solving the optimization problem for plant $\mathcal{P}_l$ is $t_1^l = \lfloor \frac{l-1}{N} \rfloor$. The controller transmits $U_l^\star(t_1^l)$ to actuator $\mathcal{A}_l$, and transmits $X_l^\star(t_1^l)$ to the network manager. Define a set $\mathcal{L}^*$, which contains the indexes of the corresponding states needed to be transmitted to the controller. The last time instant of solving the optimization problem for plant $\mathcal{P}_l$ before t is denoted by t_{last}^l. If $\mathrm{mod}(L, N) \neq 0$, then there exist $\mathrm{mod}(L, N)$ communication channels after plant $\mathcal{P}_L$ is executed at time instant $\lfloor \frac{L-1}{N} \rfloor$. Thus, add indexes $\{L - \mathrm{mod}(L, N) + 1, L - \mathrm{mod}(L, N) + 2, \ldots, L\}$ into $\mathcal{L}^*$, and then go to step 2 (select the first $N - \mathrm{mod}(L, N)$ loops). At time instant $t = \lfloor \frac{L-1}{N} \rfloor + 1$,

Step 1: If there exists l^*, $l^* \in \mathcal{L}$, such that $t - t_k^{l^*} = M$, then add the index element l^* into $\mathcal{L}^*$. The number of the elements in $\mathcal{L}^*$ is denoted by L^*. If $L^* = N$, the sensor of plant $\mathcal{P}_{l^*}$ measures the state $x_{l^*}(t)$, the network manager transmits $x_{l^*}(t)$ to the controller and sets $t_{last}^{l^*} = t, \forall l^* \in \mathcal{L}^*$, then go to step 3; else, go to step 2.

Step 2: Sensor measures the state $x_l(t)$ of plant $\mathcal{P}_l$, $\forall l \in \mathcal{L} \setminus \mathcal{L}^*$, then the network manager sorts these states according to the values $\|x_l(t) - \hat{x}_l(t|t_{last}^l)\|_{\Psi_l}^2, \forall l \in \mathcal{L} \setminus \mathcal{L}^*$, in descending order (for the same values, the network manager sorts the corresponding states according to an ascending order of indexes). After that, add the first $N - L^*$ state indexes into $\mathcal{L}^*$. Finally, the network manager transmits $x_{l^*}(t)$ to the controller and updates $t_{last}^{l^*} = t, \forall l^* \in \mathcal{L}$, then go to step 3.

Step 3: The controller transmits $U_{l^*}^\star(t_{last}^{l^*})$ and $X_{l^*}^\star(t_{last}^{l^*})$ to actuator $\mathcal{A}_{l^*}$ and the network manager, respectively, after solving the optimization problem (3) for plant $\mathcal{P}_{l^*}$, $\forall l \in \mathcal{L}^*$.

Step 4: Apply the control input $\hat{u}_l(t|t_{last}^l)$ to plant $\mathcal{P}_l$ for $[t, t+1)$, $\forall l \in \mathcal{L}$, and set $t = t + 1$, then go to step 1.

Remark 4: The differences between Algorithm 2 and Algorithm 1 are the initialization stage and the number of loops that can communicate over the networks at each time instant. The relationships between plants $\mathcal{P}_l, l = 1, 2, \ldots, L$, have been given in Remark 2, and the recursive feasibility of the optimization problem and the stability of each loop are irrelevant to N. Hence, the conditions for recursive feasibility of the optimization problems and the stability of the closed-loop system for Algorithm 2 are the same as those for Algorithm 1.

IV. Simulation Results

In this section, we will give two numerical examples to illustrate the proposed algorithms. In the first example, only one loop communicates over the networks at each time instant, while in the second example the networks can ensure the communication requirements of two loops at each time instant.

A. Only One Loop Communicates Over the Networks at Each Time Instant

In this example, we consider a control system with two plants. The parameters of plant $\mathcal{P}_1$ are $A_1 = \left[\begin{smallmatrix} 0.65 & 0.2 \\ -0.1 & 1.1 \end{smallmatrix}\right]$, $B_1 = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$, $Q_1 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $R_1 = 1$, $P_1 = \left[\begin{smallmatrix} 1.7490 & 0.1174 \\ 0.1174 & 2.2800 \end{smallmatrix}\right]$ with state feedback control gain $K_1 = [0.1 \ -1.1]$, and control input constraint $\mathcal{U}_1 = \{u_1 \in \mathbb{R} : -1 \leq u_1 \leq 1\}$. The plant $\mathcal{P}_2$ is described as $A_2 = \left[\begin{smallmatrix} 0.7 & 0.1 \\ -0.2 & 1.1 \end{smallmatrix}\right]$, $B_2 = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$, $Q_2 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $R_2 = 1$, $P_2 = \left[\begin{smallmatrix} 2.0392 & -0.0773 \\ -0.0773 & 2.2304 \end{smallmatrix}\right]$, $K_2 = [0.2 \ -1.1]$, and $\mathcal{U}_2 = \{u_2 \in \mathbb{R} : -1 \leq u_2 \leq 1\}$. For plants $\mathcal{P}_1$ and $\mathcal{P}_2$, we choose $H_l = 15$, $\varepsilon_l = 0.1$, $\alpha_l = 0.7$, $l = 1, 2$, $\Psi_1 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $\Psi_2 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, and $M = 3$. According to Theorems 1 and 2, the maximum disturbance bounds of plants $\mathcal{P}_1$ and $\mathcal{P}_2$ are determined as 5.20×10^{-4} and 5.67×10^{-4}, respectively, and we set $\rho_1 = 5.20 \times 10^{-4}$ and $\rho_2 = 5.67 \times 10^{-4}$.

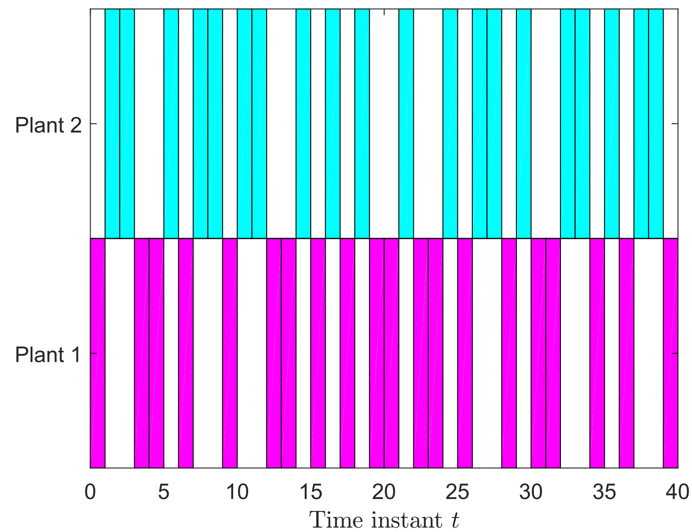

Fig. 2. Time instants of solving optimization problems of $\mathcal{P}_1$ and $\mathcal{P}_2$ calculated by Algorithm 1.

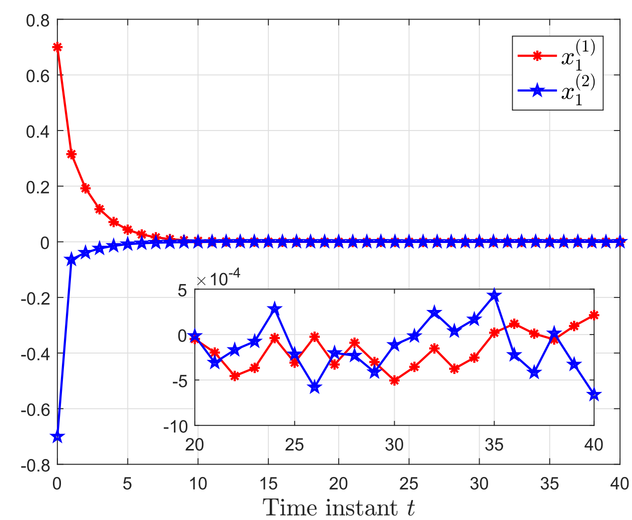

Fig. 3. State trajectory of $\mathcal{P}_1$ calculated by Algorithm 1.

The initial states of $\mathcal{P}_1$ and $\mathcal{P}_2$ are set to be $x_1(0) = (0.7 \quad -0.7)^T$ and $x_2(0) = (-0.7 \quad 0.7)^T$, respectively. Fig. 2 gives the time instants of solving optimization problems for plants $\mathcal{P}_1$ and $\mathcal{P}_2$ under Algorithm 1. We can see that there is only one loop communicating over the networks at each time instant. Figs. 3 and 4 show the state trajectories of plants $\mathcal{P}_1$ and $\mathcal{P}_2$, respectively. Because of the disturbances, the states of plants $\mathcal{P}_1$ and $\mathcal{P}_2$ converge to a neighborhood of the origin instead of the origin.

B. A Limited Number of Loops Communicate Over the Networks at Each Time Instant

In this example, the control system has three plants, and two loops can communicate over the networks at each time instant. The plants $\mathcal{P}_1$ and $\mathcal{P}_2$ are the same as the ones in the first example, the parameters of plant $\mathcal{P}_3$ are $A_3 = \left[\begin{smallmatrix} 1.1 & 0.2 \\ -0.2 & 0.7 \end{smallmatrix}\right]$, $B_3 = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$, $Q_3 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $R_3 = 1$, $P_3 = \left[\begin{smallmatrix} 2.2916 & -0.0655 \\ -0.0655 & 2.0392 \end{smallmatrix}\right]$, $K_3 = [-1.1 \ -0.2]$, and $\mathcal{U}_3 = \{u_3 \in \mathbb{R} : -1 \leq u_3 \leq 1\}$. For plant $\mathcal{P}_3$, we choose $H_3 = 15$, $\varepsilon_3 = 0.1$, $\alpha_3 = 0.7$, and $\Psi_3 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$. According to Theorems 1

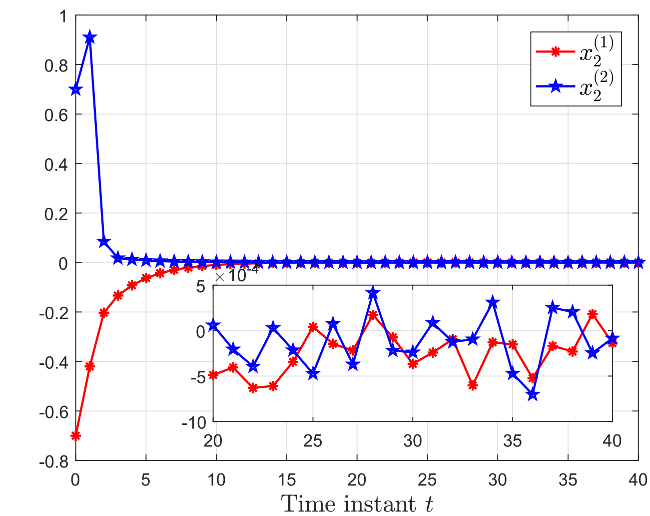

Fig. 4. State trajectory of $\mathcal{P}_2$ calculated by Algorithm 1.

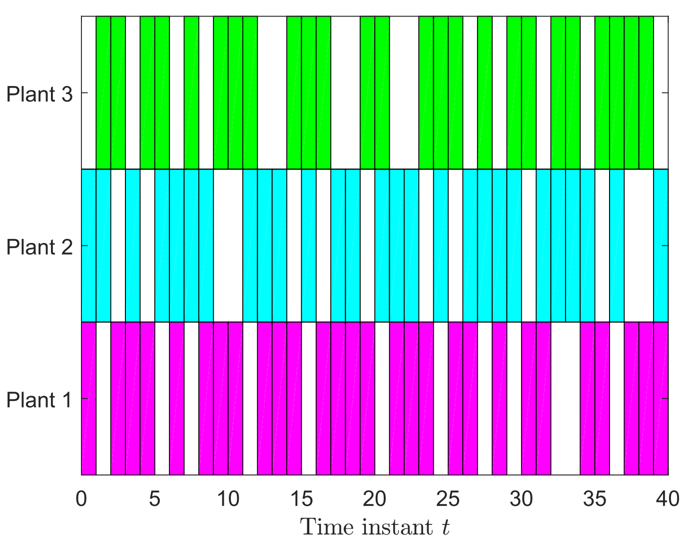

Fig. 5. Time instants of solving optimization problems of $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ calculated by Algorithm 2.

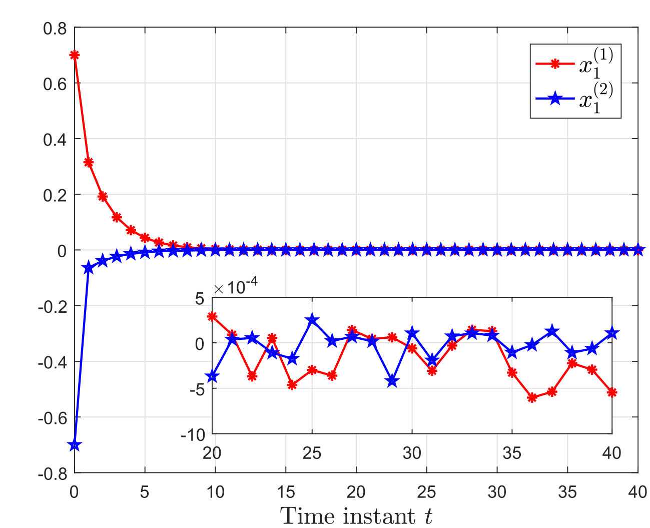

Fig. 6. State trajectory of $\mathcal{P}_1$ calculated by Algorithm 2.

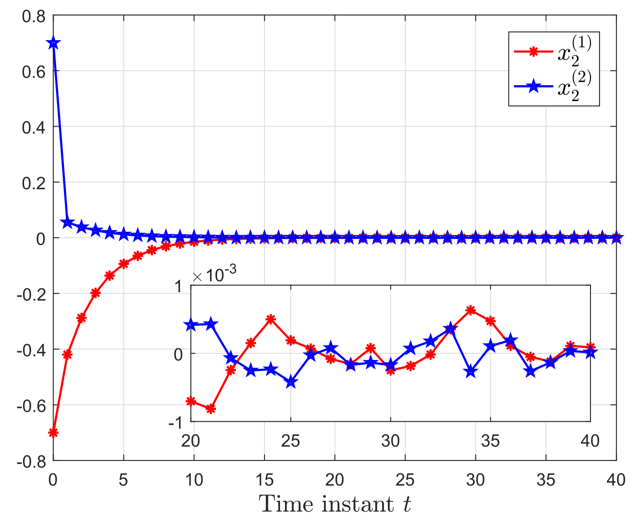

Fig. 7. State trajectory of $\mathcal{P}_2$ calculated by Algorithm 2.

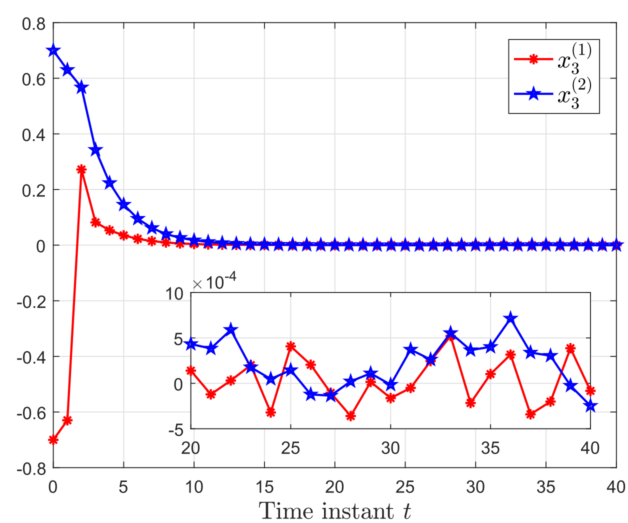

Fig. 8. State trajectory of $\mathcal{P}_3$ calculated by Algorithm 2.

and 2, the maximum disturbance bound of plant $\mathcal{P}_3$ is 5.44×10^{-4}, we set $\rho_3 = 5.44 \times 10^{-4}$.

The initial states of $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ are set to be $x_1(0) = (0.7 \ \ -0.7)^T$, $x_2(0) = (-0.7 \ \ \ 0.7)^T$, and $x_3(0) = (-0.7 \ \ \ 0.7)^T$, respectively. Fig. 5 shows the time instants of solving optimization problems for plants $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$ under Algorithm 2. As can be seen from Fig. 5, there is no conflicting transmissions. Figs. 6–8 show the system states of $\mathcal{P}_1$, $\mathcal{P}_2$, and $\mathcal{P}_3$, respectively.

V. Conclusion

In this paper, we presented algorithms of network scheduling and MPC co-design, in order to stabilize multi-loop NCS over shared bandwidth-limited communication networks. Recursive feasibility of the optimization problems and stability of all closed loops are guaranteed by appropriate conditions. The system states of all plants converge to a neighborhood of the origin without conflicting transmissions. The simulation results indicated that the proposed algorithms are effective.

REFERENCES

[1] P. Antsaklis and J. Baillieul, “Special issue on technology of networked control systems,” *Proc. IEEE*, vol. 95, no. 1, pp. 5–8, 2007.

[2] X. Zhang, Q. Han, and X. Yu, “Survey on recent advances in networked control systems,” *IEEE Trans. Ind. Inform.*, vol. 12, no. 5, pp. 1740–1752, Oct. 2016.

[3] R. A. Gupta and M. Y. Chow, “Networked control system: Overview and research trends,” *IEEE Trans. Ind. Electron.*, vol. 57, no. 7, pp. 2527–2535, Jul. 2010.

[4] W. P. M. H. Heemels, K. H. Johansson, and P. Tabuada, “An introduction to event-triggered and self-triggered control,” in *Proc. IEEE Conf. Decis. Control*, Maui, HI, USA, Dec. 10–13, 2012, pp. 3270–3285.

[5] K. Hashimoto, S. Adachi, and D. V. Dimarogonas, “Event-triggered intermittent sampling for nonlinear model predictive control,” *Automatica*, vol. 81, pp. 148–155, 2017.

[6] H. Li and Y. Shi, “Event-triggered robust model predictive control of continuous-time nonlinear systems,” *Automatica*, vol. 50, no. 5, pp. 1507–1513, 2014.

[7] K. Hashimoto, S. Adachi, and D. V. Dimarogonas, “Self-triggered model predictive control for nonlinear input-affine dynamical systems via adaptive control samples selection,” *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 177–189, Jan. 2017.

[8] H. Li, W. Yan, and Y. Shi, “Triggering and control co-design in self-triggered model predictive control of constrained systems: With guaranteed performance,” *IEEE Trans. Autom. Control*, vol. 63, no. 11, pp. 4008–4015, Nov. 2018.

[9] J. Finke, K. M. Passino, and A. G. Sparks, “Stable task load balancing strategies for cooperative control of networked autonomous air vehicles,” *IEEE Trans. Control Syst. Technol.*, vol. 14, no. 5, pp. 789–803, Sep. 2006.

[10] J. Behnamian and S. F. Ghomi, “A survey of multi-factory scheduling,” *J. Intell. Manuf.*, vol. 27, no. 1, pp. 231–249, 2016.

[11] S. Dai, H. Lin, and S. S. Ge, “Scheduling-and-control codesign for a collection of networked control systems with uncertain delays,” *IEEE Trans. Control Syst. Technol.*, vol. 18, no. 1, pp. 66–78, Jan. 2010.

[12] M. H. Mamduhi, A. Molin, and S. Hirche, “On the stability of prioritized error-based scheduling for resource-constrained networked control systems,” in *Proc. 4th IFAC Workshop Distrib. Estimation Control Networked Syst.*, 2013, pp. 356–362.

[13] M. H. Mamduhi, D. Tolić, A. Molin, and S. Hirche, “Event-triggered scheduling for stochastic multi-loop networked control systems with packet dropouts,” in *Proc. IEEE Conf. Decis. Control*, Los Angeles, CA, USA, Dec. 15–17, 2014, pp. 2776–2782.

[14] Y. Zhao, G. Liu, and D. Rees, “Integrated predictive control and scheduling co-design for networked control systems,” *IET Control Theory Appl.*, vol. 2, no. 1, pp. 7–15, Jan. 2008.

[15] E. Henriksson, D. E. Quevedo, E. G. Peters, H. Sandberg, and K. H. Johansson, “Multiple-loop self-triggered model predictive control for network scheduling and control,” *IEEE Trans. Control Syst. Technol.*, vol. 23, no. 6, pp. 2167–2181, Nov. 2015.

[16] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. Scokaert, “Constrained model predictive control: Stability and optimality,” *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.

[17] M. G. Forbes, R. S. Patwardhan, H. Hamadah, and R. B. Gopaluni, “Model predictive control in industry: Challenges and opportunities,” in *Proc. 9th Int. Symp. Adv. Control Chem. Processes*, 2015, pp. 531–538.

[18] K. Hashimoto, S. Adachi, and D. V. Dimarogonas, “A collision-free communication scheduling for nonlinear model predictive control,” in *Proc. 20th World Congress Int. Federation Autom. Control*, 2017, pp. 8939–8944.

[19] Z. Sun, L. Dai, Y. Xia, and K. Liu, “Event-based model predictive tracking control of nonholonomic systems with coupled input constraint and bounded disturbances,” *IEEE Trans. Autom. Control*, vol. 63, no. 2, pp. 608–615, Feb. 2018.