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Event-based Switching for Sampled-data Output Feedback Control: Applications to Cascade and Feedforward Control*

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Abstract— **This paper studies sampled-data output feedback control where the states are monitored by multiple sensors. Asymptotic stability conditions for given sampling intervals for each sensor are derived. Based on these results, we then propose an event-based controller switching, in which one sensor transmits its measurement to the controller with a fixed sampling rate while another sensor transmits with a send-on-delta strategy. Such a set-up is motivated by the many potential cascade and feedforward control architectures in process industry, which could enhance performance if additional wireless sensors could be added without changing existing (wired) communication schedules. Asymptotic stability conditions of the switching event-based control systems are derived. Numerical examples illustrate how our framework reduces the effect of disturbances for both cascade and feedforward PI control systems.**

I. INTRODUCTION

Control over wireless communication is of growing interest in automation industries along the recent development of wireless sensor technology. Wireless sensors enable flexible design, deployment, operation, and maintenance of industrial control systems. An important problem in industrial wireless control is how to limit the amount of information that needs to be exchanged over the network, since the system performance is critically affected by network-induced delay, packet dropout, and sensor energy shortage.

In this context, event-based control has received much attention from many researchers as a measure to reduce the communication load in networks [1]–[3]. Some extensions appeared recently. For example, time-delayed systems are considered in [4]. Event-based output feedback control with with actuator saturation is studied in [5]. Experimental validation is performed in [5], [6]. Implementation and experimental evaluation on a real industrial plant is presented in [7]–[9].

In industrial process control systems, control architectures with multiple sensors are sometimes implemented to improve control performance. For instance, classical feedforward control is added to single-loop PID control to mitigate the effect of disturbances. Furthermore, cascade control is used to reduce the effect of controlled variable deviations and thereby enable a more tightly controlled closed-loop system [10], [11]. In [12], stability conditions of dynamic output feedback control with feedforward compensation are discussed, where the feedforward controller updates its signal by event-based samplings from a sensor.

In this paper, we discuss sampled-data output feedback control systems monitored by multiple sensors, which can be considered as a general form of various popular control architectures used in process industry. We propose a novel controller switching framework utilizing event-based sampling. The controller is switched depending on the sensor measurements. First, we introduce sample-data output feedback control systems and derive stability conditions using the Lyapunov-Krasovskii functional [13]. We then propose the controller switching framework as a switched sampled-data output feedback control system. Stability conditions for the proposed control system are derived. Applications to cascade and feedforward control are then studied. It is shown in numerical examples that our framework reduces the communication load between the sensors and the controller while providing disturbance rejection.

The remainder of the paper is organized as follows. Section II describes the system. We introduce the sampled-data output feedback control systems and derive stability conditions. In Section III, we propose a controller switching framework and derive stability conditions. Section IV discusses the applications of this framework to cascade and feedforward control. We provide numerical examples in Section V. The conclusion is presented in Section VI.

Notation: Throughout this paper, $\mathbb{N}$ and $\mathbb{R}$ are the sets of nonnegative integers and real numbers, respectively. The set of n by n positive definite (positive semi-definite) matrices over $\mathbb{R}^{n\times n}$ is denoted as $\mathbb{S}^n_{++}$ ($\mathbb{S}^n_{+}$). For simplicity, we write $X > Y$ ($X \geq Y$), $X, Y \in \mathbb{S}^n_{++}$, if $X - Y \in \mathbb{S}^n_{++}$ ($X - Y \in \mathbb{S}^n_{+}$) and $X > 0$ ($X \geq 0$) if $X \in \mathbb{S}^n_{++}$ ($X \in \mathbb{S}^n_{+}$). Symmetric matrices of the form $\begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}$ are written as $\begin{bmatrix} A & B \\ \star & C \end{bmatrix}$ with $B^\top$ denoting the transpose of matrix B.

II. DYNAMIC OUTPUT FEEDBACK CONTROL UNDER SAMPLED-DATA MEASUREMENTS

In this section, we introduce a continuous-time linear system monitored by multiple sensors and controlled by sampled-data dynamic output feedback. Stability conditions are derived under bounded sampling intervals of each sensor.

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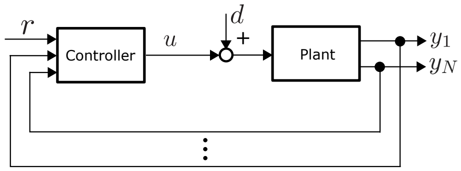

Fig. 1. Output feedback control system with two sampled-data measurements

A. System model

Consider a plant given by

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t) + \tilde{B}_p d(t), \tag{1}$$
$$y_i(t) = C_{p,i} x_p(t), \tag{2}$$

where $x_p(t) \in \mathbb{R}^{n_p}$, $u(t) \in \mathbb{R}^m$, $d(t) \in \mathbb{R}^{p_d}$ and $y_i(t) \in \mathbb{R}^{q_i}$ are the state, input, disturbance, and measurement by sensor $i \in \{1, \dots, N\}$, respectively. The matrices A_p, B_p, $\tilde{B}_p$, and $C_{p,i}$, $i = 1, \dots, N$ are real matrices of appropriate dimensions. We consider a dynamic output feedback controller

$$\dot{x}_c(t) = A_c x_c(t) + \sum_{i=1}^{N} B_{c,i} y_i(s_i(t)) + \tilde{B}_c r(t), \tag{3a}$$
$$u(t) = C_c x_c(t) + \sum_{i=1}^{N} D_{c,i} y_i(s_i(t)) + \tilde{D}_c r(t), \tag{3b}$$

that employs sampled-data measurements, where $x_c(t) \in \mathbb{R}^{n_c}$ is the controller state, $r(t) \in \mathbb{R}^{p_r}$ the reference signal, and $s_i(t) \in \mathbb{N}$ the latest time instance at time t when sensor i transmitted its measurement. The matrices A_c, $B_{c,i}$, $\tilde{B}_c$, C_c, $D_{c,i}$, $\tilde{D}_c$ $i = 1, \dots, N$ are real matrices of appropriate dimensions. The block diagram of the system is depicted in Fig. 1.

By augmenting the state $x(t) = [x_p^\top(t), x_c^\top(t)]^\top$, we have the following time-delay closed-loop system description

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} A_i x(t - \tau_i(t)) + B_d d(t) + B_r r(t), \tag{4}$$
$$y_1(t) = C_1 x(t), \quad y_2(t) = C_2 x(t), \tag{5}$$

where

$$A = \begin{bmatrix} A_p & B_p C_c \\ 0 & A_c \end{bmatrix}, \quad A_i = \begin{bmatrix} B_p D_{c,i} C_{p,i} & 0 \\ B_{c,i} C_{p,i} & 0 \end{bmatrix},$$
$$B_d = \begin{bmatrix} \tilde{B}_p \\ 0 \end{bmatrix}, \quad B_r = \begin{bmatrix} B_p \tilde{D}_c \\ \tilde{B}_c \end{bmatrix},$$
$$C_1 = \begin{bmatrix} C_{p,1} & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} C_{p,2} & 0 \end{bmatrix},$$

and $\tau_i(t) = t - s_i(k)$ with $\dot{\tau}_i(t) = 1$ for all i is time delays due to sampling, and $y(t) = [y_1(t)^\top, \dots, y_N(t)^\top]^\top$.

B. Stability conditions

We derive stability conditions for the system (4)–(5) with bounded sampling interval. We assume that any samplings satisfy $t - s_i(t) \leq h_i, \forall t > 0$. For simplicity, consider the two sensors case, i.e., $N = 2$. We have the following lemma to guarantee the stability.

Lemma 1: Assume that there exist $P, U_1, U_2 \in \mathbb{S}^n_{++}$, and some matrices P_2, P_3 such that the LMIs

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \star & \Phi_{22} + h_1 U_1 + h_2 U_2 \end{bmatrix} < 0, \tag{6}$$
$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & -h_1 P_2^\top A_1 \\ \star & \Phi_{22} + h_2 U_2 & -h_1 P_3^\top A_1 \\ \star & \star & -h_1 U_1 \mathrm{e}^{-2\alpha h_1} \end{bmatrix} < 0, \tag{7}$$
$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & -h_2 P_2^\top A_2 \\ \star & \Phi_{22} + h_1 U_1 & -h_2 P_3^\top A_2 \\ \star & \star & -h_2 U_2 \mathrm{e}^{-2\alpha h_2} \end{bmatrix} < 0, \tag{8}$$
$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & -h_1 P_2^\top A_1 & -h_2 P_2^\top A_2 \\ \star & \Phi_{22} & -h_1 P_3^\top A_1 & -h_2 P_3^\top A_2 \\ \star & \star & -h_1 U_1 \mathrm{e}^{-2\alpha h_1} & 0 \\ \star & \star & 0 & -h_2 U_2 \mathrm{e}^{-2\alpha h_2} \end{bmatrix} < 0, \tag{9}$$

where

$$\Phi_{11} = P_2^\top (A + A_1 + A_2) + (A + A_1 + A_2)^\top P_2 + 2\alpha P,$$
$$\Phi_{12} = P - P_2^\top + (A + A_1 + A_2)^\top P_3,$$
$$\Phi_{22} = -P_3 - P_3^\top,$$

are feasible. Then the closed-loop system (4) with $d(t) \equiv 0$ and $r(t) \equiv 0$, $\forall t > 0$, is exponentially stable with the decay rate $\alpha > 0$ for all sampling instants less than or equal to h_1 for sensor 1 and h_2 for sensor 2.

Remark 2: Lemma 1 can be extended to $N \geq 3$. In this case, 2^N LMIs will appear. Our assumption that $N = 2$ is reasonable as many process control loops consist of at most two sensors such as feedforward control and cascade control.

III. Event-based Control Switching

The main idea of this paper is to activate a second sensor to improve the transient response only when its output fluctuates. In this section, we propose a controller switching framework which activates the second sensor only when its measurement error goes beyond a given threshold.

A. Event-based control switching

Let us define two controllers, one of which computes the input signal using one sensor output $y_1(t)$, and the other controller uses two outputs $y_1(t)$ and $y_2(t)$. Consider the following two controllers:

$$\dot{x}_c^1(t) = A_c^1 x_c^1(t) + B_{c,1}^1 y_1(s_1(t)) + K_b^1 \phi^1(t) + \tilde{B}_c^1 r, \tag{10a}$$
$$u^1(t) = C_c^1 x_c^1(t) + D_{c,1}^1 y_1(s_1(t)) + \tilde{D}_c^1 r, \tag{10b}$$

and

$$\dot{x}_c^2(t) = A_c^2 x_c^2(t) + B_{c,1}^2 y_1(s_1(t)) + B_{c,2}^2 y_2(s_2(t)) + K_b^1 \phi^2(t) + \tilde{B}_c^2 r, \tag{11a}$$
$$u^2(t) = C_c^2 x_c^2(t) + D_{c,1}^2 y_1(s_1(t)) + D_{c,2}^2 y_2(s_2(t)) + \tilde{D}_c^2 r(t), \tag{11b}$$

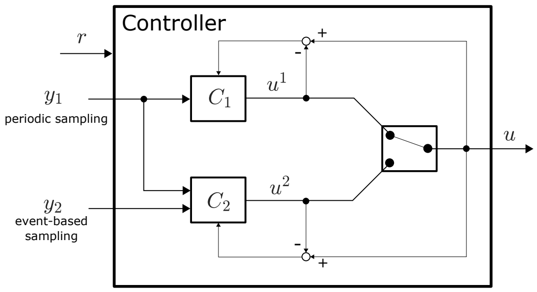

Fig. 2. The event-based switching controller. Controller C_1 computes the input signal u^1 and controller C_2 computes u^2. The input signal to the plant is chosen based on the switching rule.

where $x_c^1(t) \in \mathbb{R}^{n_c^1}, x_c^2(t) \in \mathbb{R}^{n_c^2}$ are the controller states and $\phi^i(t) \triangleq u(t) - u^i(t)$ is the control signal error. Here, $u(t)$ is the actual control signal to the actuator, which is defined by

$$u(t) = \begin{cases} u^1(t), & \text{if } \sigma(t) = 1, \\ u^2(t), & \text{if } \sigma(t) = 2, \end{cases} \tag{12}$$

where $\sigma : \mathbb{R} \to \{1, 2\}$ is the controller index function with $\sigma(t) = 1$ when controller 1 is activated and with $\sigma(t) = 2$ when controller 2 is activated. The block diagram of this switching controller is illustrated in Fig 2. The terms $K_b^i \phi^i(t)$ are called bumpless transfer and are introduced to reduce the effect of controller switching [11]. Let us note that controller (10) uses only the measurement from sensor 1, while controller (11) uses both sensors 1 and 2. Augmented by $\mathbf{x}^\top = [x_p^\top, x_c^{1\top}, x_c^{2\top}]^\top$, we obtain the hybrid system description

$$\begin{aligned} \dot{\mathbf{x}}(t) =& A^{\sigma(t)}\mathbf{x}(t) + A_1^{\sigma(t)}\mathbf{x}(s_1(t)) + A_2^{\sigma(t)}\mathbf{x}(s_2(t)) \\ &+ B_d^{\sigma(t)} d(t) + B_r^{\sigma(t)} r(t), \quad \sigma(t) \in \{1, 2\}, \quad (13) \\ y_1(t) =& C_1 \mathbf{x}(t), \quad (14) \\ y_2(t) =& C_2 \mathbf{x}(t), \quad (15) \end{aligned}$$

where

$$A^1 = \begin{bmatrix} A_p & B_p C_c^1 & 0 \\ 0 & A_c^1 & 0 \\ 0 & K_b^2 C_c^1 & A_c^2 - K_b^2 C_c^2 \end{bmatrix},$$

$$A_1^1 = \begin{bmatrix} B_p D_{c,1}^1 C_{p,1} & 0 & 0 \\ B_{c,1}^1 C_{p,1} & 0 & 0 \\ B_{c,1}^2 C_{p,2} + K_b^2 (D_{c,1}^1 - D_{c,1}^2) C_{p,1} & 0 & 0 \end{bmatrix},$$

$$A_2^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ B_{c,2}^2 C_{p,2} - K_b^2 D_{c,2}^2 C_{p,2} & 0 & 0 \end{bmatrix},$$

$$B_d^1 = \begin{bmatrix} \tilde{B}_p \\ 0 \\ 0 \end{bmatrix}, \quad B_r^1 = \begin{bmatrix} B_p \tilde{D}_c^1 \\ \tilde{B}_c^1 \\ \tilde{B}_c^2 + K_b^2 (\tilde{D}_c^1 - \tilde{D}_c^2) \end{bmatrix},$$

$$A^2 = \begin{bmatrix} A_p & 0 & B_p C_c^2 \\ 0 & A_c^1 - K_b^1 C_c^1 & K_b^1 C_c^2 \\ 0 & 0 & A_c^2 \end{bmatrix},$$

$$A_1^2 = \begin{bmatrix} B_p D_{c,1}^2 C_{p,1} & 0 & 0 \\ B_{c,1}^1 C_{p,2} + K_b^1 (D_{c,1}^2 - D_{c,1}^1) C_{p,1} & 0 & 0 \\ B_{c,1}^2 C_{p,1} & 0 & 0 \end{bmatrix},$$

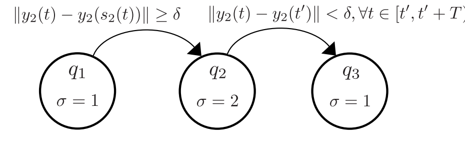

Fig. 3. Mode transition diagram among q_1, q_2, and q_3

$$A_2^2 = \begin{bmatrix} B_p D_{c,2}^2 C_{p,1} & 0 & 0 \\ K_b^1 D_{c,2}^2 D_{c,2}^2 C_{p,1} & 0 & 0 \\ B_{c,2}^2 C_{p,2} & 0 & 0 \end{bmatrix},$$

$$B_d^2 = \begin{bmatrix} \tilde{B}_p \\ 0 \\ 0 \end{bmatrix}, \quad B_r^2 = \begin{bmatrix} B_p \tilde{D}_c^2 \\ \tilde{B}_c^1 + K_b^1 (\tilde{D}_c^2 - \tilde{D}_c^1) \\ \tilde{B}_c^2 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} C_{p,1} & 0 & 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} C_{p,2} & 0 & 0 \end{bmatrix}.$$

We define a controller switching framework consisting of three modes $\mathcal{Q} \triangleq \{q_1, q_2, q_3\}$, see Fig.3. The initial states of the switching is assumed to be $q(0) = q_1$, $\mathbf{x}(0) = \mathbf{x}_0 \in \mathbb{R}^{n_p + n_c^1 + n_c^2}$, $s_1(0) = s_2(0) = 0$ where $q : \mathbb{R} \to \mathcal{Q}$ is the mode index function. Each mode is characterized by which controller is activated and when the sensor measurements are transmitted.

- In mode q_1, controller 1 is used and thus only sensor 1 transmits the measurement with every h_1 interval to the controller:

$$q_1 : \begin{cases} \sigma(t) = 1, \\ s_1(t) = \lfloor t/h_1 \rfloor h_1, \\ s_2(t) = 0. \end{cases}$$

- In mode q_2, controller 2 is used. Sensor 1 continues to transmit the measurement with every h_1 interval to the controller, but sensor 2 transmits through event-based sampling:

$$q_2 : \begin{cases} \sigma(t) = 2, \\ s_1(t) = \lfloor t/h_1 \rfloor h_1, \\ s_2(t) = \min_{t'} \{t' : (\|y_2(t) - y_2(t')\| \geq \delta \wedge \\ \qquad\qquad t - t' \geq h_{\min}) \vee t - t' = h_2\}, \end{cases}$$

  where the minimum inter-sampling time $h_{\min} < h_2$ is introduced to avoid Zeno behavior.
- In mode q_3, controller 1 is used and only sensor 1 transmits the measurement with every h_1 interval to the controller:

$$q_3 : \begin{cases} \sigma(t) = 1, \\ s_1(t) = \lfloor t/h_1 \rfloor h_1, \\ s_2(t) = s_2(t'), \end{cases}$$

  where $t' = \max_t \{t : q(t) = q_2\}$.

The mode transition from mode q_1 to q_2 occurs when $\|y_2(t) - y_2(s_2(t))\| \geq \delta$, and from q_2 to q_3 occurs at $t = t_1 + T$ when, for some t_1, $\|y_2(t) - y_2(t_1)\| < \delta, \forall t \in [t_1, t_1 + T)$.

Now, we make the following assumption.

Assumption 3: There exist $P^j, U_1^j, U_2^j \in \mathbb{S}_{++}^n$, and some matrices P_2^j, P_3^j for $j = 1, 2$ such that the LMIs (6)–(9)

hold in which the matrix variables are replaced by $P = P^j$, $U_1 = U_1^j$, $U_2 = U_2^j$, $P_2 = P_2^j$, $P_3 = P_3^j$ and

$$\Phi_{11} = P_2^\top (A^j + A_1^j + A_2^j) + (A^j + A_1^j + A_2^j)^\top P_2 + 2\alpha P$$
$$\Phi_{12} = P - P_2^\top + (A^j + A_1^j + A_2^j)^\top P_3$$
$$\Phi_{22} = -P_3 - P_3^\top.$$

Assumption 3 guarantees that both controllers (10) and (11) without switching stabilize the plant (1)–(2). The following theorem summarizes that the proposed control architecture yields a stable closed-loop system.

Theorem 4: Suppose Assumption 3 holds. The event-based switching control system defined by the plant (1)–(2) and the controllers (10)–(12) is asymptotically stable with $d(t) \equiv 0$ and $r(t) \equiv 0$, $\forall t > 0$.

Proof: First, note that $\boldsymbol{x}(t)$ converges to the origin if the system stays in mode q_1 for all $t > 0$. Then we show that, in mode q_2, there exists time instance t_1 such that $\|y_2(t) - y_2(t_1)\| < \delta$ for $t > t_1$. Due to Assumption 3, the system with controller 2 is asymptotically stable. Thus, there exists a time instance t_1' such that $y_2(t)$ never leaves the $\delta/2$-neibourhood of the origin for $t > t_1'$. Taking $t_1 > t_1'$ as the first sensor 2 sampling time after t_1', then, for $t > t_1$, we have $\|y_2(t) - y_2(t_1)\| < \delta$. This guarantees that the system goes to mode q_3 after $t = t_1 + T$. The proof completes since in mode q_3, the system with controller 1 is also asymptotically stable. ■

IV. PI Control with Event-based Sampling and Controller Switching

In this section, we apply our proposed switching control to some specific PID control architectures: cascade control and feedforward control. Both architectures are widely used in industrial process control systems to reduce the effect of disturbance. Our idea is simply to use a standard PI controller when the disturbance is not present and to activate a cascade controller or a feedforward controller when a disturbance is believed to be present.

A. Cascade control

In cascade control systems, the main (or outer) PID controller computes its control signal for the secondary (or inner) PI controller to track the reference signal. The secondary controller sends its control signal to the actuator. Corresponding to the controller switching framework, the cascade control is used in mode q_2, while single PI control is activated in mode q_1 and q_3. The block diagram of the event-based cascade control with controller switching is shown in Fig. 4.

In the cascade control, plants 1 and 2 are given by

$$\dot{x}_{p1}(t) = A_{p1} x_{p1}(t) + B_{p1} y_2(t),$$
$$\dot{x}_{p2}(t) = A_{p2} x_{p2}(t) + B_{p2} u(t) + \tilde{B}_{p2} d(t),$$

with

$$y_1(t) = C_{p1} x_{p1}(t), \quad y_2(t) = C_{p2} x_{p2}(t),$$

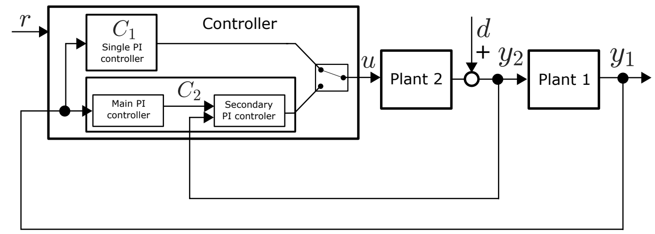

Fig. 4. The event-based cascade control system with controller switching

where $x_{p1}(t)$ and $x_{p2}(t)$ are the states of plants 1 and 2, respectively. Thus, we have

$$A_p = \begin{bmatrix} A_{p1} & B_{p1} C_{p2} \\ 0 & A_{p2} \end{bmatrix}, \quad B_p = \begin{bmatrix} 0 \\ B_{p2} \end{bmatrix}, \quad \tilde{B}_p = \begin{bmatrix} 0 \\ \tilde{B}_{p2} \end{bmatrix},$$
$$C_p^1 = \begin{bmatrix} C_{p1} & 0 \end{bmatrix}, \quad C_p^2 = \begin{bmatrix} 0 & C_{p2} \end{bmatrix}.$$

Consider PI control for both controllers. Denote $x_{c1}(t)$ and $u_1(t)$ as the main controller state and its control input, respectively. Then we have

$$\dot{x}_{c1}(t) = r(t) - y_1(s_1(t)),$$
$$u_1(t) = K_{i1}^2 x_{c1}(t) + K_{p1}^2 (r(t) - y_1(s_1(t))).$$

In the same way, we describe the secondary controller as

$$\dot{x}_{c2}(t) = u_1(t) - y_2(s_2(t)),$$
$$u^2(t) = K_{i2}^2 x_{c2}(t) + K_{p2}^2 (u_1(t) - y_2(s_2(t))),$$

where $x_{c2}(t)$ is the secondary controller state. Introducing an augmented controller state $x_c^{2\top}(t) = [x_{c1}^\top(t)\ x_{c2}^\top(t)]^\top$, we obtain a PI cascade controller as (11) with

$$A_c^2 = \begin{bmatrix} 0 & 0 \\ K_{i1}^2 & 0 \end{bmatrix}, \quad B_{c,1}^2 = \begin{bmatrix} -1 \\ -K_{p1}^2 \end{bmatrix}, \quad B_{c,2}^2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$
$$\tilde{B}_c^2 = \begin{bmatrix} 1 \\ K_{p1}^2 \end{bmatrix}, \quad C_c^2 = \begin{bmatrix} K_{p2}^2 K_{i1}^2 & K_{i2}^2 \end{bmatrix},$$
$$D_{c,1}^2 = -K_{p2}^2 K_{p1}^2, \quad D_{c,2}^2 = -K_{p2}^2, \quad \tilde{D}_c^2 = K_{p2}^2 K_{p1}^2.$$

In the same way, we obtain a single PI controller as (10) with

$$A_c^1 = 0, \quad B_{c,1}^1 = -1, \quad \tilde{B}_c^1 = 1,$$
$$C_c^1 = K_i^1, \quad D_{c,1}^1 = -K_p^1, \quad \tilde{D}_c^1 = K_p^1. \tag{16}$$

B. Feedforward control

Feedforward control is used with feedback control when disturbances can be measured. The control signal of a feedback controller is adjusted by a feedforward controller based on the disturbance measurements. The block diagram of the event-based feedforward control with controller switching is shown in Fig. 5. In Fig. 5, plant 2 can be an uncontrolled stable plant, a closed-loop system, or an independent controller located in a different place. We call the feedforward architecture a decoupler if plant 2 corresponds the other controller and $y_2(t)$ its control signal [10].

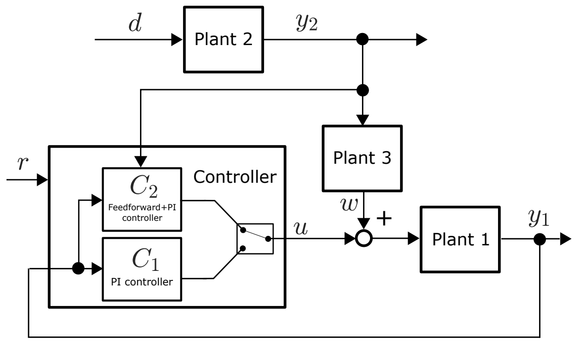

Fig. 5. The event-based feedforward control system with controller switching

The plants are given by

$$\dot{x}_{p1}(t) = A_{p1}x_{p1}(t) + B_{p1}u(t) + \tilde{B}_{p1}w(t),$$
$$\dot{x}_{p2}(t) = A_{p2}x_{p2}(t) + \tilde{B}_{p2}d(t),$$
$$\dot{x}_{p3}(t) = A_{p3}x_{p3}(t) + \tilde{B}_{p2}y_2(t),$$

with

$$y_1(t) = C_{p1}x_{p1}(t), \quad y_2(t) = C_{p2}x_{p2}(t),$$
$$w(t) = C_{p3}x_{p3}(t).$$

The feedforward controller used in mode q_2 is then described as (11) with

$$A_c^2 = 0, \quad B_{c,1}^2 = -1, \quad B_{c,2}^2 = 0, \quad \tilde{B}_c^2 = 1,$$
$$C_c^2 = K_i^2, \quad D_{c,1}^2 = -K_p^2, \quad D_{c,2}^2 = K_f^2, \quad \tilde{D}_c^2 = K_p^2,$$

where K_f^2 is a feedforward gain. In mode q_1 and q_3, the controller is given by (10) and (16).

V. Numerical Example

A. Event-based cascade control

We first illustrate the proposed event-based controller switching applied to a cascade control, where the plants are given by

$$\dot{x}_{p1}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix} x_{p1}(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} y_2(t),$$
$$\dot{x}_{p2}(t) = -2x_{p2}(t) + u(t) + d(t),$$
$$y_1(t) = \begin{bmatrix} 10 & 0 & 0 \end{bmatrix} x_{p1}(t), \quad y_2(t) = 3x_{p2}(t).$$

Lemma 1 guarantees that both controllers with the parameters $K_p^1 = 0.0119$, $K_i^1 = 0.0140$, $K_b^1 = 50$, $K_{p1}^2 = 0.015$, $K_{i1}^2 = 0.0209$, $K_{p2}^2 = 0.244$, $K_{i2}^2 = 1.8209$, and $K_b^{2\top} = [5, 5]$ stabilize the system with $h_1 = 1.5$, $h_2 = 0.3$. The parameters are obtained by applying MATLAB function `pidtune`. Then we introduce the proposed event-based controller switching with $\delta = 0.2$ and $T = 3$. Fig. 6 shows the response to the external step disturbance $d(t) = 5, \forall t \geq 5$. It can be found that the disturbance activates sensor 2, and as a result, the controller is switched to the cascade control. In mode q_2, sensor 2 takes frequent samplings at the beginning. After several periodic samplings,

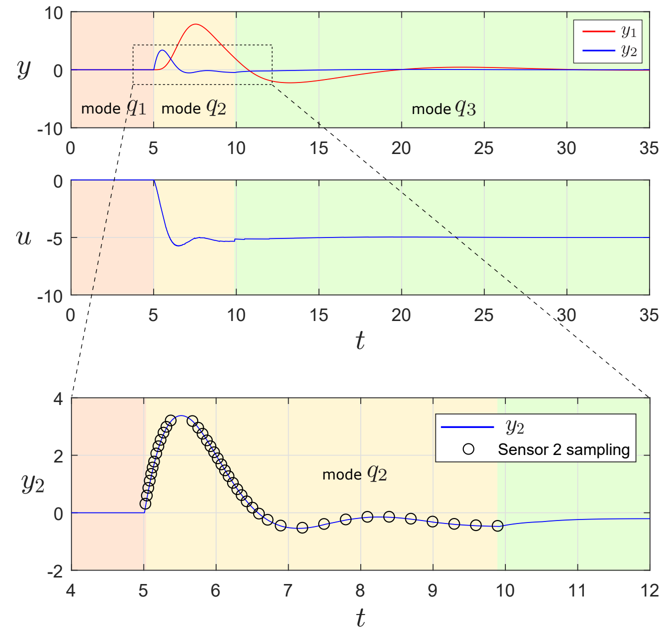

Fig. 6. The response of event-based cascade control with controller switching (red: mode q_1, yellow: mode q_2, and green: mode q_3)

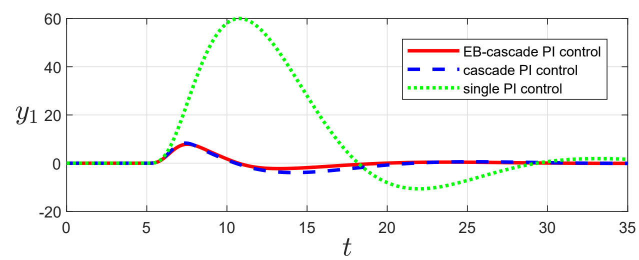

Fig. 7. Outputs for the three cases: the proposed event-based cascade control with controller switching (red solid line), cascade control with constant sampling time intervals (blue dashed line), and single PI control (green dot line).

sensor 2 is deactivated and the mode is switched to q_3 (Fig 6: bottom). Fig. 7 compares outputs $y_1(t)$ for three cases: the proposed event-based cascade control with controller switching (red solid line), cascade control with constant sampling rates with $h_1 = 1.5$, $h_2 = 0.3$, and single PI control with $h_1 = 1.5$. Apparently, cascade controllers dramatically reduce the effect of the disturbance. Furthermore, since the proposed control suspends sensor 2 samplings after the mode is switched, fewer samplings are needed compared to the cascade control with constant samplings. The proposed control takes 41 samples only in q_2, while the control with constant sampling rates requires to take 117 samplings until $t = 35$ and the total samplings will constantly increase.

B. Event-based feedforward control

Second, we show a numerical example of the proposed event-based feedforward control, where the plants are given

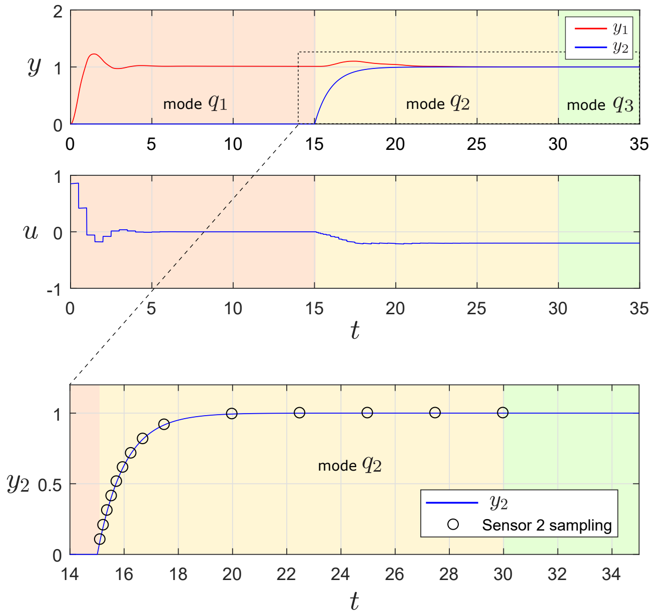

Fig. 8. The response of event-based feedforward control with controller switching (red: mode q_1, yellow: mode q_2, and green: mode q_3)

by

$$\dot{x}_{p1}(t) = \begin{bmatrix} -5 & 0 \\ 1 & 0 \end{bmatrix} x_{p1}(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} w(t),$$
$$\dot{x}_{p2}(t) = -x_{p2}(t) + d(t), \quad \dot{x}_{p3}(t) = -5x_{p2}(t) + y_2(t),$$
$$y_1(t) = \begin{bmatrix} 0 & 10 \end{bmatrix} x_{p1}(t), \quad y_2(t) = 3x_{p2}(t),$$
$$w(t) = x_{p3}(t).$$

Lemma 1 guarantees that both controllers with the parameters $K_p^1 = 0.85$, $K_i^1 = 0.0241$, $K_b^1 = 50$, $K_p^2 = 0.325$, $K_{i1}^2 = 0.288$, $K_f^2 = -0.1$, and $K_b^2 = 50$ stabilize the system with $h_1 = 0.5$, $h_2 = 2.5$. Then we introduce the proposed event-based controller switching with $\delta = 0.1$ and $T = 12.5$. Fig. 8 shows the response to the step reference signal $r(t) = 1, \forall t \geq 0$ and the step external disturbance $d(t) = 0.1, \forall t \geq 15$. The reference signal does not activate sensor 2 and the mode stays mode q_1. The disturbance occurs at $t = 15$, which results in mode switching. In mode q_2, the feedforward controller takes the corrective action based on sensor 2 measurements shown in the bottom plot of Fig 8. Sensor 2 takes frequent samples until around $t = 18$, then is deactivated at $t = 30$ after several periodic samplings. Fig. 9 compares outputs for three cases: the proposed event-based feedforward control with controller switching (red solid line), feedforward control with constant sampling rates with $h_1 = 0.8$, $h_2 = 2.5$, and single PI control with $h_1 = 0.8$. The proposed controller realizes the same step response as the single PI control which has a smaller overshoot compared to the feedforward control. It also achieves the better disturbance rejection compared to the single PI control, since the controller is switched to the one with a large I-gain and with a feedforward gain. It has even better performance than the feedforward control with constant sampling rates, since the event-based samplings can rapidly react to the disturbance.

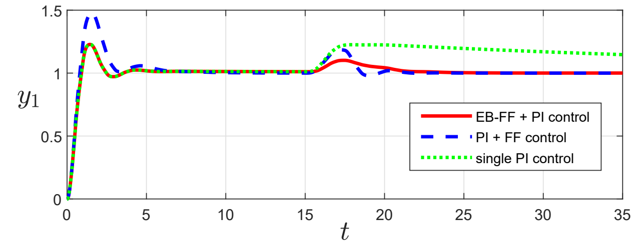

Fig. 9. Outputs for the three cases: the proposed event-based feedforward control with controller switching (event-based FF control: red solid line), feedforward control with constant sampling time intervals (FF control: blue dashed line), and single PI control (green dot line).

VI. Conclusion

In this paper, we investigated output feedback control systems with multiple sampled-data measurements. As a main result, we proposed an event-based controller switching framework. It was shown that this framework could be applied to cascade and feedforward control. Numerical examples showed that the proposed framework reacted well to disturbances with fewer samplings compared to controllers with constant sampling and without switching.

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