Energy-efficient sampling of networked control systems over IEEE 802.15.4 wireless networks

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Abstract

Self-triggered sampling is an attractive paradigm for closed-loop control over energy-constrained wireless sensor networks (WSNs) because it may give substantial communication savings. The understanding of the performance of self-triggered control systems when the feedback loops are closed over IEEE 802.15.4 WSNs is of major interest, since the communication standard IEEE 802.15.4 is the de-facto reference protocol for energy-efficient WSNs. In this paper, a new approach to control several processes over a shared IEEE 802.15.4 network by self-triggered sampling is proposed. It is shown that the sampling time of the processes, the protocol parameters, and the scheduling of the transmissions must be jointly selected to achieve a good performance of the closed-loop system and an energy-efficient utilization of the network. The challenging part of the proposed analysis is ensuring globally uniformly ultimately boundedness of the controlled processes while providing efficient scheduling of the process state transmissions. Such a scheduling is difficult when asynchronous multiple control loops share the network, because transmissions over IEEE 802.15.4 are allowed only at certain time slots. The proposed approach establishes that the joint design of self-triggered samplers and the network protocol 1) ensures globally uniformly ultimately boundedness of each control loop, 2) reduces the number of sensor transmissions, and 3) increases the sleep time of the transmitting nodes. A new dynamic scheduling problem is proposed for the joint control of each process and network protocol adaptation. An algorithm is derived, which adapts the network parameters according to the self-triggered sampler of every control loop. Numerical examples illustrate the analysis and show the benefits of the approach. It is concluded that self-triggered control strategies over WSNs ensure desired control performance, reduce the network utilization, and reduce energy consumption only if the protocol parameters are appropriately regulated.

Key words: Networked Control Systems, Self-Triggered Control, Wireless Sensor Networks, IEEE 802.15.4.

1 Introduction

Wireless Sensor Networks (WSNs) are composed by spatially distributed autonomous nodes with sensing, communication and computation functionalities. They provide self-organizing and fault tolerant functionalities, require low maintenance, and are supposed to be inexpensive and easy to deploy (Willig, 2008). Because of the benefits offered by such networks, Networked Control Systems (NCSs) over WSNs are being widely researched in many industrial and civilian applications including health care, smart grids, process control, etc. (Ploplys, Kawka and Alleyne, 2004). These benefits are achievable only if nodes of a WSN make a parsimonious use of energy, because they are powered by batteries or they harvest energy from the surrounding environment. Hence, the utilization of traditional wireless network protocols, such as for example the IEEE 802.11, where energy efficiency is not a primary issue, is impossible. To cope with the peculiarities of WSNs, the IEEE 802.15.4 networking protocol for Low Rate - Wireless Personal Area Networks (LR-WPANs) has been standardized in the last decade (IEEE 802.15.4, 2006). It is currently considered the reference networking standard for WSNs. Other communication protocols for WSNs, such as for example WirelessHART (*Wireless*HART *data sheet*, 2007) and ISA100 (ISA*100 Family of Standards*, 2009) are based on it. Nevertheless, there is not yet a systematic study of NCSs over such a protocol.

The design of NCSs over WSNs is often performed by adopting one of three general approaches: top-down, bottom-up

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or system-level design (Sangiovanni-Vincentelli, 2007). By following the first approach, the network is considered as a black box that introduces non-idealities, and the controller is designed by implicitly assuming that it does not have any influence on the network, see for example (Hespanha, Naghshtabrizi and Xu, 2007) and the references therein. Such approach has the drawback of using simple models of the network, whereby important constraints imposed by the protocols are often neglected, (Tabbara, Nesic and Teel, 2005; Tabbara, Nešič and Martins, 2008). By the second approach, the desired NCS performance is encoded in fixed specifications that must be fulfilled by the network. The design of the network is then performed according to the worst-case reliability and time delays. In this case, the network design is necessarily energy-inefficient because controllers can tolerate some degree of delays and losses, whereas high reliability and low time delays consume substantial energy. While in the top-down and the bottom-up approach control, network issues are decoupled, in the system-level design they are jointly considered. By following such an approach, a tradeoff between latency, packet loss, energy efficiency, and control performance can be found. New protocol stacks for WSNs, such as Breath (Park, Fischione, Bonivento, Johansson and Sangiovanni-Vincentelli, 2011) or TREnD (Di Marco, Park, Fischione and Johansson, 2010), have been recently developed to target such design objectives. Despite that several research consider the problem of designing IEEE 802.15.4 WSNs to ensure a certain level of reliability or a maximum time delays guarantee, (Park, Fischione and Johansson, 2010), (Di Francesco, Anastasi, Conti, Das and Neri, 2011), (Misic, Shafi and Misic, 2006), to the best of our knowledge (Tiberi, Fischione, Johansson and Di Benedetto, 2010; Tiberi, Fischione, Johansson and Di Benedetto, 2011) and the extension hereby presented, this is the first approach to the problem of system-level design of self-triggered control systems over IEEE 802.15.4 networks by placing on the same domain of analysis the dynamics of both the NCS and the protocol without any simplifying protocol assumptions.

In addition to packets dropouts, time delays, and congestions affecting NCS, in WSNs there is the problem of the sensing and energy efficiency. The radio operations, which include transmission, reception, and idle listening for messages (Texas-Instruments, 2007), give the largest contribution to the energy consumption of the wireless sensor nodes. More specifically, idle listening is the time duration in which a node keeps the radio active waiting for messages. It is alone the main cause of energy consumption (Shnayder, Hempstead, Chen, Allen and M., 2004). It follows that the reduction of the number of transmissions is not sufficient to achieve energy efficiency. To cope with energy wasting in NCSs, the strategies of event-triggered control (Tabuada, 2007; Heemels, Sandee and Van Den Bosch, 2008; Dimarogonas and Johansson, 2009; Wang and Lemmon, 2009*b*; Wang and Lemmon, 2008; Rabi, Johansson and Johansson, 2008; Henningsson, Johannensson and Cervin, 2008; Henningsson and Cervin, 2010), and self-triggered control (Velasco, Marti and Fuertes, 2003; Wang and Lemmon, 2009*a*; Anta and Tabuada, 2010; Anta and Tabuada, 2009; Araujo, Anta, Mazo, Faria, Hernandez, Tabuada and Johansson, 2011; Mazo, Anta and Tabuada, 2010; Mazo, Anta and Tabuada, 2009; Millán Gata, Orihuela, Muños de la Peña, Vivas and Rubio, 2011) have been recently proposed. They are based on sampling the state and actuating the control law only when it is needed. In the event-triggered case, the state of the system is constantly monitored, and a new sample is picked when a function of the state crosses a certain threshold. In the self-triggered case, the sampling occurs when a predicted evolution of a function of the state crosses such a triggering threshold. Thus the sampling is aperiodic and potentially leads to fewer transmissions between the process and the controller compared to conventional periodic sampling.

However, event-triggered control does not contribute to the reduction of the idle listening because nodes are enforced to keep the radio on for all the time to wait for the reception of the event-generated data. Self-triggered sampling might seem more appealing due to its predictive nature, compared to reactive event-triggered sampling, because it allows us to know in advance the next time by which the system must be sampled again. Then, between consecutive sampling instants, the network protocol can be adapted to save energy. However, while self-triggered sampling appears more suitable in a network context, it lacks robustness to uncertainties and disturbances. Since the determination of the next sampling is strictly connected to the model of the system, whenever there is a model change, it will be detected only at the next sampling instant. In this case, the controlled systems may exhibit undesirable behavior or they may even become unstable. To avoid this drawback, it is possible to design the self-triggered sampler by considering a more conservative model, but in this case the controlled system may result in a unnecessary oversampling. Hence, a proper design should provide an adaptation of the self-triggered sampler based on the detected model changes and disturbances.

In this paper, we consider a NCS composed of several control loops that share the same IEEE 802.15.4 network. We propose a distributed control strategy to reduce the energy expenditure of the network in terms of number of transmissions and idle listening periods, while ensuring globally uniformly ultimately boundedness (GUUB) of each control loop. According to the system-level design paradigm, we start our analysis from a higher level of abstraction and we refine the design as we move down the layers. We start by proposing a self-triggered sampler capable to ensure GUUB of linear systems, where the guaranteed minimum sampling interval depends on the controller and on the ultimate bound, the initial conditions, the maximum time delay introduced by the network, and the maximum amplitude of a possible external disturbance. The main difference of the proposed self-triggered sampler with respect to others proposed in literature is on the closed-loop specifications and the presence of the dynamics of a networking protocol, which have not been considered earlier. For example, the closed-loop specifications in (Wang and Lemmon, 2009*a*) are encoded in terms of the $\mathcal{L}_2$ gain, whereas in (Mazo et al., 2010) they are given in terms of a Lyapunov function decay rate. By contrast, here the closed-loop specifications are encoded through an ultimate bound. We show that the NCS performance parameter can be easily tweaked thanks to a new simple scalar inequality which captures the closed-loop system specifications and the guaranteed minimum inter-sampling interval. Robustness under external perturbations are also addressed. For instance, there are still few results which address the problem of robustness in self-triggered control. The work (Wang and Lemmon, 2009a) is limited to the case of self-triggered control with state dependent disturbance. Such work has been extended to the case of arbitrary disturbances in (Wang and Lemmon, 2010), but the result applies only to $\mathcal{H}_\infty$ controllers. The result in (Mazo et al., 2010) applies to any external disturbance and to any stabilizing controller, but it neglects the system behavior between the inter-sampling times. Moreover, in all the cited papers, information about the disturbance in the self-triggered sampler is never explicitly used. Here, an estimate of the disturbance is explicitly considered when computing the next sampling instant. This permits us to obtain less conservative inter-sampling times and to achieve a better system response, thus providing a benefit for both the network and the closed-loop response. Finally, we show that the design of a self-triggered sampler alone is insufficient for energy-efficient WSNs and that if the network dynamics are not considered, self-triggered sampling may not introduce any benefit. The IEEE 802.15.4 protocol does not allow to perform transmissions at any time, but poses several constraints on how and when communication can take place. We explicitly include the IEEE 802.15.4 protocol requirements in the design of the NCS, and we propose a dynamic network protocol adaptation to achieve energy saving, while meeting the control specifications. This approach enables us to establish a novel co-design of the IEEE 802.15.4 network and self-triggered control system.

The remainder of the paper is organized as follows: we introduce basic notation and preliminaries in Section 2, while in Section 3 we describe the IEEE 802.15.4 NCS architecture, and provide an overview of the IEEE 802.15.4 standard. We formally state the problem we aim to solve in Section 4, and in Section 5 we propose a self-triggered sampler to ensure GUUB of each control loop. This result is instrumental for the joint design of the controller and the network considered in Section 6 where we propose the system-level design of the IEEE 802.15.4 NCS. In Section 7 we validate our methodology by simulations. A discussion in Section 8 concludes the paper.

2 Notation and Preliminaries

Given a square matrix $M \in \mathbb{R}^{n\times n}$ we denote with $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ its minimum and maximum eigenvalues, respectively, and we say that M is positive definite ($M \succ 0$), if $v^T M v > 0$ for any $v \in \mathbb{R}^n$. Given a matrix $A \in \mathbb{R}^{n\times m}$, we denote $\|A\| := \sqrt{\lambda_{\max}(A^T A)}$. We indicate by $\|v\|$ the euclidean norm of a vector $v \in \mathbb{R}^n$, and by $\mathcal{B}_r := \{v : \|v\| \leq r\}$ the ball of radius r centered at the origin. For a signal $v : \mathbb{R}^+ \to \mathbb{R}^n$, we denote $\|v\|_{\mathcal{L}_p} := \left(\int_0^t \|v(s)\|^p ds\right)^{\frac{1}{p}}$, $p \in [1, +\infty)$, its $\mathcal{L}_p$-norm, with $\|v\|_{\mathcal{L}_\infty} := \operatorname{ess\,sup}_{t\in\mathbb{R}^+} \|v(t)\|$ its $\mathcal{L}_\infty$-norm and with $v_k := v(t_k)$ its realization at $t = t_k$. Given two consecutive times t_j and t_k, we denote with $\hat{v}_{k|j}$ an estimation of v_k based on measurements up till time t_j.

Given a system $\dot{x} = f(t, x)$, $x \in \mathbb{R}^n$, $x(t_0) = x_0$, $f : \mathbb{R}^+ \times \mathcal{D} \to \mathbb{R}^n$, where f is Lipschitz with respect to x and piecewise continuous with respect to t, and where $\mathcal{D} \subset \mathbb{R}^n$ is a domain that contains the origin, we say that the solutions are *Ultimately Uniformly Bounded* (UUB) if there exists three constants $a, b, T > 0$ independent of t_0 such that for all $\|x_0\| \leq a$ it holds that $\|x(t)\| \leq b$ for all $t \geq t_0 + T$, and *Globally Ultimately Uniformly Bounded* (GUUB) if the solutions are UUB for arbitrarily large a.

3 IEEE 802.15.4 NCS Architecture

We consider N independent controlled processes that share the same IEEE 802.15.4 network. We limit our attention to star topology networks, where the sensor nodes directly communicate with the central node and we consider one way feedback channel NCSs, in which there is bidirectional wireless communication only between the sensor nodes and the central node, see Fig. 1. Such architectures are highly relevant in many control applications, for instance in process industry (Samad, McLaughling and Lu, 2007; Tiberi, Lindberg and Isaksson, 2012). We assume that each sensor node is capable to measure the full state of the associated process, and we assume that the measurements are sent to the central node within a bounded time delay. The central node is wired to the controller nodes, and the controllers are wired to the actuators. We assume negligible time delay between a controller update instant and the corresponding actuator instant and we assume that the controller and the actuator of a given control loop is updated every time the controller receives a measurement from the associated sensor node. Finally we assume that the network is designed and operated according to the IEEE 802.15.4 standard (IEEE 802.15.4, 2006). The PANC Node in Fig. 1 represents the *Personal Area Network Coordinator* which is the central node that coordinates all the network operations. More details on the PANC will be given in Section 3. The PANC is directly connected to the controllers, while the nodes denoted by Node $1, \ldots,$ Node N are the sensing nodes directly connected to the processes. The dashed lines represent wireless connections, while the continuous lines represent wired connections.

3.1 Processes and controllers

The dynamics of every process is linear and of the form

$$\dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + d_i(t)\,, \tag{1}$$

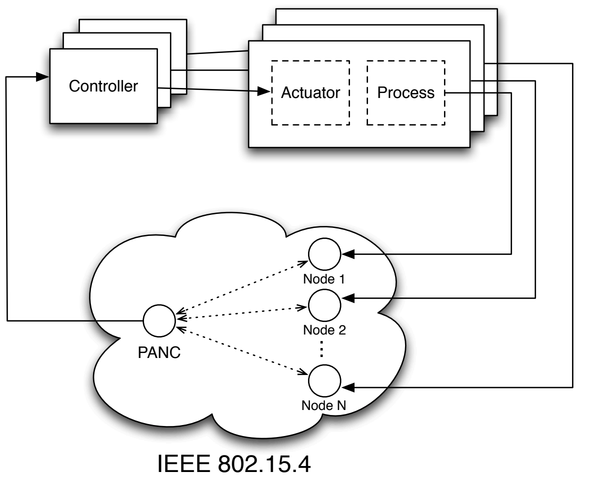

Fig. 1. A networked control system where a number of independent control loops transmit over a shared IEEE 802.15.4 network.

where $x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}, i = 1, \ldots, N$, are the states anc control, respectively, and $d_i \in \mathbb{R}^{n_i}$ is an external bounded non-measurable disturbance $\|d_i\| \leq \bar{d}$. We assume that for each control loop a controller of the form

$$u_i(t) = K_i x_i(t) \tag{2}$$

is designed so that the matrices $(A_i + B_i K_i)$ are Hurwitz for all i.

When feedback channels of several processes share a common network, the transmissions of measurements of process states cannot be continuous and istantaneous. We consider zero-order hold between two consecutive controller updates, such that the controller outputs can be written as

$$u_i(t) = K_i x_i(t_{i,k}) := K_i x_{i,k}\,, \tag{3}$$

for $t \in [t_{i,k} + \tau_{i,k}, t_{i,k+1} + \tau_{i,k+1})$, where $t_{i,k}$ is the time in which the k-th measurement of the i-th process is picked, and $\tau_{i,k}$ is the time elapsed between $t_{i,k}$ and the update instant of the corresponding controller. By using (3), (1) can be rewritten, for all $t \in [t_{i,k} + \tau_{i,k}, t_{i,k+1} + \tau_{i,k+1})$, and for all $k = 1, 2, \ldots$, as

$$\dot{x}_i(t) = (A_i + B_i K_i) x_i(t) + B_i K_i e_{i,k}(t) + d_i(t)\,, \tag{4}$$

where $e_{i,k}(t) = x_{i,k} - x_i(t)$ is the error due to the sampling. In the sequel, we assume initial delay $\tau_{i,0} = 0$ for all i and that $0 \leq \tau_{i,k} \leq \tau_{\max}$ for all i, k, where $\tau_{\max} > 0$ represent the maximum time delay introduced by the WSN.

3.2 *IEEE 802.15.4 protocol*

The IEEE 802.15.4 standard specifies the physical (PHY) and medium access control (MAC) layers of the protocol stack (IEEE 802.15.4, 2006). In each network there is a node, the PAN coordinator (PANC), that manages the operations of the entire network. We assume that the controllers are wired to the PANC. The 802.15.4 has two operating modes: the *unslotted* and the *slotted* communication mode. In the unslotted mode the nodes attempt to transmit packets according to the Carrier Sense Multiple Access/Collision Avoidance (CSMA/CA) algorithm all the time, while in the slotted mode, the nodes can transmit packets either according to CSMA/CA or according to time division multiple access (TDMA). In the slotted mode time is divided into *superframes*, which are time intervals bounded by special packets called *network beacons* sent by the PANC to all nodes of the network. The beacons contain information related to the setting of the incoming superframe. The superframe length is denoted *Beacon Interval* (BI) and satisfies

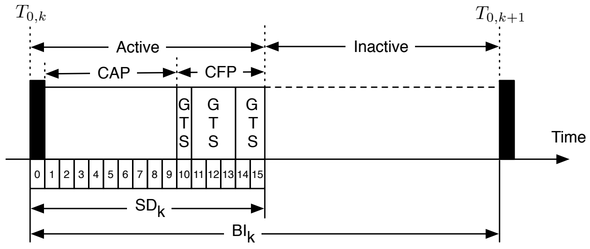

Fig. 2. Slotted IEEE 802.15.4 superframe time organization. The index $k \geq 0$ denotes the superframe k. BD_k denotes the superframe duration and BI_k denotes the beacon interval. $T_{0,k}$ is the time in which the superframe begins. During the inactive period, nodes sleep to save energy. IEEE 802.15.4 allows us to adapt the protocol parameters SD and BI to the needs of the NCS.

$$\mathrm{BI} = aBaseSuperFrameDuration \times 2^{\mathrm{BO}}\,, \tag{5}$$

where $0 \leq \mathrm{BO} \leq 14$ is called *Beacon Order* and $aBaseSuperFrameDuration$ is a parameter of the protocol fixed to 15.36 ms. By denoting $T_{0,k}$, the time in which the k-th superframe begins, we have $T_{0,k+1} - T_{0,k} = \mathrm{BI}_k$.

Fig. 2 illustrates the IEEE 802.15.4 superframe. The superframe is split into an *active* and an *inactive* period. The active period is the time interval when there can be transmissions of packets, while in the inactive period no communication is allowed and the nodes turn off the radio to save energy. The duration of the active period is called *Superframe Duration* (SD) and it is divided into 16 equally sized time slots. The SD is given by

$$\mathrm{SD} = aBaseSuperFrameDuration \times 2^{\mathrm{SO}}\,, \tag{6}$$

with $0 \leq \mathrm{SO} \leq 14$, where SO is called *Superframe Order*, and every time slot has duration $aBaseSlotDuration = \mathrm{SD}/16$. Notice that according to the IEEE standard $\mathrm{SO} \leq \mathrm{BO}$. By denoting with $T_{i,k}$ the time in which the node i performs a transmission during the k-th superframe, we further have the constraint $T_{0,k} < T_{i,k} < T_{0,k} + \mathrm{SD}_k$, where SD_k is the value of SD in the k-th supeframe. The values of SD_k and BI_k are included in the beacon packet sent by the PANC to all nodes at time $T_{0,k}$ and it cannot be modified until time $T_{0,k+1}$, i.e., until a new beacon packet is broadcast.

The active portion of the superframe is further divided in two parts: the *Contention Access Period* (CAP) and the *Contention Free Period* (CFP). During the CAP nodes contend to access the medium with the CSMA/CA algorithm, whereas in the CFP the PANC reserves *Guaranteed Time Slots* (GTSs) to nodes to transmit or receive data. Such GTSs are allocated by the PANC upon request from the nodes. A node can request the allocation of one or more GTSs, and during these time slots the node is allowed to communicate only with the PANC. The PANC can allocate maximum 7 GTSs in total in each superframe, and their scheduling is decided before the starting of the superframe. The PANC encapsulates the GTSs allocation, along with the setting of SO and BO in the beacon message. Notice that the decision about the superframe duration, the superframe length and the GTSs allocation are taken at the PANC during superframe k. The decisions and they will take effect only at the superframe $k+1$, when nodes receive the beacon. In the sequel we denote with $\omega_{i,k} \in \{0, 1, \dots, 7\}$ the time slot assigned to node i in superframe k and we assume that, every time a node is allocated, it performs a transmissions, and the associated controller is updated consequently. This means that if node i is allocated to every superframe, then after k superframes we have experienced k updates of controller i. If $\omega_{i,k} = 0$ then the node i does not have any time slot assigned in superframe k. Hence, if for a certain superframe k and a certain node i, $\omega_{i,k} \neq 0$, the time in which the node i performs a transmission in superframe k is given by

$$T_{i,k} = T_{0,k} + h_{\mathrm{CAP},k} + \omega_{i,k} \times aBaseSlotDuration\,,$$

where $h_{\mathrm{CAP},k}$ is the length of the CAP in superframe k.

A common measure of the energy efficiency of the network is given by the *duty cycle*, which is defined as

$$\mathrm{DC}_k = \frac{\mathrm{SD}_k}{\mathrm{BI}_k}\,. \tag{7}$$

For a fixed SD_k, a reduction of the duty cycle is achieved by enlarging BI_k. A reduction of the duty cycle leads to a reduction of the idle listening of the nodes, which is the main cause of energy consumption in WSNs. The *network utilization*, indicates how many nodes are allowed to transmit on the network during a superframe. We define the network utilization of the k-th superframe as the ratio of the available time slots in the k-th superframe to the used time slots in that superframe:

$$\mathrm{U}_k = \frac{\#\text{allocated GTS in superframe } k}{16}\,. \tag{8}$$

During the CAP, there is no control on the delay encountered by the packets before being transmitted, and there is no guarantee that the packets can be received successfully due to possible collisions. Therefore, in this paper, we limit our attention to the CFP. We assume that a node attached to a process is scheduled for transmission to one GTS, and, whenever a GTS is allocated, the associated node sends the full measurement to the PANC within a time slot duration $aBaseSlotDuration$. Since the PANC can allocate up to 7 GTSs per superframes, we assume that the maximum number of loops over the same network is $N \leq 7$. Furthermore, because of the simple network topology (star topology) and the utilization of the GTSs, we assume full reliability and bounded communication time delays with bound $\tau_{\max}$. We finally assume that a beacon is sent and received by all the nodes within a time equal to $aBaseTimeSlot$.

In the sequel we will show how to adapt $\mathrm{SO}_k, \mathrm{BO}_k$, and how to dynamically schedule the GTSs to reduce the average duty cycle, the number of transmissions of the nodes and to reduce the average network utilization while ensuring GUUB of each loop. To start with, we assume that the sensor nodes send their measurements to the PANC, which is in charge of performing all the computations. It is possible to distribute the computation as described in Section 6.

4 Problem Statement

The insertion of a WSN in the feedback channel of a control system introduces problems related to delayed information exchange between the sensors and the controller, and also the problem of the network energy consumption. Additional problems are introduced if the network is designed according to some specific protocol, which restricts how the communication among the nodes should be performed. The complexity in the design of the NCS further increases when the network is shared among several control loops with different requirements that should be accommodated based on the constraints imposed by the protocol.

In this paper we consider a NCS composed of several control loops that share a common IEEE 802.15.4 network. We aim at designing a control strategy to achieve a desired behavior of each loop and with energy efficient utilization of the network as stated in the following problem definition:

Problem 4.1 *Given the IEEE 802.15.4 NCS described in Section 3, we aim at*

(1) Designing a robust self-triggered sampler with respect to external disturbances to ensure GUUB of every closed-loop system;
(2) Reducing as much as possible the average duty cycle;
(3) Reducing as much as possible the number of transmissions of the nodes, i.e., reducing as much as possible the network utilization.

◁

To achieve these goals, we first propose a robust self-triggered sampler that ensures GUUB of the systems (4), where the minimum inter-sampling interval guarantee is a function of the size of the ultimate bound region, the maximum time delay, the size of the initial condition region, the maximum allowed inter-sampling time and the maximum value of the possible external disturbance. The self-triggered sampler employs a disturbance observer to achieve good reaction to disturbances without adding too much conservativeness to the inter-sampling times. Then, based on the response of the self-triggered sampler of each loop, we propose a decentralized control strategy to set $\mathrm{SO}_k, \mathrm{BO}_k$ and to dynamically schedule the GTSs allocation.

5 A Self-triggered Sampler

In this section we propose a self-triggered sampler that ensures GUUB of (4), where the next sampling time is determined by a static function of both the current and the previous measurement, the current measurement time delay and a disturbance estimate. Then, we derive a condition that ensures a certain minimum inter-sampling time guarantee based on the ultimate bound and the initial condition regions, the dynamics of the open loop and the closed loop system, the maximum time delay, the maximum disturbance and the maximum inter-sampling time. The idea we use to determine the next sampling instant is to predict when the next measurement of the state is at a distance δ from the current measurement. The difference of this sampling strategy and the Lebesgue sampling introduced in (Åström and Bernhardsson, 2002) is that Lebesgue sampling is performed in a reactive fashion, whereas the sampling rule here is performed in a predictive fashion. For the sake of notational simplicity, in the rest of this section we drop the index i of (4).

The next sampling instant given by the self-triggered control is obtained by exploiting a model of the system. We assume perfect knowledge of the pair (A, B) of every process, and we further assume that the external disturbance $d(t)$ is not measurable but we know an upper-bound. The model we use to design the self-triggered sampler, for $t \in [t_k+\tau_k, t_{k+1}+\tau_{k+1})$, has the following dynamics

$$\dot{\tilde{x}}(t) = (A+BK)\tilde{x}(t) + BK\tilde{e}_k(t) + \hat{d}_k\,, \qquad (9)$$

Where $\hat{d}_k$ is an estimate of the external disturbance $d(t)$ acting on the process.

To design our self-triggered sampler we need an upper bound of the measurement error $\|\tilde{e}_k\|$, where $\tilde{e}_k := x_k - \tilde{x}(t)$. Such an upper-bound is given by the following result.

Lemma 5.1 *Consider system* (4)*. Then, for* $t \in [t_k+\tau_k, t_{k+1}+\tau_{k+1})$*, the error* $\tilde{e}_k$ *is upper bounded as*

$$\|\tilde{e}_k\| \leq g(x_{k-1}, x_k, \hat{d}_{k-1}, \hat{d}_k, \tau_k)\,, \qquad (10)$$

where

$$\begin{aligned} g(x_{k-1}, x_k, \hat{d}_{k-1}, \hat{d}_k, \tau_k) &:= \exp(\|A\|(t-t_k-\tau_k)) \\ &\times \frac{\|Ax_k - BKx_{k-1}\| + \|\hat{d}_{k-1}\|}{\|A\|}(\exp(\|A\|\tau_k)-1) \\ &+ \frac{\|(A+BK)x_k\| + \|\hat{d}_k\|}{\|A\|}(\exp(\|A\|(t-t_k-\tau_k))-1) \end{aligned} \qquad (11)$$

◁

Proof: Since $\dot{\tilde{e}}_k = -\dot{x}$, we have that, for $t \in (t_k+\tau_k, t_{k+1}+\tau_{k+1})$

$$\frac{d}{dt}\|\tilde{e}_k\| = (\tilde{e}_k^T\,\tilde{e}_k)^{-\frac{1}{2}}\tilde{e}_k^T\,\dot{\tilde{e}}_k \leq \frac{\tilde{e}_k^T\,\dot{\tilde{e}}_k}{\|\tilde{e}_k\|} \leq \|\dot{\tilde{e}}_k\|\,.$$

It follows that

$$\begin{aligned} \|\dot{\tilde{e}}_k\| = \|-\dot{x}\| &= \|-Ax - BKx_k - \hat{d}_k\| \\ &= \|A(\tilde{e}_k - x_k) - BKx_k - \hat{d}_k\| \\ &\leq \|A\|\|\tilde{e}_k\| + \|(A+BK)x_k\| + \|\hat{d}_k\|\,. \end{aligned}$$

By using the Comparison Lemma (Khalil, 2002) we get the bound

$$\begin{aligned} \|\tilde{e}_k\| \leq& \exp(\|A\|(t-t_k-\tau_k))\|\tilde{e}_k(t_k+\tau_k)\| \\ &+ \frac{\|(A+BK)x_k\| + \|\hat{d}_k\|}{\|A\|} \\ &\times (\exp(\|A\|(t-t_k-\tau_k))-1)\,. \end{aligned} \qquad (12)$$

Next, we have to compute the value of $\|\tilde{e}_k(t_k+\tau_k)\|$. For $t \in (t_k, t_k+\tau_k)$ we have

$$\begin{aligned} \|\dot{\tilde{e}}_k\| = \|-\dot{x}\| &= \|-Ax - BKx_{k-1} - \hat{d}_{k-1}\| \\ &= \|A(\tilde{e}_k - x_k) - BKe_k - \hat{d}_{k-1}\| \\ &\leq \|A\|\|e_k\| + \|Ax_k - BKx_{k-1}\| + \|\hat{d}_{k-1}\|\,. \end{aligned} \qquad (13)$$

By taking into account that at the sampling instants $t = t_k$ it holds $\tilde{e}_k(t_k) = 0$, we have

$$\begin{aligned} \|\tilde{e}_k\| \leq& \frac{\|Ax_k - BKx_{k-1}\| + \|d_{k-1}\|}{\|A\|} \\ &\times (\exp(\|A\|(t-t_k))-1)\,, \end{aligned} \qquad (14)$$

for $t \in [t_k, t_k+\tau_k)$. Finally, because of the continuity of the error $\tilde{e}_k$ in $t = t_k+\tau_k$, by combining inequalities (12) and (14), inequality (10) follows. □

The idea behind the self-triggered sampler we propose, consists in predicting the time it takes for $\|\tilde{e}_k\|$ to go from $\|\tilde{e}_k(t_k+\tau_k)\|$ to δ. In this way, we can bound the error due to sampling $\tilde{e}_k$ through δ. However, it can happen that

$\hat{d}_{k-1} = \hat{d}_k = 0$ and $x_k = (0, \ldots, 0)^T$. Then the right-hand side of (10) is equal to zero. The consequence is that the next sampling instant goes to infinity (i.e., no more controller updates are performed). In this case, a disturbance $d(t)$ may drive the trajectories to infinity since no more controller updates are performed. Hence, an additional degree of freedom can be provided by upper-bounding the inter-sampling times with a certain $h_{\max} > 0$. By using such $h_{\max}$ we are able to prove the next result.

Theorem 5.1 *Consider the system* (4)*, let* $h_{\max}, \delta > 0$*, and consider the self-triggered sampler*

$$t_{k+1} = t_k + \min\{\gamma(x_{k-1}, x_k, \hat{d}_{k-1}, \hat{d}_k, \tau_k), h_{\max}\}\,, \quad (15)$$

where

$$\gamma(x_{k-1}, x_k, , \hat{d}_{k-1}, \hat{d}_k, \tau_k) := \frac{1}{\|A\|} \ln\left(\frac{\Psi(x_k, \hat{d}_k)}{\Xi(x_{k-1}, x_k, , \hat{d}_{k-1}, \hat{d}_k, \tau_k)}\right) + \tau_k - \tau_{\max}\,,$$

and where

$$\Psi(x_k, \hat{d}_k) := \|A\|\delta + \|(A+BK)x_k\| + \|\hat{d}_k\| \quad (16)$$

$$\begin{aligned}\Xi(x_{k-1}, x_k, \hat{d}_{k-1}, \hat{d}_k, \tau_k) :=& \Big(\|Ax_k - BKx_{k-1}\| + \|\hat{d}_{k-1}\|\Big)\\ &\times (\exp(\|A\|\tau_k) - 1)\\ &+ \|(A+BK)x_k\| + \|\hat{d}_k\|\end{aligned} \quad (17)$$

and for some fixed $\bar{d} > 0$ *such that* $\hat{d}_k$ *is such that* $\|\hat{d}_k\| \leq \bar{d}$ *for all* k*. Then, the closed-loop system is GUUB.* ◁

Proof: Consider the Lyapunov candidate $V(x) = x^T P x, P = P^T \succ 0$ such that $P(A+BK)^T + (A+BK)P = -Q, Q = Q^T \succ 0$. For $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$ the derivative of the Lyapunov candidate function along the trajectories of (4) satisfies

$$\begin{aligned}\dot{V} &\leq -x^T Q x + 2x^T P(BKe_k + d)\\ &= -x^T Q x + 2x^T P\big(BK(\tilde{e}_k + \tilde{x} - x) + d\big)\\ &\leq -\lambda_{\min}(Q)\|x\|^2 + 2\|P\|\|x\|\Big(\|BK\|(\|\tilde{e}_k\| + \|\tilde{x} - x\|) + \|d\|\Big)\\ &\leq -\lambda_{\min}(Q)\|x\|^2 + 2\|P\|\|x\|\\ &\quad \Big(\|BK\|(g(x_{k-1}, x_k, \hat{d}_{k-1}, \hat{d}_k, \tau_k) + 2\bar{d}(t - t_k - \tau_k)) + \bar{d}\Big)\,.\end{aligned} \quad (18)$$

By exploiting the continuity of V, under sampling rule (15) we can further upper bound $\dot{V}$ as

$$\dot{V} \leq -\lambda_{\min}(Q)\|x\|^2 + 2\|P\|\|x\|(\|BK\|(\delta + 2\bar{d}\,h_{\max}) + \bar{d})\,, \quad (19)$$

for all $t \geq t_0 + \tau_0 = t_0$. Now, pick any $\vartheta \in (0,1)$. We can rewrite (19) as

$$\begin{aligned}\dot{V} \leq& -(1-\vartheta)\lambda_{\min}(Q)\|x\|^2 - \vartheta\lambda_{\min}(Q)\|x\|^2\\ &+ 2\|P\|\|x\|(\|BK\|(\delta + 2\bar{d}\,h_{\max}) + \bar{d})\,.\end{aligned} \quad (20)$$

We have $\dot{V} < -(1-\vartheta)\|x\|^2$ if

$$\|x\| > \frac{2\|P\|(\|BK\|(\delta + 2\bar{d}\,h_{\max}) + \bar{d})}{\vartheta\lambda_{\min}(Q)} := \mu\,. \quad (21)$$

For the chosen Lyapunov candidate function it holds that $\lambda_{\min}(P)\|x\|^2 \leq V(x) \leq \lambda_{\max}(P)\|x\|^2$, so the system is GUUB with ultimate bound (Khalil, 2002)

$$b = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\mu\,. \quad (22)$$

□

The ultimate bound (22) can be conservative. A tighter ultimate bound can be achieved by exploiting the BIBO stability property of the sampled-data system. Under the sampling rule (15), the system (4) can be rewritten as $\dot{x} = (A+BK)x + BK(\tilde{e}_k + (x-\tilde{x})(t-t_k-\tau_k)) + d$, for $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$. Since the sampling rule enforces all the perturbations acting on the process to be bounded, i.e., $\|e_k\| \leq \delta$, $\|d(t)\| \leq \bar{d}$ and $\|x - \tilde{x}\| \leq 2\bar{d}h_{\max}$ for all $t \geq t_0$, and since $(A+BK)$ is Hurwitz and $x(t)$ is continuous, it follows that the closed-loop system is BIBO. Hence, we have $\|x\|_{\mathcal{L}_\infty} \leq \|\Phi_{cl}x_0\|_{\mathcal{L}_\infty} + \|\mathcal{H}\|_{\mathcal{L}_1}(\delta + 2\bar{d}h_{\max} + \|BK\|^{-1}\bar{d})$, where $\Phi_{cl} := \exp((A+BK)(t-t_0))$ and $\mathcal{H} := \exp((A+BK)(t-t_0))BK$ are the state transition and the impulse responses matrices of (4), respectively (Boyd and Barrat, 1991). Hence, a less conservative ultimate bound of (4) under the sampling rule (15) is given by $b = \|\mathcal{H}\|_{\mathcal{L}_1}(\delta + 2\bar{d}h_{\max} + \bar{d})$. Notice how by enlarging δ we also enlarge the ultimate bound b, but we also enlarge the inter-sampling times through (15). Hence, δ can be interpreted as a design parameter that encodes a tradeoff between the inter-sampling times and the deviation of the trajectories from the origin. Note that the sampling rule obviously does not change the Hurwitz property of the matrix $(A+BK)$.

Remark 5.1 *The stability property of the closed loop system under the sampling rule* (15) *is independent of the particular choice of the disturbance observer, but it depends on the maximum difference* $\|\hat{x} - x\|$*, which is bounded as* $\|\hat{x} - x\| \leq \bar{d}(t - t_k - \tau_k)$ *for* $t \in [t_k + \tau_k, t_{k+1} + \tau_{k+1})$*, see Figure 3. This means that we can even set* $\hat{d}_k = 0$ *for all* k*, thus obtaining fairly large inter-sampling times, and still achieving GUUB. However, in doing that, we may experience big peaks of the state trajectories if a disturbance*

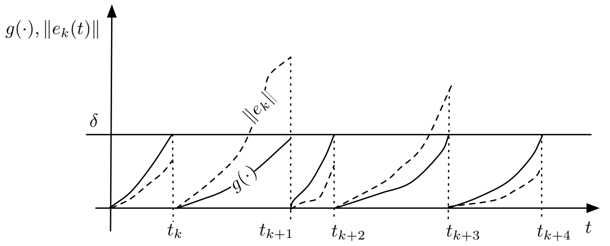

Fig. 3. The continuous line represents $g(x_{k-1}, x_k, \hat{d}_{k-1}, \hat{d}_k, \tau_k, t)$, while the dashed line represents the norm of the error $\|e_k\|$. The error estimate norm is always bounded by δ, while the error norm $\|e_k(t)\|$ can assume different values at the sampling instants $t = t_k$, depending on the disturbance $d(t)$. However, since the external disturbance $d(t)$ is bounded, then $\|e_k\|$ is also bounded.

actually enters the process, since the next time in which the system will be sampled again can be very far, and in the meanwhile the disturbance may steer the trajectories far from the origin. Alternatively, we can use a model by considering a worst-case disturbance acting over all the time, as done in (Tiberi et al., 2010). Nevertheless, the resulting self-triggered sampler would be too conservative if there are no disturbances acting on the process. Hence, the utilization of a disturbance observer permits to get tradeoff between the conservativeness of the inter-sampling times and the reactiveness to external disturbances of the controlled system. This aspect will be illustrated in the simulations in Section 7. ◁

Remark 5.2 *It is well known that a stable system can be destabilized if the sampling period of the controller is too large. This can be easily seen using our framework. If we fix a constant sampling period h, we would have that the realization of $\|e_k\|$ that corresponds to the sampling instants $t = t_k + jh, j = 1, 2, \dots,$ is not constant and equal to δ, but it will generate a sequence δ_k that represents a perturbation to the system. For sufficiently large values of h, it is easy to prove that the sequence δ_k diverges, leading to instability, while for small values of h the sequence δ_k converges to zero. In our scheme, we are instead fixing δ, which ensures BIBO stability, but it necessarily gives varying inter-sampling as per rule* (15). ◁

Theorem 5.1 does not give any information about the minimum inter-sampling time guaranteed by the self-triggered sampler (15). In addition, if $\|e_k(t_k + \tau_k)\| \geq \delta$ for some k, we would have $t_{k+1} \leq 0$. Nevertheless, since communication protocols impose a constraint on the minimum inter transmission time $h_{\min} > 0$, it is worth to find out under which condition the proposed self-triggered sampler gives $t_{k+1} - t_k > h_{\min} > 0, \forall k$. Such condition is given by the next result.

Proposition 5.1 *Consider the system* (4) *under the sampling rule* (15)*. Let* $0 \leq \tau_k \leq \tau_{\max}, 0 < h_{\min}$ *and* $M(\delta, x_0) = \|\Phi x_0\|_{\mathcal{L}_\infty} + \|\mathcal{H}\|_{\mathcal{L}_1}(\delta + 2\bar{d}h_{\max} + \bar{d})$*. If*

$$\begin{aligned}\delta > \|A\|^{-1}\Big(&((\|A\| + \|BK\|)M(\delta, x_0) + \bar{d})\\ &\times (\exp(\|A\|\tau_{\max}) - 1)\exp(\|A\|h_{\min})\\ &+ (\|A + BK\|M(\delta, x_0) + \bar{d})\\ &\times (\exp(\|A\|h_{\min}) - 1)\Big),\end{aligned} \tag{23}$$

then system (4) *is GUUB and it holds that* $t_{k+1} - t_k > h_{\min} > 0$ *for all* k. ◁

Proof: A necessary and sufficient condition to have $\gamma(x_{k-1}, x_k, \hat{d}_{k-1}, \hat{d}_k, \tau_k) > 0$ for all k is

$$\|A\|\delta > (\|Ax_k - BKx_{k-1}\| + \|\hat{d}_{k-1}\|)(\exp(\|A\|\tau_k) - 1) > 0\,, \tag{24}$$

whereby, maximizing τ_k and $\|d_{k-1}\|$, we derive the sufficient condition

$$\|A\|\delta > (\|A\| + \|BK\| + \bar{d})M(\delta, x_0)(\exp(\|A\|\tau_{\max}) - 1)\,. \tag{25}$$

Now, let us define

$$\tilde{\gamma}(x_{k-1}, x_k) := \frac{1}{\|A\|}\ln\left(\frac{\tilde{\Psi}(x_k)}{\tilde{\Xi}(x_{k-1}, x_k)}\right). \tag{26}$$

where

$$\tilde{\Psi}(x_k) := \|A\|\delta + \|(A + BK)\|\|x_k\| + \bar{d}\,, \tag{27}$$

$$\begin{aligned}\tilde{\Xi}(x_{k-1}, x_k) :=& (\|A\|\|x_k\| + \|BK\|\|x_{k-1}\| + \bar{d})\\ &\times \left(e^{\|A\|\tau_{\max}} - 1\right) + \|(A + BK)\|\|x_k\| + \bar{d}\,.\end{aligned} \tag{28}$$

If condition (25) holds, then $0 < \tilde{\gamma}(x_{k-1}, x_k) \leq \gamma(x_{k-1}, x_k, \hat{d}_{k-1}, \hat{d}_k, \tau_k)$ for all k. Hence, to find the guaranteed minimum inter-sampling interval given by (15), it is enough to solve the following optimization problem

$$\min_{x_k, x_{k-1}} \quad \tilde{\gamma}(x_{k-1}, x_k) \tag{29}$$

$$\begin{aligned}\text{s.t. } &\|x_k\| \leq M(\delta, x_0)\\ &\|x_{k-1}\| \leq M(\delta, x_0)\,.\end{aligned} \tag{30}$$

The objective function is monotonically decreasing on the decision variables over the domain specified by the constraints. It follows that this is a so called Fast-Lipschitz optimization problem (Fischione, 2011). The minimum is achieved for $\|x_{k-1}\| = \|x_k\| = M(\delta, x_0)$. By imposing that the objective function at optimum greater than $h_{\min}$, inequality (23) follows. □

The self-triggered sampler (15) encodes a tradeoff between inter-sampling times, maximum time delay, ultimate bound region, the maximum inter-sampling time, the maximum disturbance and the set of initial conditions. Given a system with a maximum time delay and a maximum external disturbance, we can tune the parameters δ and $h_{\max}$ to fulfill condition (23). However, while an increase of δ or $h_{\max}$ increases the inter-sampling times, it also increases the ultimate bound region.

Remark 5.3 *For given $A, B, K, \bar{d}, \delta$ and a desired $h_{\min}$, it is possible to compute the maximum allowable time delay to ensure $t_{k+1} - t_k > h_{\min}$ for all k by computing the inverse of* (23). *We have*

$$\tau_{\max} < \frac{1}{\|A\|} \ln \left(1 + \frac{G_1}{G_2}\right) , \tag{31}$$

where

$$G_1 := \|A\|\delta - (\|(A+BK)\|M(\delta, x_0) + \bar{d})(\exp(\|A\|h_{\min}) - 1)) \tag{32}$$

$$G_2 := \big((\|A\| + \|BK\|)M(\delta, x_0) + \bar{d}\big) \exp(\|A\|h_{\min}) \tag{33}$$

The right hand side of the previous inequality is a positive real number if, and only if, $G_1 > G_2$, and then, if, and only if $\|A\|\delta > (\|(A+BK)\|M(\delta, x_0) + \bar{d})(\exp(\|A\|h_{\min}) - 1))$. If such a condition is not verified, then condition (23) *is not verified even for $\tau_{\max} = 0$. This means that the current choice of $h_{\min}$ may be too large or δ may be to small. It is however possible to tweak the parameters δ and $h_{\min}$, or to choose a different controller K so that* (23) *is verified and $t_{k+1} - t_k > h_{\min}$ for all k.* ◁

Remark 5.4 *Since the condition* (23) *requires a bound on the initial conditions set, it seems that it is possible to achieve UUB but not GUUB. However, the bound on the initial condition required by* (23) *affects only the minimum inter-sampling time guarantee, but not the stability property of the closed-loop system as in Theorem 5.1.* ◁

Remark 5.5 *If $h_{\max}$ is chosen so that it is possible to stabilize the system with a periodic implementation with period $h_{\max}$, then it would be possible to experience less transmissions with respect to the proposed self-triggered sampler. However, it is well known that the utilization of a large sampling period would lead to a questionable system response (oscillating behavior, long transients, bad disturbance rejection, etc). On the other hand, self-triggered control is able to enlarge the sampling intervals when nothing relevant is happening, and it is also able to shrink them when it is needed, ensuring both a good system response and a good network utilization.* ◁

Finally, we design a disturbance observer. A simple disturbance observer can be designed by considering the model (9) and three consecutive measurements. For instance, given three measurements x_{k-2}, x_{k-1} and x_k at times $t = t_{k-2}, t = t_{k-1}$ and $t = t_k$ respectively, we consider a deadbeat observer given by

$$\begin{aligned}\hat{d}_k = &\Big(\Phi(t_k - t_{k-1} - \tau_{k-1})\Gamma(\tau_{k-1})\Big)^{-1} \\ &\times\Big(x_k - (\Phi(t_k - t_{k-1}) \\ &\quad - BK\Gamma(t_k - t_{k-1} - \tau_{k-1}))x_{k-1} \\ &\quad - \Phi(t_k - t_{k-1} - \tau_{k-1})BK\Gamma(\tau_{k-1})x_{k-2}\Big) ,\end{aligned} \tag{34}$$

where

$$\Phi(s) := \exp(As) , \tag{35}$$

$$\Gamma(s) := \int_0^s \exp(A(s-\sigma))d\sigma . \tag{36}$$

The disturbance observer (34) gives a first-order estimate of a constant disturbance acting for $t \in [t_{k-1}, t_k)$. The utilization of a disturbance observer does not affect the stability property of the closed-loop system, as we shown in Theorem 5.1. However, we use such observer with the only purpose of achieving a trade-off between the conservativeness of the inter-sampling times and the reactiveness of the closed-loop system with respect to external disturbances.

Remark 5.6 *The computation of $\hat{d}_k$ requires the invertibility of the matrix $\Phi(t_k - t_{k-1} - \tau k - 1)\Gamma(\tau_{k-1})$. If such a matrix is not invertible in the current coordinates, we can always find a coordinate transform T so that $T\Phi(\cdot)\Gamma(\cdot)T^{-1}$ is invertible, and estimate the disturbance $\hat{d}_k$ in the new coordinates.* ◁

Based on the self-triggered sampler (15), in the next section we show how to solve Problem 4.1. We assume that for every controlled process of the NCS a self-triggered sampler of the form (15) is available, and that there exists a proper value of δ and $h_{\max}$ to ensure $t_{k+1} - t_k > \mathrm{BI}_{\min}$ for every loop.

6 Energy-Efficient IEEE 802.15.4 NCS

In this section we investigate how the protocol parameters are selected and adapted to solve Problem 4.1. The protocol adaptation policy will be presented in two steps: first we show how to choose SO_k and BO_k assuming that all the nodes perform a transmission in each superframe. Then, we remove this assumption by proposing a dynamic GTS scheduling policy based on adaptive superframes.

Since the nodes are constrained to perform transmissions only at time $T_{i,k}$, the self-triggered samplers (15) can be rewritten as

$$t_{i,k+1} = T_{i,k} + \min\{\gamma_i(x_{i,k-1}, x_{i,k}, \hat{d}_{i,k-1}, \hat{d}_{i,k}, \tau_{i,k}), h_{i,\max}\}, \quad (37)$$

and the disturbance observer (34) becomes

$$\begin{aligned}\hat{d}_{i,k} =& \Big(\Phi_i(T_{i,k} - T_{i,k-1} - \tau_{i,k-1})\Gamma_i(\tau_{i,k-1})\Big)^{-1} \\ &\times \Big(x_k - (\Phi_i(T_{i,k} - T_{i,k-1}) \\ &\quad - B_i K_i \Gamma_i(T_{i,k} - T_{i,k-1} - \tau_{i,k-1}))x_{i,k-1} \\ &\quad - \Phi_i(T_{i,k} - T_{i,k-1} - \tau_{i,k-1}) \\ &\quad \times B_i K_i \Gamma_i(\tau_{i,k-1}) x_{i,k-2}\Big). \end{aligned} \quad (38)$$

In the sequel we use the notation $x_{i,k}$ to indicate $x_{i,k} = x_i(T_{i,k})$.

6.1 *Superframe duration and superframe length adaptation*

In this subsection we show how to set SO_k and BO_k at each superframe to achieve duty cycle reduction, under the assumption that in every superframe all the nodes are allocated to a certain time slots, i.e. $\omega_{i,k} \neq 0$ for all i, k. Provided that at a certain time $t \in [T_{0,k}, T_{0,k+1})$, all the self-triggered sampler responses are large enough, a variation of the duty cycle can be obtained by setting the $(k+1)$-th superframe BO_{k+1} such that $t_{i,k+1} \leq T_{i,k+1}$ for all i, which means that all the nodes will perform a transmission before the deadlines $t_{i,k+1}$, thus ensuring GUUB.

Unfortunately, the IEEE 802.15.4 standard does not allow us changing the k-th superframe setting at time $t \in [T_{0,k}, T_{0,k+1})$. Nevertheless, at time $t \in [T_{0,k}, T_{0,k+1})$ it is possible to decide the structure of the $(k+1)$-th superframe and to encapsulate this information in the next beacon packet. Therefore, we use the last measurement of process i $x_{i,k}$ to obtain an estimate $\hat{x}_{i,k+1}$ of the next measurement picked in a certain time slot $\omega_{i,k+1}$ in superframe $k+1$:

$$\begin{aligned}\hat{x}_{i,k+1} :=& \big(\Phi_i(T_{i,k+1} - T_{i,k}) \\ &+ B_i K_i \Gamma_i(T_{i,k+1} - T_{i,k} - \tau_{i,k})\big)x_{i,k} \\ &+ \Gamma_i(T_{i,k+1} - T_{i,k} - \tau_{i,k})\tilde{d}_{i,k} \\ &+ \Phi_i(T_{i,k+1} - T_{i,k} - \tau_{i,k}) \\ &\times \big(B_i K_i \Gamma_i(\tau_{i,k}) x_{i,k-1} + \Gamma_i(\tau_k)\tilde{d}_{i,k}\big), \end{aligned} \quad (39)$$

where $\tilde{d}_{i,k} = 0$ if we do not use any disturbance observer, $\tilde{d}_{i,k} = \hat{d}_{i,k}$ if we are using the disturbance observer, or $\tilde{d}_{i,k} = \bar{d}$ the worst-case disturbance. Note that $\hat{x}_{i,k+1}$ is a function of the next time slot $\omega_{i,k+1}$ because during superframe k is not known yet in which time slot node i will be allocated in the superframe $k+1$. We consider then the self-triggered sampler (37) in a predictor form as

$$\hat{t}_{i,k+2} = \min_{\omega_{i,k+1}} \{T_{i,k+1} + \min\{\gamma_i(x_{i,k}, \hat{x}_{i,k+1}, \tilde{d}_{i,k-1}, \tilde{d}_{i,k}, \tau_{\max}), h_{i,\max}\}\}. \quad (40)$$

We wish to remark, that in case we want to consider the worst-case disturbance for the next measurement estimate $\hat{x}_{i,k+1}$, we have to solve the following optimization problem

$$\min_{\omega_{i,k+1}, \tilde{d}_{i,k}} \{T_{i,k+1} + \min\{\gamma_i(x_{i,k}, \hat{x}_{i,k+1}, \tilde{d}_{i,k-1}, \tilde{d}_{i,k}, \tau_{\max}), h_{i,\max}\}\}. \quad (41)$$

It is possible first to minimize with respect to $\tilde{d}_{i,k}$, and then with respect to $\omega_{i,k+1}$. For a given a $\omega_{i,k+1}$, and according to (15), it follows that solving the minimization problem

$$\min_{\tilde{d}_{i,k}} \gamma_i(x_{i,k}, \hat{x}_{i,k+1}, \tilde{d}_{i,k-1}, \tilde{d}_{i,k}, \tau_{\max})$$

is equivalent to solve the maximization problem

$$\max_{\tilde{d}_{i,k}} \|(A_i + B_i K_i)\hat{x}_{i,k+1} + \tilde{d}_{i,k}\|. \quad (42)$$

By inserting (39) into (42), it is easy to see that the maximum is achieved at some point d_i^* on the boundary of $\mathcal{B}_{\bar{d}_i}$. Then, when assuming the worst-case disturbance, (41) reduces to

$$\hat{t}_{i,k+2} = \min_{\omega_{i,k+1}} \{T_{i,k+1} + \min\{\gamma_i(x_{i,k}, \hat{x}_{i,k+1}, d_i^*, d_i^*, \tau_{\max}), h_{\max}\}\}. \quad (43)$$

We are now in position to determine an energy-efficient setting of the protocol parameters SO_k and BO_k as summarized in the following result.

Theorem 6.1 *Consider the NCS over IEEE 802.15.4 as described in Section 3. Suppose that a state estimator of the form* (39) *and a self-triggered sampler of the form* (41) *for each control loop has been designed. Assume that* $\omega_{i,k} \neq 0$ *for all* i, k *and that condition* (23) *is satisfied with* $h_{i,\min} > \mathrm{BI}_{\min} > 0$ *for all* i. *Finally, let* $h_{\min} = \min_i h_{i,\min}$, $h_{\max} = \max_i h_{i,\max}$ *and* $\mathrm{BI}_{\max} \leq h_{\max}$. *Then, by setting*

$$\mathrm{SO}_k = \left\lfloor \log_2 \frac{h_{\min}}{aBaseSuperFrameDuration} \right\rfloor \forall k, \quad (44)$$

and by adapting the BO_{k+1} *with*

$$\mathrm{BO}_{k+1} = \min\left\{\widehat{\mathrm{BO}}_{k+1}, \mathrm{BO}_{\max}\right\}, \quad (45)$$

where

$$\widehat{\mathrm{BO}}_{k+1} := \left\lfloor \log_2 \frac{\hat{t}_{k+2} - (T_{0,k+1} + aBaseSlotDuration + SD)}{aBaseSuperFrameDuration} \right\rfloor \quad (46)$$

where $\hat{t}_{k+2} = \min_i \hat{t}_{i,k+2}$ *all the control loops are GUUB.* ◁

Proof: Since the standard imposes $\mathrm{SD}_k \leq \mathrm{BI}_k$ for all k, the minimum inter-transmission interval allowed by the standard is obtained for $\mathrm{BI}_k = \mathrm{SD}_k$, that in (44) is imposed to be smaller than the minimum inter sampling time required by each self-triggered samplers. Hence, the minimum inter sampling time requirement of all the self-triggered samplers meets the standard specifications.

GUUB of each loop is ensured if we adapt BI_{k+1}, so that $\hat{t}_{i,k+2} \geq T_{i,k+1} + \mathrm{BI}_{k+1}$ for all i. Since $\hat{t}_{k+2} = \min_i \hat{t}_{i,k+2}$, $T_{i,k+1} \leq T_{0,k+1} + \mathrm{SD}_k$ for all i, and since we assumed that the beacon packet takes one time slot, all the deadlines imposed by the self-triggered samplers are met if BI_{k+1} is adapted so that $\hat{t}_{k+2} \geq (T_{0,k} + aBaseSlotDuration + \mathrm{SD}_{k+1}) + \mathrm{BI}_{k+1}$. By using the previous inequality in combination with (5), then (45) follows. □

When the state of all the control loops are close to the origin, we have an enlargement of the intersampling times, and then an enlargement of the beacon interval that ends up in a reduction of the duty cycle and then a reduction of the idle listening. Theorem 6.1 describes how to set SO_k, and how to dynamically adapt BO_k to achieve a reduction of the duty cycle. If the self-triggered response of the control loop i becomes too large compared to the other self-triggered responses, then the i-th control loop is oversampled since we assumed $\omega_{i,k} \neq 0$, for all i, k. In the next section we show how to dynamically schedule the GTSs to achieve further communication savings by avoiding such unnecessary oversamplings. This means that the released time slots can be used by other nodes that implement non-time critical applications, such as monitoring applications. Hence, for a given network utilization, a proper GTSs scheduling increases the amount of information flowing through the network.

6.2 GTS scheduling

In the previous subsection we have established how to set the superframe duration and how to adapt the superframe length to reduce the idle listening of the nodes, but we also pointed out how some control loop could be oversampled due to a fixed time slot allocation. Now, the next step is to determine how time slots in the CFP are scheduled to reduce the number of transmissions from the nodes to the PANC and to reduce the network utilization.

The GTS scheduling we propose is based on the following idea. If there is a slow control loop that rises large inter sampling times, there is no need to reserve it a time slot in every superframe, but its time slot can be deallocated and reallocated later on. Since now time slots can be deallocated, it is useful to track when the last transmission has been performed. Hence, we denote the last superframe in which node i performed its last transmission

$$\ell_i(k) = \max_{0 \leq j \leq k} \{j : \omega_{i,j} \neq 0\} .$$

According to the above definition, $\ell_i(k)$ gives the last superframe in which node i has been allocated, $\ell_i(\ell_i(k) - 1)$ gives the second last superframe in which node i has been allocated, $\ell_i((\ell_i(\ell_i(k) - 1)) - 1)$ gives the third last superframe and so on.

Clearly, in the case in which all the nodes perform a transmission in every superframe, then all the controller would have experienced k updates after k superframes, and it would hold $\ell_i(k) = k$ and $\ell_i(\ell_i(k) - 1) = k - 1$ for all i, k. Nevertheless, the protocol parameter settings (37) and (44) are still valid, provided that the disturbance observer, the state estimator and the next time estimator take into account of when the nodes perfomed their last transmissions, and are thus modified as

$$\begin{aligned}
&\hat{d}_{i,\ell_i(k)} = \\
&\Big(\Phi_i(T_{i,\ell_i(k)} - T_{i,\ell_i(\ell_i(k)-1)} - \tau_{i,\ell_i(\ell_i(k)-1)}) \\
&\quad \times \Gamma_i(\tau_{i,\ell_i(\ell_i(k)-1)})\Big)^{-1} \\
&\times \Big(x_{i,\ell_i(k)} - (\Phi_i(T_{i,\ell_i(k)} - T_{i,\ell_i(\ell_i(k)-1)}) \\
&\qquad - B_i K_i \Gamma_i(T_{i,\ell_i(k)} - T_{i,\ell_i(\ell_i(k)-1)} - \tau_{i,\ell_i(\ell_i(k)-1)})) \\
&\qquad \times x_{i,\ell_i(\ell_i(k)-1)} \\
&\qquad - \Phi_i(T_{i,\ell_i(k)} - T_{i,\ell_i(\ell_i(k)-1)} - \tau_{i,\ell_i(\ell_i(k)-1)}) \\
&\qquad \times B_i K_i \Gamma_i(\tau_{i,\ell_i(\ell_i(k)-1)}) x_{i,\ell_i((\ell_i(\ell_i(k)-1))-1)}\Big) ,
\end{aligned} \quad (47)$$

$$\begin{aligned}
\hat{x}_{i,k+1|\ell_i(k)} :=& \big(\Phi_i(T_{i,k+1} - T_{i,\ell_i(k)}) \\
&+ \Gamma_i(T_{i,k+1} - T_{i,\ell_i(k)} - \tau_{i,\ell_i(k)})\big) x_{i,\ell_i(k)} \\
&+ \Gamma_i(T_{i,k+1} - T_{i,\ell_i(k)} - \tau_{i,\ell_i(k)}) \tilde{d}_{i,\ell_i(k)} \\
&+ \Phi_i(T_{i,k+1} - T_{i,\ell_i(k)} - \tau_{i,\ell_i(k)}) \\
&\times \big(B_i K_i \Gamma_i(\tau_{i,\ell_i(k)}) x_{i,\ell_i(\ell_i(k)-1)} \\
&\qquad + \Gamma_i(\tau_{\ell_i(k)}) \tilde{d}_{i,\ell_i(\ell_i(k)-1)}\big) ,
\end{aligned} \quad (48)$$

and

$$\begin{aligned}
&\hat{t}_{i,k+2|\ell_i(k)} = \min_{\omega_{i,k+1}} \{T_{i,k+1} \\
&+ \min\{\gamma_i(x_{i,\ell_i(k)}, \hat{x}_{i,k+1|\ell_i(k)}, \tilde{d}_{i,k-1}, \tilde{d}_{i,k}, \tau_{\max}), h_{i,\max}\}\} .
\end{aligned} \quad (49)$$

By considering the last measurement picked in the superframe $\ell_i(k)$ by the node i, the next time by which a new measurement must be picked again is given by

$$t_{i,\ell_i(k)+1} = T_{i,\ell_i(k)} + \min\{ \gamma_i(x_{i,\ell_i(\ell_i(k)-1)}, x_{i,\ell_i(k)}, \tilde{d}_{i,\ell_i(\ell_i(k)-1)}, \tilde{d}_{i,\ell_i(k)}, \tau_{i,\ell_i(k)}), h_{i,\max}\} . \quad (50)$$

Thus, after a superframe is adapted, we should allocate the GTSs according to the deadlines given by the previous self-triggered sampler. Nevertheless, if $t_{i,\ell_i(k)+1}$ is larger than the adapted supeframe BI_{k+1}, it appears natural to deallocate such node in the $(k+1)$-th superframe, and to reallocate it when its deadline fall into a certain beacon interval in the future. These consideration are summarized in the next result.

Proposition 6.1 *Consider the NCS over IEEE 802.15.4 as described in Section 3. Let the protocol beacon order and superframe order be as in* (44) *and in* (45)*, respectively, with* $\hat{t}_{k+2} = \min_{i,\omega_{i,k+1}}\{\hat{t}_{i,k+2|\ell_i(k)}\}$*. Let* $\mathcal{I}_{k+1} := \{i : T_{0,k+1} \le t_{i,\ell_i(k)+1} \le T_{0,k+2}\}$*. Then, by scheduling the GTSs with*

$$\begin{cases} \omega_{j,k+1} \neq 0 \textit{ for node } j : \hat{t}_{j,k+2|\ell_j(k)} = \hat{t}_{k+2} , \\ \omega_{i,k+1} \neq 0 \textit{ for all nodes } i \in \mathcal{I}_{k+1} , \\ \omega_{i,k+1} = 0 \textit{ for all other nodes} \end{cases} \quad (51)$$

all the control loops of the NCS are GUUB. ◁

This result together with Theorem 6.1 establish how the superframe duration is set, how the suprerframe length is adapted, and how the GTSs are scheduled to ensure GUUB of each loop and to reduce the energy expenditure of the nodes in terms of number of transmissions and idle listening. Moreover, since some nodes are deallocated in certain superframes, the average network utilization is also reduced with respect to the case in which $\omega_{i,k} \neq 0$ for all i,k. A sketch of the proposed protocol adaptation policy is depicted in Fig. 4.

6.3 On practical implementation and possible extensions

The implementation of the proposed protocol adaptation strategy requires the computation of the next time $t_{i,\ell_i(k)}$, the disturbance estimate $\hat{d}_{k,\ell_i(k)}$, the state estimate $\hat{x}_{i,k|\ell_i(k)}$ and the next time estimate $\hat{t}_{i,k+2|\ell_i(k)}$. All (or part of) these computations can be performed either at time $t = t_{i,k}$ (by the nodes) or at time $t = t_{i,k}+\tau_{i,k}$ (by the PANC). In the first case, the value of the time delay $\tau_{i,k}$ is unknown, but it can be replaced with its upper-bound $\tau_{\max}$. Then, the nodes directly compute the next sampling time $t_{i,\ell_i(k)}$, the state estimate $\hat{x}_{i,k|\ell_i(k)}$, the estimated next sampling time $\hat{t}_{i,k+2|\ell_i(k)}$, and then they encapsulate these values along with the current measurement into one packet. This packet is sent to the PANC that will select the appropriate BO and the GTS scheduling. That way, the computation load on the PANC is reduced, but the inter-sampling times can be slightly more conservative because a maximum communication time delay is always considered in the computation of the next sampling instant. In the second case, the value of $\tau_{i,k}$ is available at the PANC provided that the packets are time-stamped. In this second case, the inter-sampling times are less conservative, but the computation load on the PANC is larger. A sketch of a possible algorithm, where all the computations are performed by the nodes is depicted in Algorithm 1.

Regarding the number of available control GTSs in every superframe, in the IEEE 802.15.4 protocol is fixed to 7. However, this does not imply that the maximum number of the loops that can be controlled is limited to 7. For instance, it is possible to consider meta-superframes that are composed by r superframes. In that case we can consider $7r$ GTS per meta-superframe, but the minimum inter-sampling time guaranteed should be larger that $r\mathrm{BI}_{\min}$. Given that, we can still design a protocol adaptation policy based on such meta-superframes by proceeding along the same line we proposed. That way, we have no limitation on the number of controllable loops over the same network, but we should consider that the minimum inter transmission time guaranteed by the network may be fairly long.

Finally, despite we considered only wireless connection between the sensor nodes and the controllers, our framework can be easily extended in the case of *two-channel feedback*, in which there is also wireless communication between the controllers and the actuators. This is easy to do as long as we assume a constant control in the time interval $[t_k + \tau_k, t_{k+1} + \tau_{k+1})$. In this case, it is enough to consider two different time delays: for instance, let $\tau_{sc,k}$ and $\tau_{ca,k}$ the delays from sensor to controller and from controller to actuator respectively of the loop i in the superframe k. We consider two consecutive time slots allocated in one superframe for each control loop: one slot is used for the communication between the sensor and the controller, and the other one is used for the communication between the controller and the actuator. Hence, by considering piecewise constant control inputs of the form $u = Kx_k$ for $t \in [t_k + \tau_{sc,k} + \tau_{ca,k}, t_{k+1} + \tau_{sc,k+1} + \tau_{ca,k+1})$ and by proceeding with the design along the same line presented in the paper.

7 Simulations

In this section we illustrate the analysis presented in the previous section by numerical simulations. We consider $N = 3$ control loops over a IEEE 802.15.4 network and a simulation time t_{sim} of about 80 s for all the simulations. The control loops setup together with the network setup are specified below:

7.1 IEEE 802.15.4 Network

We set the minimum superframe order $\mathrm{BO}_{\min} = 1$, and the maximum superframe order $\mathrm{BO}_{\max} = 10$, for which we get

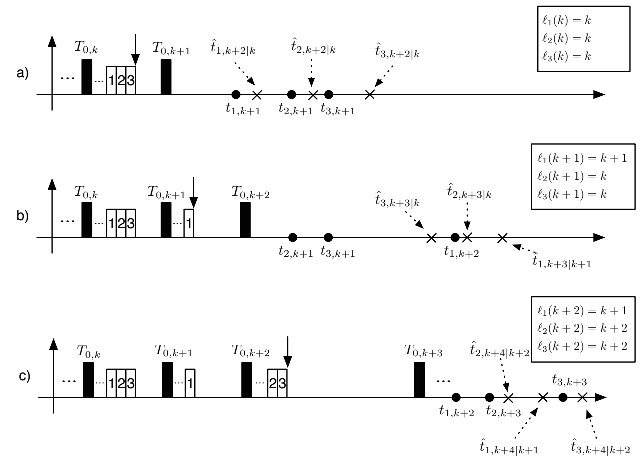

Fig. 4. Sketch of the proposed protocol adaption policy: a) At the k-th superframe, BO_{k+1} is adapted according to $\hat{t}_{1,k+2|k}$ and node 1 is scheduled in the $(k+1)$-th superframe. b) At the $(k+1)$-th superframe, the only node scheduled for transmission is node 1. BO_{k+2} is adapted according to $\hat{t}_{3,k+3|k}$ and nodes 2 and 3 are scheduled in the $(k+2)$-th superframe. c) The resulting superframe after the $(k+2)$-th adaptation. Note how the $(k+2)$-th supeframe is enlarged and how all the deadlines imposed by the self-triggered samplers are met.

Algorithm 1 Network protocol adaption with all the computations at the nodes

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init
PANC sets SO with (44);
PANC sets $\mathrm{BI}_1 = \mathrm{BI}_{\min}$;
PANC sets $\omega_{i,1} \neq 0$, $\forall 1 \leq i \leq n$;
end init
for all k **do**
  PANC sends a beacon;
  for all the nodes i s.t. $\omega_{i,k} \neq 0$ **do**
    $\ell_i(k) \leftarrow k$;
    Node i picks the measurement $x_i(T_{i,k})$;
    Node i computes $t_{i,\ell_i(k)+1}$ and $\hat{t}_{i,k+2|\ell_i(k)}$ with $\tau_{i,k} = \tau_{\max}$;
    Node i sends $x_i(T_{i,k}), t_{i,\ell_i(k)+1}$ and $\hat{t}_{i,k+2|\ell_i(k)}$ to the PANC;
    PANC updates the control laws;
  end for
  for all the nodes j s.t. $\omega_{j,k} = 0$ **do**
    PANC computes $\hat{t}_{j,k+2|\ell_i(k)}$;
    PANC sets BO_{k+1} as in (45);
    PANC sets $\omega_{i,k+1}$ as in Corollary 6.1;
  end for
end for

$\mathrm{BO}_{\min} = 30.7$ ms and $\mathrm{BO}_{\max} = 15.73$ s, and we set a maximum time delay of $\tau_{\max} = 2$ ms. We assumed that all the computation are performed by the PANC and that the packets are time-stamped.

7.2 Control Loop # 1.

We set

$$A_1 = \begin{pmatrix} -0.1 & 0.05 \\ 0.2 & 0.1 \end{pmatrix}, \qquad B_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The controller is designed to place the closed loop system eigenvalues in $\lambda_{1,1}(A_1 + B_1K_1) = -0.25, \lambda_{1,2}(A_1 + B_1K_1) = -0.18$. The initial condition are $x_{1,1}(0) = -20, x_{1,2}(0) = 15$, the triggering threshold is $\delta_1 = 2$ and the external disturbance upper-bound is $\bar{d}_1 = 0.6$. With this setting the condition (23) is satisfied with $h_{1,\min} = 33.1$ ms $\geq \mathrm{BI}_{\min}$.

7.3 Control Loop # 2.

We set

$$A_2 = \begin{pmatrix} 0.01 & 0.2 \\ 0.03 & 0 \end{pmatrix}, \qquad B_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

The controller is designed to place the closed loop system eigenvalues in $\lambda_{2,1}(A_2 + B_2K_2) = -0.15, \lambda_{2,2}(A_2 + B_2K_2) = -0.3$. The initial condition are $x_{2,1}(0) = -12, x_{2,2}(0) = 12$, the triggering threshold is $\delta_2 = 1.4$ and the external disturbance is upper-bounded wih $\bar{d}_2 = 1.2$. With this setting the condition (23) is satisfied with $h_{2,\min} = 31.6$ ms $\geq \mathrm{BI}_{\min}$.

7.4 Control Loop # 3.

We set

$$A_3 = \begin{pmatrix} 0.2 & 0.01 \\ 0.3 & -0.8 \end{pmatrix}, \qquad B_3 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

The controller is designed to place the closed loop system eigenvalues in $\lambda_{3,1}(A_3 + B_3K_3) = -0.4, \lambda_{3,2}(A_3 + B_3K_3) = -0.6$. The initial condition are $x_{3,1}(0) = -5, x_{3,2}(0) = 4$, the triggering threshold is $\delta_2 = 2.7$ and the external disturbance is upper-bounded with $\bar{d}_3 = 0.55$. With this setting the condition (23) is satisfied with $h_{3,\min} = 49.2$ ms $\geq \mathrm{BI}_{\min}$.

We first compare the simulation results by considering the cases $\tilde{d}_{i,k} = 0, \tilde{d}_{i,k} = d_i^*$ and $\tilde{d}_{i,k} = \hat{d}_{i,k}$ for all the loops, where $\hat{d}_{i,k}$ is computed with (47) and is used in both the self-triggered samplers to compute the real next-sampling time and the estimated next sampling time. The same disturbance observer has been also used in the state estimators. To test the robustness against external disturbances, we further add a disturbance $d_3(t) = (0.55 \quad 0)^T$ for $t \in [28, 32]$ s entering the loop #3. We finally compare the loops response when

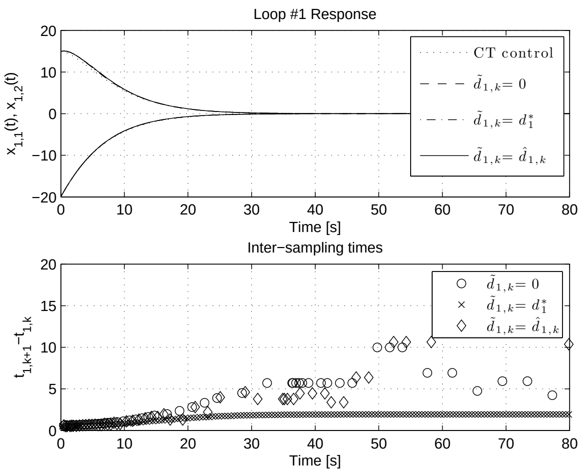

Fig. 5. Loop #1 response and inter-sampling times.

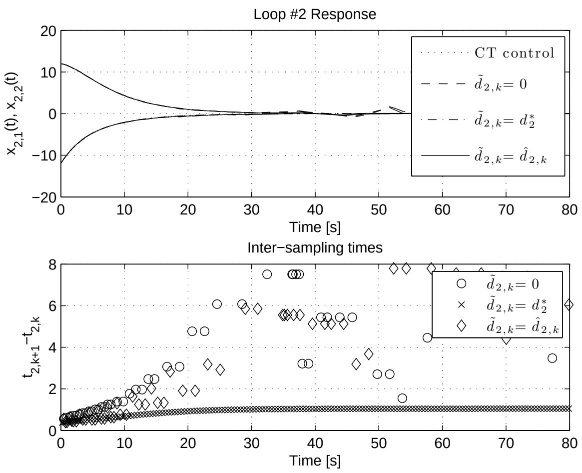

Fig. 6. Loop #2 response and inter-sampling times.

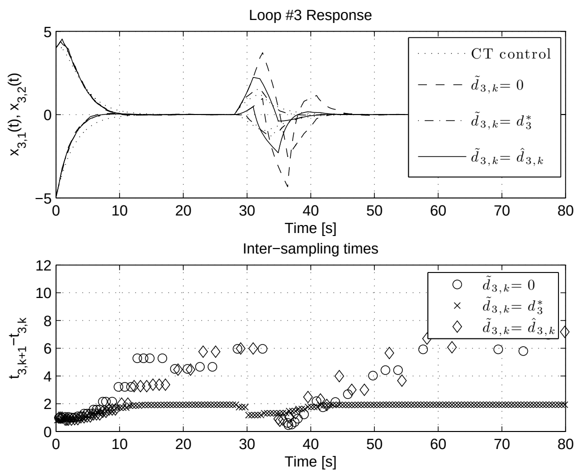

Fig. 7. Loop #3 response and inter-sampling times.

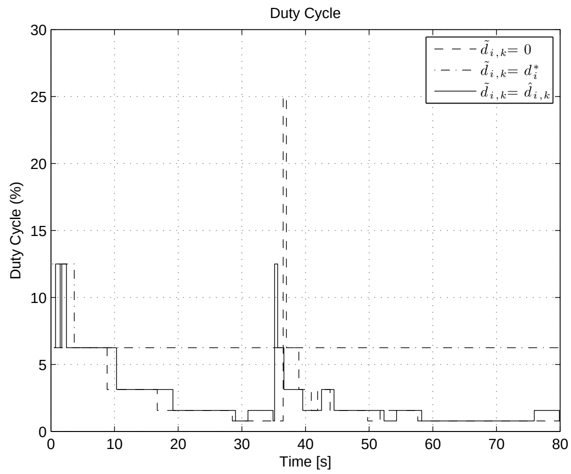

Fig. 8. Duty cycle of the network. Note that when a worst-case disturbance is considered for all the time, the duty cycle never goes under DC$\simeq 6\%$.

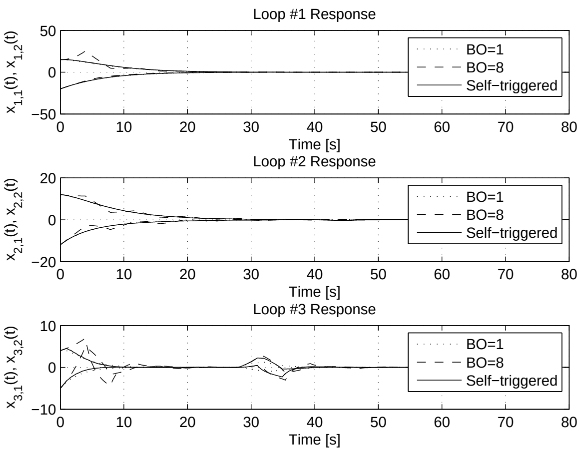

Fig. 9. Comparison of the self-triggered sampler with disturbance observer with periodic implementations with BO=1 and BO=8. Note the worst transient and the disturbance rejection when BO=8 compared to the the other two cases.

	Superfr.	Tx 1	Tx 2	Tx 3	$\mathrm{DC}_{\mathrm{avg}}$	$\mathrm{U}_{\mathrm{avg}}$
$\tilde{d}_{i,k}=0$	50	36	28	29	3.93	11.62
$\tilde{d}_{i,k}=d_i^*$	171	58	171	49	6.72	10.16
$\tilde{d}_{i,k}=\hat{d}_{i,k}$	56	33	36	31	4.21	11.16
BO = 1	2600	2600	2600	2600	100	18.75
BO = 8	20	20	20	20	0.78	18.75

Table 1
The table shows the simulation results when the proposed self-triggered control and two periodic implementations with BO=1 and BO=8 are used for a simulation time of $\sim$ 80 s. The data set includes the number of superframes, the number of transmissions of each sensor node, the average duty cycle and the average network utilization.

using different self-triggered samplers with the response obtained by the same continuous-time control and with discrete time control with periodic implementations with BO=1 and BO=8.

The results are illustrated in Figs. 5–9. All the loops are correctly controlled. In the case $\tilde{d}_{i,k} = 0$ we can appreciate how the inter-sampling times enlarge as the trajectories goes toward the origin, with the consequently reduction of the duty cycle. For this first setup, we experienced 36, 28 and 29 number of transmissions for the loop #1,#2 and #3 respectively over 56 superframes that correspond to 79.68 s of simulation time. Moreover, we have an average duty cycle $\mathrm{DC}_{\mathrm{avg}} \simeq 3.92\,\%$ and an average network utilization $\mathrm{U}_{\mathrm{avg}} = 11.62\,\%$. However, there is a big peak of the process state at time $t \simeq 33$ s, because of the disturbance entrance. This effect leads to a reduction of the inter-sampling times, and then to an abrupt increasing of the duty cycle.

To reduce the big peak experienced in the previous simulation, in this second simulation we conservatively set $\tilde{d}_{1,k} = d_1^* = (0.6 \quad 0)^T, \tilde{d}_{2,k} = d_2^* = (1.2 \quad 0)^T$ and $\tilde{d}_{3,k} = d_3^* = (0.55 \quad 0)^T$. That way, the peak in the loop #3 is reduced, but the inter-sampling times are pretty short. This leads to an increase of both the average duty cycle that result $\mathrm{DC}_{\mathrm{avg}} \simeq 6.72\,\%$, and to an increase of the number of transmissions that resulted 58, 171 and 49 for the loop #1,#2 and #3 respectively. For this simulation we have an average network utilization $\mathrm{U}_{\mathrm{avg}} = 10.16\,\%$. Such results are obtained by considering 171 superframes that correspond to 80.85 s of simulation time.

Finally, in the case where the disturbances are observed, the trajectories of the inter-sampling are similar to the case in which they are neglected, with the difference that the peak of the loop #3 is drastically reduced. It is also interesting to see how the system reaction to the disturbance is very similar to the second case and to the case with continuous-time control. Moreover, the inter-sampling times shrink correspondently to the detection of the disturbance, and the duty cycle increases. For this third simulation we experienced an average duty cycle $\mathrm{DC}_{\mathrm{avg}} \simeq 4.21\,\%$, and 33,36 and 31 number of transmissions respectively. The average network utilization is $\mathrm{U}_{\mathrm{avg}} = 11.16\,\%$. Such results are obtained by considering 57 superframes that correspond to 83.06 s of simulation time.

The network utilization and the duty cycle in the case with $\tilde{d}_{i,k} = 0$ and $\tilde{d}_{i,k} = 0$ is very similar, but the reaction to external disturbances is much better when the disturbance observer is used. On the other hand, such a disturbance can be better handled if $\tilde{d}_{i,k} = d_i^*$ for all the time. However, the number of transmissions when the worst case disturbance is considered increased considerably. Thus, the proposed self-triggered sampler appeared to be the best choice.

For the sake of completeness, we also compared the proposed self-triggered sampler with periodic implementations with BO=1 and BO=8 since for periodic implementations with BO greater than 8, some loop resulted to be unstable. The comparison of the proposed self-triggered sampler with the periodic implementation with BO=1 provided a very similar system response, but the number of transmissions, the duty cycle and the network utilization in the periodic case are much worse. A drastic reduction of both the number of transmissions and the duty cycle is achieved when BO=8, but the transient and the reaction to external disturbance is worst compared to the self-triggered sampler, as shown in Fig. 9. Hence, the best tradeoff between network performance and system response is given again by the proposed self-triggered sampler.

We finally wish to remark that the solely adaptation of the superframes would led to the reduction of the duty cycle as the trajectories of the loops move toward the origin, ensuring then an energy saving. However, without the GTSs scheduling, we would experienced a number of transmission of each loop equal to the number of superframes considered, and a fixed average network utilization of $\mathrm{U}_{\mathrm{avg}} = 18.75\,\%$ as in the periodic case. Indeed, the GTS scheduling led to a reduction of both the number of transmissions and to a reduction of the network utilization, in addition to a reduction of the duty cycle.

8 Conclusions

In this paper a novel co-design methodology of NCSs composed by several control loops sharing the same IEEE 802.15.4 network was presented. In the analysis and design of the NCS the networking protocol was explicitly considered. Based on a self-triggered sampler for each control loop, an adaptation policy of the protocol parameters to ensure GUUB of the NCS and to reduce the energy expenditure of the nodes in terms of number of transmissions and idle listening was developed. It was shown how a reduction of the number of transmissions leads to a decrease of the network utilization. A system-level design approach allowed us to capture all the essential control and networking issues within one framework. However, since any self-triggered control may cause the controlled system to run in open loop for arbitrary large times, we showed that the drawback in the proposed control strategy relies on the robustness of the closed-loop system.

We plan to extend this study by mixed event and self-triggered control strategies over hybrid MAC protocols. Additional future work includes the study of closing the loop over the contention access period of the protocol superframe, where the main challenge is represented by packets collisions of the contention access scheme. Finally, we also plan to address the problem of designing NCS over multihop networks.

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