On Setpoint Tracking and Disturbance Rejection of Event-triggered PI Control

Takuya Iwaki^{1†} and Karl Henrik Johansson¹

¹School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Stockholm, Sweden (e-mail: takya, kalle.kth.se)

Abstract: This paper studies sampled-data implementation of event-triggered PI control for continuous-time linear systems. We propose an event-triggered PI controller, in which the controller transmits its signal to the actuator when its relative value goes beyond a threshold. An exponential stability condition is derived in the form of LMIs using a Lyapunov-Krasovskii functional. It is shown that our proposed controller has the capability to track a desired constant setpoint. Furthermore, the controller can reject an uncertain disturbance by introducing an observer. A numerical example illustrates that our proposed controller reduces the communication load without performance degradation.

Keywords: PI control, event-triggered control, sampled-data systems, networked control, linear matrix inequality

1. INTRODUCTION

Control of process plants using wireless sensors and actuators is of growing interest in process automation industries [1–3]. Wireless process control offers advantages through massive sensing, flexible deployment, operation, and efficient maintenance. However, there remains an important problem, which is how to limit the amount of information that needs to be exchanged over the network, since the system performance is critically affected by network-induced delay, packet dropout, and sensor energy shortage.

In this context, event-triggered control has received a lot of attention from both academia and industry as a measure to reduce the communication load in networks [4,5]. Various event-triggered control architectures appeared recently (see the survey in [6] and the references therein). Event-triggered PID control for process automation systems is considered in some studies. For example, stability conditions of PI control subject to actuator saturation are derived in [7,8]. Event-triggered PI control for first-order systems using the PIDPLUS implementation [9] is discussed in [10]. Experimental validation is carried out in [7, 11]. Implementations on a real industrial plant is presented in [12–14].

The main objective of a PID controller is either setpoint tracking or disturbance rejection. However, the studies above mainly focus on the stability of the systems. For setpoint tracking, it is shown that the output converges to a constant setpoint when its value and the controller state are available at the sensor [10, 15], while sensors usually have no capability as a controller in process automation systems. In [16], the authors show that an event-triggered PI controller has bounded properties for setpoint tracking and disturbance rejection. Thus, the asymptotic behaviors for event-triggered control still need to be investigated.

In this paper, we study an event-triggered PI control for a time-continuous linear system. The controller updates the signal to the actuator when its relative value goes beyond a given threshold [17]. An exponential stability condition is derived using a Lyapunov-Krasovskii functional via Wirtinger's inequality [18] in the form of Linear Matrix Inequalities (LMIs). By modifying the event condition, we show that the event-triggered PI controller has a capability of setpoint tracking. Furthermore, the controller can reject an uncertain disturbance by introducing an observer. The event threshold synthesis is also proposed in this paper. A numerical example illustrates that our proposed controller reduces the communication load without performance degradation.

The remainder of the paper is organized as follows. Section 2 describes the plant and the time-trigged PI controller. An exponential stability condition for this system is derived. In Section 3, we introduce an event-trigged PI control and a stability condition is provided. Setpoint tracking and disturbance rejection are discussed in Section 4. We provide a numerical example in Section 5. The conclusion is presented in Section 6.

Notation

Throughout this paper, \mathbb{R} is the set of real numbers. The set of n by n positive definite (positive semi-definite) matrices over $\mathbb{R}^{n \times n}$ is denoted as \mathbb{S}_{++}^n (\mathbb{S}_{+}^n). For simplicity, we write X > Y ($X \ge Y$), $X, Y \in \mathbb{S}_{++}^n$, if $X - Y \in \mathbb{S}_{++}^n$ ($X - Y \in \mathbb{S}_{+}^n$) and X > 0 ($X \ge 0$) if $X \in \mathbb{S}_{++}^n$ ($X \in \mathbb{S}_{+}^n$). Symmetric matrices of the form $\begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}$ are written as $\begin{bmatrix} A & B \\ * & C \end{bmatrix}$ with B^\top denoting the transpose of B.

2. TIME-TRIGGERED PI CONTROL

In this section, we introduce a continuous-time linear plant and a time-triggered PI controller. An exponential stability condition is derived. The block diagram of the system is shown in Fig. 1.

[†] Takuya Iwaki is the presenter of this paper.



Fig. 1 Block diagram of the event-triggered PI control system. The event trigger is introduced in Section 3.

2.1. System model

Consider a plant given by

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t) + B_d d,$$
 (1)

$$y(t) = C_p x_p(t), \tag{2}$$

where $x_p(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$, $d \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the state, input, the constant disturbance, and output, respectively. We assume that the sensor samples and transmits its measurement every h time interval. The timetriggered PI controller, which updates its state and control signal every h time interval, is given by

$$\dot{x}_c(t) = r - y(t_k), \quad t \in [t_k, t_{k+1}),$$
(3)

$$u(t) = K_i x_c(t_k) + K_p(r - y(t_k)),$$
(4)

where $x_c(t) \in \mathbb{R}$ is the controller state, $r \in \mathbb{R}$ the constant reference signal, and $t_k, k = 0, 1, 2, ...$, is the time of transmission k of the sensor, i.e., $t_{k+1} - t_k = h$ for all t > 0.

By augmenting the state $x(t) = [x_p^{\top}(t), x_c^{\top}(t)]^{\top} \in \mathbb{R}^{n+1}$, we have the following closed-loop system description

$$\dot{x}(t) = Ax(t) + A_1 x(t_k) + B_D d + B_R r, \quad t \in [t_k, t_{k+1})$$
(5)

with

$$A = \begin{bmatrix} A_p & 0\\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -B_p K_p C_p & B_p K_i\\ -C_p & 0 \end{bmatrix},$$
$$B_D = \begin{bmatrix} B_d\\ 0 \end{bmatrix}, \quad B_R = \begin{bmatrix} B_p K_p\\ 1 \end{bmatrix}.$$

2.2. Stability condition of time-triggered PI control

We derive a stability condition of the system (5). **Theorem 1** Consider the plant (1)–(2) and the controller (3)–(4). Given $K_p, K_i \in \mathbb{R}$, and decay rate $\alpha > 0$, assume that there exist $P, W \in \mathbb{S}^{n+1}_{++}$, such that

$$\Phi \triangleq \begin{bmatrix} \Phi_{11} & PA_1 & A^{\top}Q \\ * & -\frac{\pi^2}{4}W & (A+A_1)^{\top}Q \\ * & * & -Q \end{bmatrix} < 0$$
(6)

where $\Phi_{11} \triangleq P(A + A_1) + (A + A_1)^\top P + 2\alpha P$ and $Q \triangleq h^2 e^{2\alpha h} W$. Then the closed-loop system (5) is exponentially stable with decay rate α . **Proof:** See Appendix A.

3. EVENT-TRIGGERED PI CONTROL

In this section, we discuss the event-triggered control introduced in [17,19]. We derive a stability condition and propose how to tune the event threshold for this setting.

3.1. System model of event-triggered PI control

Consider a plant given by

$$\dot{x}_p(t) = A_p x_p(t) + B_p \tilde{u}(t) + B_d d, \tag{7}$$

$$y(t) = C_p x_p(t), \tag{8}$$

where $\tilde{u}(t)$ is the event-triggered control signal. We assume that $\tilde{u}(t)$ is updated by checking the event condition

$$(u(t_k) - \tilde{u}(t_{k-1}))^2 > \sigma u^2(t_k)$$
(9)

at every sampling time t_k , k = 0, 1, ..., where $\sigma \in [0, 1)$ is a relative threshold. Thus, the event-triggered control signal is given by

$$\tilde{u}(t) = \begin{cases} u(t_k), & t \in [t_k, t_{k+1}), & \text{if (9) is true,} \\ \tilde{u}(t_{k-1}), & t \in [t_k, t_{k+1}) & \text{if (9) is false} \end{cases}$$

with $\tilde{u}_0 = u(t_0)$. Define the control signal error as

$$v(t) \triangleq \tilde{u}(t) - u(t)$$

= $\tilde{u}(t_k) - u(t_k), \quad t \in [t_k, t_{k+1}).$

Then the closed-loop system is given by

 $\dot{x}(t) = Ax(t) + A_1x(t_k) + Bv(t) + B_Dd + B_Rr$ (10) with

$$B = \begin{bmatrix} B_p \\ 0 \end{bmatrix}.$$

3.2. Stability conditions of event-triggered PI control

We have the following stability condition of the system (10) with d = r = 0.

Theorem 2 Consider the plant (1)–(2) with d = 0, the controller (3)–(4) with r = 0, and the event condition (9). Given $K_p, K_i \in \mathbb{R}$, and decay rate $\alpha > 0$, assume that there exist $P, W \in \mathbb{S}^{n+1}_{++}, w > 0$, and $\sigma > 0$, such that

$$\Psi \triangleq \begin{bmatrix} & | PB \ w\sigma K^{\top} \\ \Phi \ | 0 \ w\sigma K^{\top} \\ & | QB \ 0 \\ * & * & | -w \ 0 \\ * & * & | 0 \ -w\sigma \end{bmatrix} < 0$$
(11)

where $K = \begin{bmatrix} -K_p C_p & K_i \end{bmatrix}$. Then the closed-loop system (10) is exponentially stable with decay rate α . **Proof:** See Appendix B.

3.3. Event threshold tuning

Using (11), we can tune the event threshold σ to give a minimum communication load satisfying a given stability margin α .

Corollary 1 Given $K_p, K_i \in \mathbb{R}$, and $\alpha > 0$, if the semidefinite programming problem (SDP):

$$\sigma^* \stackrel{\Delta}{=} \max \sigma$$
 (12a)

s.t.
$$\Psi < 0$$
, (12b)



Fig. 2 Block diagram of the event-triggered PI control system for setpoint tracking and disturbance rejection.

is feasible, then the closed-loop system (10) under the event condition (9) with σ^* is exponentially stable with decay rate α .

4. SETPOINT TRACKING AND DISTURBANCE REJECTION OF EVENT-TRIGGERED PI CONTROL

Theorem 2 provides the stability condition of the event-triggered PI control with d = r = 0. In this case, the state converges to the origin. When $r \neq 0$ or $d \neq 0$, however, each element of the state converges possibly non-zero values even if the event-triggered controller successfully stabilizes the plant. This requires us to modify the event condition.

In this section, we discuss the setpoint tracking of the event-triggered control, i.e., the case $r \neq 0$. Then we consider the disturbance rejection, $d \neq 0$. The block diagram of the proposed system is shown in Fig 2.

4.1. Setpoint tracking

We have the following result on the system (10) with d = 0.

Theorem 3 Consider the plant (1)–(2) with d = 0, the controller (3)–(4), and the event condition

$$(u(t_k) - \tilde{u}(t_{k-1}))^2 > \sigma(u(t_k) - Kx_e)^2$$
(13)

where $x_e \triangleq -(A + A_1)^{-1}B_R r$. Given $K_p, K_i \in \mathbb{R}$, and decay rate $\alpha > 0$, assume that there exist $P, W \in \mathbb{S}^{n+1}_{++}$, w > 0, and $\sigma > 0$, such that $\Psi < 0$. Then $y(t) \to r$ as $t \to \infty$

Proof: Suppose that $\Psi < 0$. Then $\Phi_{11} < 0$ and therefore $A + A_1$ is Hurwitz and non-singular. We apply a coordinate transformation $\bar{x}(t) = x(t) - x_e$. Then the system (9) can be written as

$$\dot{\bar{x}}(t) = A\bar{x}(t) + A_1\bar{x}(t_k) + Bv(t).$$

By Theorem 2, this system is exponentially stable with the event condition

$$(\bar{u}(t_k) - \tilde{\bar{u}}(t_{k-1}))^2 > \sigma \bar{u}^2(t_k)$$

where $\bar{u}(t_k) = K\bar{x}(t_k) = u(t_k) - Kx_e$. This completes the proof.

4.2. Disturbance rejection

Theorem 3 implies that the event trigger needs to compute the steady-state input. However, it cannot be obtained for uncertain disturbance d. The idea to tackle this problem is to introduce an observer. Consider an augmented plant

$$\dot{x}_a(t) = A_a x_a(t) + B_a \tilde{u}(t), \tag{14}$$

$$y(t) = C_a x_a(t), \tag{15}$$

where $x_a(t) = [x_p^{\top}(t), d]^{\top} \in \mathbb{R}^{n+1}$ with

$$A_a = \begin{bmatrix} A_p & B_d \\ 0 & 0 \end{bmatrix}, \quad B_a = \begin{bmatrix} B_p \\ 0 \end{bmatrix}, \quad C_a = \begin{bmatrix} C_p & 0 \end{bmatrix}.$$

For the system (14)–(15), we introduce an observer with sampled-data implementation

$$\hat{x}_{a}(t) = A_{a}\hat{x}_{a}(t_{k}) + B_{a}\tilde{u}(t) + L(y(t_{k}) - C_{a}\hat{x}(t_{k}))$$
(16)

where $\hat{x}_a^{\top}(t) = [\hat{x}_p(t), \hat{d}(t)]^{\top}$ is the estimation of $x_a(t)$, $L = [L_p^{\top}, L_d] \in \mathbb{R}^{n+1}$ the observer gain. Denoting $e_p(t) \triangleq x_p(t) - \hat{x}_p(t)$ and $e_d(t) \triangleq d - \hat{d}(t)$ as the estimation errors, we have

$$\begin{split} \dot{e}_{p}(t) &= A_{p}x_{p}(t) - A_{p}x_{p}(t_{k}) \\ &+ (A_{p} - L_{p}C_{p})e_{p}(t_{k}) + B_{d}e_{d}(t_{k}), \\ \dot{e}_{d}(t) &= -L_{d}C_{p}e_{p}(t_{k}). \end{split}$$

By augmenting the state

$$\mathbf{x}(t) \triangleq \begin{bmatrix} x_p(t) \\ e_p(t) \\ e_d(t) \\ x_c(t) \end{bmatrix} \in \mathbb{R}^{2n+2},$$

we have the following closed-loop system description

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{A}_1\mathbf{x}(t_k) + \mathbf{B}v(t) + \mathbf{B}_D\hat{d}(t_k) + \mathbf{B}_R r, \quad t \in [t_k, t_{k+1}) \quad (17)$$

with

We are now ready to present the stability condition with the constant disturbance d.

Theorem 4 Consider the plant (1)–(2), the observer (16), the controller (3)–(4), and the event condition

$$(u(t_k) - \tilde{u}(t_{k-1}))^2 > \sigma(u(t_k) - \mathbf{K}\mathbf{x}_e(t_k))^2$$
(18)

where $\mathbf{x}_e(t_k) \triangleq -(\mathbf{A} + \mathbf{A}_1)^{-1}(\mathbf{B}_D \hat{d}(t_k) + \mathbf{B}_R r)$ and $\mathbf{K} \triangleq [-K_p C_p, 0, 0, K_i]$. Given $K_p, K_i \in \mathbb{R}, L \in \mathbb{R}^{n+1}$

and decay rate $\alpha > 0$, assume that there exist $P, W \in \mathbb{S}^{2n+2}_{++}$, w > 0, and $\sigma > 0$, such that

 $\Xi \triangleq$

$$\begin{bmatrix} \Xi_{11} & P\mathbf{A}_1 & \mathbf{A}^{\top}Q & P\mathbf{B} & w\sigma\mathbf{K}^{\top} \\ * & -\frac{\pi^2}{4}W & (\mathbf{A} + \mathbf{A}_1)^{\top}Q & 0 & w\sigma\mathbf{K}^{\top} \\ * & * & -Q & Q\mathbf{B} & 0 \\ * & * & * & -w & 0 \\ * & * & * & -w & 0 \\ \end{bmatrix} < 0,$$

where $\Xi_{11} \triangleq P(\mathbf{A} + \mathbf{A}_1) + (\mathbf{A} + \mathbf{A}_1)^\top P + 2\alpha P$. Then the closed-loop system (17) is exponentially stable with decay rate α . Furthermore, $y(t) \to r$ as $t \to \infty$ for any constants r and d.

Proof: This can be shown as well as Theorem 2 and Theorem 3.

Corollary 2 Given $K_p, K_i \in \mathbb{R}, L \in \mathbb{R}^{n+1}$, and $\alpha > 0$, if the SDP:

$$\max \ \sigma \tag{19a}$$

s.t.
$$\Xi < 0,$$
 (19b)

is feasible, then the closed-loop system (17) under the event condition (18) with σ^* is exponentially stable with decay rate α .

5. NUMERICAL EXAMPLE

In this section, we provide a numerical example to illustrate our theoretical results. Consider a first-order linear system

$$\dot{x}_p(t) = 0.1x_p(t) + 0.2\tilde{u}(t-\eta) + 0.1d,$$
(20)

$$y(t) = x_p(t). \tag{21}$$

By solving SDP (19) with $K_p = 2.20$, $K_i = 0.31$, $L_p = 1.0$, $L_d = 2.0$, the sampling interval h = 0.2, the decay rate $\alpha = 0.04$, we obtain the event thresholds $\sigma^* = 0.277$. The SDP can be solved effectively by YALMIP toolbox [20]. To evaluate the system performance, we use the Integral of the Absolute Error (IAE) which is calculated as

$$IAE = \int_0^{+\infty} |r - y(t)| dt$$

We consider a reference signal $r(t) = 1, \forall t \ge 0$ and a disturbance $d(t) = -2, \forall t \ge 80$. The numerical results for two strategies: the proposed event-triggered PI control (ET-control, red solid line) and the conventional sampled-data PI control without event-triggering (SD-control, blue dashed line) are shown in Table 1 and Fig. 3. It can be found that the event-triggered controller compensates for the disturbance d and the output converges to r = 1 as well as the conventional PI controller with slight performance degradation. In fact, the IAE for the event-triggered controller and the conventional controller is 8.52 and 8.37, respectively. The third plot in Fig. 3 shows the time instances of the control signal updates. We can see, as well as Table 1, that the communications between the controller and the actuator are

	Comm.	Comm.	IAE
	until $t = 160$	Reduction	
ET-control	1676	47.7%	8.52
SD-control	3202	0%	8.37

Table 1 Number of communications, their reductions,
and the IAE for each strategy.



Fig. 3 Responses to the setpoint r(t) = 1, $\forall t \ge 0$ and the disturbance d(t) = -2, $\forall t \ge 80$ of the two cases: Event-triggered PI control (ET-control, red solid line) and sampled-data PI control without event-triggering (SD-control, blue dashed line). The third plot shows the event generation at the event-triggered controller.

performed only 75 times until t = 160. Including the communications between the sensor and the controller, the proposed controller reduces the communications by 47.7% compared to the conventional PI controller.

6. CONCLUSION

In this paper, we investigated the event-triggered PI control for the time-continuous liner systems, where the controller updated its input signal when its relative value went beyond a given threshold. An exponential stability condition was derived. Furthermore, it was shown that the proposed controller has a capability of setpoint tracking and disturbance rejection. The event threshold synthesis was also proposed. Future work includes the extension to a PID controller for uncertain systems.

APPENDIX

A. PROOF OF THEOREM 1

Before presenting the proof, we introduce the following lemma.

Lemma 1 [21] Let $z : [a, b] \to \mathbb{R}^n$ be an absolutely continuous function with a square integrable first order derivative such that z(a) = 0 or z(b) = 0. Then for any $\alpha > 0$ and $W \in \mathbb{S}_{++}^n$, the following inequality holds:

$$\int_{a}^{b} e^{2\alpha\xi} z^{\top}(\xi) W z(\xi) d\xi$$

$$\leq \mathrm{e}^{2|\alpha|(b-a)} \frac{4(b-a)^2}{\pi^2} \int_a^b \mathrm{e}^{2\alpha\xi} \dot{z}^\top(\xi) W \dot{z}(\xi) \mathrm{d}\xi.$$

Now, we derive the stability condition of the system (5). Consider the functional

$$V = V_0 + V_W \tag{22}$$

where

$$V_{0} \triangleq x(t)^{\top} P x(t),$$

$$V_{W} \triangleq h^{2} e^{2\alpha h} \int_{t_{k}}^{t} \dot{x}(s)^{\top} W \dot{x}(s) ds$$

$$- \frac{\pi^{2}}{4} \int_{t_{k}}^{t} e^{-2\alpha(t-s)} \delta(s)^{\top} W \delta(s) ds,$$

with $\delta(t) \triangleq x(t_k) - x(t)$. Using Lemma 1 and $t - t_k \leq h$, we have $V_W \geq 0$. We take the derivatives of each term:

$$\begin{split} \dot{V}_0 &+ 2\alpha V_0 \\ &= x^\top(t) P \dot{x}(t) + \dot{x}^\top(t) P x(t) + 2\alpha x^\top(t) P x(t), \\ &= x^\top(t) \big(P(A + A_1) + P(A + A_1)^\top + 2\alpha P \big) x(t) \\ &+ x^\top(t) P A_1 \delta(t) + \delta^\top(t) A_1^\top P x(t), \end{split}$$

and

$$\dot{V}_W + 2\alpha V_W = h^2 \mathrm{e}^{2\alpha h} \dot{x}^\top(t) W \dot{x}(t) - \frac{\pi^2}{4} \delta^\top(t) W \delta(t).$$

Thus, we have

$$\dot{V} + 2\alpha V \le \phi^{\top} \left(\begin{bmatrix} \Phi_{11} & PA_1 \\ * & -\frac{\pi^2}{4}W \end{bmatrix} + \begin{bmatrix} A^{\top}Q \\ (A+A_1)^{\top}Q \end{bmatrix} Q^{-1} \begin{bmatrix} QA & Q(A+A_1) \end{bmatrix} \right) \phi < 0$$

where $\phi \triangleq [x^{\top}(t), \delta^{\top}(t)]^{\top}$. The proof completes by Schur complements.

B. PROOF OF THEOREM 2

First, note that by the event condition (9), for some $w \ge 0$, we have

$$w\sigma u^2(t_k) - wv^2(t) \ge 0.$$

Introducing the functional (22) gives

$$\begin{split} \dot{V} + 2\alpha V &\leq \phi^{\top} \begin{bmatrix} \Phi_{11} & PA_1 \\ * & -\frac{\pi^2}{4}W \end{bmatrix} \phi \\ &+ x^{\top}(t)PBv(t) + v^{\top}(t)B^{\top}Px(t) \\ &+ \dot{x}^{\top}(t)Q\dot{x}(t) + w\sigma u^2(t_k) - wv^2(t) \\ &= \psi^{\top} \begin{bmatrix} \Phi_{11} & PA_1 & PB \\ * & -\frac{\pi^2}{4}W & 0 \\ -\frac{\pi^2}{4}W & 0 \\ \frac{\pi^2}{4}W & 0 \\$$

where $\psi = [x^{\top}(t), \delta^{\top}(t), v^{\top}(t)]^{\top}$. Since $u(t_k) = Kx(t_k)$ and by Schur complements, we have that $\dot{V} + 2\alpha V < 0$ if $\Psi < 0$.

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