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Brief paper

A PDE approach to deployment of mobile agents under leader relative position measurements☆

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ABSTRACT

We study the deployment of a first-order multi-agent system over a desired smooth curve in 2D or 3D space. We assume that the agents have access to the local information of the desired curve and their relative positions with respect to their closest neighbors, whereas in addition a leader is able to measure his relative position with respect to the desired curve. For the case of an open $\mathcal{C}^2$ curve, we consider two boundary leaders that use boundary instantaneous static output-feedback controllers. For the case of a closed $\mathcal{C}^2$ curve we assume that the leader transmits his measurement to other agents through a communication network. The resulting closed-loop system is modeled as a heat equation with a delayed (due to the communication) boundary state, where the state is the relative position of the agents with respect to the desired curve. By choosing appropriate controller gains (the diffusion coefficient and the gain multiplying the leader state), we can achieve any desired decay rate provided the delay is small enough. The advantage of our approach is in the simplicity of the control law and the conditions. Numerical example illustrates the efficiency of the method.

1. Introduction

Most of the existing work on multi-agent systems (MAS) consider interconnected agents modeled using ordinary differential equations (ODEs) or difference equations, and design the control for each agent depending either on global or local information. Besides these studies, there has been some work using partial differential equations (PDEs) to describe the spatial dynamics of multi-agent systems, e.g., Freudenthaler and Meurer (2016), Fridman (2014a), Frihauf and Krstic (2011), Qi, Vazquez and Krstic (2015) and Servais, d'Andréa Novel, and Mounier (2014). This approach is especially powerful when the number of the agents is large. One of the advantages of using PDE models for MAS is to reduce a high-dimensional ODE system to a single PDE. Furthermore, for a desired PDE model, the corresponding performance and the control protocol for the individual agents can be designed by proper PDE discretization. In principle, this procedure is independent with respect to the number of agents, provided this number is large enough.

In this paper, we consider a formation control problem which is referred to as formation transition or deployment. This can be seen as a combination of a displacement-based and position-based method. Each agent measures the relative positions (displacements) of its neighboring agents with respect to a global coordinate system. The desired formation is specified by the desired relative positions between pair of agents. Then the agents, without any knowledge of their positions, achieve the desired formation by actively controlling the relative positions of their neighboring agents. As pointed out in Oh, Park, and Ahn (2015), in order to move the agents to prescribed absolute positions, there should be a small number of agents able to measure their absolute positions. For existing ODE methods we refer to Garcia de Marina, Jayawardhana, and Cao (2017) and Tanner, Jadbabaie, and Pappas (2007) and the references within. Here we review some related work on deployment using PDE models. In Frihauf and Krstic (2011) and Qi, Vazquez et al. (2015), the agents' dynamics is modeled by reaction–advection–diffusion PDEs. By using the backstepping approach to boundary control, the agents are deployed onto families of planar curves and 2D manifolds, respectively. In Qi, Pan and Qi (2015), the authors consider the formation tracking problem using complex-valued PDE with an input-to-state stability (ISS) type of convergence.

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In Meurer and Krstic (2011), finite-time deployment of MAS into a planar formation is studied, via predefined spatial–temporal paths, using a leader–follower architecture, i.e., boundary control. For details of this method we refer to Meurer (2012). The same problem of deployment into planar curves using boundary control is considered in Freudenthaler and Meurer (2016) and Servais et al. (2014) by using non-analytic solutions and a modified viscous Burger's equation, respectively. In Pilloni, Pisano, Orlov, and Usai (2016), the authors proposed a boundary control law for a MAS, which is modeled as the heat equation, to achieve state consensus.

The main contribution of the paper is that we propose a framework for the deployment of mobile agents onto arbitrary open or closed $\mathcal{C}^2$ curves under the assumption that only leader measures his absolute position and by using simple static output-feedback control. In this framework we assume that a small number of agents, which will be referred as leaders, are able to measure their absolute positions. More precisely, the leaders calculate their relative position with respect to the desired curve. We consider two scenarios: deployment on open or closed $\mathcal{C}^2$ curves. In the first scenario leaders use simple boundary controllers proportional to their relative positions with respect to the desired curve. Note that in this scenario it is difficult to guarantee the convergence in the presence of small time-varying delays in the relative leaders' positions (that appear e.g. due to network-based measurements). In the second scenario, leaders send the value of their relative positions to all the agents by using a communication network which results in time-varying delay due to sampling and communication (Fridman, 2014a). The other agents, which are referred as followers, have access only to the local information of the desired curve and relative positions with respect to their neighbors. Since in the second scenario the desired formation is a closed curve, the MAS is modeled as a diffusion equation with periodic boundary condition. The method used in this paper is based on Fridman and Blighovsky (2012) and Selivanov and Fridman (2018) which deal with Dirichlet and mixed boundary conditions. We derive linear matrix inequality (LMI) conditions for convergence with a desired convergence rate. Compared to the ODE MAS with communication delay, e.g., Li, Chen, and Liu (2013), the LMI conditions derived in this paper are simpler with lower dimension. This paper is an extended version of the conference paper (Wei, Fridman, Selivanov, & Johansson, 2019) where the results were presented for the second scenario only and part of the proof was omitted.

The paper is organized as follows. In Section 2, some useful inequalities are recalled. The MAS deployment problem using sampled control is formulated in Section 3. In Section 4, we consider a simple boundary control protocol for deployment into arbitrary smooth open curves. The main results, i.e., deployment on the closed $\mathcal{C}^2$ curve, are included in Section 5, where we derive LMI conditions for the desired decay rate with communication delay. Simulations are presented in Section 6. The paper is concluded in Section 7.

Notations. With $\mathbb{R}_-, \mathbb{R}_+, \mathbb{R}_{\geq 0}$ and $\mathbb{R}_{\leqslant 0}$ we denote the sets of negative, positive, non-negative, and non-positive real numbers, respectively. $\|\cdot\|_p$ denotes the ℓ_p-norm and the ℓ_2-norm is denoted as $\|\cdot\|$ without a subscript. $\mathcal{L}_2(a,b)$ is the Hilbert space of square integrable functions $\phi(\xi), \xi \in [a,b]$ with the corresponding norm given as $\|\phi\|_{\mathcal{L}_2} = \sqrt{\int_a^b z^2 d\xi}$. $\mathcal{H}^1(a,b)$ is the Sobolev space of absolutely continuous scalar functions $\phi : [a,b] \to \mathbb{R}$ with $\frac{d\phi}{d\xi} \in \mathcal{L}_2(a,b)$. $\mathcal{H}^2(a,b)$ is the Sobolev space of scalar functions $\phi : [a,b] \to \mathbb{R}$ with absolutely continuous $\frac{d\phi}{d\xi}$ and with $\frac{d^2\phi}{d\xi^2} \in \mathcal{L}_2(a,b)$.

2. Preliminaries: some inequalities

Recall the following Wirtinger-type inequality (Hardy, Littlewood, & Pólya, 1952): let $\phi \in \mathcal{H}^1(a,b)$ be a scalar function with $\phi(a) = 0$ or $\phi(b) = 0$. Then,

$$\int_a^b \phi^2(\xi)d\xi \leq \frac{4(b-a)^2}{\pi^2}\int_a^b \left(\frac{d\phi}{d\xi}\right)^2 d\xi. \tag{1}$$

Lemma 1 (*Halanay's Inequality, Fridman, 2014a; Halanay, 1966*). *Let $0 < \delta_1 < 2\delta_0$ and let $V : [t_0 - \tau_M, \infty) \to [0,\infty)$ be an absolutely continuous function that satisfies*

$$\dot V(t) \leq -2\delta_0 V(t) + \delta_1 \sup_{-\tau_M \leq \theta \leq 0} V(t+\theta), \quad t \geq t_0. \tag{2}$$

Then

$$V(t) \leq \exp(-2\delta(t-t_0)) \sup_{-\tau_M \leq \theta \leq 0} V(t_0+\theta), \ t \geq t_0,$$

where $\delta > 0$ is the unique positive solution of

$$\delta = \delta_0 - \frac{\delta_1 \exp(2\delta\tau_M)}{2}. \tag{3}$$

The following lemma, which is an extension of Sobolev's inequality, will be useful for this paper.

Lemma 2 (*Kang & Fridman, 2018*). *Let $\phi \in \mathcal{H}^1(0,1)$ be a scalar function. Then*

$$\max_{x\in[0,1]} \phi^2(x) \leq 2\int_0^1 \phi^2(\xi)d\xi + \int_0^1 \phi_\xi^2(\xi)d\xi.$$

3. Problem formulation

We consider a group of N agents that move in the space $\mathbb{R}^j$ for $j \in \{2,3\}$. Denote $\mathcal{I} = \{1,\dots,N\}$. The dynamics of each agent is given as

$$\dot x_i^m = u_i^m, \quad m \in \{1,\dots,j\}, \quad j = 2,3. \tag{4}$$

where $i \in \mathcal{I}$, $x_i^m \in \mathbb{R}$ and u_i^m are components of the position and control for ith agent, respectively. The aim is to deploy the agents on a given $\mathcal{C}^2$ curve $\gamma : [0,a] \to \mathbb{R}^j$, where $0 < a \leq 2\pi$.

Let us denote the line graph with N vertices as $\mathcal{G}_\ell = \{\mathcal{V},\mathcal{E}\}$, where $\mathcal{V} = \{v_1,\dots,v_N\}$ is the vertex set and $\mathcal{E} = \{(v_i, v_{i+1}), i = 1,\dots,N-1\}$ is the edge set. As a typical formation control procedure, one assigns N points on the curve, denoted as $\gamma(h),\dots,\gamma(Nh)$ where $h = a/N$. For simplicity we omit the superscript m in multi-agent description. Then the following displacement-based protocol (Oh et al., 2015)

$$\begin{aligned} \dot x_1(t) =& \varphi\frac{x_2(t)-x_1(t)}{h^2} - \varphi\frac{\gamma(2h)-\gamma(h)}{h^2} \\ \dot x_i(t) =& \varphi\frac{x_{i-1}(t)+x_{i+1}(t)-2x_i(t)}{h^2} \\ &- \varphi\frac{\gamma((i-1)h)+\gamma((i+1)h)-2\gamma(ih)}{h^2} \\ & i = 2,\dots,N-1, \\ \dot x_N(t) =& \varphi\frac{x_{N-1}(t)-x_N(t)}{h^2} - \varphi\frac{\gamma((N-1)h)-\gamma(Nh)}{h^2} \end{aligned} \tag{5}$$

where $x_i = [x_i^1,\dots,x_i^j]^\top$ and $\varphi > 0$, guarantees that all agents converge to the formation

$$E := \{x \in \mathbb{R}^{Nj} \mid x_i - x_j = \gamma(ih) - \gamma(jh), i,j \in \mathcal{I}\}, \tag{6}$$

which is the desired curve up to constant translations. The parameter φ is a controller gain. Indeed, the dynamics of the error $e := x - \gamma$ obeys the following consensus protocol

$$\dot e = -(L \otimes I_j)e \tag{7}$$

where L is the weighted Laplacian of the graph $\mathcal{G}_c$ with edge weights φ/h^2. As suggested in Ferrari-Trecate, Buffa, and Gati (2006), system (7) can be considered as the discretization in the spatial variable of the heat equation

$$e_t(\alpha, t) = \varphi e_{\alpha\alpha}(\alpha, t), \quad \alpha \in (0, a). \tag{8}$$

Thus the model (5), when N is large, is an approximation of

$$x_t(\alpha, t) = \varphi(x_{\alpha\alpha}(\alpha, t) - \gamma_{\alpha\alpha}(\alpha)). \tag{9}$$

It can be seen that system (9) cannot drive the agents onto the desired curve γ, but up to a constant translation. In fact, $x^* = \gamma + c$ is an equilibrium of system (9) for any constant c. This is consistent with the displacement-based formation control in Oh et al. (2015). In order to solve this problem, we shall consider two different scenarios to guarantee the convergence to the desired curve.

In both of the scenarios, we assign leader agents who can measure the absolute positions of themselves and of their targets.

In the first scenario, we consider the case that the agents $x(0, t)$ and $x(a, t)$ are the leaders of the system and controlled by

$$\begin{aligned} x_t(0, t) &= \kappa(\gamma(0) - x(0, t)) \\ x_t(a, t) &= \kappa(\gamma(a) - x(a, t)) \end{aligned} \tag{10}$$

where $\kappa > 0$ is the controller gain. The other agents only have access to their relative positions with respect to their nearest neighbors. Now the closed-loop of (9) and (10) will be referred to as systems with boundary control. Note that the ODE version of this model corresponds to the model in Section 5.5.1 in Oh et al. (2015).

By defining $e(\alpha, t) = x(\alpha, t) - \gamma(\alpha)$, the error dynamics is given as (8) with boundary condition

$$\begin{aligned} e_t(0, t) &= -\kappa e(0, t) \\ e_t(a, t) &= -\kappa e(a, t), \end{aligned} \tag{11}$$

and subject to absolutely continuous initial condition $e(\alpha, 0)$ with respect to $\alpha \in (0, a)$. The stability of this system will be analyzed in Section 4.

In the second scenario, which is the main contribution of the paper, we consider closed curve γ, i.e., γ is defined over $[0, a] = [0, 2\pi]$, and is $\mathcal{C}^2$. Then it is natural to consider the multi-agent system with periodic boundary condition

$$\begin{aligned} x(0, t) &= x(2\pi, t) \\ x_\alpha(0, t) &= x_\alpha(2\pi, t). \end{aligned} \tag{12}$$

Furthermore, we assume, without loss of generality, that the leader is located at $\alpha = \pi$ and it can measure $x(\pi, t) - \gamma(\pi)$ and send this information to the other agents through a communication network which results in a bounded time-varying input delay. The closed-loop system is given as

$$x_t = \varphi(x_{\alpha\alpha} - \gamma_{\alpha\alpha}) - K(x(\pi, t_k - \eta_k) - \gamma(\pi)), \tag{13}$$

where $t \in [t_k, t_{k+1})$, $\varphi > 0$, $K > 0$, t_k is the updating time of the controller with $\lim_{k\to\infty} t_k = \infty$, and η_k is the uniformly bounded network-induced delay. Note that the closed-loop system (13) suggests a unified control protocol for all the agents. So one does not have to design the actuators of the agents separately. This is not the case for the first approach where the leaders are controlled separately using boundary conditions. Parameters φ and K are the control gains. By using the time-delay approach to networked control systems (Fridman, 2014a, Chapter 7), we denote $\tau(t) = t - t_k + \eta_k$. Then the system (13) can be presented as

$$x_t = \varphi(x_{\alpha\alpha} - \gamma_{\alpha\alpha}) - K(x(\pi, t - \tau(t)) - \gamma(\pi)). \tag{14}$$

Here $\tau(t) \leq \tau_M$, where τ_M is the sum of the maximum transmission interval and maximum network-induced delay. We shall refer to (14) with boundary condition (12) as the system with periodic boundary condition. In this paper, we set $t_0 = 0$. In this case, the error dynamics is given as

$$\begin{aligned} e_t &= \varphi e_{\alpha\alpha} - Ke(\pi, t - \tau(t)) \\ &= \varphi e_{\alpha\alpha} - K[e(\alpha, t - \tau(t)) - \int_\pi^\alpha e_\zeta(\zeta, t - \tau(t))d\zeta]. \end{aligned} \tag{15}$$

with boundary condition

$$\begin{aligned} e(0, t) &= e(2\pi, t) \\ e_\alpha(0, t) &= e_\alpha(2\pi, t). \end{aligned} \tag{16}$$

Consider the initial condition for (15), (16) as

$$e(\alpha, t) = e(\alpha, 0), t < 0. \tag{17}$$

The stability of this system will be analyzed in Section 5.

In this paper, we design sufficient conditions for the system (14), with delay bound τ_M, to achieve exponential stabilization (with any desirable decay rate for small enough $\tau_M \geq 0$).

4. Deployment onto open curves

In this section, we consider the first scenario, i.e., system (9) with boundary control (10). In the following proposition, we prove that the deployment of the agents to the curve γ is guaranteed with any desirable decay rate.

Proposition 3. *Given a positive scalar δ, choose $\kappa > \delta$ and $\varphi > \frac{8\delta a^2}{(1-\delta)\pi^2}$. Then the system (8) under the boundary conditions (11) is exponentially stable with a decay rate δ meaning that the following inequality holds,*

$$\begin{aligned} &\frac{2a\kappa\delta}{\kappa - \delta} e^2(0, t) + \int_0^a [\kappa e^2(\alpha, t) + \varphi e_\alpha^2(\alpha, t)]d\alpha \\ &\leq \exp(-2\delta t)[\frac{2a\kappa\delta}{\kappa - \delta} e^2(0, 0) + \int_0^a [\kappa e^2(\alpha, 0) + \varphi e_\alpha^2(\alpha, 0)]d\alpha]. \end{aligned}$$

Proof. Since in the system (8) the components of e are decoupled, here we prove the case that $e : [0, a] \times \mathbb{R}_{\geq 0} \to \mathbb{R}$. Consider the Lyapunov functional

$$V(e) = q_1 e^2(0, t) + \int_0^a [\kappa e^2(\alpha, t) + \varphi e_\alpha^2(\alpha, t)]d\alpha \tag{18}$$

with time derivative

$$\begin{aligned} \dot{V} &= -2q_1\kappa e^2(0, t) + 2\int_0^a \kappa e e_t + \varphi e_\alpha e_{\alpha t} d\alpha \\ &= -2q_1\kappa e^2(0, t) - 2\varphi\kappa \int_0^a e_\alpha^2 d\alpha - 2\varphi \int_0^a e_t e_{\alpha\alpha} d\alpha \end{aligned}$$

where the last equality follows from integration by parts. By Lemma 2 in Selivanov and Fridman (2018) with $\nu = 2$, we have $\int_0^a e^2 d\alpha \leq C_1 e^2(0, t) + C_2 \int_0^a e_\alpha^2 d\alpha$, where $C_1 = 2a$ and $C_2 = \frac{8a^2}{\pi^2}$ are constant parameters. Hence,

$$\begin{aligned} \dot{V} + 2\delta V \leq &-2q_1(\kappa - \delta)e^2(0, t) - 2\varphi(\kappa - \delta)\int_0^{2\pi} e_\alpha^2 d\alpha \\ &+ 2\delta\kappa C_1 e^2(0, t) + 2\delta\kappa C_2 \int_0^{2\pi} e_\alpha^2 d\alpha. \end{aligned}$$

By setting $q_1 = \frac{\delta\kappa C_1}{1-\delta}$, we have

$$\dot{V} + 2\delta V \leq -(2\varphi(\kappa - \delta) - 2\delta\kappa C_2)\int_0^{2\pi} e_\alpha^2 d\alpha.$$

Then for $\varphi > \frac{\delta\kappa C_2}{\kappa-\delta} = \frac{8\delta\kappa a^2}{(\kappa-\delta)\pi^2}$, we obtain $\dot V + 2\delta V \le 0$. Hence the conclusion follows. □

Remark 1. In Frihauf and Krstic (2011), the authors considered a linear reaction–advection–diffusion equation. Differently from Frihauf and Krstic (2011), where the deployment can only be achieved to a family of curves, our protocol can be applied to arbitrary $\mathcal{C}^2$ curves. Our main advantage is in the simplicity of the control law. Note that adding $m-1$ leaders at $a\ell/m$, $\ell = 1,\ldots,m-1$, points, we can reduce the condition of Proposition 3 to $\varphi > \frac{8\delta(a/m)^2}{(1-\delta)\pi^2}$ that allows to reduce the value of φ.

Remark 2. Deployment to arbitrarily time-varying $\mathcal{C}^2$ curves was introduced in Qi, Pan et al. (2015), where a practical stability of the error equation was guaranteed. In the present paper, we achieve exponential convergence of the error to zero with any desired decay rate by employing the simplest static output-feedback.

Remark 3. To deploy the agents to time-varying curves, namely $\gamma(\alpha,t)$ instead of $\gamma(\alpha)$, the authors in Meurer (2012) and Meurer and Krstic (2011) employed Burgers equation with time-varying parameters. However, only some families of curves were feasible. One can verify in a straightforward manner that the result derived in this section can be extended to the case with an arbitrary smooth time-varying curve $\gamma(\alpha,t)$ provided the value of $\gamma_t(ih,t)$ is available to each agent. In fact, the dynamics of the agents is governed by

$$x_t(\alpha,t) = \varphi(x_{\alpha\alpha}(\alpha,t) - \gamma_{\alpha\alpha}(\alpha,t)) + \gamma_t(\alpha,t)$$

with the leaders controlled by

$$x_t(0,t) = \kappa(\gamma(0,t) - x(0,t)) + \gamma_t(0,t)$$

$$x_t(a,t) = \kappa(\gamma(a,t) - x(a,t)) + \gamma_t(a,t).$$

Remark 4. The algorithm developed in this section is valid for the deployment also in 1D space, which corresponds to deployment of the agents from an initial interval to a desired one.

5. Network-based deployment onto closed curves

As already mentioned, in the first scenario, it is difficult to guarantee the convergence in the presence of small time-varying delays in the relative leaders' positions. In this section, we consider the second scenario, where the agent at π is the leader who can measure $x(\pi,t) - \gamma(\pi)$ and send this information to the other agents under a bounded communication delay. The system is given as (14) with boundary condition (12).

We start with the well-posedness of the system (15) with periodic boundary condition (16) is analyzed as follows. Consider the initial condition $e(\cdot,0) \in X$, where

$$X = \{w \in \mathcal{H}^1(0,2\pi) \mid w(0) = w(2\pi)\}$$

is the state space with the $\mathcal{H}^1$-norm. The system (15), (16) can be presented in the form

$$\dot\zeta(t) + \mathcal{A}\zeta(t) = -K\zeta(\pi, t_k - \eta_k) =: f,\ t \in [t_k, t_{k+1}) \tag{19}$$

where $\zeta(t) = e(\cdot,t)$ and

$$\mathcal{A} : D(\mathcal{A}) \to \mathcal{L}^2(0,2\pi)$$
$$\mathcal{A}w = -w'',$$

where w'' denotes the second order derivative, is a linear operator on the Hilbert space

$$D(\mathcal{A}) = \{w \in \mathcal{H}^2(0,2\pi) \mid w(0) = w(2\pi), \tag{20}$$

$$w_\alpha(0) = w_\alpha(2\pi)\} \tag{21}$$

with the inner product $\langle u, v\rangle_{D(\mathcal{A})} = \langle \mathcal{A}u, \mathcal{A}v\rangle_{\mathcal{L}^2}$.

A strong solution of (19) on $[0,T]$ is a function

$$\zeta \in \mathcal{L}^2(0,T; D(\mathcal{A})) \cap \mathcal{C}([0,T]; X), \tag{22}$$

such that $\dot\zeta \in \mathcal{L}^2(0,T;\mathcal{L}^2(0,2\pi))$ and (19) holds almost everywhere on $[0,T]$. By arguments of Fridman and Blighovsky (2012) and Selivanov and Fridman (2018), (19) has a unique strong solution for the initial condition $\zeta(0) = e(\cdot,0) \in X$ and all $t \ge 0$. Moreover, for $e(\cdot,0) \in D(\mathcal{A})$ this solution is in $\mathcal{C}^1([t_k,t_{k+1}); X)$ for all $k = 0,1,\ldots$ (Fridman & Bar Am, 2013).

In the rest part of this section, we derive LMI conditions for desired decay rate with given system parameters φ, K, and τ_M for system (14) with bounded delay and periodic boundary condition (12).

The main result is given as follows.

Theorem 4. *Consider the boundary-value problem (15) with boundary condition (16). Given positive tuning parameters δ_0 and δ_1 satisfying $\delta_1 < 2\delta_0$ and φ, K, let there exist positive scalars p_1, p_2, p_3, r, g and s satisfying the following LMIs*

$$\delta_0 p_3 \le p_2, \quad \Phi \le 0, \quad \begin{bmatrix} r & s \\ s & r \end{bmatrix} \ge 0, \tag{23}$$

where

$$\begin{aligned}
&\Phi_{12} = p_1 - p_2, && \Phi_{11} = g + 2\delta_0 p_1 - r\exp(-2\delta_0\tau_M),\\
&\Phi_{22} = \tau_M^2 r - 2p_3, && \Phi_{14} = -p_2K + (r-s)\exp(-2\delta_0\tau_M),\\
&\Phi_{15} = p_2K, && \Phi_{13} = \exp(-2\delta_0\tau_M)s,\\
&\Phi_{24} = -p_3K, && \Phi_{33} = -(g+r)\exp(-2\delta_0\tau_M),\\
&\Phi_{25} = p_3K, && \Phi_{34} = (r-s)\exp(-2\delta_0\tau_M),\\
&\Phi_{55} = -\frac{\delta_1 p_3\varphi}{4}, && \Phi_{44} = -2(r-s)\exp(-2\delta_0\tau_M) - \delta_1 p_1.
\end{aligned}$$

Then the system (15), initialized with $e(\alpha,t) = e(\alpha,0) \in X$, $\forall t < 0$, is exponential stable with a decay rate δ, where δ is the unique solution to (3), i.e., the following inequality holds

$$C_2 \max_{\alpha\in[0,2\pi]} e^2(\alpha,t) \tag{24}$$

$$\le p_1\int_0^{2\pi} e^2(\alpha,t)d\alpha + p_3\int_0^{2\pi} \varphi e_\alpha^2(\alpha,t)d\alpha \tag{25}$$

$$\le C\exp(-2\delta t)\Big[p_1\int_0^{2\pi} e^2(\alpha,0)d\alpha + p_3\int_0^{2\pi}\varphi e_\alpha^2(\alpha,0)d\alpha\Big] \tag{26}$$

with some positive constant C_2 and C. Moreover, given any $\delta > 0$ and $K > \delta_0$, LMIs (23) are always feasible for large enough φ and small enough τ_M.

Proof. Consider the Lyapunov–Krasovskii functional (Fridman & Blighovsky, 2012)

$$\begin{aligned}
V(t) = &p_1\int_0^{2\pi} e^2(\alpha,t)d\alpha + p_3\int_0^{2\pi}\varphi e_\alpha^2(\alpha,t)d\alpha\\
&+ \int_0^{2\pi}\Big[\tau_M r\int_{-\tau_M}^0\int_{t+\theta}^t \exp(2\delta_0(s-t))e_s^2(\alpha,s)dsd\theta\\
&+ g\int_{t-\tau_M}^t \exp(2\delta_0(s-t))e^2(\alpha,s)ds\Big]d\alpha.
\end{aligned} \tag{27}$$

For the strong solution of (19), the functional V is well-defined and continuous. Differentiating V in time, almost for all $t \geq 0$ we have

$$\begin{aligned}
&\dot{V} + 2\delta_0 V \\
=&2\delta_0 p_1 \int_0^{2\pi} e^2(\alpha,t) + 2\delta_0 p_3\varphi \int_0^{2\pi} e_\alpha^2(\alpha,t)d\alpha \\
&+ 2p_1 \int_0^{2\pi} e_t e d\alpha + 2p_3\varphi \int_0^{2\pi} e_\alpha e_{\alpha t} d\alpha \\
&+ \tau_M^2 r \int_0^{2\pi} e_t^2(\alpha,t)d\alpha \qquad (28)\\
&- \tau_M r \int_0^{2\pi}\int_{-\tau_M}^0 \exp(2\delta_0\theta)e_t^2(\alpha,t+\theta)d\theta d\alpha \\
&+ g\int_0^{2\pi} [e^2(\alpha,t) - \exp(-2\delta_0\tau_M)e^2(\alpha,t-\tau_M)]d\alpha.
\end{aligned}$$

Note that the derivative $e_{\alpha t}$ is defined in the distributional sense, where $e_{t\alpha} = e_{\alpha t}$ almost for all α and t (cf. Remark A.1 of Fridman and Bar Am (2013)). Thus, almost for all $t \geq 0$

$$\int_0^{2\pi} e_\alpha e_{t\alpha} d\alpha = \int_0^{2\pi} e_\alpha e_{\alpha t} d\alpha. \qquad (29)$$

Note also that

$$\begin{aligned}
&-\int_0^{2\pi} \tau_M r \int_{-\tau_M}^0 [\exp(2\delta_0\theta)e_t^2(\alpha,t+\theta)]d\theta d\alpha \\
=&-\tau_M r \int_0^{2\pi}\int_{t-\tau_M}^t \exp(2\delta_0(s-t))e_s^2(\alpha,s)dsd\alpha.
\end{aligned}$$

By applying Jensen's inequality (Gu, Chen, & Kharitonov, 2003, Proposition B.8) and further Park inequality (Lemma 1 in Fridman (2014b)), we have

$$\begin{aligned}
&-\tau_M r \int_0^{2\pi}\int_{t-\tau_M}^t \exp(2\delta_0(s-t))e_s^2(\alpha,s)dsd\alpha \\
\leq& -\frac{\tau_M}{\tau_M - \tau(t)} r\exp(-2\delta_0\tau_M)\int_0^{2\pi}[\int_{t-\tau_M}^{t-\tau(t)} e_s(\alpha,s)ds]^2 d\alpha \\
&-\frac{\tau_M}{\tau(t)} r\exp(-2\delta_0\tau_M)\int_0^{2\pi}[\int_{t-\tau(t)}^{t} e_s(\alpha,s)ds]^2 d\alpha \\
\leq& -\exp(-2\delta_0\tau_M)\int_0^{2\pi} \xi^\top \begin{bmatrix} r & s \\ s & r\end{bmatrix}\xi d\alpha, \qquad (30)
\end{aligned}$$

where the parameter s is chosen such that

$$\begin{bmatrix} r & s \\ s & r\end{bmatrix} \geq 0,$$

and $\xi^\top := [e(\alpha,t) - e(\alpha,t-\tau), e(\alpha,t-\tau) - e(\alpha,t-\tau_M)]$.

We further employ the descriptor method (Fridman, 2001), where the left-hand side of the following equation

$$\begin{aligned}
D :=&2\int_0^{2\pi} [p_2 e + p_3 e_t][-e_t + \varphi e_{\alpha\alpha} - Ke(\pi, t-\tau(t))]d\alpha \\
=&0 \qquad (31)
\end{aligned}$$

is added to $\dot{V} + 2\delta_0 V$. Integrating by parts we have

$$\begin{aligned}
2\varphi\int_0^{2\pi} p_3 e_t e_{\alpha\alpha} d\alpha &= -2p_3\varphi\int_0^{2\pi} e_\alpha e_{t\alpha} d\alpha \\
2p_2\varphi\int_0^{2\pi} e e_{\alpha\alpha} d\alpha &= -2p_2\varphi\int_0^{2\pi} e_\alpha^2 d\alpha. \qquad (32)
\end{aligned}$$

Then, from (28)–(32) we find

$$\begin{aligned}
&\dot{V} + D + 2\delta_0 V \\
\leq& 2\delta_0 p_1 \int_0^{2\pi} e^2(\alpha,t)d\alpha + 2\delta_0 p_3\varphi\int_0^{2\pi} e_\alpha^2(\alpha,t)d\alpha \\
&+ 2p_1\int_0^{2\pi} e_t e d\alpha - \exp(-2\delta_0\tau_M)\int_0^{2\pi}\xi^\top \begin{bmatrix} r & s \\ s & r\end{bmatrix}\xi d\alpha \\
&+ \tau_M^2 r\int_0^{2\pi} e_t^2(\alpha,t)d\alpha - 2p_2\varphi\int_0^{2\pi} e_\alpha^2 d\alpha \\
&+ g\int_0^{2\pi}[e^2(\alpha,t) - \exp(-2\delta_0\tau_M)e^2(\alpha,t-\tau_M)]d\alpha \\
&+ 2\int_0^{2\pi}[p_2 e + p_3 e_t]\big[-e_t - Ke(\pi,t-\tau(t))\big]d\alpha \\
=&\int_0^{2\pi}\eta^\top\bar{\Phi}\eta d\alpha - (2p_2\varphi - 2\delta_0 p_3\varphi)\int_0^{2\pi} e_\alpha^2(\alpha,t)d\alpha
\end{aligned}$$

where

$$\begin{aligned}
&\bar{\Phi}_{11} = g + 2\delta_0 p_1 - r\exp(-2\delta_0\tau_M), && \bar{\Phi}_{12} = p_1 - p_2, \\
&\bar{\Phi}_{14} = -p_2 K + (r-s)\exp(-2\delta_0\tau_M), && \bar{\Phi}_{15} = p_2 K, \\
&\bar{\Phi}_{13} = \exp(-2\delta_0\tau_M)s, && \bar{\Phi}_{22} = \tau_M^2 r - 2p_3, \\
&\bar{\Phi}_{34} = (r-s)\exp(-2\delta_0\tau_M), && \bar{\Phi}_{25} = p_3 K, \\
&\bar{\Phi}_{33} = -(g+r)\exp(-2\delta_0\tau_M), && \bar{\Phi}_{24} = -p_3 K, \\
&\bar{\Phi}_{44} = -2(r-s)\exp(-2\delta_0\tau_M), && \bar{\Phi}_{55} = 0,
\end{aligned}$$

$\eta := [e, e_t, e(\alpha,t-\tau_M), e(\alpha,t-\tau(t)), f]^\top$ and $f := e(\alpha,t-\tau(t)) - e(\pi,t-\tau(t))$.

To further apply Halanay's inequality, we note that

$$\begin{aligned}
&\dot{V} + 2\delta_0 V - \delta_1 \sup_{\theta\in[-\tau_M,0]} V(t+\theta) \\
\leq&\dot{V} + 2\delta_0 V - \delta_1 V(t-\tau(t)) \\
\leq&\dot{V} + 2\delta_0 V - \delta_1\int_0^{2\pi} p_1 e^2(\alpha,t-\tau(t)) \\
&+ p_3\varphi e_\alpha^2(\alpha,t-\tau(t))d\alpha \\
\leq&\int_0^{2\pi}\eta^\top\bar{\Phi}\eta d\alpha - (2p_2\varphi - 2\delta_0 p_3\varphi)\int_0^{2\pi} e_\alpha^2(\alpha,t)d\alpha \\
&-\delta_1\int_0^{2\pi} p_1 e^2(\alpha,t-\tau)d\alpha \\
&-\delta_1 p_3\varphi\int_0^{2\pi} e_\alpha^2(\alpha,t-\tau(t))d\alpha \\
\leq&\int_0^{2\pi}\eta^\top\bar{\Phi}\eta d\alpha - (2p_2\varphi - 2\delta_0 p_3\varphi)\int_0^{2\pi} e_\alpha^2(\alpha,t)d\alpha \\
&-\delta_1\int_0^{2\pi} p_1 e^2(\alpha,t-\tau)d\alpha - \frac{\delta_1 p_3\varphi}{4}\int_0^{2\pi} f^2 d\alpha \\
\leq&\int_0^{2\pi}\eta^\top\Phi\eta d\alpha - (2p_2\varphi - 2\delta_0 p_3\varphi)\int_0^{2\pi} e_\alpha^2(\alpha,t)d\alpha
\end{aligned}$$

where $\Phi = \bar{\Phi} + \mathrm{diag}(0,0,0,-\delta_1 p_1, -\frac{\delta_1 p_3\varphi}{4})$. This implies that, if $p_2 \geq \delta_0 p_3$ and $\Phi \leq 0$, then (2) holds and Halanay's inequality yields

$$V(t) \leq \exp(-2\delta t)\sup_{\theta\in[-\tau_M,0]} V(\theta). \qquad (33)$$

Finally, since the initial condition is set to be $e(\alpha,t) = e(\alpha,0)$, $\forall t < 0$, we have

$$\begin{aligned}
&\sup_{\theta\in[-\tau_M,0]} V(\theta) \qquad (34)\\
=&p_1\int_0^{2\pi} e^2(\alpha,0)d\alpha + p_3\int_0^{2\pi}\varphi e_\alpha^2(\alpha,0)d\alpha \\
&+ g\int_0^{2\pi}\int_{-\tau_M}^0 \exp(2\delta_0 s)e^2(\alpha,0)dsd\alpha \qquad (35)\\
=&C(p_1\int_0^{2\pi} e^2(\alpha,0)d\alpha + p_3\int_0^{2\pi}\varphi e_\alpha^2(\alpha,0)d\alpha), \qquad (36)
\end{aligned}$$

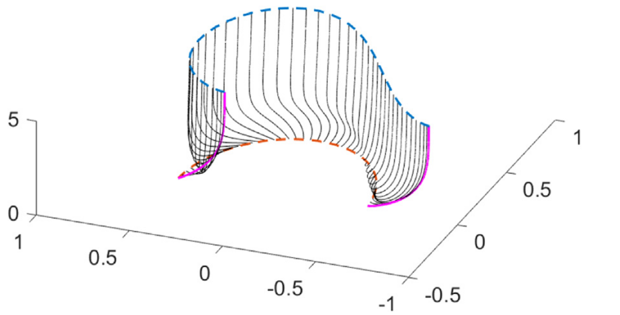

Fig. 1. Deployment of the agents according to the system (9) with boundary control (10) . (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

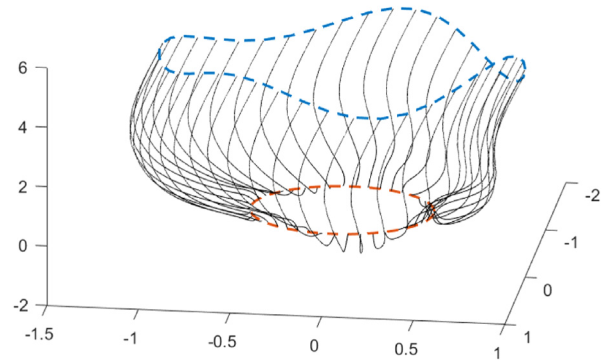

Fig. 2. Deployment of the agent according to the system (14) with periodic boundary condition (12).

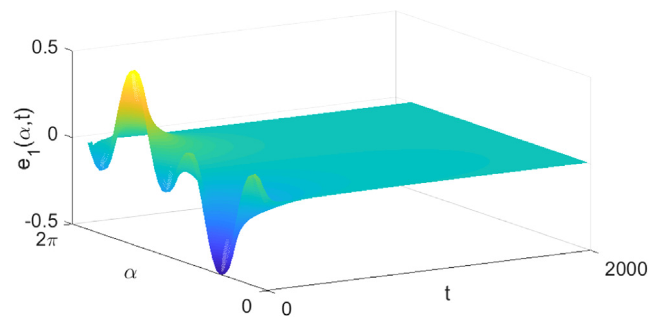

Fig. 3. The error $e_1(\alpha, t)$ of the simulation given in Fig. 2.

where constant C does not depend on the initial condition, which implies the inequality (26). The inequality (25) is implied by Lemma 2 with positive constant C_2.

Denote by Ψ the matrix Φ with the deleted last column and row and $\delta_1 = 0$. Then $\Psi < 0$ guarantees via the descriptor method that the system

$$\dot{x}(t) = -Kx(t-\tau)$$

is exponentially stable with a decay rate δ_0 (cf. (4.23) in Fridman (2014a)). Moreover, given any $\delta_0 > 0$ and $K > \delta_0$ by arguments of Fridman (2014a) it can be shown that $\Psi < 0$ is always feasible for small enough τ_M. Given any $\delta > 0$ and choosing $\delta_1 = 0.1\delta$ and $\delta_0 = \delta + 2\delta_1$ and $K > \delta_0$, we find further p_1, p_2, p_3, r and s that solve $\Psi < 0$ for small τ_M. Then, applying Schur complements to the last column and row of Φ, we conclude that $\Phi < 0$ for large enough φ. The latter implies that the system is exponentially stable with a decay rate δ. □

Remark 5. It was shown in Wei et al. (2019) that for any $\delta > 0$, the choice of $K > \delta$ and $\phi \geq K^2/(K-\delta)$ guarantees the exponential convergence of (15) with a decay rate $\delta > 0$ for $\tau_M = 0$.

Remark 6. If there are several leaders (e.g. at $\pi/2$ and $3/2\pi$), then in (13), we can use $-K(x(\pi/2, t_k - \eta_k) - \gamma(\pi/2))$ for $x \in [0, \pi)$, and $-K(x(1.5\pi, t_k - \eta_k) - \gamma(1.5\pi))$ for $x \in [\pi, 2\pi]$, that allows to reduce the gain φ (Fridman & Blighovsky, 2012).

6. Simulations

In this section, we present the simulative results of the proposed control laws in Sections 4 and 5. In all the simulations, we consider a multi-agent system with $N = 45$ agents. In all the figures of the deployment, the blue dashed lines are the desired formation, and the red dashed lines are the initial positions of the agents which are set to be $(0.5 * \sin(i\frac{a}{N}), 0.5 * \cos(i\frac{a}{N}), 0), i = 1, \dots, N$, where $a = \pi$ in the case of open desired curve and $a = 2\pi$ in the case of the closed one. The black solid lines are the trajectories of the agents.

In the first example, we present the simulation of the system (9) with boundary control (10). For the system parameters, we set $\varphi = 10, K = 1$. The desired curve in $\mathbb{R}^3$ is parameterized in the interval $[0, \pi]$ as $(0.8\sin^3(\alpha), 0.05(13\cos(\alpha) - 5\cos(2\alpha) - 2\cos(3\alpha) - \cos(4\alpha)), 5)$. For any δ such that $\varphi > \frac{8\delta}{1-\delta}$, we have the system is exponential converging with a decay rate δ. Here we take $\delta = 0.1$. The simulation is given in Fig. 1. In this example, the initial positions of the agents only occupy an arc in the red dashed circle. The two magenta solid lines are the trajectories of the leaders.

For the closed curves, namely with periodic boundary condition, we present an example with the desired curve being parameterized in the interval $[0, 2\pi]$ as $(\sin^3(\alpha), \cos\alpha, \cos(2\alpha)\sin(2\alpha) + 5)$. For the system parameters, we set $\varphi = 10, K = 3$. We choose $\delta_0 = 2.5$. Furthermore, the parameter δ_1 in Halanay's inequality is set to be equal to $1.5\delta_0$ which is less than $2\delta_0$. The LMI conditions (23) are satisfied by $p_1 = 0.34, p_2 = 1.18, p_3 = 0.46, r = 10, g = 0.06, s = 0.32$ and $\tau_M = 0.12$ which can be verified numerically, e.g., by CVX (Grant & Boyd, 2014). This guarantees the desired decay rate $\delta = 0.60$. The performance of (14) is given in Fig. 2 and the error of the first dimension is plotted in Fig. 3.

7. Conclusion

In this paper, we considered the deployment of the first-order multi-agents onto a desired smooth curve. The model is motivated by the displacement-based multi-agent formation control algorithm (Oh et al., 2015). For the open curves, we proposed a simple static output-feedback boundary control mechanism. The main part of the paper is devoted to the case with closed curves, where we assumed that the agents have access to the local information of the desired curve and their relative positions with respect to their closest neighbors, whereas a leader is able to measure its relative position with respect to the desired curve and transmit it to other agents through communication network. It was proved that, based on LMI conditions, by choosing appropriate controller gains, any desired decay rate can be achieved provided the delay is small enough. More precisely, exponential convergence to any closed $\mathcal{C}^2$ curve is guaranteed.

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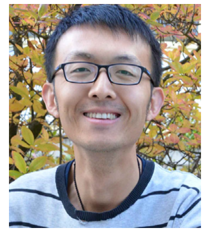

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