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# Asymptotic and finite-time almost global attitude tracking: representations free approach

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*Abstract*— **In this paper, the attitude tracking problem is considered using the rotation matrices. Due to the inherent topological restriction, it is impossible to achieve global stability with any continuous attitude control system on $SO(3)$. Hence in this work, we propose some control protocols that achieve almost global tracking asymptotically and in finite time, respectively. In these protocols, no world frame is needed and only relative state information are requested. For the closed-loop systems, Filippov solutions and non-smooth analysis techniques are adopted to handle the discontinuities.**

*Index Terms*— **Agents and autonomous systems, Attitude tracking, Nonlinear systems**

## I. INTRODUCTION

Originally motivated by aerospace developments in the middle of the last century [3], [11], the rigid body attitude control problem has continued to attract attention with many applications such as aircraft attitude control [1], [21], spacial grabbing technology of manipulators [15], target surveillance by unmanned vehicles [17], and camera calibration in computer vision [14]. Furthermore, the configuration space of rigid-body attitudes is the compact non-Euclidean manifold $SO(3)$, which poses theoretical challenges for attitude control [2]. The coordination of multiple attitudes is of high interest both in academic and industrial research, e.g., [6], [18], [20].

Here we review some related existing work. As attitude systems evolves on $SO(3)$—a compact manifold without a boundary—there exists no continuous control law that achieves global asymptotic stability [4]. Hence one has to resort to some hybrid or discontinuous approaches. In [12], an almost global attitude tracking control system based on an alternative attitude error function is proposed. This attitude error function is not differentiable at certain attitudes and employs the Frobenius attitude difference, and the resulting control input is not continuous. In [13], one tracking protocol is proposed for unmanned aerial vehicle (UAV), again using Frobenius state differences. So far, finite-time attitude tracking problems are studied in different settings. One closely related work is [7], where finite-time attitude synchronization was investigated in a leader-follower architecture, namely all the followers tracking the attitude of the leader.

In this paper, we shall focus on the attitude tracking problem, based on the rotation matrices in $SO(3)$. First, based on geodesic direction between two rotation matrices, two controllers which achieve asymptotic and finite-time convergence, respectively, are proposed. Similarly, two more controllers yielding asymptotic and finite-time tracking are designed when the Frobenius difference between two rotation matrices, i.e., relative attitude, is available to the follower. All the controllers designed in this paper only need the relative state information without world frame and achieve almost global tracking. For the finite-time tracking case, since these control schemes are discontinuous, nonsmooth analysis is employed throughout the paper.

The structure of the paper is as follows. In Section II, we review some results for the special orthogonal group $SO(3)$. Section III presents the problem formulation of the attitude tracking. The main results of the stability analysis of the finite-time convergence are presented in Section IV, where two types of controllers, using geodesic and Frobenius state differences, respectively, are proposed to achieve almost global tracking. Then, in Section V, the paper is concluded.

**Notations.** With $\mathbb{R}_-, \mathbb{R}_+, \mathbb{R}_{\geqslant 0}$ and $\mathbb{R}_{\leqslant 0}$ we denote the sets of negative, positive, non-negative, non-positive real numbers, respectively. The rotation group $SO(3) = \{R \in \mathbb{R}^{3\times 3} : RR^\top = I, \det R = 1\}$. The vector space of real $n$ by $n$ skew symmetric matrices is denoted as $\mathfrak{so}(3)$. The vectors $\mathbb{1}_n$ and $\mathbf{0}_n$ represents a $n$-dimensional column vector with each entry being 1 and 0, respectively. We denote

$$
\begin{aligned}
E_1 &= \operatorname{diag}[-1,-1,1] \\
E_2 &= \operatorname{diag}[-1,1,-1] \\
E_3 &= \operatorname{diag}[1,-1,-1],
\end{aligned}
$$

respectively.

## II. PRELIMINARIES

In this section, we briefly review some essentials about rigid body attitudes [19]. For the definitions related to Filippov solutions, we refer to [8].

The tangent space at a point $R \in SO(3)$ is

$$T_R SO(3) = \{R\omega : \omega \in \mathfrak{so}(3)\}. \tag{1}$$

For $SO(3)$, two exponential maps are needed, namely Riemannian exponential at the point $R$ and Lie group exponential, denoted $\exp_R$ and $\exp$ respectively.

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For any $p = [p_1, p_2, p_3]^\top \in \mathbb{R}^3$ and $\hat{p} \in \mathfrak{so}(3)$ given as

$$\hat{p} := \begin{pmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{pmatrix},$$

Rodrigues' formula is the right-hand side of

$$\exp(\hat{p}) = \begin{cases} I_3 + \frac{\sin(\|p\|)}{\|p\|}\hat{p} + \frac{1-\cos(\|p\|)}{\|p\|^2}(\hat{p})^2, & \text{if } \|p\| \neq 0, \\ I_3, & \text{if } \|p\| = 0. \end{cases} \tag{2}$$

The matrix $\exp(\hat{p})$ is the rotation matrix through an angle $\|p\|$ anticlockwise about the axis $p$.

Next lemma follows from Euler's Rotation Theorem.

**Lemma 1.** *The exponential map*

$$\exp : \mathfrak{so}(3) \to SO(3) \tag{3}$$

*is surjective.*

The Riemannian exponential map $\exp_R : T_R SO(3) \to SO(3)$ is defined as

$$\exp_{R_1}(v) = \gamma(1)$$

where

$$\gamma(t) = R_1(R_1^\top R_2)^t, \quad 0 \leqslant t \leqslant 1$$

is the length of the shortest geodesic curve that connect $R_1$ and $R_2$, and $\gamma'(0) = v$. The relation between these exponential maps is $\exp_R(R\omega) = R\exp(\omega)$ for any $RW \in T_R SO(3)$.

The principle logarithm for a matrix $R \in SO(3)$ is defined as

$$\log(R) = \begin{cases} \frac{\theta}{2\sin(\theta)}(R - R^\top), & \text{if } \theta \neq 0, \\ \mathbf{0}, & \text{if } \theta = 0 \end{cases} \tag{4}$$

where $\theta = \arccos(\frac{\mathrm{tr}(R)-1}{2})$. We define $\log(I_3)$ as the zero matrix in $\mathbb{R}^{3\times 3}$. Note that (4) is not defined for $\theta = \pi$.

There are three commonly used metrics in $SO(3)$. A straightforward one is Frobenius (chordal) metric

$$\begin{aligned} d_F(R_1, R_2) &= \|R_1 - R_2\|_F \\ &= \sqrt{6 - \mathrm{tr}(R_1^\top R_2) - \mathrm{tr}(R_2^\top R_1)}, \end{aligned}$$

which is Euclidean distance of the ambient space $\mathbb{R}^{3\times 3}$. Another metric employs the Riemannian structure, namely the Riemannian (geodesic) metric

$$d_R(R_1, R_2) = \frac{1}{\sqrt{2}}\|\log(R_1^{-1}R_2)\|_F.$$

The third one is hyperbolic metric defined as $d_H(R_1, R_2) = \|\log(R_1) - \log(R_2)\|_F$.

One important relation between $SO(3)$ and $\mathbb{R}^3$ is that the open ball $B_\pi(I)$ in $SO(3)$ with radius $\pi$ around the identity, which is almost the whole $SO(3)$, is diffeomorphic to the open ball $B_\pi(0)$ in $\mathbb{R}^3$ via the logarithmic and the exponential map defined in (4) and (2).

In the remainder of this section, we define *Filippov set-valued map*. Let $f$ be a map from $\mathbb{R}^m$ to $\mathbb{R}^n$ and let $2^{\mathbb{R}^n}$ denote the collection of all subsets of $\mathbb{R}^n$. The Filippov set-valued map of $f$, denoted $\mathcal{F}[f] : \mathbb{R}^m \to 2^{\mathbb{R}^n}$, is defined as

$$\mathcal{F}[f](x) := \bigcap_{\delta>0} \bigcap_{\mu(S)=0} \overline{\mathrm{co}}\{f(B(x,\delta)\backslash S)\},$$

where $S$ is a subset of $\mathbb{R}^m$, $\mu$ denotes the Lebesgue measure, $B(x,\delta)$ is the ball centered at $x$ with radius $\delta$ and $\overline{\mathrm{co}}\{\mathcal{X}\}$ denotes the convex closure of a set $\mathcal{X}$. If $f$ is continuous at $x$, then $\mathcal{F}[f](x)$ contains only the point $f(x)$.

## III. PROBLEM FORMULATION

In this paper we consider attitude tracking problem. The basic model can be considered as two agent where the follower tracks the attitude of the target. We denote the world frame as $\mathcal{F}_w$, the instantaneous body frame of the target and the follower as $\mathcal{F}_r$ and $\mathcal{F}_1$, respectively. Let $R_r(t), R_1(t) \in SO(3)$ be the attitude of $\mathcal{F}_r$ and $\mathcal{F}_1$ relative to $\mathcal{F}_w$ at time $t$.

Recall that the tangent space at a point $R \in SO(3)$ is

$$T_R SO(3) = \{R\omega : \omega \in \mathfrak{so}(3)\}.$$

Then the kinematics of the two attitudes are given by [19]

$$\dot{R} = \mathrm{diag}(R_r, R_1)\omega \tag{5}$$

where

$$\begin{aligned} R &= [R_r^\top, R_1^\top]^\top, \\ \omega &= [\omega_r^\top, \omega_1^\top]^\top, \end{aligned}$$

where $\omega_1$ is the control input to design. Notice that $\omega_r, \omega_1$ are skew-symmetric matrices in $\mathfrak{so}(3)$.

By asymptotic and finite time attitude tracking we mean that for the multi-agent system (5), the absolute rotations of agent 1 track the rotation of the target in the world frame $\mathcal{F}_w$ asymptotically and in finite time, respectively. In other words,

$$\begin{aligned} &R_1 \to R_r, \text{ as } t \to \infty, \text{ and} \\ \exists T > 0, \text{ s.t. } &R_1 \to R_r, \text{ as } t \to T, \end{aligned}$$

respectively.

## IV. MAIN RESULT: SINGLE AGENT TRACKING

In this section, we first assume that the desired velocity $\omega_r(t) \in \mathfrak{so}(3)$ and the geodesic difference are available to the agent 1. Here we present two controllers as

$$\omega_{1,a} = \log(R_1^{-1}R_r) + \omega_r, \tag{6}$$

$$\omega_{1,f} = \frac{1}{\|\log(R_1^{-1}R_r)\|_F}\log(R_1^{-1}R_r) + \omega_r, \tag{7}$$

which will be proved to achieve asymptotic and finite-time tracking, respectively.

As discontinuities are introduced if the controller (7) is employed, we shall understand the trajectories in the sense of Filippov, namely an absolutely continuous function $x(t)$ satisfying the differential inclusion

$$\begin{aligned}\begin{bmatrix}\dot{R}_r\\ \dot{R}_1\end{bmatrix} \in &\begin{bmatrix}R_r\omega_r\\ \mathcal{F}[R_1\omega_{1,f}]\end{bmatrix}\\ =:&\,\mathcal{F}_1\end{aligned} \tag{8}$$

for almost all time, where we used Theorem 1(5) in [16].

**Theorem 2.** *Consider system* (5). *Assume the system initialized without singularity, i.e.,* $\arccos(\frac{\mathrm{tr}(R_r^\top(0)R_1(0))-1}{2}) \neq \pi$. *Then*

1) *the singularity is avoided for all time for both controller* (6) *and* (7)*;*
2) *the attitude* $R_1$ *tracks* $R_r$ *exponentially and in finite time, respectively, by* (6) *and* (7)*. For* (7)*, the conclusion holds for all the solutions.*

*Proof.* The proof is divided into two parts, one for each controller (6) and (7).

Part I: In this part, we prove that by using controller (6), the asymptotic tracking is achieved and the singularity is avoided. We can write the closed-loop as

$$\begin{aligned}\dot{R}_r &= R_r\omega_r\\ \dot{R}_1 &= R_1(\log(R_1^{-1}R_r)+\omega_r)\end{aligned}$$

Notice that the singularity only happens at $\theta = \arccos(\frac{\mathrm{tr}(R_r^\top R_1)-1}{2}) = \pi$, hence we only need to show that $\theta(t) \in [0,\pi)$ for all $t \geqslant 0$. Notice that

$$\begin{aligned}\frac{\partial\theta}{\partial R_r} &= \frac{-1}{\sqrt{1-\Delta^2}}\frac{\partial\Delta}{\partial R_r} = \frac{-1}{2\sqrt{1-\Delta^2}}R_1,\\ \frac{\partial\theta}{\partial R_1} &= \frac{-1}{\sqrt{1-\Delta^2}}\frac{\partial\Delta}{\partial R_1} = \frac{-1}{2\sqrt{1-\Delta^2}}R_r,\end{aligned} \tag{9}$$

where $\Delta = \frac{\mathrm{tr}(R_r^\top R_1)-1}{2}$. Then we have

$$\begin{aligned}\dot{\theta}(t) =& \,\mathrm{tr}(\frac{\partial^\top\theta}{\partial R_r}\dot{R}_r + \frac{\partial^\top\theta}{\partial R_1}\dot{R}_1)\\ =& \frac{-1}{2\sqrt{1-\Delta^2}}\frac{\theta}{\sin(\theta)}\,\mathrm{tr}\left(I - R_r^\top R_1 R_r^\top R_1\right) \leqslant 0\end{aligned}$$

where the last inequality is based on the fact that $R_r^\top R_1 R_r^\top R_1 \in SO(3)$. This proves that if the singularity is avoid at the initialization, i.e., $\theta(0) < \pi$, then it is avoided along the trajectory.

Then consider the Lyapunov function $W(R_r, R_1) = d_R^2(R_r, R_1) = \frac{1}{2}\|\log(R_r^\top R_1)\|_F^2$, and we have

$$\begin{aligned}\frac{\partial W}{\partial R_r} &= -R_r\log(R_r^\top R_1)\\ \frac{\partial W}{\partial R_1} &= -R_1\log(R_1^\top R_r)\end{aligned}$$

and

$$\begin{aligned}\dot{W}(t) =& \,\mathrm{tr}(\frac{\partial^\top W}{\partial R_r}\dot{R}_r + \frac{\partial^\top W}{\partial R_1}\dot{R}_1)\\ =& -\mathrm{tr}\left(\log^\top(R_1^\top R_r)\log(R_1^\top R_r)\right)\\ =& -2W.\end{aligned}$$

Hence by LaSalle-Yoshizawa Theorem (see e.g., [5]), the follower tracks the attitude of the target exponentially.

Part II: In this part we prove that the finite-time tracking can be achieved by controller (7) and the singularity is avoided. The proof is similar to Part I. Hence we only provide the sketch.

For this case, we need to consider differential inclusion (8) since the discontinuity is present. Notice that the function $W$ and $\theta$ is $\mathcal{C}^1$, hence regular. Then for $\theta \neq 0$, i.e., $R_1^\top R_r \neq I$, we have

$$\begin{aligned}\mathcal{L}_{\mathcal{F}_1}\theta =& \Big\{\frac{-1}{2\sqrt{1-\Delta^2}}\frac{\theta}{\sin(\theta)}\frac{1}{\|\log(R_1^\top R_r)\|_F}\\ &\mathrm{tr}\left(I - R_r^\top R_1 R_r^\top R_1\right)\Big\}\\ \subset& \,\mathbb{R}_-.\end{aligned}$$

By the fact that $\theta$ is $\mathcal{C}^1$ continuous, hence $\theta(R_r(t), R_1(t))$ is absolutely continuous and $\dot{\theta}(t)$ exists almost everywhere which belongs to $\mathcal{L}_{\mathcal{F}_1}\theta$. Then

$$\theta(t) = \int_0^t \dot{\theta}(\tau)d\tau + \theta(0) \leqslant \theta(0),$$

which indicate the singularity is avoided.

Next, we prove the finite-time tracking. Consider the error $V := W^\alpha$ with $\alpha > \frac{1}{2}$. Then the set-valued derivative is given as

$$\mathcal{L}_{\mathcal{F}_1}V = \begin{cases}\{-\alpha\sqrt{2}V^\beta\}, & \text{if } R_1^\top R_r \neq I\\ \{0\}, & \text{if } R_1^\top R_r = I\end{cases}$$

where $\beta = \frac{2\alpha-1}{2\alpha} \in (0,1)$. Notice that

$$\{(R_r, R_1) \mid 0 \in \mathcal{L}_{\mathcal{F}_1}V\} = \{(R_r, R_1) \mid V = 0\},$$

and $\dot{V}$ exists when $V \neq 0$, and $\dot{V}$ exists almost everywhere when $V = 0$ (by the fact that $V$ is $\mathcal{C}^1$, hence regular) and $\dot{V} \subset \mathcal{L}_{\mathcal{F}_1}V = \{0\}$. In other words, we have

$$\dot{V} = -\alpha\sqrt{2}V^\beta, \text{ for } V \neq 0$$

with $\beta \in (0,1)$, which implies that $V$ converge to the origin in finite time (see, e.g., [10], [9]). Hence we the follower tracks the attitude of the target in finite time.

□

In the controller (6) and (7), it is assumed that the geodesic state difference is available. In the rest part of this section, we show that the same conclusion as in Theorem 2 can be derived for the controller with Frobenius difference, which is relative information as well, i.e.,

$$\omega_{1,a} = R_1^\top R_r - R_r^\top R_1 + \omega_r, \tag{10}$$

$$\omega_{1,f} = \frac{1}{\|R_1 - R_r\|_F}\left(R_1^\top R_r - R_r^\top R_1\right) + \omega_r, . \tag{11}$$

**Corollary 3.** *Consider system* (5). *Assume the system initialized without singularity, i.e.,* $\arccos(\frac{\mathrm{tr}(R_r^\top(0)R_1(0))-1}{2}) \neq \pi$. *Then*

1) *the singularity is avoided for all time for both controller* (10) *and* (11)*;*
2) *the attitude $R_1$ tracks $R_r$ exponentially and in finite time, respectively, by* (10) *and* (11)*. For* (11)*, the conclusion holds for all the solutions.*

*Proof.* Here the proof is similar to the one of Theorem 2, hence we only provide the sketch. Here the proof is again divided into two parts.

Part I: First, by (9), we have

$$\begin{aligned}\dot{\theta}(t) =&\operatorname{tr}(\frac{\partial^\top \theta}{\partial R_r}\dot{R}_r + \frac{\partial^\top \theta}{\partial R_1}\dot{R}_1)\\ =&\frac{-1}{2\sqrt{1-\Delta^2}}\operatorname{tr}\left(R_r^\top R_1(R_1^\top R_r - R_r^\top R_1)\right)\\ =&\frac{-1}{2\sqrt{1-\Delta^2}}\operatorname{tr}\left(I - R_r^\top R_1 R_r^\top R_1\right)\\ \leqslant&0.\end{aligned}$$

Hence the singularities are avoided along the trajectory, i.e., the rotation matrices $R_r(t)^\top R_1(t) \neq E_i, i = 1, 2, 3$ if the equality does not hold for $R_r(0)^\top R_1(0)$.

Then consider the Lyapunov function $W(R_r, R_1) = \frac{1}{2}d_F^2(R_r, R_1) = 3 - \operatorname{tr} R_r^\top R_1$, then

$$\begin{aligned}\dot{W}(t) =& -\operatorname{tr}(R_r^\top \dot{R}_1 + \dot{R}_r^\top R_1)\\ =& -\operatorname{tr}\left(I - R_r^\top R_1 R_r^\top R_1\right)\\ \leqslant&0.\end{aligned}$$

Hence by LaSalle-Yoshizawa Theorem (see e.g., [5]), the follower tracks the attitude of the target asymptotically. Moreover, as the $\theta \to 0$ asymptotically, there exists $T$ such that for any $t \geqslant T$, we have

$$\operatorname{tr}(R_r^\top R_1 R_r^\top R_1) \leqslant \operatorname{tr} R_r^\top R_1.$$

Hence for $t \leqslant T$, $\dot{W} \leqslant -W$. This implies the convergence is in fact exponential.

Part II: The conclusion for controller (11) can be derived similar to the proof of Theorem 2, by using the Lyapunov function $V = W^\alpha$ for $\alpha \in (\frac{1}{2}, \infty)$. □

**Remark 1.** *For the finite-time tracking controller* (7) *and* (11)*, one closely related work is [7]. Compare the result here to the one in Section III in [7], which assumes that the absolute attitude, the bounded velocity, the bounded acceleration of the target are available to the follower, the advantages of our controllers are that the control laws are very intuitive, that we do not assume that the desired velocity is bounded, and that only relative measurement is needed, i.e., the geodesic and Frobenius difference.*

## V. CONCLUSION

In this paper, we consider the asymptotic and finite-time attitude tracking problem. Based on the geodesic state difference, one asymptotic and finite-time tracking protocols are proposed. These protocols stabilize the system almost globally, i.e., the state of the follower tracks the attitude of the target if the system is initialized without singularity. For the finite-time controller, the solution of the closed-loop system is understood in the sense of Filippov. Similar protocols, asymptotic and finite-time one, are proposed if the Frobenius state differences are available. Future topics include estimation of the reference velocity using internal model principle, and tracking protocols using adaptive control mechanisms e.g., prescribed performance control.

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