Brief paper

Coordinated output regulation of heterogeneous linear systems under switching topologies

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A R T I C L E   I N F O

Article history:
Received 9 May 2013
Received in revised form 15 June 2014
Accepted 9 December 2014
Available online 11 February 2015

Keywords:
Heterogeneous linear dynamic systems
Coordinated output regulation
Switching communication topology

A B S T R A C T

In this paper, we construct a framework to describe and study the coordinated output regulation problem for multiple heterogeneous linear systems. Each agent is modeled as a general linear multiple-input multiple-output system with an autonomous exosystem which represents the individual offset from the group reference for the agent. The multi-agent system as a whole has a group exogenous state which represents the tracking reference for the whole group. Under the constraints that the group exogenous output is only locally available to each agent and that the agents have only access to their neighbors’ information, we propose observer-based feedback controllers to solve the coordinated output regulation problem using output feedback information. A high-gain approach is used and the information interactions are allowed to be switching over a finite set of networks containing both graphs that have a directed spanning tree and graphs that do not. Simulations are shown to validate the theoretical results.

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1. Introduction

Coordinated control of multi-agent systems has recently drawn large attention due to its broad applications in physical, biological, social, and mechanical systems (Bai, Arcak, & Wen, 2011; Chopra & Spong, 2009; Cortes, Martinez, & Bullo, 2006; Meng, Dimarogonas, & Johansson, 2014; Meng et al., 2013; Tanner, Jadbabaie, & Pappas, 2007). The key idea of a coordination algorithm is to realize a global emergence using only local information interactions (Jadbabaie, Lin, & Morse, 2003; Olfati-Saber, Fax, & Murray, 2007). The coordination problem of a single-integrator network has been fully studied with an emphasis on the system robustness to the input time delays and switching communication topologies (Blondel, Hendrickx, Olshevsky, & Tsitsiklis, 2005; Jadbabaie et al., 2003; Olfati-Saber et al., 2007; Ren & Beard, 2005), discrete-time dynamical models (Moreau, 2005; You & Xie, 2011), nonlinear couplings (Lin, Francis, & Maggiore, 2007), convergence speed (Cao, Morse, & Anderson, 2008), and leader–follower tracking (Shi, Hong, & Johansson, 2012). The coordination of multiple general linear dynamic systems has recently been studied. For example, the authors of Wieland, Kim, and Allgöwer (2011) generalize the coordination of multiple single-integrator systems to the case of multiple linear time-invariant high-order systems. For a network of neutrally stable systems and polynomially unstable systems, the author of Tuna (2009) proposes a design scheme for achieving synchronization. The case of switching communication topologies is considered in Scardovi and Sepulchre (2009) and a so-called consensus-based observer is proposed to guarantee leaderless synchronization of multiple identical linear dynamic systems under a jointly connected communication topology. Similar problems are also considered in Ni and Cheng (2010) and Wang, Cheng, and Hu (2008), where a frequently connected communication topology is studied in Wang et al. (2008) and an assumption on the neutral stability is imposed in Ni and Cheng (2010). The authors of Li, Duan, Chen, and Huang (2010) propose a neighbor-based observer to solve the synchronization problem for general linear time-invariant systems. In addition, the classical Laplacian matrix is generalized in Yang, Roy, Wan, and Saberi (2011) to a so-called interaction matrix and a D-scaling approach is used to stabilize this interaction.
matrix. Synchronization of multiple heterogeneous linear systems has been investigated under both fixed and switching communication topologies [Alvergue, Pandey, Gu, & Chen, 2013; Grip, Yang, Saberi, & Stoorvogel, 2012; Lunze, 2012; Wieland, Sepulchre, & Allgöwer, 2011]. In Grip et al. (2012), a high-gain approach is proposed to dominate the non-identical dynamics of the agents. The cases of frequently connected and jointly connected communication topologies are studied in Kim, Shim, Back, and Seo (2013) and Vengertsev, Kim, Shim, and Seo (2010), respectively, where a slow switching condition and a fast switching condition are presented. Recently, the generalizations of coordination of multiple linear dynamic systems to the cooperative output regulation problem are studied in Ding (2013), Kim, Shim, and Seo (2011), Su and Huang (2012), Wang, Hong, Huang, and Jiang (2012) and Xiang, Wei, and Li (2009). In addition, the study on the synchronization of homogeneous and heterogeneous networks with nonlinear couplings is considered in Cao, Chen, and Li (2008), Cao, Wang, and Sun (2007) and He, Du, Qian, and Cao (2013).

In this paper, we generalize the classical output regulation problem of a single linear system to the coordinated output regulation problem of multiple heterogeneous linear systems. We consider the case where each agent has an individual offset and simultaneously there is a group tracking reference. The individual offset and the group reference are generated by autonomous systems (i.e., systems without inputs). Each individual offset is available to its corresponding agent while the group reference can be obtained only through constrained communication among the agents, i.e., the group reference trajectory is available to only a subset of the agents. Our goal is to find an observer-based feedback controller for each agent such that the output of each agent converges to a given trajectory determined by the combination of the individual offset and the group reference. Motivated by the approach in Grip et al. (2012), we propose a unified observer to solve the coordinated output regulation problem of multiple heterogeneous general linear systems, where the open-loop poles of the agents can be exponentially unstable and the dynamics are allowed to be different both with respect to dimensions and parameters. This relaxes the common assumption of identical dynamics [Li et al., 2010; Ni & Cheng, 2010; Scardovi & Sepulchre, 2009; Su & Huang, 2012; Tuna, 2009; Vengertsev et al., 2010; Xiang et al., 2009], or open-loop poles at most polynomially unstable (Ni & Cheng, 2010; Scardovi & Sepulchre, 2009; Su & Huang, 2012; Wieland, Sepulchre et al., 2011), or relative degree and minimum phase requirement (Kim et al., 2011). In addition, in this work, the information interaction is allowed to be switching from a graph set containing both a directed spanning tree set and a disconnected graph set. This extends the existing works considering fixed communication topologies (Grip et al., 2012; Kim et al., 2011; Li et al., 2010; Tuna, 2009; Wang et al., 2012).

The remainder of the paper is organized as follows. In Section 2, we give some basic definitions on the network model. In Section 3, we formulate the problem of coordinated output regulation of multiple heterogeneous linear systems. We then propose the state feedback control law with a unified observer design in Section 4. Numerical studies are carried out in Section 5 to validate our design and a brief concluding remark is drawn in Section 6.

2. Network model

We use graph theory to model the communication topology among agents. A directed graph G consists of a pair (V, E), where V := {v1, v2, ..., vn} is a finite, nonempty set of nodes and E ⊆ V × V is a set of ordered pairs of nodes. An edge (vi, vj) denotes that node vj can obtain information from node vi. All neighbors of node vi are denoted as Ni := {vj|(vi, vj) ∈ E}. For an edge (vi, vj) in a directed graph, vi is the parent node and vj is the child node.

A directed path in a directed graph is a sequence of edges of the form (v1, v2), (v2, v3), ..., A directed tree is a directed graph, where every node has exactly one parent except for one node, called the root, which has no parent, and the root has a directed path to every other node. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all other nodes.

For a leader–follower graph G := (V, E), we have V = {v0, v1, v2, ..., vn}, E ⊆ V × V, where v0 is the leader and v1, v2, ..., vn denote the followers. The leader–follower adjacency matrix $\mathbf{A} = [a_{ij}]$ is defined such that $a_{ij}$ is positive if $(v_i, v_j) \in E$ while $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$, $i = 0, 1, 2, ..., n$, and the leader has no parent, i.e., $a_{0i} = 0$, $i = 1, 2, ..., n$. The leader–follower “grounded” Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n \times n}$ associated with $\mathbf{A}$ is defined as $l_{ij} = \sum_{k=0}^{n} a_{jk}$ if $i \neq j$.

We assume that the leader–follower communication topology $G_{\sigma(t)}$ is time-varying and switched from a finite set $\{G_{k}\}_{k \in \mathbb{R}}$, where $\Gamma = \{1, 2, ..., \delta\}$ is an index set and $\delta \in \mathbb{N}$ indicates its cardinality. We impose the technical condition that $G_{\sigma(t)}$ is right continuous, where $\sigma: [t_0, \infty) \rightarrow \Gamma$ is a piecewise constant function of time, i.e., $G_{\sigma(t)}$ remains constant for $t \in [t_{\ell}, t_{\ell+1})$, $\ell = 0, 1, 2, ..., \delta$. In addition, we assume that $\text{inf}(t_{\ell+1} - t_\ell) \geq \tau > 0$, $\ell = 0, 1, 2, ..., \delta$, with $\lim_{\tau \rightarrow \infty} t_\ell = \infty$, where $\tau$ is a constant known as the dwell time (Liberzon & Morse, 1999). Let the sets $\{\Gamma_{\ell}\}_{\ell \in \mathbb{R}}$ be the leader–follower adjacency matrices and leader–follower ground Laplacian matrices associated with $\{G_{k}\}_{k \in \mathbb{R}}$, respectively. Consequently, the time-varying leader–follower adjacency matrix and time-varying leader–follower ground Laplacian matrix are defined as $\tilde{\mathbf{A}}_{\sigma(t)} = [a_{ij}(t)]$ and $\tilde{\mathbf{L}}_{\sigma(t)} = [l_{ij}(t)]$.

Other notations in this paper: $\lambda_{\text{min}}(P)$ and $\lambda_{\text{max}}(P)$ denote, respectively, the minimum and maximum eigenvalues of a real symmetric matrix $P$. $P^T$ denotes the transpose of $P$. $I_n$ denotes the $n \times n$ identity matrix, and diag(A1, A2, ..., An) denotes a block diagonal matrix with the main diagonal blocks matrices. A square matrix $A$ is called a Hurwitz matrix if every eigenvalue of $A$ has strictly negative real part.

3. Problem formulation

3.1. Agent dynamics

Suppose that we have $n$ agents modeled by the linear multiple-input multiple-output (MIMO) systems for each $v_i \in V$:

$$\dot{x}_i = A_i x_i + B_i u_i,$$

where $x_i \in \mathbb{R}^{n_i}$ is the state agent, $u_i \in \mathbb{R}^{m_i}$ is the control input, $A_i \in \mathbb{R}^{n_i \times n_i}$, $B_i \in \mathbb{R}^{n_i \times m_i}$, and $n_i$ and $m_i$ are positive integers, for all $v_i \in V$.

Also suppose that there is an individual autonomous exosystem for each $v_i \in V$:

$$\dot{\omega}_i = S_i \omega_i,$$

where $\omega_i \in \mathbb{R}^{n_i}$, $S_i \in \mathbb{R}^{n_i \times n_i}$, and $q_i$ is a positive integer, for all $v_i \in V$.

In addition, there is a group autonomous exosystem for the multi-agent system as a whole:

$$\dot{x}_0 = A_0 x_0,$$

where $x_0 \in \mathbb{R}^{n_0}$, $A_0 \in \mathbb{R}^{n_0 \times n_0}$, and $n_0$ is a positive integer.

3.2. Available information for agents

For the individual autonomous exosystem tracking, available output information for each agent $v_i \in V$ is $y_i = C_i x_i + C_{ei} \omega_i$, where $y_i \in \mathbb{R}^{p_i}$, $C_i \in \mathbb{R}^{p_i \times n_i}$, $C_{ei} \in \mathbb{R}^{p_i \times q_i}$, and $p_i$ is a positive integer.
For the group autonomous exosystem tracking, only neighbor-based output information is available due to the constrained communication. This means that not all the agents have access to $y_0$. The available information is the neighbor-based sum of each agent’s own output relative to that of its’ neighbors, i.e.,

$$
\zeta_i = \sum_{j=0}^{n} a_j(t)(y_{i0} - y_{j0}) \quad \text{is available for each agent } v_i \in V,
$$

with $a_i(t)$, $i = 0, 1, \ldots, n$, $j = 0, 1, \ldots, n$, is entry $(i,j)$ of the adjacency matrix $\bar{A}(t)$ associated with $G_{\sigma(t)}$ defined in Section 2 at time $t$. $\zeta_i \in R^p$, $i = 1, 2, \ldots, n$, $y_{i0}$ is represented by $y_{i0} = \mathcal{C}_0 x_0$, where $\mathcal{C}_0 \in R^{p2 \times n}$, $i = 1, 2, \ldots, n$, $\mathcal{C}_0 \in R^{p2 \times n}$, $\nu_i \in R^p$, $i = 0, 1, \ldots, n$, and $p_2$ is a positive integer. Also, the relative estimation information is available using the same communication topologies, i.e., $\tilde{v}_i = \sum_{j=0}^{n} a_j(t)(\tilde{y}_i - \tilde{y}_j)$ is available for each agent $v_i \in V$, where $\tilde{y}_i$ is an estimate produced internally by each agent $v_i \in V$, $\tilde{v}_i \in R^p$, $i = 1, 2, \ldots, n$ and $\tilde{y}_i \in R^p$, $i = 0, 1, \ldots, n$, which will be given explicitly in Section 4.

4. Coordinated output regulation with unified observer design

4.1. Redundant modes

Before designing the state feedback control and distributed observer, we need first to remove the redundant modes that have no effect on $y_{i0}$ and $y_{i0} - y_{j0}$. We impose the following assumptions on the structure of the systems.

**Assumption 3.**
- $(A_i, C_{ii})$, $i = 1, 2, \ldots, n$ is observable.
- $(A_0, C_0)$, $i = 1, 2, \ldots, n$ is observable.

We write the state and output of each agent in the compact form:

$$
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i
\end{bmatrix}
= 

\begin{bmatrix}
A_i & B_i \\
C_{ii} & D_{ii}
\end{bmatrix}
\begin{bmatrix}
x_i \\
y_i
\end{bmatrix} +

\begin{bmatrix}
\bar{A}_{i2} & \bar{B}_{i2} & \bar{C}_{i2}
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_i \\
\tilde{y}_i - y_{i0}
\end{bmatrix}
+ 

\begin{bmatrix}
B_i \\
0
\end{bmatrix} u_i,
$$

Given that Assumption 3 is satisfied, we can perform the state transformation given in Step 1 of Gripp et al. (2012) by considering $\omega_i$ and $x_0$ together. We construct a new state $\tilde{x}_i = W_i \tilde{x}_i$ with the dynamics

$$
\begin{bmatrix}
\dot{x}_i \\
\dot{y}_i
\end{bmatrix}
= 

\begin{bmatrix}
\bar{A}_{i2} & \bar{B}_{i2} & \bar{C}_{i2}
\end{bmatrix}
\begin{bmatrix}
\tilde{x}_i \\
\tilde{y}_i - y_{i0}
\end{bmatrix} + 

\begin{bmatrix}
B_i \\
0
\end{bmatrix} u_i
$$

where $e_{ii} = y_{i0} - y_{j0}$, and the details of $W_i, \mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i$ are given in Gripp et al. (2012). It was shown that the pair $(\bar{A}_i, \bar{C}_i)$ is observable and the eigenvalues of $\bar{A}_{i2}$ are a subset of the eigenvalues of $S_i$ and $A_0$, $i = 1, 2, \ldots, n$.

4.2. Regulated state feedback control law

We now design a controller to regulate $\epsilon_i$ to zero for each agent based on the state information $x_i = [\tilde{x}_i, \tilde{y}_i]$, where $x_i \in R^n$. We impose the following assumptions on the structure of the systems.

**Assumption 4.**
- $(A_i, B_i, D_{ii})$ is stabilizable, $i = 1, \ldots, n$.
- $(A_i, B_i, D_{ii})$ is right-invertible, $i = 1, \ldots, n$.
- $(A_i, B_i, D_{ii})$ has no invariant zeros in the closed right-half complex plane that coincide with the eigenvalues of $S_i$ or $A_0$, $i = 1, \ldots, n$.

**Lemma 1.** Let Assumption 4 hold. Then, the regulator equations (6) are solvable and the state-feedback controller $u_i = F_i(x_i -$
is a positive constant to be determined, and \( \mathcal{P} = \mathcal{P}^T \) is a positive definite matrix satisfying
\[
\mathcal{P} + \mathcal{P}^T - 2 \mathcal{P} \epsilon \begin{bmatrix} I_1 & 0 \\ 0 & \sqrt{\mathcal{P}} \end{bmatrix} \mathcal{P} + I_2 \mathcal{P} = 0,
\]
where \( \theta = \min_{k \in \Gamma_1} \beta_k, \beta_k \) is a positive constant satisfying \( \beta_k < \min \eta (\lambda (L_k)) \), \( k \in \Gamma_1 \), and \( \min \eta (\lambda (L_k)) \) denotes the minimum value of all the real parts of the eigenvalues of \( L_k \). Note that the existence of \( \mathcal{P} \) is due to the fact that \( (\mathcal{P}, \begin{bmatrix} I_1 & 0 \\ 0 & \sqrt{\mathcal{P}} \end{bmatrix}) \) is observable.

**Lemma 2.** All the eigenvalues of \( L_k \) are in the closed right-half plane and those on the imaginary axis are simple, where \( L_k \) is associated with \( \mathcal{G}_k \), defined in Section 2, for some \( \mathcal{G}_k \in [\mathcal{G}_k]_{k \in \Gamma_1} \).
- Furthermore, all the eigenvalues of \( L_k \) are in the open right-half plane for \( \mathcal{G}_k \in [\mathcal{G}_k]_{k \in \Gamma_1} \).

**Proof.** See Theorem 4.29 in Qu (2009) and Lemma 1.6 in Ren and Cao (2011).

**Lemma 3.** Let Assumptions 1–3 hold and assume that \( \kappa \geq \alpha/(4-2 \max(0.1, \max_{j \in \Gamma} \mathcal{P}^j)) \), where \( \alpha \in \mathbb{R}^+, \mathcal{P} \) and \( \theta \) are given by (11). Then, there exists an \( \epsilon^* \in (0, 1] \) such that, if \( \epsilon \in (0, \epsilon^*) \), \( \lim_{t \to \infty} (\xi(t) - \tilde{\xi}(t)) = 0 \), i.e., 1, 2, . . . , \( n \) for systems (8).

**Proof.** Note that for all \( i = 1, 2, \ldots, n \), \( \sum_{j=0}^{n} a_{ij}(t)(y_{ij} - y_{ij}) = \sum_{j=1}^{n} l_j(t)(y_{ij} - \hat{y}_{ij}) = \sum_{j=1}^{n} l_j(t)(y_{ij} - \hat{y}_{ij}) \).

Define \( \tilde{\chi}_i = \chi_i - \tilde{\chi}_i \). It then follows from (7) and (8) that for all \( i = 1, 2, \ldots, n \),
\[
\begin{aligned}
\tilde{\chi}_i &= (\mathcal{A} + \mathcal{L}) \chi_i - S(\epsilon) \mathcal{P}^T \left[ \sum_{j=1}^{n} l_j(t)(y_{ij} - \hat{y}_{ij}) \right],
\end{aligned}
\]
where \( l_j(t), i = 1, \ldots, n, j = 1, \ldots, n \), is the (i, j)th entry of the adjacency matrix \( L(t) \) associated with \( \mathcal{G}_k(t) \) defined in Section 2 at time \( t \). It follows that \( \tilde{\chi}_i = (\mathcal{A} + \mathcal{L}_k) \tilde{\chi}_i - S(\epsilon) \mathcal{P}^T \left[ \sum_{j=1}^{n} l_j(t)(y_{ij} - \hat{y}_{ij}) \right], i = 1, 2, \ldots, n \). By introducing \( \tilde{\xi}_i = (\mathcal{A} + \mathcal{L}_k) \tilde{\xi}_i - \mathcal{P}^T \left[ \sum_{j=1}^{n} l_j(t)(y_{ij} - \hat{y}_{ij}) \right] \), \( i = 1, 2, \ldots, n \), where \( \mathcal{L}_k = \left[ \epsilon^* l_j(t)_{i,j} \right] = O(\epsilon) \).

Note that \( \left[ \epsilon^* \tilde{\xi}_i \right] = \epsilon^* \tilde{\xi}_i \), for all \( i = 1, 2, \ldots, n \). The overall dynamics can be written as
\[
\begin{aligned}
\tilde{\xi}_k &= \left( I_n \otimes \mathcal{A} + \mathcal{L}_k - \left( I_n \otimes \mathcal{P}^T \right) \right) \left( I_n \otimes \tilde{\xi}_k \right) + \left( I_n \otimes \left[ \begin{array}{ccc} 0 & \epsilon \mathcal{P}^T \mathcal{P} \end{array} \right] \right) \left( I_n \otimes \tilde{\xi}_k \right),
\end{aligned}
\]
where \( \tilde{\xi}_k \in [\tilde{\xi}_1, \tilde{\xi}_2, \ldots, \tilde{\xi}_n]^T \) and \( \mathcal{L}_k = \text{diag}(\mathcal{L}_k, \mathcal{L}_k, \ldots, \mathcal{L}_k) \).

Note that \( -L_k, k \in \Gamma_1 \) is a Hurwitz matrix according to Lemma 2. Therefore, we can always guarantee that \( -L_k + \beta_k I_n \) is also a Hurwitz matrix by choosing \( \beta_k \) sufficiently small. In particular, we choose \( \beta_k \) as a positive constant satisfying \( \beta_k < \min \eta (\lambda (L_k)) \), \( k \in \Gamma_1 \). Then, we define the piecewise Lyapunov function candidate \( V_k = \tilde{\xi}_k^T (I_n \otimes \mathcal{P}^T) \tilde{\xi}_k \), where \( P_k \) is a positive definite matrix satisfying
\[
P_k (-L_k + \beta_k I_n) + (-L_k + \beta_k I_n) P_k = -I_n < 0, \quad k \in \Gamma_1, \quad P_k (-L_k + \beta_k I_n) P_k \leq 0, \quad k \in \Gamma_1,
\]
where the second inequality is due to Lemma 2.
It then follows that for all $k \in \Gamma_i$,

$$V_k \leq 2\xi^T (P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1})_t) \xi + 2\xi^T (P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1})_t) L_t \xi$$

$$- 2\xi^T (P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1}L_k)_t) \xi$$

$$\leq \xi^T (P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1}L_k)_t) \xi + 2\xi^T (P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1}L_k)_t) L_t \xi$$

where we have used (11) and the fact that $0 \leq \lambda_k$, $k \in \Gamma_i$. It then follows that $V_k \leq \frac{1}{2} \lambda_k V_k$, $\forall k \in \Gamma_i$, if $\|L_t\|_{\infty} \leq \frac{\lambda_{\min}(P_k) \lambda_{\min}(\varphi_{\omega_1, \omega_2})}{\lambda_{\max}(P_k) \lambda_{\max}(\varphi_{\omega_1, \omega_2})}$, $\forall k \in \Gamma_i$.

On the other hand, for all $k \in \Gamma_i$, we have that

$$V_k \leq 2\xi^T (P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1})_t) \xi + 2\xi^T (P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1})_t) L_t \xi$$

where we have used (11). Note that $\lambda_{\max}(P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1}L_k)_t) \xi + 2\xi^T (P_k \otimes (\varphi_{\omega_1, \omega_2}^{-1}L_k)_t) \xi \leq \lambda_{\max}(P_k) \lambda_{\max}(\varphi_{\omega_1, \omega_2})$, where $\lambda_k = 2 \max(\theta, 1) \lambda_{\max}(\varphi_{\omega_1, \omega_2})$, $\forall k \in \Gamma_i$.

Following the similar analysis as that in Liberzon and Morse (1999) and Zhai, Hu, Yasuda, and Michel (2000), we let $\sigma = p_j$ on $[t_j^{-1}, t_j]$ for $p_j \in \Gamma_i$. Then, for any $t$ satisfying $t_0 < t_1 < \cdots < t_{\ell} < t < t_{\ell+1}$, define $V = \xi^T (P_{\sigma(t)} \otimes (\varphi_{\omega_1, \omega_2}^{-1})) \xi$ for (12). We have that, $\forall \xi \in [t_j^{-1}, t_j)$.

$$V(\xi) \leq e^{-\frac{1}{2} \lambda_k(\xi-\xi)} V(t_{j-1})$$

$$\leq e^{-\frac{1}{2} \lambda_k(\xi-\xi)} V(t_{j-1}) - \lambda_k \xi$$

$$\leq e^{-\frac{1}{2} \lambda_k(\xi-\xi)} V(t_{j-1})$$

We then know that $V(t) \leq 0 \lim_{t \to t_0} V(t)$. Thus, it follows that $V(\xi) \leq e^{-\frac{1}{2} \lambda_k(\xi-\xi)} V(t_{j-1})$, where $\rho$ denotes times of switching during $[t_0, t]$. Given that $\lambda \geq \kappa^* = \frac{2 \max(\theta, 1) \lambda_{\max}(\varphi_{\omega_1, \omega_2})}{\epsilon}$ for some $\lambda \in (0, \lambda^*)$, it follows from Assumption 2 that $T_{q_j}^\rho(t) \geq \kappa^* T_{q_j}(t)$ for all $t \geq t_0$. This implies that $\lambda^* T_{q_j}^\rho(t) - \lambda^* T_{q_j}^\rho(t) \leq -\lambda(T_{q_j}^\rho(t) + T_{q_j}^\rho(t))$, for all $t \geq t_0$ and we therefore know that

$$V(t) \leq e^{\frac{1}{2} \lambda_k(\xi-\xi)} V(t_{j-1})$$

we then have that $\kappa^* = \frac{2 \max(\theta, 1) \lambda_{\max}(\varphi_{\omega_1, \omega_2})}{\epsilon}$, and

$$V(t) \leq e^{-\frac{1}{2} \lambda_k(\xi-\xi)} V(t_{j-1})$$

Furthermore, set $\lambda = \frac{\alpha^{2 \max(\theta, 1) \lambda_{\max}(\varphi_{\omega_1, \omega_2})}}{1-\alpha}$, and

$$V(t) \leq e^{-\frac{1}{2} \lambda_k(\xi-\xi)} V(t_{j-1})$$

It follows that if $\epsilon < \frac{\alpha^{2 \max(\theta, 1) \lambda_{\max}(\varphi_{\omega_1, \omega_2})}}{1-\alpha}$ we have for (12) that $\|x(t)\|_{\infty} \leq e^{-\frac{1}{2} \lambda_k(\xi-\xi)} V(t_{j-1})$, where $c^* = \frac{\lambda_{\max}(\varphi_{\omega_1, \omega_2})}{\epsilon \lambda_{\min}(P_k) \lambda_{\min}(\varphi_{\omega_1, \omega_2})}$. Thus, it follows that $\lambda_{\max}(\varphi_{\omega_1, \omega_2}) < \min_{k \in \Gamma} \frac{\lambda_{\max}(\varphi_{\omega_1, \omega_2})}{\epsilon \lambda_{\min}(P_k) \lambda_{\min}(\varphi_{\omega_1, \omega_2})}$. Then that $\lim_{t \to \infty} \chi_i(t) = 0$, $i = 1, 2, \ldots, n$. From the unified observer design, we then have that

$$\hat{x}_i = (T_i^\rho T_i^{-1}) \hat{\chi}_i = \frac{1}{2} T_i^\rho T_i^{-1} \chi_i = \frac{1}{2} T_i^\rho \chi_i, \quad i = 1, 2, \ldots, n,$$

which will be used in the control design.

4.5. Main results

In this section, we show that the observer architecture introduced in the previous sections provide an asymptotically stable closed-loop system, as presented in Theorem 1. The observer-based controller is

$$u_i = F_i \hat{x}_i + (I_i - F_i P_i) \hat{x}_0,$$

where $P_i$ and $I_i$ are the solutions of the regulator equations (6), and $\hat{x}_0$ and $\hat{x}_0$ can be obtained from (8) and (13). Theorem 1. Let Assumptions 1–4 hold and assume that $\kappa^* = \frac{2 \max(\theta, 1) \lambda_{\max}(\varphi_{\omega_1, \omega_2})}{\epsilon}$, where $\alpha \in (0, 1)$, $\theta$ and $\varphi$ are given by (11). Then, there exists $\epsilon^* \in (0, 1)$ such that, $t \in (0, \epsilon^*)$, (14) ensures that $\lim_{t \to \infty} c_i(t) = 0$, $i = 1, 2, \ldots, n$, for the multi-agent system (1)-(4).

Proof. Follows from Lemmas 1 and 3, and the separation principle.
5. Simulation results

In this section, we illustrate the theoretical results. Consider a network of three agents. We assume that the adjacency matrix $\tilde{A}_{\ell(t)}$ associated with $\overline{\mathcal{G}}_{\ell(t)}$ is switching periodically. Denote $\ell = 0, 20, 40, \ldots, \tilde{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, when $t \in \{\ell, \ell + 6\}$, $\tilde{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, when $t \in \{\ell + 6, \ell + 12\}$, $\tilde{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, when $t \in \{\ell + 12, \ell + 18\}$, $\tilde{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, when $t \in \{\ell + 18, \ell + 20\}$.

The dynamics of the agents are described by $A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $C_1 = C_2 = D_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $C_2 = D_2 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$. The dynamics of the individual autonomous exosystems are given by $S_i = 0$, $C_{ui} = D_{ui} = -1$, $i = 1, 2, 3$, and $\omega_1(0) = -2$, $\omega_2(0) = -4$, and $\omega_3(0) = -6$. The dynamics of the group autonomous exosystem are given by $A_0 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $C_0 = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, $D_0 = -C_0$.

Following the design scheme proposed in Section 4, for the solutions of regulator equations (6), we have that $F_1 = \begin{bmatrix} -1 & -4.5 & -6 \\ 0 & 0.0690 & 0.1724 \end{bmatrix}$ for agent $v_1$, $F_2 = \begin{bmatrix} -2 & -6 \\ 0 & 0.4 \end{bmatrix}$ for agent $v_2$, $F_3 = \begin{bmatrix} 0 & 0.5 \\ 0 & 0.6 \end{bmatrix}$ for agent $v_3$. We also have $\varepsilon = 0.2$ for (8) and $\theta = 0.1$ for (11).

Figs. 2 and 3 show, respectively, the state convergence and the error convergence of system (1), (2), and (3) under the observer-based controller (14). We see that coordinated output regulation is realized even when there exists multiple heterogeneous dynamics and the information interactions are switching. This agrees with the result in Theorem 1.

6. Conclusions

This paper studied the coordinated output regulation problem of multiple heterogeneous linear systems. We first formulated the coordinated output regulation problem and specified the information that is available for each agent. A high-gain based distributed observer and an individual observer were introduced for each agent and observer-based controllers were designed to solve the problem. The information interactions among the agents and the group autonomous exosystem were allowed to be switching over a finite set of networks containing both graphs having a spanning tree and graphs having not. Simulations were given to validate the theoretical results. Future directions include relaxing the dwell-time assumption.

References


