Proceedings of the
47th IEEE Conference on Decision and Control
Cancun, Mexico, Dec. 9-11, 2008

Estimation over Heterogeneous Sensor Networks

Henrik Sandberg, Maben Rabi, Mikael Skoglund, and Karl Henrik Johansson

Abstract— **Design trade-offs between estimation performance, processing delay and communication cost for a sensor scheduling problem is discussed. We consider a heterogeneous sensor network with two types of sensors: the first type has low-quality measurements, small processing delay and a light communication cost, while the second type is of high quality, but imposes a large processing delay and a high communication cost. Such a heterogeneous sensor network is common in applications, where for instance in a localization system the poor sensor can be an ultrasound sensor while the more powerful sensor can be a camera. Using a time-periodic Kalman filter, we show how one can find an optimal schedule of the sensor communication. One can significantly improve estimation quality by only using the expensive sensor rarely. We also demonstrate how simple sensor switching rules based on the Riccati equation drives the filter into a stable time-periodic Kalman filter.**

I. Introduction

The resource limimitations of wireless sensor networks is an important issue in the design of emerging applications [1], [2]. The need for minimizing the communication of individual sensor nodes poses interesting challenges for estimation and control strategies [3]. In this paper we consider a novel networked estimation problem for a situation in which two types of sensors with different resource demands share the same network.

As a motivating example, consider the problem of tracking a mobile object using observations from two types of sensors. The sensors communicate their data to a central node that perform the processing. The first type of sensors are proximity sensors with low-quality measurements, small processing delay and a light communication cost. The second type of sensor is a camera with high-quality measurements, but large processing delay and high communication cost. Given a TDMA scheme for the wireless network, an important problem now is how each sensor should be used at each time instant. Some sensor measurements with not much information are available almost immediately at the tracking station. On the other hand, there are some high quality sensor measurements of which only a delayed and intermittent version is available to the tracking station.

The main contribution of the paper is a design trade-off between estimation performance, processing delay and communication cost for a sensor scheduling problem with heterogeneous sensors. We show how optimal periodic sensor schedules can be found by means of a search over a finite set. We also show that by switching with a certain period between two relatively poorly performing sensors, one can achieve strictly better estimates. The switching between the sensors can be decided off-line. We also show how sensor switching can be done on-line using covariance estimates from Riccati equation for the off-line problem. This switching rule is shown to give a stable periodic switching schedule.

Sensor selection problems have been studied extensively, e.g., [4]. Our approach is novel in that we incorporate communication cost in the cost criterion together with processing delays. See [5] for another recently studied problem. The motivation for our formulation comes from the trade-off one need to do in new systems utilizing wireless networks. Our solution relies on the studied periodic prediction problem [6], [7].

The outline of the paper is as follows. Section II presents the model of the considered system and formulates a performance criterion that takes both the estimation quality and communication cost into account. By utilizing results on periodic Riccati equation for the periodic prediction problem, we present a solution to the optimal filtering problem. Section III illustrates the result on a sensing problem for the random walk. We then show in Section IV how the solution can be implemented by letting the sensor switching be based on how much increase in accuracy one can get from using one particular sensor. This covariance-based sensor switching is then applied to the random walk problem in Section V. The paper is concluded in Section VI.

II. Optimal Periodic Sensor Schedules and Filters

In this section, it is assumed a priori that the high-quality sensor is used once every N-th sample. In between the high-quality samples, the low-quality sensor is used in every sample. How the period N should be chosen is one of the main problems in the paper, and will be discussed in the following.

We define the sets $T_{hq}(N)$ and $T_{lq}(N)$ as follows

$$\begin{aligned} T_{hq}(N) &:= \{N-1, 2N-1, 3N-1, \ldots\} \\ &= \{k \geq 0 | (k+1) \mod N = 0\}, \\ T_{lq}(N) &:= \{0, 1, \ldots, N-2, N, \ldots\} \\ &= \{k \geq 0 | (k+1) \mod N \neq 0\}, \end{aligned}$$

where the period is $N \geq 1$ and k is the discrete time index. That is, when $k \in T_{hq}(N)$ the high-quality sensor is used, and when $k \in T_{lq}(N)$ the low-quality sensor is used.

This work was supported by the Swedish Research Council and the Swedish Foundation for Strategic Research.

H. Sandberg, M. Rabi, M. Skoglund, and K. H. Johansson are with the ACCESS Linnaeus Centre, School of Electrical Engineering, Royal Institute of Technology, 100 44 Stockholm, Sweden. {hsan, mabenr, skoglund, kallej}@ee.kth.se

978-1-4244-3124-3/08/$25.00 ©2008 IEEE

It is also assumed that the plant we measure is linear,

$$x(k+1) = Ax(k) + Bw(k), \qquad k \geq 0, \tag{1}$$
$$y_1(k) = C_1 x(k) + v_1(k), \qquad k \in T_{lq}(N), \tag{2}$$
$$y_2(k) = C_2 x(k-d) + v_2(k), \qquad k \in T_{hq}(N), \tag{3}$$

with state vector $x(k) \in \mathbb{R}^n$, Gaussian white process noise $w(k) \in \mathbb{R}^m$, measurements $y_1(k), y_2(k) \in \mathbb{R}^p$, and Gaussian white measurement noises $v_1(k), v_2(k) \in \mathbb{R}^p$. The covariance of the process noise is $\mathbf{E}w(k)w(k')^T = W\delta(k-k')$, and the covariances of the measurement noises $\mathbf{E}v_1(k)v_1(k')^T = \Sigma\delta(k-k')$, and $\mathbf{E}v_2(k)v_2(k')^T = \sigma\delta(k-s)$. It is assumed that the high-quality sensor measurement $y_2(k)$ is more accurate than $y_1(k)$, i.e., $\sigma \ll \Sigma$, but it is delayed with d samples because of an assumed higher processing time. It is assumed that the delay of the low-quality sensor can be neglected. Note that $y_1(k)$ is not defined when $k \in T_{hq}(N)$ and $y_2(k)$ is not defined when $k \in T_{lq}(N)$.

Remark 1: It is also possible to let the dimensions of the measurements $y_1(k)$ and $y_2(k)$ to be different, i.e., $y_1(k) = C_1 x(k) + v_1(k) \in \mathbb{R}^{p_1}$ and $y_2(k) = C_2 x(k-d) + v_2(k) \in \mathbb{R}^{p_2}$, with $p_1 \neq p_2$.

Next, we derive the optimal filter for the model (1)–(3).

A. Performance criterion

We here introduce a performance criterion for the estimation problem that makes the trade-off between communication cost for the high-quality sensor and estimation quality explicit. As estimation quality criterion, we choose the average trace of the covariance of the estimation error over the time interval $[0,k]$, $p_{\text{av}}(k,N) := \frac{1}{k+1}\sum_{i=0}^{k} \operatorname{trace} P(i,N)$ where

$$P(k,N) := \mathbf{E}\left[(x(k)-\hat{x}(k))(x(k)-\hat{x}(k))^T \,|Y_{k-1}(N)\right], \tag{4}$$

and $Y_{k-1}(N)$ are all the measurements up until time $k-1$, $Y_{k-1}(N) := \{y_1(k'), y_2(k'')| k', k'' \leq k-1; k' \in T_{lq}(N); k'' \in T_{hq}(N)\}$, using N-periodic high-quality measurements. Here, $\hat{x}(k)$ denotes an estimate of $x(k)$, based on the available measurements in $Y_{k-1}(N)$. For notational convenience, we will often suppress the argument N in $P(k,N)$ when it is clear what the period N is, i.e., $P(k) := P(k,N)$.

The average trace of the error covariance, $p_{\text{av}}(k,N)$, is a measure of how accurately we know the state and takes the information from both sensors into account. We hope to decrease $p_{\text{av}}(k,N)$ by a proper choice of the period N.

The performance criterion $V(k,N)$ we want to minimize is now defined by the sum of an average communication cost and the average error covariance,

$$V(k,N) := \frac{\lambda}{N} + p_{\text{av}}(k,N). \tag{5}$$

There is a communication cost λ associated with each measurement $y_2(k)$. The unit for λ is chosen to be the unit for estimation quality. The average communication cost per time sample is λ/N, and without any requirement for estimation quality, $N = \infty$ is optimal. When only the high-quality sensor is used, the performance is $V(k,1) = \lambda + p_{\text{av}}(k,1)$, and when only the low-quality sensor is used, the performance is $V(k,\infty) = p_{\text{av}}(k,\infty)$. Thus the communication cost λ is a measure of how much better (measured in resulting average error covariance) than the low-quality sensor the high-quality sensor must be for us to prefer to use it ($V(k,1) < V(k,\infty)$).

We would like to minimize the criterion (5) with respect to sensor cycle period N and the error covariance, i.e., $\min_N \min_{P(0),\ldots,P(k)} V(k,N)$, subject to the model (1)–(3) for all times k. The minimization problem can be solved in two steps,

$$\begin{aligned} &\min_N \left(\frac{\lambda}{N} + \min_{P(0),\ldots,P(k)} p_{\text{av}}(k,N)\right) \\ &= \min_N \left(\frac{\lambda}{N} + p^*_{\text{av}}(k,N)\right) \end{aligned} \tag{6}$$

Intuitively, one would think that a small period N always decreases the estimation error p_{av} since more high-quality measurements are used, at the price of increasing the communication cost λ/N. However, the situation is a little bit more complicated than that, as we shall see.

In the next subsection, we characterize the optimum $p^*_{\text{av}}(k,N)$. We return to the outer minimization over N in the subsection II-C.

B. Minimizing the average estimation error

Here, we focus on minimizing the term $p_{\text{av}}(k,N)$ in the performance criterion $V(k,N)$, for fixed period N. It is well known that the optimal predictor for a linear system (a predictor that minimizes $\operatorname{trace} P(k,N)$ for all k) is given by the Kalman filter, see, [8], for example. In order to apply the Kalman filter to the model (1)–(3), the model has to be rewritten to accommodate for the time delay d. Introduce a new state vector $\bar{x}$ by

$$\bar{x}(k) = \begin{bmatrix} x(k) & x(k-1) & \ldots & x(k-d)\end{bmatrix}^T \in \mathbb{R}^{n(d+1)}. \tag{7}$$

Then the model (1)–(3) can be rewritten as

$$\bar{x}(k+1) = \bar{A}\bar{x}(k) + \bar{B}w(k), \tag{8}$$
$$\bar{y}(k) = \bar{C}(k)\bar{x}(k) + \bar{v}(k), \tag{9}$$

where

$$\bar{A} = \begin{bmatrix} A & 0 & \ldots & 0 & 0 \\ I_n & 0 & & 0 & 0 \\ 0 & I_n & & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & I_n & 0 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$\bar{C}(k) = \begin{cases} \begin{bmatrix} C_1 & 0 & \ldots & 0 & 0 \end{bmatrix}, & k \in T_{lq}(N), \\ \begin{bmatrix} 0 & 0 & \ldots & 0 & C_2 \end{bmatrix}, & k \in T_{hq}(N), \end{cases}$$

$$\mathbf{E}\bar{v}(k)\bar{v}(k+k')^T =: \bar{V}(k)\delta(k'),$$

$$\bar{V}(k) = \begin{cases} \Sigma, & k \in T_{lq}(N), \\ \sigma, & k \in T_{hq}(N). \end{cases}$$

The system (8)–(9) is a *linear time-periodic system* of period N. The periodicity comes from the periodic sensing. The minimal possible covariance $\bar{P}^*(k)$ (* denotes minimal) of the estimation error satisfies a time-varying recursive Riccati equation of the form [6], [8]

$$\bar{P}^*(k+1) = \bar{A}[\bar{P}^*(k) - \bar{P}^*(k)\bar{C}(k)^T \times[\bar{C}(k)\bar{P}^*(k)\bar{C}(k)^T + \bar{V}(k)]^{-1}\bar{C}(k)\bar{P}^*(k)]\bar{A}^T + \bar{B}W\bar{B}^T \quad (10)$$

where

$$\bar{P}(k) = \mathbf{E}\left[(\bar{x}(k) - \hat{\bar{x}}(k))(\bar{x}(k) - \hat{\bar{x}}(k))^T \,|Y_{k-1}(N)\right] =: \begin{bmatrix} \bar{P}_{0,0}(k) & \dots & \bar{P}_{0,d}(k) \\ \vdots & \ddots & \vdots \\ \bar{P}_{d,0}(k) & \dots & \bar{P}_{d,d}(k) \end{bmatrix} \in \mathbb{R}^{n(d+1)\times n(d+1)},$$

is the covariance of the estimation error of the state $\bar{x}(k)$, and

$$\bar{P}_{e,f}(k) := \mathbf{E}\left[(x(k-e) - \hat{x}(k-e)) \times (x(k-f) - \hat{x}(k-f))^T \,|Y_{k-1}(N)\right], \quad (11)$$

$0 \le e, f \le d$, and $\bar{P}_{e,f}(k) = \bar{P}_{f,e}(k)^T$. This means that the covariance of any other filter, $\bar{P}(k)$, is greater than that of the Kalman filter, $\bar{P}^*(k)$. That is, $\bar{P}(k) - \bar{P}^*(k)$ is positive semidefinite ($\bar{P}(k) - \bar{P}^*(k) \ge 0$), see [8, Section 3.2]

The time-varying Kalman filter that achieves the optimal accuracy $\bar{P}^*(k)$ is given by

$$\hat{\bar{x}}(k+1) = (\bar{A} - \bar{K}(k)\bar{C}(k))\hat{\bar{x}}(k) + \bar{K}(k)\bar{y}(k),$$
$$\bar{K}(k) = \bar{A}\bar{P}^*(k)\bar{C}(k)^T(\bar{C}(k)\bar{P}^*(k)\bar{C}(k)^T + \bar{V}(k))^{-1}. \quad (12)$$

It is known that $\bar{P}^*(k)$ converges to an N-periodic trajectory in steady-state, under weak assumptions on the system.

Lemma 1: Assume that (A, C_1) or (A, C_2) are detectable, and that (A, B) is stabilizable. If $\bar{P}^*(0)$ is symmetric and positive semidefinite, then $\bar{P}^*(k)$ converges to a unique symmetric and positive semidefinite N-periodic solution $\bar{P}^*_{\text{per}}(k)$ as $k \to \infty$, where $\bar{P}^*_{\text{per}}(k)$ satisfies (10) with the boundary conditions $\bar{P}^*_{\text{per}}(0) = \bar{P}^*_{\text{per}}(N) \ge 0$.

Proof: If (A, B) is stabilizable, it is straightforward to show that $(\bar{A}, \bar{B})$ is stabilizable. To show that the periodic system $(\bar{A}, \bar{C}(k))$ is H-detectable, [9], requires more work. H-detectability means that for each eigenvalue $\bar{\lambda}$ of $\bar{A}^N$ such that $|\bar{\lambda}| > 1$, the conditions

$$\bar{A}^N\bar{\eta} = \bar{\lambda}\bar{\eta}, \quad \bar{\eta} \in \mathbb{C}^{n(d+1)}, \quad (13)$$
$$\bar{C}(k)\bar{A}^k\bar{\eta} = 0, \quad k \in [0, N-1], \quad (14)$$

imply $\bar{\eta} = 0$. The condition (13) means that $\bar{\eta}$ has the structure $\bar{\eta} = \begin{bmatrix}\lambda^d\eta & \lambda^{d-1}\eta & \dots & \eta\end{bmatrix}^T$, where $A\eta = \lambda\eta$ and $\lambda^N = \bar{\lambda}$. Notice that $|\lambda| > 1$. The condition (14) now reduces to $C_1\eta = 0$ and $C_2\eta = 0$. Because of the assumption of the detectability of (A, C_1) or (A, C_2), it follows that $\eta = 0$. This implies that $\bar{\eta} = 0$, and H-detectability of $(\bar{A}, \bar{C}(k))$ follows. Since $(\bar{A}, \bar{B})$ is stabilizable and $(\bar{A}, \bar{C}(k))$ is detectable, the existence of an attractive N-periodic solution of the Riccati equation follows by Theorem 7 in [6]. ∎

Remark 2: If $N = \infty$, (A, C_1) should be detectable, and if $N = 1$, (A, C_2) should be detectable.

Using the lemma, we can derive the following theorem.

Theorem 1: Assume that (A, C_1) or (A, C_2) are detectable, and (A, B) is stabilizable. If $\bar{P}^*(0)$ is symmetric and positive semidefinite, then it holds that

(i) the minimal estimation error covariance $P^*(k, N)$, see (4), is given by the $(1, 1)$-block of $\bar{P}^*(k)$ ($\bar{P}^*_{0,0}(k)$);
(ii) the optimal covariance $P^*(k, N)$ converges to a periodic solution $P^*_{\text{per}}(k, N)$, i.e., $P^*_{\text{per}}(k, N) = P^*_{\text{per}}(k + N, N)$, $\forall k$, and $P^*(k, N) \to P^*_{\text{per}}(k, N)$, as $k \to \infty$;
(iii) the optimal average estimation quality is given by $p^*_{\text{av}}(k, N) = \frac{1}{k+1}\sum_{i=0}^{k} \operatorname{trace} P^*(i, N)$;
(iv) the optimal average estimation quality converges to a constant $p^*_{\text{av}}(N)$ as $k \to \infty$, and $p^*_{\text{av}}(N) := \lim_{k\to\infty} p^*_{\text{av}}(k, N) = \frac{1}{N}\sum_{i=1}^{N} \operatorname{trace} P^*_{\text{per}}(i, N)$.

Proof: (i) By inspection it is seen that $P^*(k, N)$ is the $(1, 1)$-block of $\bar{P}(k)$, that is $\bar{P}_{0,0}(k)$. By the optimality of the Kalman filter, $\bar{P}_{0,0}(k)$ is minimal. (ii) By Lemma 1, $\bar{P}(k)$ (and its components) converges to a periodic trajectory. (iii) When $P(k, N)$ is minimized, $\operatorname{trace} P(k, N)$ is also minimized, since $P(k, N)$ is symmetric positive semidefinite. We have that $\operatorname{trace} P^*(i, N) \le \operatorname{trace} P(i, N)$, $i = 0, \dots, k$, for all admissible $P(i, N)$. Hence $\sum_{i=0}^{k} \operatorname{trace} P^*(i, N) \le \sum_{i=0}^{k} \operatorname{trace} P(i, N)$. (iv) Follows by the periodicity of $P^*(k, N)$, see (ii). ∎

The practical value of Theorem 1 is that it shows how to compute the optimal value $p^*_{\text{av}}(k, N)$ that is needed to solve the optimization problem (6). Because $p^*_{\text{av}}(k, N)$ converges to a constant $p^*_{\text{av}}(N)$ for large k, we will usually only discuss this limiting value. To compute it, we need to solve a periodic Riccati equation for each period N. One can compute the periodic solutions $\bar{P}^*(k)$ and $\bar{K}^*(k)$ by just iterating (10) because of the global convergence property in Lemma 1, but more efficient methods are available, see for example [7]. There are two cases of special interest: $p^*_{\text{av}}(1)$ and $p^*_{\text{av}}(\infty)$. Both these cases collapse into time-invariant problems, and correspond to the cases when the high-quality or the low-quality sensor, respectively, is used all the time.

One may think that $p^*_{\text{av}}(N)$ is an increasing function of N. The argument could be as follows: The high-quality sensor is more accurate, and the more often it is used (N small), the better estimation quality we get. This is not always correct, however. The reason is the time delay d of the high-quality measurement. The high-quality sensor is accurate, but its information can be old. If the process noise into the system is sufficiently large (W large), then we can get that $p^*_{\text{av}}(1) > p^*_{\text{av}}(\infty)$. That is, using the low-quality sensor all the time gives better estimates than using the high-quality sensor all the time. One could ask if it then is useful at all to have a high-quality sensor. Maybe somewhat surprisingly the answer is yes, as we shall show in the random walk example in Section III. It turns out that by switching with a suitable period N between two sensors that are poor by themselves, we can improve the estimation quality.

C. Minimizing the performance criterion

At time k, the optimal sensor cycle period is given by

$$N^*(k) = \arg\min_N \left(\frac{\lambda}{N} + p^*_{\text{av}}(k, N) \right), \tag{15}$$

where $p^*_{\text{av}}(k, N)$ was characterized in Theorem 1. It is clear that $1 \le N^*(k) \le k+1$, so that (15) is a simple minimization problem over a finite set.

The steady-state optimal period N^* for the sensor schedule is given by

$$N^* = \arg\min_N \left(\frac{\lambda}{N} + p^*_{\text{av}}(N) \right) =: \arg\min_N V^*(N), \tag{16}$$

where $p^*_{\text{av}}(N)$ was characterized in Theorem 1. The limiting value $V^*(\infty)$ is easy to compute, since $p^*_{\text{av}}(\infty)$ is a time-invariant problem. We have the following proposition that helps to convert the infinite-dimensional problem (16) into a finite problem.

Proposition 1: There exists a finite period $N_{\text{mono}} \ge 1$ such that the sequence $V^*(N) - p^*_{\text{av}}(\infty)$, $N \in \{N_{\text{mono}}, N_{\text{mono}}+1, \ldots\}$ is monotone and converges to zero.

In the examples we have considered, $V^*(N)$ quickly converges to $p^*_{\text{av}}(\infty)$ so that it has been easy to find a period N_{mono}. Then we can reduce (16) into a minimization problem over a finite set $N \in \{1, 2, \ldots, N_{\text{mono}}, \infty\}$.

Even though we have not been able to prove that there is always a unique global minimum N^*, this has been the case for the numerical examples considered. More details and figures are given in the random walk example in Section III.

D. General multi-sensor case

The presented method can be extended to the case where more than two sensors are available. Consider, for example, the case where two high-quality sensors, with periods N_1 and N_2, and one low-quality sensor is available. Then one additional measurement equation $y_3(k)$ is added to (2)–(3). The new system can again be written as a periodic system (8)–(9). An upper bound on the period is $N_1 N_2$. One needs to be careful about the times when the high-quality measurements coincide. One solution is to use both measurements (dimension of $\bar{y}(k)$ can vary). As before we can define a performance criterion

$$V(k, N_1, N_2) := \frac{\lambda_1}{N_1} + \frac{\lambda_2}{N_2} + p_{\text{av}}(k, N_1, N_2),$$

for communication costs λ_1 and λ_2. The minimum of the criterion can be found by first finding $p^*_{\text{av}}(k, N_1, N_2)$ for fixed N_1 and N_2 using the Riccati equation (10). Again, the optimal periods, N_1^* and N_2^*, can be found from a search over a finite set, although this set will grow exponentially with the number of sensors available.

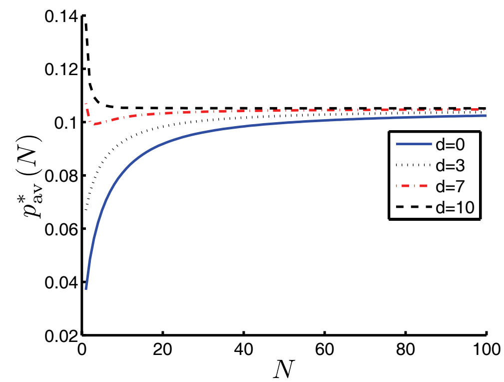

Fig. 1. The function $p^*_{\text{av}}(N)$ for four different delays d.

III. Example: The Random Walk

As an example for a plant, we use the random walk in one dimension:

$$\begin{aligned} x(k+1) &= x(k) + w(k), & x(k) &\in \mathbb{R}, \\ y_1(k) &= x(k) + v_1(k), & k &\in T_{lq}(N), \\ y_2(k) &= x(k-d) + v_2(k), & k &\in T_{hq}(N). \end{aligned}$$

This means the system matrices are chosen as $A = B = C_1 = C_2 = 1$. For this simple system, we can express the optimal $P^*(k) \in \mathbb{R}$ as

$$P^*(k+1) = P^*(k) + W - \frac{P^*(k)^2}{P^*(k) + \Sigma}, \quad k \in T_{lq}(N), \tag{17}$$

$$P^*(k+1) = P^*(k) + W - \frac{\bar{P}^*_{0,d}(k)^2}{\bar{P}^*_{d,d}(k) + \sigma}, \quad k \in T_{hq}(N), \tag{18}$$

which is the the (1,1)-block of (10) and where we have used that $\bar{P}^*_{0,0}(k) = P^*(k)$. As seen, it is only when $k \in T_{hq}(N)$ that more information than $\bar{P}^*_{0,0}(k)$ is needed from $\bar{P}^*(k)$.

Next, we compute $p^*_{\text{av}}(k, N)$ and $p^*_{\text{av}}(N)$ for the random walk for various delays, when the variance of the process noise is $W = 0.01$, and the measurement noise is $\sigma = 0.1$ and $\Sigma = 1.0$. That is, the high-quality sensor is a factor 10 more accurate than the low-quality sensor. The function $p^*_{\text{av}}(N)$ is shown in Fig. 1. As can be seen, it is not at all the case that decreasing N always yields a more accurate average estimate, at least for large delays d. In fact, in the case $d = 7$, there is even a minima at $N = 3$ in the curve. This means that switching between the two sensors with period $N = 3$ yields a strictly better estimate than the sensors can produce by themselves. It is also seen that all the curves converge to the same value $p^*_{\text{av}}(\infty)$. This is because the high-quality sensor is not used at all when $N = \infty$, and the low-quality sensor has no delay. How the covariance $P^*(k, N)$, $N = 1, 3, \infty$, and $p^*_{\text{av}}(k, 3)$ evolve over time for a fixed delay of $d = 7$ is shown in Fig. 2. As expected from Theorem 1, $P^*(k, N)$ converges to periodic trajectories, and the average function $p^*_{\text{av}}(k, 3)$ converges to a constant. In this case, a high-quality

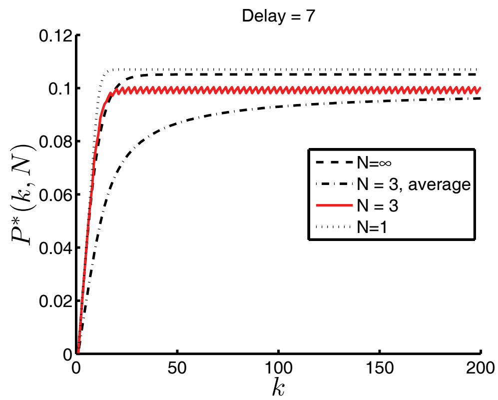

Fig. 2. The function $P^*(k, N)$ for different periods N and $p_{\text{av}}(k, 3)$ when $d = 7$.

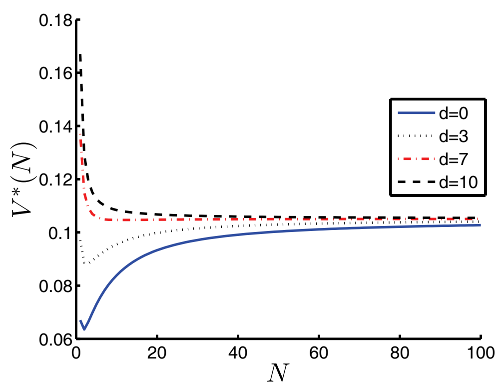

Fig. 3. The performance cost $V^*(N) = \lambda/N + p^*_{\text{av}}(N)$, $\lambda = 0.03$, as function of period N for four different delays d.

measurement every third sample decreases the average and instantaneous error covariance.

Next, we turn to finding the optimal sensor cycle period N^*. In Fig. 3, the optimal performance cost $V^*(N)$ for the same example as in Fig. 1 is shown when the cost per use of the high-quality sensor is $\lambda = 0.03$. As seen, there is a unique optimum N^* in all cases. As suggested in Proposition 1, the curves are monotonic for large N. The results are summarized in Table I. It is clear that putting the communication cost to zero, $\lambda = 0$, will in most cases result in the extreme choices $N^* = 1$ or $N^* = \infty$, although not always (see $d = 7$ in Fig. 1 for example). This means only one sensor is used all the time. It is interesting to notice that if there is a communication cost $\lambda > 0$ associated with the high-quality measurement, often nontrivial optimal periods $1 < N^* < \infty$ result.

IV. Covariance-Based Sensor Switching

In the previous section, the sensor switching was periodic by assumption. In this section, we instead let the sensor switching be based on how much increase in accuracy one can get from using one particular sensor. We call this *covariance-based switching*. Let us compare the two expressions (17) and (18) for the optimal error covariance for the random walk. We see that for a given $\bar{P}^*(k)$, the

TABLE I

The optimal sensor cycle periods for various communication costs λ and sensor delay d.

λ	d	N^*	V^*	λ	d	N^*	V^*
0	0	1	0.0370	0.03	0	2	0.0635
0	3	1	0.0670	0.03	3	3	0.0883
0	7	3	0.0992	0.03	7	11	0.1047
0	10	∞	0.1051	0.03	10	∞	0.1051

high-quality measurement results in a smaller $P^*(k+1)$ only if

$$\frac{\bar{P}^*_{0,d}(k)^2}{\bar{P}^*_{d,d}(k) + \sigma} - \frac{P^*(k)^2}{P^*(k) + \Sigma} > 0. \tag{19}$$

The idea with covariance-based switching is to iterate the recursive Riccati equation (10), and use the high-quality sensor only when a condition like (19) is true. More generally, we define a switch schedule $s(k)$ and a continuous switch function $f : \mathbb{R}^{n(d+1)\times n(d+1)} \to \mathbb{R}$ such that

$$s(k) = \begin{cases} 1, & f(\bar{P}^*(k)) \le 0 \\ 2, & f(\bar{P}^*(k)) > 0 \end{cases} \tag{20}$$

and the following $\bar{C}$ and $\bar{V}$ matrices are used in the Riccati equation (10)

$$\bar{C}(k) = \begin{cases} \begin{bmatrix} C_1 & 0 & \dots & 0 & 0 \end{bmatrix}, & s(k) = 1, \\ \begin{bmatrix} 0 & 0 & \dots & 0 & C_2 \end{bmatrix}, & s(k) = 2, \end{cases}$$

$$\bar{V}(k) = \begin{cases} \Sigma, & s(k) = 1, \\ \sigma, & s(k) = 2. \end{cases}$$

For a given initial covariance $\bar{P}^*(0)$, the schedule $s(k)$ can be computed on-line.

Remark 3: If $d = 0$ and $\sigma < \Sigma$, then (19) is always true when $\bar{P}^*(k) > 0$. To avoid that the high-quality sensor always is used, and to account for its communication cost, we can again introduce a communication cost λ and modify (19) to $\bar{P}^*_{0,d}(k)^2/(\bar{P}^*_{d,d}(k) + \sigma) - \lambda - P^*(k)^2/(P^*(k) + \Sigma) > 0$.

The covariance-based schedules $s(k)$ are robust to small perturbations in the covariance $\bar{P}^*(k)$. Also, periodic schedules are locally attracting as explained in the following theorem.

Theorem 2: Assume that (A, C_1) or (A, C_2) are detectable, and that (A, B) is stabilizable. Assume furthermore that there is an N-periodic switch schedule $s(k)$ such that the corresponding unique periodic solution $\bar{P}^*_{\text{per}}(k)$ satisfies (20) with strict inequalities for $k = 1, \dots, N$. Then $\bar{P}^*_{\text{per}}(k)$ is a locally attracting periodic solution for a continuous filter with switch function f, and the switch schedule $s(k)$ remains unchanged under small perturbations of the covariance.

Proof: Since the inequalities involving f are strictly satisfied, and f is continuous, there are open neighborhoods around every $\bar{P}^*_{\text{per}}(k)$ that result in the same switching signal $s(k)$. Denote the radius of the largest open ball that is contained in all these neighborhoods by R.

Now make a symmetric perturbation to $\bar{P}^*_{\text{per}}(0)$ and call the perturbed covariance $\bar{P}^*(0)$, such that $\|\bar{P}^*_{\text{per}}(0) - \bar{P}^*(0)\|_2 \le \rho$, for a positive constant $\rho > 0$ to be fixed later.

If the covariance $\bar{P}^*(0)$ is iterated forward in time using the periodic Riccati equation (10) with $\bar{C}_{\text{per}}(k)$ and $\bar{V}_{\text{per}}(k)$ (corresponding to the N-periodic schedule $s(k)$) we obtain a sequence $\bar{P}^*(k)$. Denote the Kalman gain corresponding to $\bar{P}^*_{\text{per}}(k)$ and $\bar{P}^*(k)$ by $\bar{K}_{\text{per}}(k)$ and $\bar{K}(k)$, respectively. Also define $\bar{A}_{\text{per}}(k) = \bar{A} - \bar{K}_{\text{per}}(k)\bar{C}_{\text{per}}(k)$, and $\bar{A}(k) = \bar{A} - \bar{K}(k)\bar{C}_{\text{per}}(k)$. Then it holds that both $\bar{A}_{\text{per}}(k)$ and $\bar{A}(k)$ are exponentially stable, see [6, Theorem 5], since the periodic system is detectable and stabilizable. Furthermore, it holds that the the difference $\Delta P(k) = \bar{P}^*_{\text{per}}(k) - \bar{P}^*(k)$ satisfies the equation

$$\begin{aligned}\Delta P(k+1) &= \bar{A}_{\text{per}}(k)\Delta P(k)\bar{A}(k)^T \\ &= \bar{A}_{\text{per}}(k)\Delta P(k)\bar{A}_{\text{per}}(k)^T + O(\|\Delta P(k)\|_2^2),\end{aligned} \tag{21}$$

see [6, Lemma 3]. Since, $\bar{A}_{\text{per}}(k)$ is exponentially stable, there are constants $\kappa > 0$, $0 \leq \lambda < 1$, $\rho' > 0$ such that for all $\|\Delta P(0)\|_2 < \rho'$ it holds that $\|\Delta P(k)\|_2 \leq \kappa\lambda^k\|\Delta P(0)\|_2$.

Now we choose the radius ρ such that $\rho \leq \rho'$ and $\rho < R/\kappa$. Then it follows that $\|\Delta P(k)\|_2 \leq \kappa\lambda^k\rho < R, \quad k \geq 0$. The radius ρ ensures that that the perturbed covariance $\bar{P}^*(k)$ always remains within the ball of radius R around $\bar{P}^*_{\text{per}}(k)$, and the switching signal $s(k)$ remains unchanged. This means that the perturbed solution $\bar{P}^*(k)$ is also a solution to the covariance-switched filter. Furthermore, it converges exponentially fast to the periodic solution, as shown in (21). This concludes the proof. ■

V. Example Revisited

In Fig. 4, a simulation of the same random walk as in Section III with delay $d = 7$ is made using a switched Kalman filter based on (19). The covariance-based filter converges to a 3-periodic schedule after 5 samples: Two high-quality measurements followed by a low-quality measurement. The periodic filter from Section II-B with $N = 3$ is also used for comparison. Interestingly, the a priori periodic filter gives a lower error covariance. Here, it is better to use two low-quality measurements followed by a high-quality measurement. The reason for this is the long time delay of $d = 7$. The condition (19) only tries to make $P^*(k)$ one step ahead small. Then the covariance-based scheduler does not find the globally optimal periodic schedule.

The 3-periodic solution that is used for comparison does not give rise to its own switch schedule when (19) is applied to it. Then it cannot be a periodic solution of the covariance-based filter. This can also be seen because the two filters have the same state at time $k = 0$ but the solutions diverge into two different periodic trajectories. As predicted by Theorem 2, the 3-periodic schedule that the covariance-based filter reaches is locally stable, which can be verified by simulations.

An interesting problem for future work is to design switch functions f that guarantee a small average of the error covariance and good performance. From this single example it is clear that one needs to look more than one step ahead when the sensors have time delays. Such switch functions could be useful to find good switch schedules automatically.

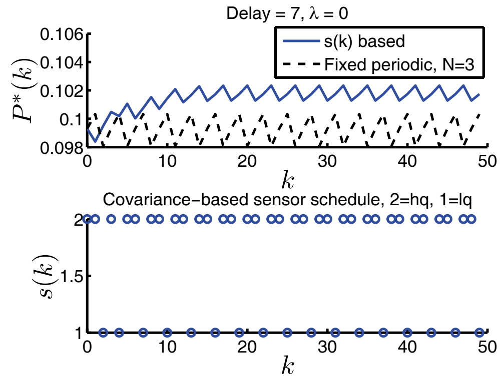

Fig. 4. In the upper plot, the error covariance $P^*(k)$ for a filter using the covariance-based schedule (19) and a fixed periodic schedule from the example in Section III are shown. In the lower plot, the covariance-based sensor schedule $s(k)$ is shown. It quickly converges to a period of three.

This could save computational efforts if one compares to the method in Section II-C.

VI. Conclusions

We have considered the problem of scheduling two heterogeneous sensors. One sensor was inaccurate and the other sensor was accurate, but could have a long time delay. As a tool, time-periodic Kalman filters were used. It was shown how optimal periodic sensor schedules can be found by means of a search over a finite set. It was also shown that by switching with a certain period between two relatively poorly performing sensors, one can achieve strictly better estimates. We also showed how simple sensor switching rules can be derived from the Riccati equation and that they can be used to obtain stable periodic switching schedules. An interesting problem for future research is to develop methodology for how to design switching rules that find optimal periodic schedules.

References

[1] I. F. Akyildiz, W. Su, Y. Sankarasubramaniam, and E. Cayirci, "Wireless sensor networks: a survey," *Computer Networks*, vol. 38, no. 4, 2002.

[2] D. Culler, D. Estrin, and M. Srivastava, "Overview of wireless sensor networks," *IEEE Computer*, Aug 2004, special Issue in Sensor Networks.

[3] P. Antsaklis and J. Baillieul, "Special issue on technology of networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, 2007.

[4] M. Athans, "On the determination of optimal costly measurement strategies for linear stochastic systems," *Automatica*, vol. 8, pp. 397–412, 1972.

[5] W. Wu and A. Arapostathis, "Optimal control of markovian systems with observations cost: Models and lqg controls," in *Proceedings of American Control Conference, Portland*, 2005, pp. 294–299.

[6] S. Bittanti, P. Colaneri, and G. De Nicolao, "The difference periodic Riccati equation for the periodic prediction problem," *IEEE Transactions on Automatic Control*, vol. 33, no. 8, pp. 706–712, Aug. 1988.

[7] A. Varga, "On solving discrete-time periodic Riccati equations," in *Proceedings of the IFAC World Congress*, 2005.

[8] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. Dover Publications, 2005.

[9] S. Bittanti and P. Bolzern, "Stabilizability and detectability of linear periodic systems," *Systems & Control Letters*, vol. 6, no. 2, pp. 141–145, July 1985.