Proceedings of the 38th Chinese Control Conference
July 27-30, 2019, Guangzhou, China

Distributed Parameter Estimation Under Event-triggered Communications

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Abstract: In this paper, we study a distributed parameter estimation problem with an asynchronous communication protocol over multi-agent systems. Different from traditional time-driven communication schemes, in this work, data can be transmitted between agents intermittently rather than in a steady stream. First, we propose a recursive distributed estimator based on an event-triggered communication scheme, through which each agent can decide whether the current estimate is sent out to its neighbors or not. With this scheme, considerable communications between agents can be effectively reduced. Then, under mild conditions including a collective observability, we provide a design principle of triggering thresholds to guarantee the asymptotic unbiasedness and strong consistency. Furthermore, under certain conditions, we reveal that, with probability one, for every agent the time interval between two successive triggering instants goes to infinity as time goes to infinity. Finally, we provide a numerical simulation to validate the theoretical results of this paper.

Key Words: Distributed parameter estimation, Event-triggered communications, Strong consistency

1 Introduction

As one of the hottest research topics over the last decade, multi-agent systems have attracted a lot of attention of researchers around the world due to their broad applications in sensor networks, cyber-physical systems, computer games, transportation, etc. With the development of network technology and the increasing of data amount, distributed learning and estimation protocols without requiring a data center are becoming more and more popular.

Distributed parameter estimation over multi-agent systems is on the problem of estimation or learning of an unknown parameter based on data transmission between neighboring agents. Numerous practical applications, such as temperature monitoring, weather prediction and environmental exploration, can be cast into distributed parameter estimation problems. Due to environmental complexity, the estimation problem is usually modeled under stochastic frameworks, where measurements of each agent are polluted by random noises. In [1–4], the distributed parameter estimation problems are investigated with respect to the estimation properties including consistency and asymptotic normality. The distributed parameter estimation problems over random networks and imperfect communication channels are studied in [5, 6]. The connection between graph topologies and estimation performance in terms of asymptotic variances are analyzed in the above literature.

Design and analysis of communicaton schemes between agents is an essential research topic of networked estimation and control. Due to the limitations of channel capacity and energy resources, traditional time-driven communication schemes may not be suitable to some practical applications, such as wireless sensor networks. Thus, in the existing literature, there have been a few results which consider event-triggered communication schemes. Event-triggered measurement scheduling problems are well studied in [7–13]. In these literature, the parameter estimation or filtering problems are investigated under centralized frameworks, where a center can process the transmitted messages to obtain estimates for a parameter or state vector. Distributed filtering problems with event-triggered communications are investigated in [14–16], where the messages of state estimates or covariance bounds are transmitted to other agents intermittently. However, to the best knowledge of authors, the distributed parameter estimation problems with event-triggered communications have not been well studied in the existing literature. The main difficulty is to design and analyze triggering conditions so as to reduce communication frequency between agents with guaranteed estimation properties.

In this paper, we study the distributed parameter estimation problem with event-triggered intermittent communications between neighboring agents. The contributions of this work are two fold. First, we propose an event-triggered communication scheme, through which each agent can decide whether the current estimate is sent out to its neighbors or not. With this scheme, redundant communications between agents can be effectively reduced. Second, under mild conditions, for the considered distributed estimator, we study the main estimation properties including asymptotic unbiasedness and strong consistency. Besides, we reveal that, for every agent the time interval between two successive triggering instants goes to infinity as time goes to infinity in the sense of almost sure, which means that the communication frequency between any two neighboring agents is tremendously reduced if the terminal time is sufficiently large. It should be noted that the main difference between the event-triggered framework proposed in this work and the existing

The work is supported by Knut & Alice Wallenberg foundation, and by Swedish Research Council.

literature is that our triggering threshold will go to zero as time goes to infinity, which is necessary to guarantee the asymptotic convergence of estimates in mild collective observability conditions.

The remainder of the paper is organized as follows: Section 2 is on preliminaries and problem formulation. Section 3 considers the event-triggered communication scheme and some main asymptotic estimation properties. Section 4 provides a numerical simulation. The conclusion of this paper is given in Section 5.

1.1 Notations

The superscript "T" represents the transpose. $\mathbb{R}^{n\times m}$ is the set of real-valued matrices with n rows and m columns. I_n stands for the n-dimensional square identity matrix. $\mathbf{1}_N$ stands for the N-dimensional vector with all elements being one. $\mathbb{E}\{x\}$ denotes the mathematical expectation of the stochastic variable x, and $\mathrm{blockdiag}\{\cdot\}$ represent the diagonalizations of block elements. Additionally, $A \otimes B$ is the Kronecker product of A and B. $\|x\|$ is the norm of a vector x, and $\|A\|$ is the induced norm, i.e., $\|A\| = \sup_{\|x\|=1} \|Ax\|$, where $A \in \mathbb{R}^{n\times m}$. The mentioned scalars, vectors and matrices of this paper are all real-valued.

2 Preliminaries and Problem Formulation

In this section, we provide some necessary graph preliminaries and then formulate the problem studied in this work.

2.1 Graph Preliminaries

In this paper, the communication between agents of a network is modeled as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, which consists of the set of nodes $\mathcal{V}$, the set of edges $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$, and the adjacency matrix $\mathcal{A} = [a_{i,j}]$. $\mathcal{A}$ is a symmetry matrix consisting of one and zero. If $a_{i,j} = 1, j \neq i$, there is an edge $(i, j) \in \mathcal{E}$, which means that node i can exchange information with node j, and node j is called a neighbor of node i. For node i, its neighbor set is denoted by $\{j \in \mathcal{V}|(i, j) \in \mathcal{E}\} \triangleq \mathcal{N}_i$. We suppose that the graph has no self-loop, which means $a_{i,i} = 0$ for any $i \in \mathcal{V}$. $\mathcal{G}$ is called connected if for any pair nodes (i_1, i_l), there exists a path from i_1 to i_l consisting of edges $(i_1, i_2), (i_2, i_3), \ldots, (i_{l-1}, i_l)$. Besides, we denote $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{L}$ is called Laplacian matrix and $\mathcal{D}$ is called degree matrix which is a diagonal matrix consisting of numbers of neighboring nodes. For detailed definitions, the readers are referred to [17]. On the connectivity of a graph, the following theorem holds.

Theorem 2.1. *[17] The graph $\mathcal{G}$ is connected if and only if $\lambda_2(\mathcal{L}) > 0$, where $\lambda_2(\mathcal{L})$ is the second smallest eigenvalue of $\mathcal{L}$.*

2.2 Problem Setup

Consider an unknown parameter vector $\theta \in \mathbb{R}^M$ is observed by $N > 0$ agents with the following model

$$y_i(t) = H_i\theta + v_i(t), i = 1, 2, \cdots, N, \tag{1}$$

where $y_i(t) \in \mathbb{R}^{m_i}$ is the measurement vector, $v_i(t) \in \mathbb{R}^{m_i}$ is the zero-mean independent and identically distributed (i.i.d.) measurement noise with covariance R_i, and $H_i \in \mathbb{R}^{m_i\times M}$ represents the known measurement matrix of agent i. The noise covariance matrix of all agents is R_v, i.e., $R_v = \mathbb{E}\{V(t)V(t)^T\}$ with $V(t) = \left[v_1^T(t), \ldots, v_N^T(t)\right]^T$. Note that we simply require the temporal independence of measurement noises, thus the noises of agents could be spatially correlated.

Assume that $x_i(t)$ is the estimate of agent i at time t for the parameter vector θ. In [2], the following estimator is studied

$$\begin{aligned} x_i(t+1) =& x_i(t) + \beta(t) \sum_{j\in\mathcal{N}_i} (x_j(t) - x_i(t)) \\ &+ \alpha(t)KH_i^T(y_i(t) - H_i x_i(t)), \end{aligned} \tag{2}$$

where $\alpha(t)$ and $\beta(t)$ are time-varying steps satisfying certain conditions. K is the parameter to be designed.

To reduce limitation of energy consumption and alleviate burden of communication channels, we focus on studying event-triggered communication scheme, in which the state estimates of each agent will not be consistently transmitted.

In the following of this paper, we focus on solving the problems as follows. 1) How to design a fully distributed event-triggered communication scheme for each agent? 2) What conditions are required to guarantee essential estimation properties, including asymptotic unbiasedness and strong consistency for each agent? 3) How does the event-triggered communication scheme contribute to reducing the communication frequency of agents with guaranteed properties?

3 Main results

In this section, we will propose an event-triggered communication scheme and analyze the main estimation properties of a recursive distributed estimator based on the triggering scheme.

3.1 Event-triggered communication scheme

In this subsection, we consider design an event-triggered scheme for each agent, which can decide whether the current estimate is sent out to its neighbors or not. Let t_k^i be the kth triggering time of the ith agent, and it is the latest triggering time of agent i. Then, we denote the triggering event $\mathbf{E}_i(t)$

$$\mathbf{E}_i(t) : \left\|x_i(t) - x_i(t_k^i)\right\| > \frac{1}{(t+1)^{\rho_i}}, i \in \mathcal{V}, \tag{3}$$

where ρ_i is a positive scalar addressed in the following, and $x_i(t_k^i)$ is the latest state estimate sent out by agent i at time t_k^i.

Let $\rho_0 = \min\{\rho_j, \forall j \in \mathcal{V}\}$, and the random indicator variable $\gamma_i(t)$ be

$$\gamma_i(t) = \begin{cases} 0, & \text{if } \mathbf{E}_i(t) \text{ occurs}, \\ 1, & \text{otherwise}. \end{cases} \tag{4}$$

Note that the distribution of $\gamma_i(t)$ influences the communication frequency of the whole multi-agent system. If $\mathbb{P}(\gamma_i(t) = 0) = 0$, $\forall i \in \mathcal{V}$, $t \in \mathbb{N}$, then the communications between agents will not happen almost surely. And if $\mathbb{P}(\gamma_i(t) = 0) = 1$, $\forall i \in \mathcal{V}$, $t \in \mathbb{N}$, the communication scheme is equivalent to the time-driven one almost surely.

Based on the triggering scheme (3) - (4), for agent i, we propose the following event-triggered distributed parameter estimator

$$x_i(t+1) = x_i(t) + \alpha(t)H_i^T(y_i(t) - H_i x_i(t)), \quad (5)$$
$$+ \beta(t) \sum_{j \in \mathcal{N}_i} \left(\gamma_j(t) x_j(t_k^j) + (1 - \gamma_j(t)) x_j(t) - x_i(t) \right),$$

where $\alpha(t)$ and $\beta(t)$ are time-varying steps addressed in Assumption 3.4.

Remark 3.1. The sequence of random variables $\{\gamma_j(t), j \in \mathcal{N}_i, i \in \mathcal{V}\}_{t=0}^{\infty}$ is not transmitted between neighbors. Agent i can obtain $\gamma_j(t), j \in \mathcal{N}_i$ through judging whether it receives the message from agent j at time t. In other words, if agent i receives the message from its neighbor agent j at time t, then $\gamma_j(t) = 0$, otherwise, $\gamma_j(t) = 1$ which essentially means that there is no communication from agent j to agent i at time t. To achieve the estimator (5), each agent should reserve the latest state estimates which its neighboring agents sent out. If the new estimates of neighbors are received, the stored ones will be updated.

Remark 3.2. Different from existing results [7–16], the triggering threshold $\frac{1}{(t+1)^{\rho_i}}$ goes to zero as t goes to infinity. If the threshold does not go to zero as time goes to infinity and the collective observability condition (see Assumption 3.2) without agent i is not satisfied, the estimates of all agents except agent i will not converge to the true parameter vector.

3.2 Performance Analysis

For convenience, we provide the following notations.

$$\begin{cases} \Theta = \mathbf{1}_N \otimes \theta \\ Y(t) = \left[y_1^T(t), \ldots, y_N^T(t)\right]^T \\ X(t) = \left[x_1^T(t), \ldots, x_N^T(t)\right]^T \\ X(t_k) = \left[x_1^T(t_k^1), \ldots, x_N^T(t_k^N)\right]^T \\ \bar{D}_H = \text{blockdiag}\left\{H_1^T, \ldots, H_N^T\right\} \\ D_H = \text{blockdiag}\left\{H_1^T H_1, \ldots, H_N^T H_N\right\}. \end{cases} \quad (6)$$

The following assumptions are needed in this paper.

Assumption 3.1. The graph is connected, i.e., $\lambda_2(\mathcal{L}) > 0$.

Assumption 3.2. The observation system (1) is collectively observable, i.e., $G = \sum_{i=1}^N H_i^T H_i$ is of full rank.

Assumption 3.3. There exists a positive scalar $\epsilon_1 > 0$, such that $\mathbb{E}\{\|V(t)\|^{2+\epsilon_1}\} < \infty$.

Assumption 3.4. The steps in (5) are set with $\alpha(t) = \frac{a}{(t+1)^{\tau_1}}$ and $\beta(t) = \frac{b}{(t+1)^{\tau_2}}$, where $a, b > 0$, $0 < \tau_2 \leq \tau_1 \leq 1$. Besides, $\tau_1 > \max\{\tau_2 + \frac{1}{2+\epsilon_1}, 0.5\}$.

Remark 3.3. Assumption 3.1 is a standard condition of distributed estimation and control for multi-agent systems. Assumption 3.2 is a collective observability condition, which is satisfied even if any local observability condition is not satisfied. Assumption 3.3 is on the moment condition of noises. Assumption 3.4 provides feasible design conditions of the steps in (5).

On the triggering scheme in (4), if $\gamma_i(t) = 0$, the agent i will send its estimate $x_i(t)$ to its neighbors, who then update the stored estimate with $x_i(t_k^i) = x_i(t)$. Thus, we rewrite (5) in the following form

$$x_i(t+1) = x_i(t) + \beta(t) \sum_{j \in \mathcal{N}_i} (x_j(t) - x_i(t))$$
$$+ \alpha(t) H_i^T (y_i(t) - H_i x_i(t)) \quad (7)$$
$$+ \beta(t) \sum_{j \in \mathcal{N}_i} (x_j(t_k^j) - x_j(t)).$$

where $x_j(t_k^j)$ is the latest received estimate from agent j by agent i. If agent i communicates with its neighbors at each time (e.g., typical time-driven communications), then the last term in the right side of (7) will be zero.

Considering the notations in (6), we obtain the compact form of (7) in the following

$$X(t+1) = X(t) - \beta(t)(\mathcal{L} \otimes I_M) X(t)$$
$$+ \alpha(t) \bar{D}_H (Y(t) - \bar{D}_H^T X(t)) \quad (8)$$
$$+ \beta(t)(\mathcal{A} \otimes I_M)(X(t_k) - X(t)).$$

We have the following lemma on the error between transmitted estimate vector $X(t_k)$ and current estimate vector $X(t)$.

Lemma 3.1. *Consider (8), then there exists a scalar $\bar{m} > 0$, such that*

$$\|X(t_k) - X(t)\| \leq \frac{\bar{m}}{(t+1)^{\rho_0}}. \quad (9)$$

The following two lemmas are useful to further analysis.

Lemma 3.2. *Under Assumption 3.1 and Assumption 3.2, $\mathcal{L} \otimes I_M + D_H$ is a positive definite symmetry matrix. Furthermore, there exists a constant matrix $M_0 \in \mathbb{R}^{NM \times NM}$ and a sufficiently large integer T, such that for any $t > T$,*

$$\alpha(t) M_0 \leq \beta(t)(\mathcal{L} \otimes I_M) + \alpha(t) D_H < I_{N \times M}.$$

Proof. The proof is similar to Lemma 6 of [2]. □

Lemma 3.3. *(Lemma 6, [3]) Consider a scalar sequence $\{z(t)\}$ satisfying*

$$z(t+1) = (1 - r_1(t)) z(t) + r_2(t),$$

with initial value $z(0) \geq 0$, where $r_1(t) = \frac{a_1}{(t+1)^{\delta_1}}$ and $r_2(t) = \frac{a_2}{(t+1)^{\delta_2}}$, with $0 \leq r_1(t) \leq 1$, $a_1 > 0$, $a_2 > 0$, $0 \leq \delta_1 \leq 1$, $\delta_2 \geq 0$, and $\delta_1 < \delta_2$. Then

- *if $\delta_1 < 1$, for all $\delta_0 \in [0, \delta_2 - \delta_1)$, we have*

$$\lim_{t \to \infty} (t+1)^{\delta_0} z(t) = 0. \quad (10)$$

- *if $\delta_1 = 1$ and $a_1 > \delta_0$, (10) holds.*

On the estimator (5), the asymptotic unbiasedness is studied in the following theorem.

Theorem 3.1. *(Asymptotically Unbiased) If $\rho_0 > \tau_1 - \tau_2$, the estimate sequence $\{x_i(t)\}$ by (5) is asymptotically unbiased with respect to the true parameter θ, i.e., $\lim_{t \to \infty} \mathbb{E}\{x_i(t)\} = \theta, \forall i \in \mathcal{V}$.*

Proof. According to (8), we have

$$\begin{aligned}X(t+1) =& X(t) - \beta(t)(\mathcal{L} \otimes I_M)X(t) \\ &+ \alpha(t)D_H(\Theta - X(t)) \\ &+ \alpha(t)\bar{D}_H V(t) \\ &+ \beta(t)(\mathcal{A} \otimes I_M)(X(t_k) - X(t)).\end{aligned} \tag{11}$$

Let $\tilde{X}(t) = X(t) - \Theta$ and $\bar{X}(t) = X(t_k) - X(t)$. By $(\mathcal{L} \otimes I_M)\Theta = 0$, we have

$$\begin{aligned}&\tilde{X}(t+1) \\ =&\tilde{X}(t) - \beta(t)(\mathcal{L} \otimes I_M)\tilde{X}(t) - \alpha(t)D_H\tilde{X}(t) \\ &+ \alpha(t)\bar{D}_H V(t) + \beta(t)(\mathcal{A} \otimes I_M)\bar{X}(t) \\ =&\left(I_{M\times N} - \beta(t)(\mathcal{L} \otimes I_M) - \alpha(t)D_H\right)\tilde{X}(t) \\ &+ \alpha(t)\bar{D}_H V(t) + \beta(t)(\mathcal{A} \otimes I_M)\bar{X}(t).\end{aligned} \tag{12}$$

Taking expectation on both sides of (12), we have

$$\begin{aligned}&\mathbb{E}\{\tilde{X}(t+1)\} \\ =&\left(I_{M\times N} - \beta(t)(\mathcal{L} \otimes I_M) - \alpha(t)D_H\right)\mathbb{E}\{\tilde{X}(t)\} \\ &+ \beta(t)(\mathcal{A} \otimes I_M)\,\mathbb{E}\{\bar{X}(t)\}.\end{aligned} \tag{13}$$

According to Lemma 3.2, there exists a sufficiently large integer T, such that for any $t > T$,

$$\alpha(t)M_0 \leq \beta(t)(\mathcal{L} \otimes I_M) + \alpha(t)D_H < I_{N\times M}.$$

Then, for $t > T$, taking norm operator on both sides of (13) yields

$$\begin{aligned}&\left\|\mathbb{E}\{\tilde{X}(t+1)\}\right\| \\ \leq& \left\|(I_{M\times N} - \beta(t)(\mathcal{L} \otimes I_M) - \alpha(t)D_H)\right\| \left\|\mathbb{E}\{\tilde{X}(t)\}\right\| \\ &+ \beta(t)\left\|(\mathcal{A} \otimes I_M)\right\| \left\|\mathbb{E}\{\bar{X}(t)\}\right\| \\ \leq&(1 - \alpha(t)m_0)\left\|\mathbb{E}\{\tilde{X}(t)\}\right\| + \beta(t)MN\left\|\mathbb{E}\{\bar{X}(t)\}\right\|,\end{aligned} \tag{14}$$

where $m_0 = \lambda_{min}(M_0)$.

Recall $\beta(t) = \frac{b}{(t+1)^{\tau_2}}$, then there exists a constant scalar $m_1 > 0$, such that $\beta(t)\left\|\mathbb{E}\{\bar{X}(t)\}\right\| \leq \frac{m_1}{(t+1)^{\tau_2+\rho_0}}$. As a result, from (14), we have

$$\begin{aligned}&\left\|\mathbb{E}\{\tilde{X}(t+1)\}\right\| \\ \leq&(1 - \alpha(t)m_0)\left\|\mathbb{E}\{\tilde{X}(t)\}\right\| + \frac{MNm_1}{(t+1)^{\tau_2+\rho_0}} \\ =&(1 - \frac{am_0}{(t+1)^{\tau_1}})\left\|\mathbb{E}\{\tilde{X}(t)\}\right\| + \frac{MNm_1}{(t+1)^{\tau_2+\rho_0}}.\end{aligned} \tag{15}$$

Without losing generality, here we suppose $am_0 > \rho_0 + \tau_2 - \tau_1$. Otherwise, we can obtain a sufficiently large m_0 by increasing t and maintaining the value $\frac{am_0}{(t+1)^{\tau_1}}$. Due to $\rho_0 > \tau_1 - \tau_2$, Lemma 3.3 and (15), $\left\|\mathbb{E}\{\tilde{X}(t+1)\}\right\|$ goes to zero as t goes to infinity. □

We can see from Theorem 3.1 that the initial estimation biases of agents can be removed by the estimator (5) as time goes to infinity.

Lemma 3.4. *Under Assumptions 3.1 - 3.4, if $\rho_0 > 0.5 - \tau_2$, there exists a finite random variable $R > 0$, such that*

$$\mathbb{P}\left(\sup_{i\in\mathbb{N}}\|X(t)\| \leq R\right) = 1.$$

Proof. Due to page limitation, the proof is omitted. □

To study the convergence of estimates in (5), first we introduce a centralized estimator with strong consistency, i.e., the estimate sequence converges to the true parameter almost surely. Then, we prove the estimates of (5) can reach consensus, and the consensus value can asymptotically converge to the estimates of the centralized estimator. Thus, the strong consistency of estimates in (5) can be proved.

Definition 3.1. (Centralized Linear Estimator) A centralized linear estimator $\{x_c(t)\}$ has the following form

$$x_c(t+1) = x_c(t) + \frac{\alpha_c(t)}{N}\sum_{i=1}^{N} H_i^T\left(y_i(t) - H_i x_c(t)\right), \tag{16}$$

where $\alpha(t) = \frac{a_c}{(t+1)^{\tau_c}}$ for some $a_c > 0$ and $\tau_c \geq 0$.

Lemma 3.5. *[2] For the centralized linear estimator given in Definition 3.1, we have the following results*

1) The estimate sequence $\{x_c(t)\}$ is of strong consistency with respect to θ, i.e.,

$$\mathbb{P}\left(\lim_{t\to\infty} x_c(t) = \theta\right) = 1. \tag{17}$$

2) Let $\alpha_c(t) = \frac{a_c}{(t+1)}$ with $a_c > \frac{N}{2\lambda_{min}(G)}$. Then the sequence $\{\sqrt{t+1}(x_c(t) - \theta)\}$ is asymptotically normal, i.e.,

$$\sqrt{t+1}(x_c(t) - \theta) \Rightarrow \mathcal{N}(0, S_c),$$

where

$$\begin{aligned}S_c &= \frac{a_c^2}{N^2}\int_0^\infty e^{\Sigma_1 v} S_1 e^{\Sigma_1^T v} dv \\ \Sigma_1 &= -\frac{a_c}{N}G + \frac{1}{2}I_M \\ S_1 &= (\boldsymbol{1} \otimes I_M)^T \bar{D}_H R_v \bar{D}_H^T(\boldsymbol{1} \otimes I_M).\end{aligned}$$

Define $x_{avg}(t) = \frac{1}{N}(\mathbf{1}_N \otimes I_M)^T X(t)$. In the following lemma, we provide conditions such that the estimates of agents reach consensus.

Lemma 3.6. *Let Assumptions 3.1 - 3.4 hold. Then, for any*

$$0 \leq \tau_0 < \min\{\rho_0, \tau_1 - \tau_2 - \frac{1}{2+\epsilon_1}\},$$

we have

$$\mathbb{P}\left(\lim_{t\to\infty}(t+1)^{\tau_0}\|x_i(t) - x_{avg}(t)\| = 0\right) = 1, \forall i \in \mathcal{V}.$$

Proof. Due to page limitation, the proof is omitted. □

Next, we show that the consensus value, i.e., the average estimates, will converge to the estimates of the centralized estimator in (16).

Lemma 3.7. *Let Assumptions 3.1 - 3.4 hold. Suppose that* $\{x_c(t)\}$ *is the centralized estimates given in Definition 3.1 with* $\tau_c = \tau_1$, $a_c = a$. *If* $\rho_0 > \tau_1 - \tau_2$, *then for any*

$$0 \leq \tau_0 < \min\{\tau_1 - \tau_2 - \frac{1}{2+\epsilon_1}, \rho_0 + \tau_2 - \tau_1\},$$

we have

$$\mathbb{P}\left(\lim_{t\to\infty}(t+1)^{\tau_0}\|x_{avg}(t) - x_c(t)\| = 0\right) = 1.$$

Proof. Due to page limitation, the proof is omitted. □

The strong consistency of the estimator (5) is provided in the following theorem.

Theorem 3.2. *(Strong Consistency) Consider the algorithm (5). Let Assumptions 3.1 - 3.4 hold. If* $\rho_0 > \tau_1 - \tau_2$, *the estimate sequence* $\{x_i(t)\}$ *is of strong consistency with respect to* θ, *i.e.,*

$$\mathbb{P}\left(\lim_{t\to\infty} x_i(t) = \theta\right) = 1, \forall i \in \mathcal{V}. \tag{18}$$

Proof. According to Lemmas 3.5 - 3.7, taking $\tau_0 = 0$, the conclusion holds. □

In the next theorem, we provide the convergence speed that the estimates by (5) converge to the estimates of centralized estimator in (16).

Theorem 3.3. *(Centralized Approximation) Let the algorithm (5) share the same parameter setting as the centralized estimator* $\{x_c(t)\}$ *in that* $\tau_1 = \tau_c$. *Assume that Assumptions 3.1 - 3.4 hold, and if* $\tau_1 = 1$, *further suppose*

$$a > \frac{N\tau_0}{\lambda_{min}(G)}. \tag{19}$$

Then, if $\rho_0 > \tau_1 - \tau_2$, *for each* τ_0 *subject to*

$$0 \leq \tau_0 < \min\{\tau_1 - \tau_2 - \frac{1}{2+\epsilon_1}, \rho_0 + \tau_2 - \tau_1\},$$

we have

$$\mathbb{P}\left(\lim_{t\to\infty}(t+1)^{\tau_0}\|x_i(t) - x_c(t)\| = 0\right) = 1, \forall i \in \mathcal{V}. \tag{20}$$

Proof. According to Lemma 3.6, Lemma 3.7, the conclusion holds. □

Communication frequency is essential to the research of event-triggered distributed estimation. In the following theorem, the triggering interval of the defined event in (3) is investigated in the sense of infinite time.

Theorem 3.4. *(Triggering Interval) Let* t_k^i *be kth triggering instant of agent* i, *Assumptions 3.1 - 3.4 hold and agents share the same threshold, i.e.,* $\rho_i = \rho_0$. *If*

$$\rho_0 < \tau_1 - \frac{1}{2+\epsilon_1}, \tag{21}$$

then for each agent, the time interval between two successive triggering instants goes to infinity, i.e.,

$$\mathbb{P}\left(\lim_{k\to\infty}(t_{k+1}^i - t_k^i) = \infty\right) = 1, \forall i \in \mathcal{V}. \tag{22}$$

Proof. See the proof in the extended version [18]. □

4 Numerical Simulation

In this section, we provide a numerical simulation to testify the effectiveness of the distributed estimator based on event-triggered communication scheme proposed in this paper.

Consider an undirected network with four agents. The adjacency matrix of the network is $\mathcal{A} = \begin{pmatrix} 0&1&0&1\\ 1&0&1&0\\ 0&1&0&1\\ 1&0&1&0 \end{pmatrix}$. The true parameter vector is supposed to be $\theta = [-1, 2]^T$. The observation matrices and the initial parameter estimates of these agents have the following forms

$$H_1 = [1,0]^T, H_2 = [0,1]^T$$
$$H_3 = [1,1]^T, H_4 = [1,2]^T$$
$$x_1(0) = [10,20]^T, x_2(0) = [10,-10]^T$$
$$x_3(0) = [10,-20]^T, x_2(0) = [20,-10]^T.$$

We consider the time sequence $t = 0, 1, \ldots, 10000$. Let $\alpha(t) = \frac{1}{(t+1)^{0.7}}$, $\beta(t) = \frac{1}{(t+1)^{0.5}}$ and $\rho_i(t) = \frac{1}{(t+1)^{0.6}}$, for $i = 1, 2, 3, 4$. The noises of each agent are supposed to be i.i.d and Gaussian. The noises of agents are spatially independent. The distribution of measurement noises is zero-mean with variance 0.01.

Under the above setting, by employing the distributed estimator (5) with triggering scheme (4) and the centralized estimator (16), we obtain simulation results in Fig. 1, Fig. 2 and Fig. 3. We see from Fig. 1 that the sequence of average estimates is asymptotically convergent to the true parameter vector of the system. By Fig. 2, the consistency of the estimator for each agent is shown. Besides, we see that the centralized estimator has faster convergence speed, since it utilizes all measurements. The triggering time instants satisfying the triggering scheme (4) during the whole estimation process is plotted in Fig. 3 with communication rate[1] 1.175%. Thus, the communication frequency of the agents has been tremendously reduced with guaranteed convergence properties.

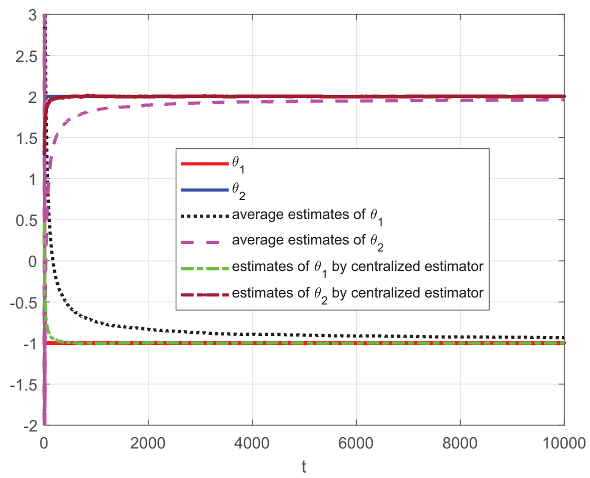

Fig. 1: Parameter estimates of agents (average) and data center (centralized)

[1]Communication rate is the ratio of whole triggering time instants over the whole time-driven communication time instants

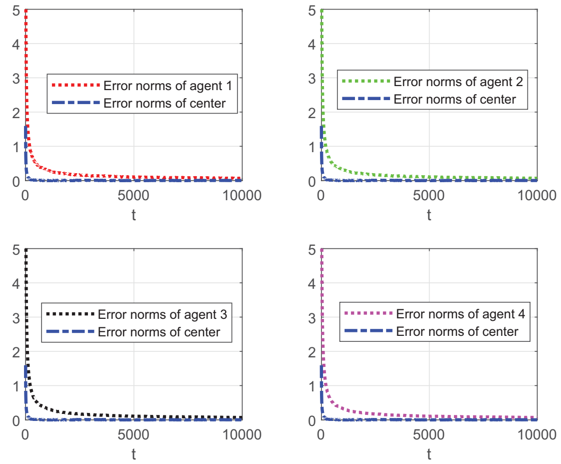

Fig. 2: Estimation error norms of agents and data center

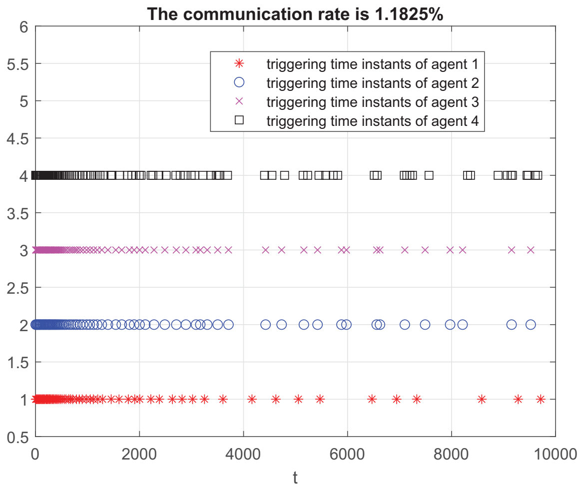

Fig. 3: Triggering time instants and communication rate

5 Conclusion

In this paper, a distributed parameter estimation problem with intermittent communications was studied. First, we proposed an event-triggered communication scheme for each agent, by comparing a decaying threshold with the difference between the current estimate and the latest one sent out to neighboring agents. Then, we analyze some main estimation properties including asymptotic unbiasedness and strong consistency. We also showed that, with probability one, for every agent the time interval between two successive triggering instants goes to infinity as time goes to infinity.

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