A Scenario-Based Distributed Stochastic MPC for Building Temperature Regulation

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Abstract— **In this paper, we focus on the temperature regulation of rooms in buildings. By using the dynamic model of the thermal process, weather condition, occupancy and so on, a Stochastic Model Predictive Control (SMPC) problem is formulated to keep the temperature of rooms within a comfortable range with a predefined probability while consuming less energy. The temperature regulation problem in this paper is an optimal control problem of a linear system with additive uncertainty. To overcome the computational burden caused by the large number of rooms, a subgradient-based dual decomposition method is used to solve the SMPC problem in a distributed manner. Simulation results show the effectiveness of our results.**

I. Introduction

From the United Nations environment programme report [12], buildings account for 40 percent of energy consumption and resources and one third of greenhouse gas emissions. Thus it is attractive to reduce the energy cost of buildings. One of the most promising directions is to optimize the energy efficiency of heating, ventilation and air conditioning (HVAC) system because it composes one third to half of building energy usage [1].

Model Predictive Control (MPC) is a very popular method to optimize the energy efficiency of an HVAC system. This method is to formulate an optimization problem based on the system dynamic model and current measurement at each time step. Only the first control action will be implemented and the optimization problem will be reformulated and solved again when the next measurement comes. It should be noted that though at each time step an open-loop optimal control problem is solved, the whole system is closed loop since the optimal solution is a function of current measurement. It is superior to traditional control strategies due to its optimal nature and the ability to handle input and output constraints. A lot of scholars have proposed MPC approaches to reduce the energy cost of HVAC systems while keeping the temperature within a comfortable region, see, e.g. [16], [15] and so on.

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Because of the uncertainty of weather conditions and occupancy, the future evolution of indoor temperature process cannot be predicted precisely. To deal with this uncertainty, some robust optimization methods are used, see, e.g. [9] and [23]. By assuming that the uncertainty is bounded, the optimization problem is solved for the worst case. However, this assumption and the resulted worst case solution are rather conservative for building temperature regulation problem because extremely bad weather conditions do not happen very often.

To achieve less conservative solutions to the optimization problem with uncertainty, SMPC problems are formulated in [21], [14] and [17]. In these articles, the weather and occupancy uncertainties are modeled by Gaussian random variables which are unbounded. Also some violation situations of constraints are allowed with small probability which leads to probabilistic constraints. In [21] and [14], nonlinear SMPC problems are formulated and are solved by a sequential quadratic programming method. Sparsity of the subproblem is used to derive fast iteration algorithms and disturbance feedback is introduced to improve performance.

In real applications, weather condition and human occupancy are not necessarily of Gaussian distribution. However, without the specific distribution assumption, the stochastic problem cannot be solved explicitly [5]. Randomized Model Predictive Control (RMPC) provides an approximate solution to the SMPC without specific distribution assumption. This approach is based on a scenario-based optimization technique [3], [4]. The main idea of this approach is to use samples of the involved random variables to substitute them so that the probabilistic constraints are converted into deterministic ones. The number of samples is determined by the number of decision variables, the confidence level and the violation probability of the constraints. Due to its ability to handle random variables with general distributions, RMPC has been used in building temperature and HVAC system control in recent years, see, e.g. [22], [27] and [26].

When the number of rooms and prediction and control horizons become large, the size of centralized MPC grows fast and the MPC problem turns out to be computationally intractable. Especially when RMPC is used, due to that the large number of scenarios should be evaluated, it becomes more challenging to attenuate the computational burden. In [13], a nonlinear optimization problem is formulated and solved through tailed sequential quadratic programming. Then the subproblem is decomposed further by a subgradient method. The system dynamics serve as equality constraints and the decision variables include not only control sequences but also system states. In [19], a distributed algorithm is used to search the Nash equilibrium. However, this solution may not be optimal.

In this paper, we propose an algorithm which combines the RMPC technique, Lagrangian relaxation [11] and sub-gradient decomposition together to solve the multi-room temperature regulation problem with random disturbances. Through this method, the probabilistic constraints which contain random disturbances like weather condition and occupancy are handled by a scenario-based optimization technique while the computational burden is distributed over multiple low cost processors so that the implementation becomes more economic.

The paper is organized as follows. In Section II, a simplified model for control design is introduced. Then a centralized optimization problem is formulated in Section III. A distributed algorithm is outlined in Section IV. Simulation results are given in Section V and conclusions are drawn in Section VI.

II. System Modeling

In this section, we introduce a simplified physical model of indoor temperature dynamics and then derive a control oriented linear time-invariant model based on that.

Because of the complex behavior of air flow and heat transfer process, the indoor temperature dynamics should be described by a time-varying nonlinear partial differential equation which is not suitable for control and optimization. Therefore similar to those works in [8], [24] and [15], we make the following assumptions to simplify the modeling.

- The air in each room and outdoor environment is well mixed immediately so that the temperature distributions are uniform.
- The heat capacity of air is assumed to be constant.

An undirected graph is used to describe the communication topology among rooms. An undirected graph is defined as a set $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V}$ is the set of all nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of all edges. We treat room i as the node i in graph $\mathcal{G}$. If room i and j are adjacent, edge (i, j) is in $\mathcal{E}$. The set of all neighbors of room i is defined as $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$.

The indoor temperature dynamics for one single room is considered as a Resistive-Capacitive (RC) system [7] and the temperature of the whole building is considered as a network of RC systems. Each node in this network is a room and its state represents its temperature. Cooling is considered as control input to each node while weather condition and human activity are modeled as a random disturbance to each node.

Under the above assumptions, we have the following equation to describe the dynamics of indoor temperature:

$$m_i c_{pa} \dot{\tau}_i = Q_{out} + cN_{people,i} + \sum_{j \in \mathcal{N}_i} Q_{ji} + Q_{cooling,i}, \quad i = 1, \dots, N, \quad (1)$$

where m_i is the air amount of room i, c_{pa} is the heat capacity of air, Q_{out} is the heat coming from outdoor environment which is random, $N_{people,i}$ is the number of people in room i, c is the average heat generated by one person, Q_{ji} is the heat flow from room j to room i and $Q_{cooling,i}$ is the heat flow caused by the HVAC system to keep the indoor temperature of the ith room in a comfortable region, N is the number of all rooms. By using $R_{ij} = R_{ji}$ to model the heat resistance between room i and j, $R_{out,i}$ to model the heat resistance between room i and outdoor environment, the equation (1) can be written as follows:

$$\begin{aligned} m_i c_{pa} \dot{\tau}_i &= \frac{(\tau_{out} - \tau_i)}{R_{out,i}} + cN_{people,i} \\ &+ \sum_{j \in \mathcal{N}_i} \frac{(\tau_j - \tau_i)}{R_{ji}} + \dot{m}_{vent,i} c_{pa} (\tau_{sa,i} - \tau_i) \quad (2) \end{aligned}$$

where $\dot{m}_{vent,i}$ is the ventilation mass flow of the i-th room and $\tau_{sa,i}$ is the temperature of supply air in the i-th room.

It can be observed that system (2) is bilinear in the state τ_i and control input $\dot{m}_{vent,i}$. A computationally intractable problem will arise if we directly use this model. To linearize it, [15] uses the sequential quadratic programming method which linearizes the nonlinear system along its state trajectory. However, in this scenario-based approach, the number of possible state trajectories is equal to the number of scenarios which is usually very large. So it is not practical to linearize the model along every possible state trajectory. We use u_i in this case to substitute the nonlinear term $\dot{m}_{vent,i} c_{pa} (\tau_{sa,i} - \tau_i)$ and discretize the system by trapezoidal rule with $\Delta T = 15$ minutes. Then we get a linear discrete-time system model as follows:

$$\begin{aligned} \tau_i(k+1) &= a_{ii} \tau_i(k) + b_i u_i(k) + \sum_{j \in \mathcal{N}_i} a_{ji} \tau_j(k) \\ &+ c_i w_i(k) \quad (3) \end{aligned}$$

where a_{ii}, b_i, a_{ji} and c_i are constants related to m_i, c_{pa}, $R_{out,i}$, c, R_{ji}; $w_i(k)$ is a random variable which models disturbance to room i from weather condition and human activity.

III. Randomized MPC formulation

In this section, we formulate the SMPC problem based on LTI model (3). To formulate this problem, it is necessary to estimate the distribution of random disturbances like weather condition and occupancy. Copula is one of the most popular tools which can be used to give such an estimation. Therefore first of all, a brief introduction to copula is given.

A copula $C(y_1, \dots, y_n)$ is a multivariate distribution whose marginal distributions of $y_1, \dots, y_n$ are all uniform. Since the marginal distributions are fixed in a copula, it is used to describe the multivariate dependence structure. Given a random variable X with cumulative distribution F_X which is continuous, the random variable under a probability integral transform defined as $Y = F_X(X)$ has a uniform distribution on $[0, 1]$. Combined this fact and the concept of copula, we can estimate the marginal distributions and dependence structure of a group of random variables separately. Suppose that we want to estimate the multivariate distribution $F(x_1, \dots, x_n)$. The basic procedure is that:

- Collect empirical sample data from experiment.
- Estimate the empirical marginal distributions of $x_1, \ldots, x_n$ as $F_{x_1}, \ldots, F_{x_n}$.
- Use probability integral transforms $y_i = F_{x_i}(x_i)$, $i = 1, \ldots, n$ to transform the collected data into points in $[0,1]^n$.
- Estimate the dependence structure $C(y_1, \ldots, y_n)$ of y_i, $i = 1, \ldots, n$ by using some well established copula families whose parameters describe the dependence among those random variables.
- Generate samples $(y_{1,j}, \ldots, y_{n,j})$, $j = 1, \ldots, N_s$ from the joint distribution $C(y_1, \ldots, y_n)$ where N_s is the number of scenarios.
- Use inverse probability integral transforms $x_i = F^{-1}(y_i)$ to get scenarios $(x_{1,j}, \ldots, x_{n,j})$, $j = 1, \ldots, N_s$ from generated $(y_{1,j}, \ldots, y_{n,j})$.

For more details of copula, please refer to [6], [20], [25] and references therein.

Let N_c be the prediction horizon. $\tau_i(k|t)$, $u_i(k|t)$ and $w_i(k|t)$ are used to denote the predicted temperature of room i at time t, the optimal control action to room i and the predicted disturbance to room i at the k-th time instant from t. Since the system is time invariant, we just use $\tau_i(k)$, $u_i(k)$ and $w_i(k)$ instead of $\tau_i(k|t)$, $u_i(k|t)$ and $w_i(k|t)$.

Let $T_i = [\tau_i(1), \ldots, \tau_i(N_c)]^T$, $U_i = [u_i(0), \ldots, u_i(N_c - 1)]^T$, $W_i = [w_i(0), \ldots, w_i(N_c - 1)]^T$ and $W = [W_1^T, \ldots, W_N^T]^T$. Then the future trajectory of the temperature of room i can be written as:

$$\begin{aligned} T_i &= A_i\tau_i(0) + B_iU_i + C_iW_i + \sum_{j\in\mathcal{N}_i} A_{ji}\tau_j(0) \\ &+ \sum_{j\in\mathcal{N}_i} D_{ji}T_j, \quad i = 1, \ldots, N, \end{aligned} \tag{4}$$

where matrices A_i, B_i, A_{ji} and D_{ji} can be easily constructed by using (3) recursively.

The above equations can be reorganized as:

$$\underbrace{\begin{bmatrix} I & -D_{21} & \ldots & -D_{N1} \\ -D_{12} & I & \ldots & -D_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ -D_{1N} & -D_{2N} & \ldots & I \end{bmatrix}}_{D} \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \begin{bmatrix} A_1\tau_1(0) + B_1U_1 + C_1W_1 + \sum A_{j1}\tau_j(0) \\ A_2\tau_2(0) + B_2U_2 + C_2W_2 + \sum A_{j2}\tau_j(0) \\ \vdots \\ A_N\tau_N(0) + B_NU_N + C_NW_N + \sum A_{jN}\tau_j(0) \end{bmatrix}$$

By taking the inverse of matrix D, the future temperature trajectory can be solved in the following equation:

$$\begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix} = \underbrace{\begin{bmatrix} K_{11} & K_{21} & \ldots & K_{N1} \\ K_{12} & K_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ K_{1N} & \ldots & \ldots & K_{NN} \end{bmatrix}}_{K=D^{-1}} \times \begin{bmatrix} A_1\tau_1(0) + B_1U_1 + C_1W_1 + \sum A_{j1}\tau_j(0) \\ A_2\tau_2(0) + B_2U_2 + C_2W_2 + \sum A_{j2}\tau_j(0) \\ \vdots \\ A_N\tau_N(0) + B_NU_N + C_NW_N + \sum A_{jN}\tau_j(0) \end{bmatrix} \tag{5}$$

$T^i_{\max}$ and $T^i_{\min} \in \mathbb{R}^{N_c}$ are used to denote comfort region of temperature in the ith room and serve as constraints in optimization. These constraints can be greatly relaxed during off time. Denote $F = [I; -I]$, $s = [-s_{11}^T, \ldots, -s_{1N}^T, s_{21}^T, \ldots, s_{2N}^T]^T$ where s_{1i} and $s_{2i} \in \mathbb{R}^{N_c}$ are used to relax the upper and lower bounds of temperature of room i respectively, $T_{con} = [(T^1_{\max})^T, \ldots, (T^N_{\max})^T, -(T^1_{\min})^T, \ldots, -(T^N_{\min})^T]^T$ and $T = [T_1^T, T_2^T, \ldots, T_N^T]^T$. The elements of all s_{1i} and s_{1i} are nonnegative. The temperature constraints can be expressed as

$$FT \leq T_{con}$$

By substituting (5) into the above equation, we have

$$FK \begin{bmatrix} B_1U_1 \\ B_2U_2 \\ \vdots \\ B_NU_N \end{bmatrix} \leq T_{con} - FKQ$$

where

$$Q = \begin{bmatrix} A_1\tau_1(0) + C_1W_1 + \sum A_{j1}\tau_j(0) \\ A_2\tau_2(0) + C_2W_2 + \sum A_{j2}\tau_j(0) \\ \vdots \\ A_N\tau_N(0) + C_NW_N + \sum A_{jN}\tau_j(0) \end{bmatrix}$$

In the case where the problem is infeasible, slack variable s is introduced to relax the temperature constraints further as follows:

$$FK \begin{bmatrix} B_1U_1 \\ B_2U_2 \\ \vdots \\ B_NU_N \end{bmatrix} + s \leq T_{con} - FKQ \tag{6}$$

Considering that the energy cost for air-conditioning of a group of rooms is the sum of the energy cost in each room, we define the cost function as:

$$G(U, s) = \sum_{i=1}^{N} [c^T U_i + p^T(s_{1i} + s_{2i})]$$

where c^T contains the power of AC and the electricity charge, p is used to penalize the slack variables and $U = [U_1^T, \ldots, U_N^T]^T$.

By denoting (6) as $g(U,s) \le f(Q)$ where Q contains all the disturbance W_i, the probabilistic constraints optimization problem is formulated as follows:

Problem 3.1:

$$\begin{aligned} \min_{U,s} \quad & G(U,s)=\sum_{j=1}^{N}[c^T U_j + p^T(s_{1j}+s_{2j})] \\ s.t. \quad & P(g(U,s)\le f(Q)) \ge 1-\alpha, \\ & FU_j \le U_{con}, \quad j=1,\dots,N \\ & 0\le s_{1j} \le s_{\max}, \quad j=1,\dots,N \\ & 0\le s_{2j} \le s_{\max}, \quad j=1,\dots,N \end{aligned}$$

where $U_{con}=[\mathbf{1}^T,\mathbf{0}^T]^T$.

Assume that $W^i=[W_1^{iT},\dots,W_N^{iT}]^T$, $i=1,\dots,N_s$ are N_s scenarios generated from the estimated distribution of W and N_s satisfies that $N_s \ge \frac{2}{\alpha}(\ln\frac{1}{\beta}+d)$ where $\beta\in(0,1)$ is the confidence level of the solution and d is the number of decision variables [3], [4]. Problem 3.1 can be approximated by the following deterministic linear programming:

Problem 3.2:

$$\begin{aligned} \min_{U,s} \quad & G(U,s)=\sum_{j=1}^{N}[c^T U_j + p^T(s_{1j}+s_{2j})] \\ s.t. \quad & g(U,s)\le f(Q_i), \quad i=1,\dots,N_s \\ & FU_j \le U_{con}, \quad j=1,\dots,N \\ & 0\le s_{1j} \le s_{\max}, \quad j=1,\dots,N \\ & 0\le s_{2j} \le s_{\max}, \quad j=1,\dots,N \end{aligned}$$

where Q_i is constructed by using W^i to substitute W in Q.

Denote $Y = \min_i(T_{con}-FKQ_i) = [(Y_u^1)^T,\dots,(Y_u^N)^T,(Y_l^1)^T,\dots,(Y_l^N)^T]^T$ where min applies element-wise to a vector. The linear programming 3.2 can be rewritten as:

Problem 3.3:

$$\begin{aligned} \min_{U,s} \quad & G(U,s)=\sum_{j=1}^{N}[c^T U_j + p^T(s_{1j}+s_{2j})] \\ s.t. \quad & FK\begin{bmatrix} B_1U_1 \\ B_2U_2 \\ \vdots \\ B_NU_N \end{bmatrix} + s \le Y, \\ & FU_j \le U_{con}, \quad j=1,\dots,N \\ & 0\le s_{1j} \le s_{\max}, \quad j=1,\dots,N \\ & 0\le s_{2j} \le s_{\max}, \quad j=1,\dots,N \end{aligned}$$

In the centralized optimization Problem 3.3, there are $3N*N_c$ decision variables. The complexity of this problem will grow rapidly when the number of rooms and prediction horizons are large. Thus it is necessary to design some distributed algorithm to reduce the computational burden.

IV. Distributed optimization algorithm

In this section, a distributed algorithm based on Lagrangian relaxation [11] and duel decomposition is proposed.

By Lagrangian relaxation method, Problem 3.3 can be rewritten as:

Problem 4.1:

$$\begin{aligned} \max_{\lambda}\min_{U,s} \quad & \sum_{j=1}^{N}[c^T U_j + p^T(s_{1j}+s_{2j})] \\ & +\sum_{j=1}^{N}\lambda_j^T(H_j[U^T,s_{1j}^T,s_{2j}^T]^T-[Y_u^{jT},Y_l^{jT}]^T) \\ s.t. \quad & FU_j \le U_{con}, \quad j=1,\dots,N \\ & 0\le s_{1j} \le s_{\max}, \quad j=1,\dots,N \\ & 0\le s_{2j} \le s_{\max}, \quad j=1,\dots,N \\ & \lambda_j \ge 0, \quad j=1,\dots,N \end{aligned}$$

where

$$H_j=\begin{bmatrix} K_{1j}B_1 & \dots & K_{Nj}B_N & -I & 0 \\ -K_{1j}B_1 & \dots & -K_{Nj}B_N & 0 & I \end{bmatrix}$$

and $\lambda=[\lambda_1^T,\dots,\lambda_N^T]^T$.

$H_j[U^T,s_{1j}^T,s_{2j}^T]^T-[Y_u^{jT},Y_l^{jT}]^T$ is the subgradient of λ_j. Note that for fixed Lagrangian variable λ, Problem 4.1 is separable. By taking advantage of this special structure, Problem 4.1 is decomposed into N subproblems and the jth subproblem is:

Problem 4.2:

$$\begin{aligned} \min_{U_j,s_{1j},s_{2j}} \quad & M_j[U_j^T,s_{1j}^T,s_{2j}^T]^T \\ s.t. \quad & FU_j \le U_{con}, \\ & 0\le s_{1j} \le s_{\max}, \\ & 0\le s_{2j} \le s_{\max}, \end{aligned}$$

where

$$M_j = \begin{bmatrix} c^T+\sum_{i=1}^{N}\lambda_i^T E_{ij} & p^T+\lambda_j^T\begin{bmatrix} -I \\ 0 \end{bmatrix} & p^T+\lambda_j^T\begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix}$$

and $E_{ij}=\begin{bmatrix} K_{ji}B_j \\ -K_{ji}B_j \end{bmatrix}$.

When the original problem has a nonlinear convex objective function and constraints, then it can be solved by subgradient decomposition method efficiently, e.g. [13]. However, in this linear programming case, even though the optimal dual variables are found, the optimal primal variable can not be constructed directly due to the linearity of the objective function. The solutions of the subproblems 4.2 are just the extreme points provided bounded feasible sets. Obviously, in most situations, those extreme points are either infeasible or not optimal to the original problem. This lack of coordinability in linear programming has also been reported and discussed in [2], [11]. In [11], the author proposed an average strategy to construct the primal optimal solution motivated by the fact that the optimal solution to the original problem is a nontrivial convex combination of the extreme points of the subproblems.

By combining the algorithm proposed in Theorem 3 in [11] and dual decomposition, the Problem 4.1 is solved as follows:

At the beginning of this algorithm, λ^1 is initialized as $\mathbf{1}$ and X_j^0 are set to be $\mathbf{0}$, $j=1,\dots,N$. Give constants a, $c>0$ and $b\ge 0$ and threshold $\epsilon_2>0$. Let $p=1$:

- Step 1: The value of dual variable λ is set as λ^p and each λ_j^p is broadcast in the communication network.
- Step 2: Based on given dual variable λ, subproblems 4.2 are constructed and solved by its corresponding optimizer. The primal variables are constructed as $[(U_j^p)^T, (s_{1j}^p)^T, (s_{2j}^p)^T]^T = \frac{\sum_{l=1}^p X_j^l}{p}$.
- Step 3: Break out and set $[(U_j)^T, (s_{1j})^T, (s_{2j})^T]^T = [(U_j^p)^T, (s_{1j}^p)^T, (s_{2j}^p)^T]^T$ when $||(U_j^p)^T - (U_j^{p-1})^T|| < \epsilon_2$; else go to Step 4.
- Step 4: Each U_j^p is broadcast in the communication network.
- Step 5: Each λ_j^{p+1} is updated by $\lambda_j^{p+1} = \mathcal{P}[\lambda_j^p + \alpha_p(H_j[U^T, s_{1j}^T, s_{2j}^T]^T - [(Y_u^j)^T, (Y_l^j)^T]^T)]_+$ where $\alpha_p = \frac{a}{b+cp}$ and $\mathcal{P}[*]_+$ is the projection of $*$ to the positive orthant.
- Step 6: Set $p = p + 1$ and go to Step 1.

Remark 4.1: The selection of the stpesize α_p is critical to guarantee the convergence of the algorithm. In [11], the authors prove that if the stepsize is chosen as in Step 5, then $\frac{\sum_{l=1}^p X_j^l}{p}$ will converge to the optimal solution of the original problem as $p \to \infty$.

V. Simulation results

In this case study we consider a network of 10 rooms. It is assumed that the HVAC system is working in the cooling mode. The connection topology of rooms is $\mathcal{G} = \{\{1, \ldots, 10\}, \{(1,2), (2,3), \ldots, (9,10)\}\}$. The parameters of rooms are given by $a_{ii} = 0.64$, $b_i = -3$, $a_{ji} = 0.1$, $c_i = 0.26$ when $i = 1$ or 10, $c_i = 0.16$ when $i = 2, \ldots, 9$. The comfortable region of temperature is set to be [22°C 23°C] during the working time while [20°C 25°C] during the off time. The temperature constraints are labeled by solid blue lines in the simulation results.

The algorithms used in this section are coded in Matlab® and run on a PC with Intel Core Duo i5-2400 CPU 3.10GHz. By solving the centralized MPC problem directly, the control sequences and temperature evolution are shown in Fig. 1 and Fig. 2 with the cost of 264.3. The average computational time is 39 seconds. Fig. 3 shows the temperature evolution under the distributed algorithm with the cost of 268.7. One can see that under closed-loop implementation, the performance loss is negligible. The average computational time for the distributed algorithm is 9 seconds. Fig. 4 and Fig. 5 show the temperature controlled by PID controllers and the corresponding control sequences. It is observed that the upper bound is violated. The cost is 298.5 which is larger than all of the MPC results above. It can be also observed that by using MPC controllers, the temperature variation is much smaller and the control action is smoother. These properties are good for indoor thermal comfort and prolonging the life of devices.

The energy saving of the proposed MPC algorithm is not surprising. From the simulation, we can see that under MPC, the temperature trajectory is very close to the upper bound of the temperature constraint. Since we assume that the HVAC system is working in the cooling mode, being

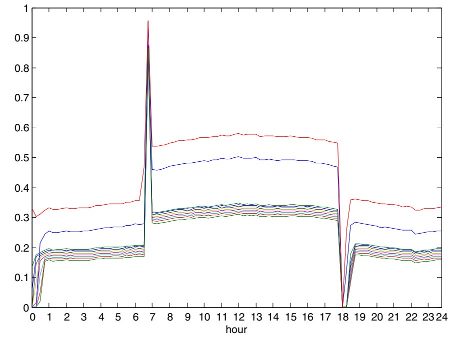

Fig. 1. Centralized control sequence

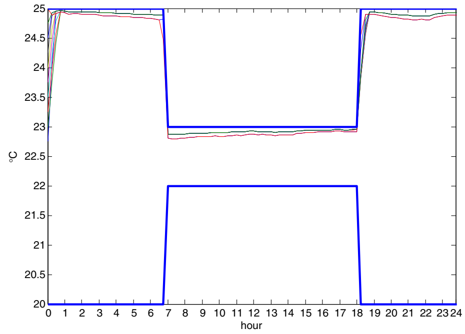

Fig. 2. Indoor temperature controlled by centralized MPC

close to the upper bound of the temperature constraint means saving energy. The ideal situation is to track the upper bound perfectly. However, due to the uncertain disturbance, perfectly tracking is impossible. Since we do not know the upper bound of the disturbance, robust MPC [10] can not be directly used. Samples may be used to estimate the bound of the disturbance and this was discussed in [18].

VI. Conclusions

In this study, we have considered the temperature regulation problem for a group of rooms. To reduce the conservativeness of robust MPC, SMPC has been formulated. By using scenario-based approach, this stochastic optimization problem has been approximated by a deterministic one. Then a Lagrangian relaxation based algorithm has been proposed to solve this optimization problem in a distributed manner.

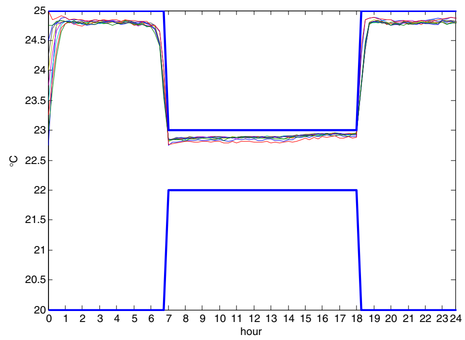

Fig. 3. Indoor temperature controlled by distributed MPC

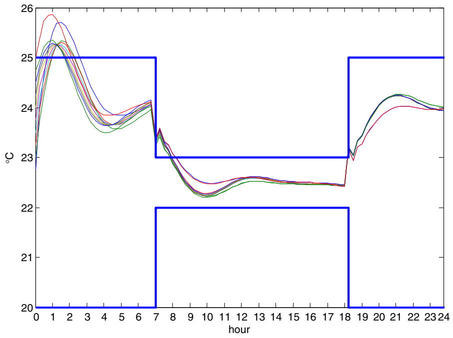

Fig. 4. Indoor temperature controlled by PID

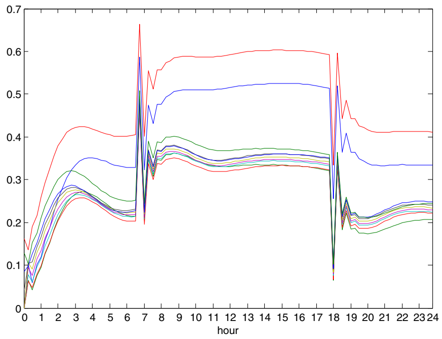

Fig. 5. Control sequence of PID

Finally, simulation results have demonstrated the superior performance of the algorithm we proposed.

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