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Brief paper

Targeted agreement of multiple Lagrangian systems[☆]

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ABSTRACT

In this paper, we study the targeted agreement problem for a group of Lagrangian systems. Each system observes a convex set as its local target and the objective of the group is to reach a generalized coordinate agreement towards these target sets. Typically, the generalized coordinate represents position or angle. We first consider the case when the communication graphs are fixed. A control law is proposed based on each system's own target sensing and information exchange with neighbors. With necessary connectivity, the generalized coordinates of multiple Lagrangian systems are shown to achieve agreement in the intersection of all the local target sets while generalized coordinate derivatives are driven to zero. We also discuss the case when the intersection of the local target sets is empty. Exact targeted agreement cannot be achieved in this case. Instead, we show that approximate targeted agreement can be guaranteed if the control gains are properly chosen. In addition, when communication graphs are allowed to be switching, we propose a model-dependent control algorithm and show that global targeted agreement is achieved when joint connectivity is guaranteed and the intersection of local target sets is nonempty. Simulations are given to validate the theoretical results.

1. Introduction

Distributed control of multi-agent systems has been extensively studied during the last decade. The key idea of distributed control is to realize a collective task for the overall system by using only neighboring information exchange (Jadbabaie, Lin, & Morse, 2003; Olfati-Saber, Fax, & Murray, 2007). Such systems rely on communication and thus raises a natural question on the influence of communication link failures. Therefore, the analysis of distributed algorithms executed over switching communication graphs has been investigated, for both continuous-time (Olfati-Saber et al., 2007) and discrete-time models (Blondel, Hendrickx, Olshevsky, & Tsitsiklis, 2005). The extension to the case of nonlinear multi-agent dynamics was studied in Lin, Francis, and Maggiore (2007), Tang, Gao, Zou, and Kurths (2013) and Yang, Meng, Shi, Hong, and Johansson (2016). The motivation of such studies is the fact that in many practical problems the agent dynamics are inherently nonlinear, e.g., Vicsek's model and Kuramoto's model.

As an important special class of nonlinear systems, distributed control of multiple Lagrangian systems has drawn a great deal of attention recently. Compared with for instance single integrator dynamics, a Lagrangian model can be used to accurately describe mechanical systems, such as mobile robots, autonomous vehicles, robotic manipulators, and rigid bodies. Therefore, the study on the distributed control of multiple Lagrangian systems is more applicable to applications including spacecraft formation flying and relative attitude keeping and control of multiple unmanned aerial vehicles, just to name a few. In particular, the author of Ren (2009) proposed distributed model-independent consensus algorithms for multiple Lagrangian systems in a leaderless setting. The coordination problem of multiple mechanical systems with safety guarantees was studied in Chopra, Stipanovi, and Spong (2008). The control laws were proposed to achieve both velocity synchronization and collision avoidance. The case of time-varying

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leader was studied in Chung and Slotine (2009), where the nonlinear contraction analysis was introduced to obtain globally exponential convergence results. The connectivity maintenance problem was studied for multiple nonholonomic robots in Dimarogonas and Kyriakopoulos (2008) and finite-time cooperative tracking algorithms were presented in Khoo, Xie, and Man (2009) over graphs that are quasi-strongly connected. Distributed containment control was proposed in Mei, Ren, and Ma (2012) where a sliding mode based strategy was introduced to estimate the leaders' generalized coordinate derivative information. The authors of Meng, Dimarogonas, and Johansson (2014) considered a leader–follower coordinated tracking problem for multiple Lagrangian systems. A chattering-free algorithm with adaptive coupling gains was developed such that the tracking errors between the followers and the leader are driven to zero. The influence of communication delays was discussed in Abdessameud, Polushin, and Tayebi (2014) and Nuno, Ortega, Basanez, and Hill (2011). Sufficient conditions for reaching synchronization of multiple Lagrangian systems were established for the case of fixed and unknown delays in Nuno et al. (2011), and for the case of discontinuous time-varying delays in Abdessameud et al. (2014). Region-based shape control was studied in Cheah, How, and Slotine (2009) and Haghighi and Cheah (2012), where a group of robots modeled by Lagrangian dynamics are driven into a desired region while guaranteeing collision avoidance. A similar problem was studied in Yan, Chen, and Sun (2012) where a multi-level architecture was proposed so that the robots not only converge into the desired region, but also form a desired shape.

In this paper, we study a targeted agreement problem for a group of cooperative Lagrangian systems. The dynamics of each agent is modeled by a Lagrangian equation and each agent observes a convex set as its local target. The objective is to ensure that the generalized coordinate derivatives of all the agents converge to zero and the generalized coordinates of all the agents reach an agreement towards these target sets. Typically, generalized coordinates represent positions, angles, and so on and generalized coordinate derivatives represent velocities, angular velocities and so on. The applications of the targeted agreement problem of Lagrangian systems include region-based motion control of multiple mobile robots (Cheah et al., 2009; Haghighi & Cheah, 2012; Yan et al., 2012) and cooperative target grasping for multiple robotic manipulators (Erhart & Hirche, 2013). The solution of this problem is leveraging a projected agreement algorithm for the distributed optimization problem of single integrator networks (Nedic, Ozdaglar, & Parrilo, 2010; Shi, Johansson, & Hong, 2013). The contributions of this paper are three-fold. First, we propose a controller ensuring global targeted agreement over fixed graphs. By applying LaSalle's Invariance Principle, we show that all systems not only reach an agreement, but also converge to the intersection of the local target sets. Second, we consider the situation when the intersection of the local target sets is empty. We show that instead of exact targeted agreement, approximate targeted agreement can be achieved in the sense that agreement and the particular target set tracking are achieved up to an arbitrary accuracy if control gains are properly chosen. Third, the case of switching graphs is studied and a model-dependent control algorithm is proposed to guarantee global targeted agreement over the network with joint connectivity. The major efforts of this part are to show that the states of the closed-loop system remain bounded and to properly use the converging-input converging-state property of consensus algorithms over networks with directed joint connectivity. A brief version of this work has been published in Meng, Yang, Shi, Dimarogonas, Hong, and Johansson (2014).

2. Preliminaries

2.1. Convex analysis

Denote $\|\cdot\|$ the Euclidean norm. For any nonempty set $\mathcal{S} \subseteq \mathbb{R}^m$, we use $d(x, \mathcal{S}) = \inf_{y\in\mathcal{S}}\|x-y\|$ to describe the distance between $x \in \mathbb{R}^m$ and $\mathcal{S}$. Obviously, $d(x, \mathcal{S}) = 0$, for $x \in \mathcal{S}$. A set $\mathcal{S} \subset \mathbb{R}^m$ is said to be convex if $(1-\zeta)x+\zeta y \in \mathcal{S}$ when $x \in \mathcal{S}, y \in \mathcal{S}$, and $0 \le \zeta \le 1$. Let $\mathcal{S}$ be a convex set. The convex projection of any $x \in \mathbb{R}^m$ onto $\mathcal{S}$ is denoted by $P_{\mathcal{S}}(x) \in \mathcal{S}$ satisfying $\|x - P_{\mathcal{S}}(x)\| = d(x, \mathcal{S})$. We also know that $d^2(x, \mathcal{S})$ is continuously differentiable for all $x \in \mathbb{R}^m$, and its gradient can be explicitly obtained by Aubin (1991):

$$\nabla d^2(x, \mathcal{S}) = 2(x - P_{\mathcal{S}}(x)), \tag{1}$$

where ∇ denotes the gradient. Also, it is easy to see that

$$(P_{\mathcal{S}}(x) - x)^{\mathrm{T}}(P_{\mathcal{S}}(x) - y) \le 0, \quad \forall y \in \mathcal{S}. \tag{2}$$

In addition,

$$\|P_{\mathcal{S}}(x) - P_{\mathcal{S}}(y)\| \le \|x - y\| \;\; \forall x, y \in \mathbb{R}^m. \tag{3}$$

2.2. Graph theory

An undirected graph $\mathcal{G}$ consists of a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is a finite, nonempty set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of unordered pairs of nodes. An arc $\{j, i\} \in \mathcal{E}$ denotes that node i, j can obtain each other's information mutually. All neighbors of node i are denoted $\mathcal{N}_i := \{j : \{j, i\} \in \mathcal{E}\}$. A path between i_1 and i_k is a sequence of arcs of the form $\{i_1, i_2\}, \{i_2, i_3\}, \ldots, \{i_{k-1}, i_k\}$. An undirected graph $\mathcal{G}$ is connected if each node has an undirected path to any other node. The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ associated with the graph $\mathcal{G}$ is defined such that a_{ij} is positive if $\{j, i\} \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. We assume $a_{ij} = a_{ji}$, for all $i, j \in \mathcal{V}$. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with A is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

2.3. Dini derivatives

Let $D^+V(t, x(t))$ be the upper Dini derivative of $V(t, x(t))$ with respect to t, i.e., $D^+V(t, x) = \limsup_{\eta\to 0^+} \frac{V(t+\eta, x(t+\eta)) - V(t, x(t))}{\eta}$. The following lemma is useful for our analysis.

Lemma 1 (Danskin, 1966). *Suppose for each $i \in \mathcal{V}$, $V_i : \mathbb{R} \times \mathbb{R}^m \to \mathbb{R}$ is continuously differentiable. Let $V(t, x) = \max_{i\in\mathcal{V}} V_i(t, x)$, and let $\widetilde{\mathcal{V}}(t) = \{i \in \mathcal{V} : V_i(t, x(t)) = V(t, x(t))\}$ be the set of indices where the maximum is reached at time t. Then, $D^+V(t, x(t)) = \max_{i\in\widetilde{\mathcal{V}}(t)} \dot{V}_i(t, x(t))$.*

2.4. Problem definition

Consider a network with n agents labeled by $\mathcal{V} = \{1, 2, \ldots, n\}$. The dynamics of agent $i \in \mathcal{V}$ is described by the Lagrangian equation

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i = \tau_i, \tag{4}$$

where $q_i \in \mathbb{R}^m$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{m\times m}$ is the $m \times m$ inertia (symmetric) matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i$ is the Coriolis and centrifugal terms, and $\tau_i \in \mathbb{R}^m$ is the control force. The dynamics of a Lagrangian system satisfies the following properties (Spong, Hutchinson, & Vidyasagar, 2006): 1. $M_i(q_i)$ is positive definite and bounded for any $q_i \in \mathbb{R}^m$. More specifically, there exist positive constants $k_{\overline{M}}$ and $k_{\underline{M}}$ such that $k_{\underline{M}} I_m \le M_i(q_i) \le k_{\overline{M}} I_m$. 2. $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew symmetric. 3. $C_i(q_i, \dot{q}_i)$ is bounded with respect to q_i and linearly bounded with respect to $\dot{q}_i$. More specifically, there is positive constant k_C such that $\|C_i(q_i,\dot{q}_i)\| \le k_C\|\dot{q}_i\|$.

We consider the targeted agreement problem for a group of Lagrangian systems. Each agent $i \in \mathcal{V}$ observes its own target set $\mathcal{X}_i \subset \mathbb{R}^m$. The objective is to ensure that the generalized coordinate derivatives of all the agents converge to zero and all the agents reach their target sets and simultaneously reach agreement with other agents on their generalized coordinates. Note that this targeted agreement problem is different from the coordinated tracking problem (Meng et al., 2014), since we consider a set target objective instead of a point target objective. It is also different from the containment control problem (Ji, Ferrari-Trecate, Egerstedt, & Buffa, 2008; Mei et al., 2012) or the target-aggregation problem (Shi & Hong, 2009), since every agent has its own target set. We assume that each agent observes the boundary points of its target set and obtains the relative distance information between the target set and itself.

We impose an assumption on the target sets.

Assumption 1. $\mathcal{X}_1, \mathcal{X}_2, \dots, \mathcal{X}_n$ are compact convex sets.

Note that Assumption 1 has been extensively used in the literature. We next introduce global targeted agreement, where all the agents not only reach an agreement, but also converge to the intersection of all $\mathcal{X}_i$, $i \in \mathcal{V}$, denoted by, $\mathcal{X}_0 = \bigcap_i \mathcal{X}_i$.

Definition 1. Multi-agent system (4) with a given control law τ_i, for all $i \in \mathcal{V}$, achieves global targeted agreement if for all $q_i(t_0) \in \mathbb{R}^m$, $\dot{q}_i(t_0) \in \mathbb{R}^m$, $i \in \mathcal{V}$,

(1) $\lim_{t\to\infty} d(q_i(t), \mathcal{X}_0) = 0$, $\forall i \in \mathcal{V}$, where $\mathcal{X}_0 = \bigcap_{i=1}^n \mathcal{X}_i$,
(2) $\lim_{t\to\infty}(q_i(t) - q_j(t)) = 0$, $\forall i, j \in \mathcal{V}$,
(3) $\lim_{t\to\infty}\dot{q}_i(t) = 0$, $\forall i \in \mathcal{V}$.

3. Fixed communication graphs

Let an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ define the communication of state information between the agents.

The following model-independent control law is proposed for all $i \in \mathcal{V}$:

$$\tau_i = -k_i\dot{q}_i - \alpha_i(q_i - P_{\mathcal{X}_i}(q_i)) - \beta\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j), \tag{5}$$

where $k_i > 0$ denotes generalized coordinate derivative damping, $q_i - P_{\mathcal{X}_i}(q_i)$ the relative distance between q_i to the set $\mathcal{X}_i$, $\alpha_i > 0$ the gain for the target set projection control, $\beta > 0$ the gain for the cooperative control, and $a_{ij} > 0$ is the (i, j) entry of the adjacency matrix A associated with the graph $\mathcal{G}$, which marks the strength of the information flow between i and j.

3.1. Exact targeted agreement

Theorem 1. *Suppose that Assumption 1 holds and the fixed communication graph $\mathcal{G}$ is connected. Then the multi-agent system (4) with (5) achieves global targeted agreement if and only if $\mathcal{X}_0$ is nonempty.*

Proof. (Sufficiency.) Note that the closed-loop system can be written as $\dot{q}_i = \dot{q}_i$, $\ddot{q}_i = M_i^{-1}(q_i)(C_i(q_i,\dot{q}_i)\dot{q}_i - k_i\dot{q}_i - \alpha_i(q_i - P_{\mathcal{X}_i}(q_i)) - \beta\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j))$, $i \in \mathcal{V}$. Choose state variable $x = [q^{\mathrm{T}}, \dot{q}^{\mathrm{T}}]^{\mathrm{T}} = [q_1^{\mathrm{T}}, q_2^{\mathrm{T}}, \dots, q_n^{\mathrm{T}}, \dot{q}_1^{\mathrm{T}}, \dot{q}_2^{\mathrm{T}}, \dots, \dot{q}_n^{\mathrm{T}}]^{\mathrm{T}}$. By using the properties of Lagrangian dynamics (Section 2.4) and noting that $P_{\mathcal{X}_i}(q_i)$ is a globally Lipschitz continuous function (from (3)), we know that (4)–(5) is an autonomous system with form $\dot{x} = \bar{f}(x)$ and $\bar{f}(x)$ is Lipschitz continuous. Then, consider the following Lyapunov function:

$$V(x) = \frac{1}{2}\sum_{i=1}^n \dot{q}_i^{\mathrm{T}} M_i(q_i)\dot{q}_i + \frac{1}{2}\sum_{i=1}^n \alpha_i\|q_i - P_{\mathcal{X}_i}(q_i)\|^2 + \frac{\beta}{4}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i} a_{ij}\|q_i - q_j\|^2. \tag{6}$$

Based on the properties of Lagrangian dynamics (Section 2.4) and Assumption 1, it follows that $V(x)$ is radially unbounded, i.e., $V(x) \to \infty$ as $\|x\| \to \infty$. Therefore, the set $\Omega_c = \{x \in \mathbb{R}^{2mn} | V(x) \le c\}$ is bounded for all $c = V(x(t_0))$. The derivative of V along (4)–(5) is

$$\begin{aligned}
\dot{V} &= \sum_{i=1}^n \dot{q}_i^{\mathrm{T}}\left(\frac{1}{2}\dot{M}_i(q_i)\dot{q}_i + M_i(q_i)\ddot{q}_i\right) + \sum_{i=1}^n \alpha_i\dot{q}_i^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) \\
&\quad + \frac{\beta}{2}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j)^{\mathrm{T}}(\dot{q}_i - \dot{q}_j) \\
&= \sum_{i=1}^n \dot{q}_i^{\mathrm{T}}\left(-k_i\dot{q}_i - \alpha_i(q_i - P_{\mathcal{X}_i}(q_i)) - \beta\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j)\right) \\
&\quad + \sum_{i=1}^n \alpha_i\dot{q}_i^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) + \beta\sum_{i=1}^n \dot{q}_i^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) \\
&= -\sum_{i=1}^n k_i\dot{q}_i^{\mathrm{T}}\dot{q}_i \le 0,
\end{aligned}$$

where we have used (1) to derive the first equality, and the fact that $a_{ij} = a_{ji}$ and the second property of Lagrangian dynamics to derive the second equality. Note that $V(x)$ is continuously differentiable for all $x \in \mathbb{R}^{2nm}$ from Section 2.1. We take $\Omega = \Omega_c$ as the positively invariant compact set. Then, based on LaSalle's Invariance Principle, we know that every solution of (4)–(5) converges to the set $\mathcal{M}$, where $\mathcal{M} = \{q_i \in \mathbb{R}^m, \dot{q}_i \in \mathbb{R}^m, \forall i \in \mathcal{V} \mid \dot{q} = 0$, and $q, \dot{q}$ are subject to (4)–(5)$\}$. Let $x(t)$ be a solution that belongs to $\mathcal{M}$. Then, we know that $\dot{q} \equiv 0 \Rightarrow \alpha_i(q_i - P_{\mathcal{X}_i}(q_i)) + \beta\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) \equiv 0$, $\forall i \in \mathcal{V}$.

Pick any $q_0 \in \mathcal{X}_0$. Such a q_0 exists since Assumption 1 holds and $\mathcal{X}_0$ is nonempty. Thus, it follows that for all $i \in \mathcal{V}$, $\beta(q_i - q_0)^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) + \alpha_i(q_i - q_0)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) \equiv 0$. We then know that $\beta\sum_{i=1}^n(q_i - q_0)^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) + \alpha_i\sum_{i=1}^n(q_i - q_0)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) \equiv 0$. It also follows that $\sum_{i=1}^n(q_i - q_0)^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) = \frac{1}{2}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i} a_{ij}\|q_i - q_j\|^2 \ge 0$ by noting that $a_{ij} = a_{ji}$. Also, we know from (2) that for all $i \in \mathcal{V}$, $(P_{\mathcal{X}_i}(q_i) - q_0)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) \ge 0$. It then follows that $(q_i - q_0)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) = \|q_i - P_{\mathcal{X}_i}(q_i)\|^2 + (P_{\mathcal{X}_i}(q_i) - q_0)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) \ge \|q_i - P_{\mathcal{X}_i}(q_i)\|^2$. This shows that $\sum_{i=1}^n(q_i - q_0)^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) \equiv 0$, and $\|q_i - P_{\mathcal{X}_i}(q_i)\| \equiv 0$, $\forall i \in \mathcal{V}$. Note that the above analysis holds for all $x(t_0) \in \mathbb{R}^{2nm}$. Therefore, we know from LaSalle's Invariance Principle and the fact that $\mathcal{G}$ is connected that for all $q_i(t_0) \in \mathbb{R}^m$, $\dot{q}_i(t_0) \in \mathbb{R}^m$, $i \in \mathcal{V}$, $\lim_{t\to\infty}\dot{q}_i(t) = 0$, $\lim_{t\to\infty}(q_i(t) - P_{\mathcal{X}_i}(q_i(t))) = 0$, $\forall i \in \mathcal{V}$, and $\lim_{t\to\infty}(q_i(t) - q_j(t)) = 0$, $\forall i, j \in \mathcal{V}$. It then follows that for all $i \in \mathcal{V}$ and $l \in \mathcal{V}$,

$$\begin{aligned}
\|q_i - P_{\mathcal{X}_l}(q_i)\| &\le \|q_i - q_l\| + \|q_l - P_{\mathcal{X}_l}(q_l)\| \\
&\quad + \|P_{\mathcal{X}_l}(q_l) - P_{\mathcal{X}_l}(q_i)\| \\
&\le 2\|q_i - q_l\| + \|q_l - P_{\mathcal{X}_l}(q_l)\|,
\end{aligned} \tag{7}$$

where we have used (3). This implies that $\lim_{t\to\infty}(q_i(t) - P_{\mathcal{X}_l}(q_i(t))) = 0$, $\forall i \in \mathcal{V}$ and $l \in \mathcal{V}$. Therefore, $\lim_{t\to\infty} d(q_i(t), \mathcal{X}_0) = 0$, $\forall i \in \mathcal{V}$. This shows that global targeted agreement is achieved.

(Necessity.) It follows directly from the fact that $\mathcal{X}_0$ nonempty is a necessary condition such that the first part of Definition 1 can be achieved. ■

3.2. Weighted distance optimization

The assumption that the intersection of $\mathcal{X}_i$, for all $i \in \mathcal{V}$, is nonempty is a necessary condition for Theorem 1. In this section, we discuss the situation when this assumption does not hold. Obviously, global targeted agreement in the sense of Definition 1 cannot be achieved since self-targeted tracking control and cooperative control are conflicting to each other. Instead, we first show that overall weighted distance optimization can be achieved.

Definition 2. Multi-agent system (4)–(5) achieves weighted distance optimization if there exists $q^* = [q_1^*, q_2^*, \ldots, q_n^*] \in \operatorname{argmin} F(q)$ such that $\lim_{t\to\infty} q_i(t) = q_i^*$, and $\lim_{t\to\infty}\dot{q}_i(t) = 0, \forall i \in \mathcal{V}$, where $F(q) = \frac{1}{2}\sum_{i=1}^n \alpha_i\|q_i - P_{\mathcal{X}_i}(q_i)\|^2 + \frac{\beta}{4}\sum_{i=1}^n\sum_{j\in\mathcal{N}_i} a_{ij}\|q_i - q_j\|^2$.

Theorem 2. *Suppose that Assumption 1 hold, the fixed communication graph $\mathcal{G}$ is connected and $\mathcal{X}_0$ is empty. Then the multi-agent system (4)– (5) achieves weighted distance optimization.*

Proof. By using the Lyapunov function (6) in the proof of Theorem 1, we know that every solution of (4)–(5) converges to the largest invariant set in $\mathcal{M}$ (defined in the proof of Theorem 1). Then, based on (4) with (5), we know that $\mathcal{M}_1 = \{q_i \in \mathbb{R}^m, \dot{q}_i \in \mathbb{R}^m, \ \forall i \in \mathcal{V} \mid \dot{q} = 0, \alpha_i(q_i - P_{\mathcal{X}_i}(q_i)) + \beta\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) = 0, \ \forall i \in \mathcal{V}\} \supseteq \mathcal{M}$ and $\lim_{t\to\infty}\dot{q}_i(t) = 0, \ \forall i \in \mathcal{V}$.

Note that $\alpha_i(q_i - P_{\mathcal{X}_i}(q_i)) + \beta\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) = -\nabla_{q_i}F(q)$, for all $i \in \mathcal{V}$. It also follows that $\mathcal{M}_2 = \{q_i \in \mathbb{R}^m, \dot{q}_i \in \mathbb{R}^m, \ \forall i \in \mathcal{V} \mid \dot{q} = 0, \ q \in \operatorname{argmin} F\} \supseteq \mathcal{M}$. Therefore, according to LaSalle's Invariance Principle, it follows that weighted distance optimization is achieved. ■

We next consider the scenario when certain local set is more important than others and the objective is to guarantee all agents approaching a pre-given local set $\mathcal{X}_l$.

Definition 3. Given $l \in \mathcal{V}$, the multi-agent system (4)–(5) achieves approximate targeted agreement to the set $\mathcal{X}_l$ if for any given $\varepsilon > 0$, there exist control gains $\beta(\varepsilon)$, $\alpha_i(\varepsilon)$, $k_i(\varepsilon)$, $i \in \mathcal{V}$, such that $\limsup_{t\to\infty} d(q_i(t), \mathcal{X}_l) \le \varepsilon$, $\limsup_{t\to\infty}\|q_i(t) - q_j(t)\| \le \varepsilon$, $\lim_{t\to\infty}\dot{q}_i(t) = 0, \forall i, j \in \mathcal{V}$.

Proposition 1. *Suppose that Assumption 1 holds, the fixed communication graph $\mathcal{G}$ is connected and $\mathcal{X}_0$ is empty. Then the multi-agent system (4)–(5) achieves approximate targeted agreement.*

Proof. Following the proof of Theorem 2, we next consider a given $l \in \mathcal{V}$ and fix control gains $k_i > 0$, for all $i \in \mathcal{V}$, and $\alpha_i > 0$, for all $i \in \mathcal{V} \setminus \{l\}$. We next show that $\limsup_{t\to\infty} d(q_i(t), \mathcal{X}_l) \le \varepsilon$, $i \in \mathcal{V}$. First, a global minimum of the function $F_l(q_{l^o}) = \frac{1}{2}\sum_{i\in\mathcal{V}\setminus\{l\}}\alpha_i\|q_i - P_{\mathcal{X}_i}(q_i)\|^2$ can be found as $F_l^* = F_l(q_{l^o}^*) = 0$, where $q_{l^o} = [q_1, \ldots, q_{l-1}, q_{l+1}, \ldots, q_n]$ and $q_{l^o}^* \in \mathcal{X}_1 \times \cdots \times \mathcal{X}_{l-1} \times \mathcal{X}_{l+1} \cdots \times \mathcal{X}_n$. Based on the fact that $\mathcal{X}_i$ is bounded for all $i \in \mathcal{V}$, it follows that there exists a constant $Z^* > 0$ such that $F_l(q_{l^o}) > 0$ for all $\|q_{l^o}\| > Z^*$. Therefore, we know that $F(q) > 0$ for all $\|q_{l^o}\| > Z^*$. This implies that the global minimum of F can be reached only when $\|q_{l^o}\| \le Z^*$. We next define $\zeta = \sup\{\|q_i - q_{0i}\|, \ \forall q_{0i} \in \mathcal{X}_i, \ \forall i \in \mathcal{V} \setminus \{l\}, \ \|q_{l^o}\| \le Z^*\}$. It is obvious that ζ is finite. Also, based on the fact that $a_{ij} = a_{ji}$, it follows that $\alpha_i(q_i - P_{\mathcal{X}_i}(q_i)) + \beta\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) \equiv 0, \ \forall i \in \mathcal{V} \Rightarrow \sum_{i=1}^n \alpha_i(q_i - P_{\mathcal{X}_i}(q_i)) \equiv 0$.

We thus know that by choosing $2\varepsilon^{-1}\zeta\sum_{i\in\mathcal{V}\setminus\{l\}}\alpha_i \le \alpha_l \le 2\varepsilon^{-1}\nu\zeta\sum_{i\in\mathcal{V}\setminus\{l\}}\alpha_i$, $\mathcal{M}_3 = \{q_i \in \mathbb{R}^m, \dot{q}_i \in \mathbb{R}^m, \ \forall i \in \mathcal{V} \mid \dot{q} = 0, \ \|q_l - P_{\mathcal{X}_l}(q_l)\| \le \frac{1}{2}\varepsilon\} \supseteq \mathcal{M}$, where $\nu > 1$ is a given positive constant. Next, we define $\omega = n^{-1}\sum_i^n q_i$. On the set $\mathcal{M}_2$, we know that $\sum_{i=1}^n \alpha_i(q_i - \omega)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) + \beta\sum_{i=1}^n(q_i - \omega)^{\mathrm{T}}\sum_{j\in\mathcal{N}_i} a_{ij}(q_i - q_j) \equiv 0$. It follows from Theorem 3 of Olfati-Saber et al. (2007) that $|\beta\varpi^{\mathrm{T}}(L \otimes I_m)\varpi| \ge \beta\lambda_2\varpi^{\mathrm{T}}\varpi$, where λ_2 denotes the smallest non-zero eigenvalue of Laplacian matrix L, $\varpi_i = q_i - \omega$, for all $i \in \mathcal{V}$, and $\varpi = [\varpi_1^{\mathrm{T}}, \varpi_2^{\mathrm{T}}, \ldots, \varpi_n^{\mathrm{T}}]^{\mathrm{T}}$. Also, we know that $\left|\sum_{i=1}^n \alpha_i(q_i - \omega)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i))\right| \le \frac{1}{2}\varpi^{\mathrm{T}}\varpi + \frac{1}{2}\sum_{i=1}^n \alpha_i^2\left|q_i - P_{\mathcal{X}_i}(q_i)\right|^2 \le \frac{1}{2}\varpi^{\mathrm{T}}\varpi + \frac{(1+4\nu^2)\zeta^2}{2}\left(\sum_{i\in\mathcal{V}\setminus\{l\}}\alpha_i\right)^2$. It thus follows that $\sum_{i=1}^n\|q_i - \omega\|^2 \le \frac{(1+4\nu^2)\zeta^2\left(\sum_{i\in\mathcal{V}\setminus\{l\}}\alpha_i\right)^2}{2\beta\lambda_2 - 1}$.

Therefore, we know that for all $i, j \in \mathcal{V}$, $\|q_i - q_j\| \le \frac{\zeta\sqrt{1+4\nu^2}\left(\sum_{i\in\mathcal{V}\setminus\{l\}}\alpha_i\right)}{\sqrt{2\beta\lambda_2 - 1}}$. It then follows that by choosing $\beta \ge \frac{(1+4\nu^2)\zeta^2\left(\sum_{i\in\mathcal{V}\setminus\{l\}}\alpha_i\right)^2}{2\lambda_2\varepsilon^2} + \frac{1}{2\lambda_2}$, we can guarantee that $\|q_i - q_j\| \le \frac{1}{2}\varepsilon$, for all $i, j \in \mathcal{V}$. We then know that on the set $\mathcal{M}_2 \bigcap \mathcal{M}_3$, for all $j \in \mathcal{V}$, $\|q_j - P_{\mathcal{X}_l}(q_l)\| \le \|q_l - P_{\mathcal{X}_l}(q_l)\| + \|q_j - q_l\| \le \varepsilon$. This shows that for a given $l \in \mathcal{V}$ and any given $\varepsilon > 0$, by properly choosing α_l and β, we can guarantee that $\limsup_{t\to\infty} d(q_i(t), \mathcal{X}_l) \le \varepsilon, \ \forall i \in \mathcal{V}$, $\limsup_{t\to\infty}\|q_i(t) - q_j(t)\| \le \varepsilon, \ \forall i, j \in \mathcal{V}$, and $\lim_{t\to\infty}\dot{q}_i(t) = 0, \ \forall i \in \mathcal{V}$. Therefore, the desired conclusion follows. ■

4. Switching communication graphs

One potential issue is communication link failure. Link failure becomes even more important when we consider controlling multiple vehicles with limited power. Therefore, it is necessary to consider the case of switching communication graphs. We associate the switching communication with a time-varying graph $\mathcal{G}_{\sigma(t)} = (\mathcal{V}, \mathcal{E}_{\sigma(t)})$, where $\sigma : [t_0, +\infty) \to \mathcal{P}$ is a piecewise constant function and $\mathcal{P}$ is a finite set of all possible graphs. $\mathcal{G}_{\sigma(t)}$ remains constant for $t \in [t_\ell, t_{\ell+1})$, $\ell = 0, 1, \ldots$ and switches at $t = t_\ell$, $\ell = 1, \ldots$. In addition, we assume that $\inf_\ell(t_{\ell+1} - t_\ell) \ge \tau_d > 0$, $\ell = 1, \ldots$, where τ_d is a constant. This dwell time assumption is extensively used in the analysis of switched systems (Liberzon & Morse, 1999). The joint graph of $\mathcal{G}_{\sigma(t)}$ during time interval $[t_1, t_2)$ is defined by $\mathcal{G}_{\sigma(t)}([t_1, t_2)) = \bigcup_{t\in[t_1,t_2)}\mathcal{G}(t) = (\mathcal{V}, \bigcup_{t\in[t_1,t_2)}\mathcal{E}(t))$. Moreover, j is a neighbor of i at time t whenever $\{j, i\} \in \mathcal{E}_{\sigma(t)}$, and $\mathcal{N}_i(\sigma(t))$ represents the set of agent i's neighbors at time t.

Definition 4. $\mathcal{G}_{\sigma(t)}$ is uniformly jointly connected if there exists a constant $T > 0$ such that $\mathcal{G}([t, t + T))$ is connected for any $t \ge t_0$.

The switching communication graph makes the targeted agreement problem much more complex. We assume therefore that the system parameters are available and propose the following control law:

$$\tau_i = C_i(q_i, \dot{q}_i)\dot{q}_i - kM_i(q_i)\dot{q}_i - \alpha_i M_i(q_i)(q_i - P_{\mathcal{X}_i}(q_i)) - \beta M_i(q_i)\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\sigma(t))(q_i - q_j), \ \forall i \in \mathcal{V}, \tag{8}$$

where $k > 0$ denotes generalized coordinate derivative damping, $\alpha_i > 0$ the gain for the target set projection control, $\beta > 0$ the gain for the cooperative control, and $a_{ij}(p)$ is the (i, j) entry of the adjacency matrix A_p associated with graph $\mathcal{G}_p$, for all $p \in \mathcal{P}$.

Remark 1. The exact system model information is used in the control law (8). Such a model-dependent control algorithm is called an inverse dynamics controller. It has been extensively used in the control of Lagrangian systems (Spong et al., 2006). Note that the proposed algorithm is implementable due to that the time-varying matrix $M_i(q_i)$ is always positive definite, which does not hold for general nonlinear systems.

Theorem 3. *Suppose that Assumption 1 holds, $\mathcal{X}_0$ is nonempty, and k is sufficiently large. Then the multi-agent system (4) with (8) achieves global targeted agreement if the communication graph $\mathcal{G}_{\sigma(t)}$ is uniformly jointly connected.*

We prove Theorem 3, with the help of a series of lemmas.

Note that the closed-loop system of (4) and (8) can be written as

$$\ddot{q}_i = -k\dot{q}_i - \beta\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\sigma(t))(q_i - q_j) - \alpha_i(q_i - P_{\mathcal{X}_i}(q_i)). \tag{9}$$

Lemma 2. *Suppose that Assumption 1 holds, $\mathcal{X}_0$ is nonempty, and k is sufficiently large. For the multi-agent system (4) with (8), it follows that $\lim_{t\to\infty}\left(q_i(t) - P_{\mathcal{X}_i}(q_i(t))\right) = 0$, and $\lim_{t\to\infty}\dot{q}_i(t) = 0$, for all $i \in \mathcal{V}$.*

Proof. Note that the Lyapunov function (6) proposed in the proof of Theorem 1 is not valid here due to the switching graph. Instead, by picking any $q_0 \in \mathcal{X}_0$, we propose the following Lyapunov function: $V = \frac{1}{2}\sum_{i=1}^n \frac{1}{\beta}\dot{q}_i^{\mathrm{T}}\dot{q}_i + \sum_{i=1}^n \frac{1}{\beta}(q_i - q_0)^{\mathrm{T}}\dot{q}_i + \sum_{i=1}^n \frac{k}{2\beta}\|q_i - q_0\|^2 + \frac{1}{2}\sum_{i=1}^n \frac{\alpha_i}{\beta}\|q_i - P_{\mathcal{X}_i}(q_i)\|^2$, where we choose $k > 1$ to guarantee V positive definite. The derivative of V along (9) is

$$\begin{aligned}
\dot{V} = &\sum_{i=1}^n \frac{1}{\beta}\dot{q}_i^{\mathrm{T}}\Bigg(-k\dot{q}_i - \beta\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\sigma(t))(q_i - q_j)\\
&- \alpha_i(q_i - P_{\mathcal{X}_i}(q_i))\Bigg) + \sum_{i=1}^n \frac{\alpha_i}{\beta}\dot{q}_i^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i))\\
&+ \sum_{i=1}^n \frac{(q_i - q_0)^{\mathrm{T}}}{\beta}\Bigg(-k\dot{q}_i - \beta\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\sigma(t))(q_i - q_j)\\
&- \alpha_i(q_i - P_{\mathcal{X}_i}(q_i))\Bigg) + \sum_{i=1}^n \frac{\|\dot{q}_i\|^2}{\beta} + \sum_{i=1}^n \frac{k}{\beta}(q_i - q_0)^{\mathrm{T}}\dot{q}_i\\
= &-\sum_{i=1}^n \frac{k-1}{\beta}\|\dot{q}_i\|^2 - \sum_{i=1}^n \dot{q}_i^{\mathrm{T}}\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\sigma(t))(q_i - q_j)\\
&- \sum_{i=1}^n \frac{\alpha_i}{\beta}(q_i - q_0)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) - \sum_{i=1}^n (q_i - q_0)^{\mathrm{T}}\\
&\times \sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\sigma(t))(q_i - q_j).
\end{aligned}$$

It follows that $\dot{V} \le -\begin{bmatrix} q(t) & \dot{q}(t)\end{bmatrix}\left(\begin{bmatrix} L_{\sigma(t)} & \frac{L_{\sigma(t)}}{2} \\ \frac{L_{\sigma(t)}}{2} & K\end{bmatrix} \otimes I_m\right) \times \begin{bmatrix} q(t) \\ \dot{q}(t)\end{bmatrix} - \sum_{i=1}^n \frac{\alpha_i}{\beta} d^2(q_i(t), \mathcal{X}_i) - \sum_{i=1}^n \frac{1}{\beta}\|\dot{q}_i(t)\|^2$, where $K = \frac{k-2}{\beta} I_n$, $q = [q_1^{\mathrm{T}}, q_2^{\mathrm{T}}, \ldots, q_n^{\mathrm{T}}]^{\mathrm{T}}$, $\dot{q} = [\dot{q}_1^{\mathrm{T}}, \dot{q}_2^{\mathrm{T}}, \ldots, \dot{q}_n^{\mathrm{T}}]^{\mathrm{T}}$, $L_{\sigma(t)}$ is the Laplacian matrix associated with graph $\mathcal{G}_{\sigma(t)}$ at time t, and we have used the fact that $(q_i - q_0)^{\mathrm{T}}(q_i - P_{\mathcal{X}_i}(q_i)) \ge \|q_i - P_{\mathcal{X}_i}(q_i)\|^2$ based on (2).

It is easy to show that L_p is symmetric and positive semi-definite, for all $p \in \mathcal{P}$. It follows that L_p can be diagonalized as $L_p = \Gamma_p^{-1}\Lambda_p\Gamma_p$, where Γ_p is a real orthogonal matrix, $\Lambda_p = \mathrm{diag}\{\lambda_p^1, \lambda_p^2, \ldots, \lambda_p^n\}$ and $\lambda_p^i \ge 0$ for all $i \in \mathcal{V}$ and all $p \in \mathcal{P}$. We then know that $F_p = \begin{bmatrix} \Gamma_p^{-1} & 0 \\ 0 & \Gamma_p^{-1}\end{bmatrix} P_p \begin{bmatrix} \Gamma_p & 0 \\ 0 & \Gamma_p\end{bmatrix}$, where $F_p = \begin{bmatrix} L_p & \frac{L_p}{2} \\ \frac{L_p}{2} & K\end{bmatrix}$ and $P_p = \begin{bmatrix} \Lambda_p & \frac{\Lambda_p}{2} \\ \frac{\Lambda_p}{2} & K\end{bmatrix}$. It then follows that the eigenvalue μ_p of P_p are the solutions of $\mu_p^2 - (\lambda_p^i + \frac{k-2}{\beta})\mu_p + \frac{k-2}{\beta}\lambda_p^i - \frac{1}{4}(\lambda_p^i)^2 = 0$ for all $p \in \mathcal{P}$. Thus, F_p is positive semi-definite for all $p \in \mathcal{P}$ if k is chosen such that $k \ge 2 + \frac{1}{4}\beta\max_{p\in\mathcal{P}}\{\lambda_{\max}(L_p)\}$. Since $\lambda_{\max}(L_p)$ can be bounded by $\lambda_{\max}(L_p) \le 2\max_{i\in\mathcal{V}}\sum_{j\in\mathcal{N}_i(p)} a_{ij}(p)$ based on inequality (12) of Olfati-Saber et al. (2007), we know that it is sufficient to choose $k \ge 2 + \frac{(n-1)a^*\beta}{2}$ such that F_p is positive semi-definite for all $p \in \mathcal{P}$, where $a^* = \max_{p\in\mathcal{P}}\max_{i,j\in\mathcal{V}} a_{ij}(p)$.

We let $\sigma = p_l$ on $[t_{l-1}, t_l)$ for $p_l \in \mathcal{P}$. Then, for any t satisfying $t_0 < t_1 < \cdots < t_l < t < t_{l+1}$, we have $\dot{V} \le -\begin{bmatrix} q(t) & \dot{q}(t)\end{bmatrix}\left(\begin{bmatrix} L_{p_l} & \frac{L_{p_l}}{2} \\ \frac{L_{p_l}}{2} & K\end{bmatrix} \otimes I_m\right)\begin{bmatrix} q(t) \\ \dot{q}(t)\end{bmatrix} - \sum_{i=1}^n \frac{\alpha_i}{\beta} d^2(q_i(t), \mathcal{X}_i) - \sum_{i=1}^n \frac{1}{\beta}\|\dot{q}_i(t)\|^2$. Since the selection of V is independent of σ and F_{p_l} is positive semi-definite for all $p_l \in \mathcal{P}$ given $k \ge 2 + \frac{(n-1)a^*\beta}{2}$, we know that for all $t \ge t_0$,

$$\dot{V} \le -\sum_{i=1}^n \frac{\alpha_i}{\beta} d^2(q_i, \mathcal{X}_i) - \sum_{i=1}^n \frac{1}{\beta}\|\dot{q}_i\|^2 \le 0. \tag{10}$$

Therefore, q_i and $\dot{q}_i$, $\forall i \in \mathcal{V}$, are bounded. We also know that (10) implies that $\int_{t_0}^{\infty}\left(\sum_{i=1}^n \frac{\alpha_i}{\beta} d^2(q_i(t), \mathcal{X}_i) + \sum_{i=1}^n \frac{1}{\beta}\|\dot{q}_i(t)\|^2\right)\mathrm{d}t \le V(t_0)$ is bounded.

Therefore, from (9) and that q_i and $\dot{q}_i$, $\forall i \in \mathcal{V}$ are bounded, we know that

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\sum_{i=1}^n \frac{\alpha_i}{\beta} d^2(q_i(t), \mathcal{X}_i) + \sum_{i=1}^n \frac{1}{\beta}\|\dot{q}_i(t)\|^2\right)$$

is bounded $\forall t \ge t_0$. Then, based on Barbalat's lemma (Khalil, 2002), we can show that

$$\lim_{t\to\infty}\left(\sum_{i=1}^n \frac{\alpha_i}{\beta} d^2(q_i(t), \mathcal{X}_i) + \sum_{i=1}^n \frac{1}{\beta}\|\dot{q}_i(t)\|^2\right) = 0.$$

Therefore, $\lim_{t\to\infty} d(q_i(t), \mathcal{X}_i) = 0$, and $\lim_{t\to\infty}\dot{q}_i(t) = 0$, for all $i \in \mathcal{V}$. Finally, we know that $\lim_{t\to\infty}\left(q_i(t) - P_{\mathcal{X}_i}(q_i(t))\right) = 0$, for all $i \in \mathcal{V}$. ■

We next define $x_i = q_i$, $x_{n+i} = q_i + \frac{1}{\beta}\dot{q}_i$, for all $i \in \mathcal{V}$. After some manipulations, (9) can be rewritten as

$$\dot{x}_i = -\beta(x_i - x_{n+i}), \tag{11a}$$

$$\dot{x}_{n+i} = -\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\sigma(t))(x_{n+i} - x_{n+j}) + \delta_i(t), \tag{11b}$$

where $i \in \mathcal{V}$, and $\delta_i = (1 - \frac{k}{\beta})\dot{q}_i + \frac{1}{\beta}\sum_{j\in\mathcal{N}_i(\sigma(t))} a_{ij}(\sigma(t))(\dot{q}_i - \dot{q}_j) - \frac{\alpha_i}{\beta}(q_i - P_{\mathcal{X}_i}(q_i))$, for all $i \in \mathcal{V}$. Note that Lemma 2 implies that $\lim_{t\to\infty}\delta_i(t) = 0$, for all $i \in \mathcal{V}$. We next present two lemmas on the connectivity of (11) and on the converging-input converging-state property of the agreement algorithm.

Consider (11) as a multi-agent system with node set $\overline{\mathcal{V}} = \{1, 2, \ldots, 2n\}$. We associate this system with a graph $\overline{\mathcal{G}}_{\sigma(t)} = (\overline{\mathcal{V}}, \overline{\mathcal{E}}_{\sigma(t)})$ and the corresponding neighbor set $\overline{\mathcal{N}}_i(\sigma(t))$ and adjacency matrix $\overline{A}_{\sigma(t)}$, where the connections and weights for agents $\{n+1, n+2, \ldots, 2n\}$ are defined by $\mathcal{E}_{\sigma(t)}$ and $A_{\sigma(t)}$. In addition, there exists arcs $a_{i(i+n)}(t) = \beta > 0$, for all $i = 1, 2, \ldots, n$ and all $t \ge t_0$. Note that $\overline{\mathcal{G}}_{\sigma(t)}$ is switching from the sets of directed graphs. We define that the directed graph $\overline{\mathcal{G}}$ is quasi-strongly connected if there exists a node $i \in \overline{\mathcal{V}}$ such that there exists a directed path from i to any other node. In addition, the switching graph $\overline{\mathcal{G}}_{\sigma(t)}$ is said to be uniformly jointly quasi-strongly connected if there exists a constant $T > 0$ such that $\overline{\mathcal{G}}([t, t+T))$ is quasi-strongly connected for any $t \ge t_0$.

It is not hard to verify the following lemma.

Lemma 3. *Suppose $\mathcal{G}_{\sigma(t)}$ is uniformly jointly connected with a uniform constant T. Then, $\overline{\mathcal{G}}_{\sigma(t)}$ is uniformly jointly quasi-strongly connected with the same constant T.*

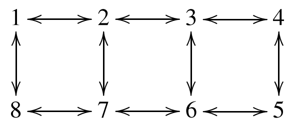

Fig. 1. The fixed communication graph $\mathcal{G}$.

The converging-input converging-state property of the agreement algorithm over networks with joint connectivity has been shown in Proposition 4.10 of Shi and Johansson (2013), which is restated in the following lemma.

Lemma 4. *Consider a network of* $2n$ *nodes with the communication graph* $\overline{\mathcal{G}}_{\sigma(t)}$*. The dynamics of node i is given by*

$$\dot{x}_i = -\sum_{j\in\overline{\mathcal{N}}_i(\sigma(t))} a_{ij}(\sigma(t))(x_i - x_j) + \delta_i(t),$$

for all $i \in \overline{\mathcal{V}}$*, where* δ_i *is a piecewise continuous function. Suppose* $\overline{\mathcal{G}}_{\sigma(t)}$ *is uniformly jointly quasi-strongly connected and* $\lim_{t\to\infty}\delta_i(t) = 0$ *for all* $i \in \overline{\mathcal{V}}$*. Then,* $\lim_{t\to\infty}(x_i(t) - x_j(t)) = 0, \ \forall i, j \in \overline{\mathcal{V}}$*.*

Proof of Theorem 3. Based on Lemmas 2–4, we know that $\lim_{t\to\infty}\dot{q}_i(t) = 0$, $\lim_{t\to\infty}(q_i(t) - P_{\mathcal{X}_i}(q_i(t))) = 0$, $\forall i \in \mathcal{V}$, and $\lim_{t\to\infty}(q_i(t) - q_j(t)) = 0$. $\forall i, j \in \mathcal{V}$. Following (7) in the proof of Theorem 1, it follows that $\lim_{t\to\infty} d(q_i(t), \mathcal{X}_0) = 0$, $\forall i \in \mathcal{V}$. This shows that global targeted agreement is achieved.

5. Simulation verifications

Assume that there are eight agents ($n = 8$) in the group with system dynamics given by $\begin{bmatrix} M_{11,i} & M_{12,i} \\ M_{21,i} & M_{22,i} \end{bmatrix} \begin{bmatrix} \ddot{q}_{ix} \\ \ddot{q}_{iy} \end{bmatrix} + \begin{bmatrix} C_{11,i} & C_{12,i} \\ C_{21,i} & C_{22,i} \end{bmatrix} \begin{bmatrix} \dot{q}_{ix} \\ \dot{q}_{iy} \end{bmatrix} = \begin{bmatrix} \tau_{ix} \\ \tau_{iy} \end{bmatrix}$, $i = 1, 2, \ldots, 8$, where $M_{11,i} = \theta_{1i} + 2\theta_{2i}\cos q_{iy}$, $M_{12,i} = M_{21,i} = \theta_{3i} + \theta_{2i}\cos q_{iy}$, $M_{22,i} = \theta_{3i}$, $C_{11,i} = -\theta_{2i}\sin q_{iy}\dot{q}_{iy}$, $C_{12,i} = -\theta_{2i}\sin q_{iy}(\dot{q}_{ix} + \dot{q}_{iy})$, $C_{21,i} = \theta_{2i}\sin q_{iy}\dot{q}_{ix}$, $C_{22,i} = 0$. Choose $\theta_{1i} = 1.301$, $\theta_{2i} = 0.256$, $\theta_{3i} = 0.096$, $i = 1, 2, \ldots, 8$. We assume that the available target sets of all the agents are disks. The radius of the disks are $rl_i = 3$, $i = 1, 2 \ldots, 8$. Denote the coordinates of the center points as $l_i = [l_{ix}, l_{iy}]^{\mathrm{T}} \in \mathbb{R}^2$, $i = 1, 2 \ldots, 8$ and $l_1 = l_3 = l_7 = [1.5, 1.5]^{\mathrm{T}}$, $l_2 = l_6 = l_8 = [0, -3]^{\mathrm{T}}$, and $l_4 = l_5 = [1.5, -3]^{\mathrm{T}}$. The initial states of the agents are given by $q_1(0) = [-8, 8]^{\mathrm{T}}$, $q_2(0) = [6.4, 12]^{\mathrm{T}}$, $q_3(0) = [-8, -8]^{\mathrm{T}}$, $q_4(0) = [6, -8]^{\mathrm{T}}$, $q_5(0) = [-8.8, -4]^{\mathrm{T}}$, $q_6(0) = [4.8, -12]^{\mathrm{T}}$, $q_7(0) = [-4, -8]^{\mathrm{T}}$, $q_8(0) = [3.2, -12]^{\mathrm{T}}$, $\dot{q}_1(0) = [-0.4, 0.4]^{\mathrm{T}}$, $\dot{q}_2(0) = [0.8, -0.8]^{\mathrm{T}}$, $\dot{q}_3(0) = [2.8, -2.8]^{\mathrm{T}}$, and $\dot{q}_4(0) = [1.6, -1.6]^{\mathrm{T}}$, $\dot{q}_5(0) = [-1.2, 0.8]^{\mathrm{T}}$, $\dot{q}_6(0) = [1.6, -0.4]^{\mathrm{T}}$, $\dot{q}_7(0) = [1.6, -2]^{\mathrm{T}}$, and $\dot{q}_8(0) = [0.8, -0.8]^{\mathrm{T}}$. The control parameters of (5) are chosen as $k_i = 1$, $\alpha_i = 1$, for all $i \in \mathcal{V}$, and $\beta = 1$. The communication graph $\mathcal{G}$ is given in Fig. 1. Also, the weight of adjacency matrix A associated with $\mathcal{G}$ is chosen to be 1, for all $i, j \in \mathcal{V}$. For the multi-agent system (4) with (5), snapshots of generalized coordinates are shown in Fig. 2. We see that all the generalized coordinates of agents converge to a common point in the intersection set of all target set $\mathcal{X}_i$, for all $i \in \mathcal{V}$. This shows that global targeted agreement is achieved.

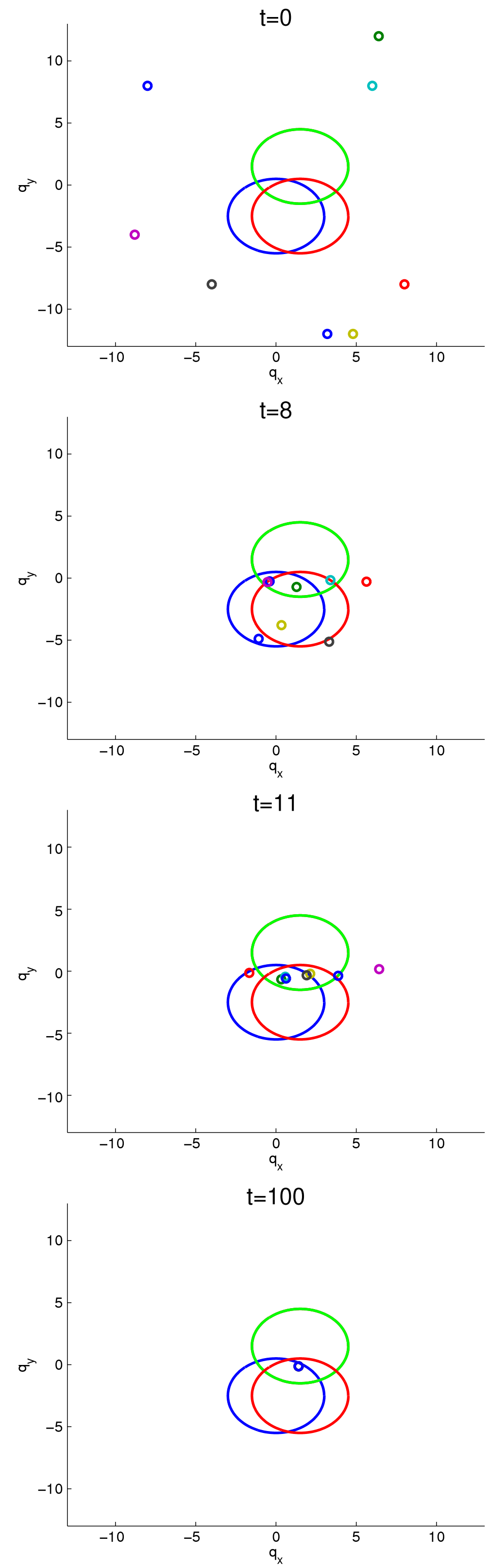

Fig. 2. Snapshots of the generalized coordinates of the multi-agent system. The small circles denote the generalized coordinates of the agents and the large circles are target sets. As indicated by the plots, global targeted agreement is achieved.

6. Conclusions

In this paper, we studied the targeted agreement problem for a group of cooperative Lagrangian systems. The objective was to drive generalized coordinated derivatives to converge to zero and achieve generalized coordinate agreement for the all the agents while the final generalized coordinate of each agent was restricted by its target set. Under a necessary condition that the intersection of all the target sets is nonempty, we first proposed a control algorithm that achieved global targeted agreement, i.e., all the

Lagrangian systems achieve agreement in the intersection of all the target sets. The assumption that the intersection of all the target sets is nonempty was later removed. Instead of exact targeted agreement for such a case, we showed that approximate targeted agreement can be achieved by properly choosing control gains. In addition, the case of switching communication graphs was considered using a model-dependent control algorithm, which guaranteed global targeted agreement over the network with joint connectivity. Simulations were given to validate the theoretical results. Future works include the study of moving targets and directed communication graphs.

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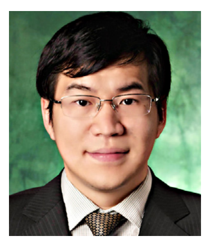

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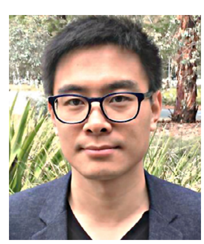

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