Encoder–Decoder Design for Event-Triggered Feedback Control over Bandlimited Channels

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***Abstract*— Bandwidth limitations and energy constraints set severe restrictions on the design of control systems that utilize wireless sensor and actuator networks. It is common in these systems that a sensor node needs not be continuously monitored, but communicates to the controller only at certain instances when it detects a disturbance event. In this paper, such a scenario is studied and particular emphasis is on efficient utilization of the shared communication resources. Encoder–decoder design for an event-based control system with the plant affected by pulse disturbances is considered. A new iterative procedure is proposed which can jointly optimize encoder–decoder pairs for a certainty equivalent controller. The goal is to minimize a design criterion, in particular, a linear quadratic cost over a finite horizon. The algorithm leads to a feasible design of time-varying non-uniform encoder–decoder pairs. Numerical results demonstrate significant improvements in performance compared to a system using uniform quantization.**

I. INTRODUCTION

The demands for sharing resources efficiently in large networked systems are continuously increasing. In many situations, there is a challenging conflict between the amount of transmitted data and the response time. In particular for emerging distributed control applications, limits imposed on the available signaling bandwidth from communication channels can severely restrict closed-loop performance and even destabilize the system. Networked control based on limited sensor and actuator information has therefore been a research topic that has attracted attention during the past decade. There are two main research directions: one is concerned with the minimum communication rate required to fulfill certain criteria, e.g., [2], [13], [15]. The other concern system performance for particular classes of encoder–decoders, e.g., [5], [7], [8], [10], [16]. Most of this literature is on extensions of the traditional sampled feedback control loop. For control systems utilizing wireless sensors and actuator networks, it seems reasonable to consider asynchronous control instead. This approach appears to have been received little attention in the literature on control with limited data rate.

Event-triggered control strategies have great potential in many cases to be more efficient than conventional time-triggered (or sampled) control [1]. To optimally utilize the communication resources, it is desirable to let each control loop communicate less frequently. How this should be done in general is largely unexplored and only preliminary results are available, e.g., [1], [14]. In this paper, we propose a new control strategy combining the approaches from event-triggered and quantized control. It is shown that for an interesting class of systems, which are affected by rarely occurring disturbances drawn from a known probability distribution, it is possible to achieve a good control performance with limited control attention and sensor communication. The focus here is on how to encode sensor data efficiently. Related problems on encoding control commands for motion control have been studied in [4], [6], [9].

The main contribution of the paper is a practical synthesis technique for jointly optimizing the encoder and the decoder subject to a given probability density function (pdf) of the plant disturbance. As illustrated in a motivating example, this is an important problem in event-based control that can be used when a large set of wireless sensor nodes need to limit their individual access to the communication medium. Previous work on control with limited communication has mainly focused on uniform quantizers, while our main contribution is to introduce the use of optimized (generally non-uniform) encoders. In a standard approach, the encoder and decoder are initialized by the current realization of the initial observation. As the system evolves, the encoder–decoder pair are adjusted according to the available measurements and the common agreements between the encoder and the decoder. A reason for choosing a uniform quantizer is that it is convenient to implement. However, for applications with high communication cost (like sensor networks), it is natural to study optimized encoder–decoder pairs that can provide a more efficient use of the limited communication resources.

The paper is organized as follows. A motivating example based on a wireless sensor networks is presented in Sec. II. Sec. III defines the control system with encoder, decoder, and communication channel. The problem statement, which concerns a linear quadratic (LQ) objective over finite horizon, is presented in Sec. IV. The joint encoder–decoder design and a proposed training procedure are described in Sec. V. Sec. VI presents numerical results and discusses some implementation issues. Finally, the conclusions are given in Sec. VII.

II. WIRELESS SENSOR NETWORK EXAMPLE

Consider the wireless networked control system in Fig. 1. It consists of a large number of sensor nodes that can connect through a shared wireless medium to a control node. The control command is executed through a common actuator. Suppose that the sensors are spatially distributed over a large

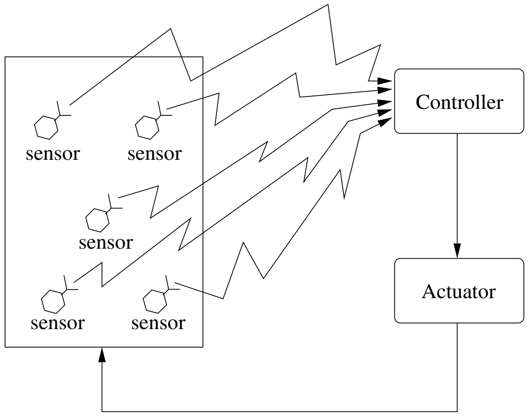


Fig. 1. Control system which utilizes data from wireless sensor network. The results in this paper can be applied to optimize the use of the wireless medium.

area and that they measure the states of a field. The field is affected by local disturbances, which at each time instance are affecting at most one of the sensors. The actuator is supposed to be able to counteract the disturbance by a suitable action determined by the controller. In order to efficiently utilize the communication resources, it is desirable that at each disturbance event the corresponding sensor transmits only a few symbols and that each symbol consists of a few bits. Under the assumption that an estimate of the probability density function of the magnitude of the disturbance at each sensor location is known, we can optimize the encoder–decoder pair for each sensor transmission together with the control law. A reasonable control objective when doing this is to keep the state measured by each sensor close to an equilibrium by using a small amount of control actuation.

The resulting decentralized control strategy is as follows. Let the control command corresponding to a specific sensor be zero as long as the corresponding state is close to zero. When the sensor detects that the state is outside an ε-interval around the origin, the sensor reading is encoded and transmitted to the controller node. The message is decoded at the controller node and a control command is derived and actuated to counteract the specific disturbance acting on the transmitting sensor. The controller and the encoder–decoder pair are designed based on a finite-horizon linear quadratic criterion. If disturbance events are rare, the design is mainly dependent on the distribution of the magnitude of the disturbance. A partial solution is therefore to solve the a linear quadratic (LQ) problem with uncertainty only in the initial condition. The plant dynamics will however propagate the influences of consecutive disturbances. A class of time-varying non-uniform encoder–decoders are derived in the paper, which together with a simple state feedback control is suitable to handle the situation.

III. PRELIMINARIES

Consider a control system with a communication channel depicted in Fig. 2. For completeness we describe in this section a general version of the system; while we will later focus on a special case. Let $x_a^b = \{x_a, \ldots, x_b\}$ denote the

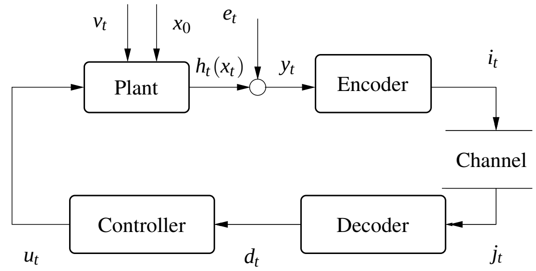


Fig. 2. Feedback control system with channel constraints

evolution of a discrete-time signal x_t from $t = a$ to $t = b$. The plant is given as the scalar system

$$\begin{aligned} x_{t+1} &= \theta_t(x_{t-M_x}^t, u_t) + v_{t+1} \\ y_t &= h_t(x_t) + e_t, \end{aligned} \tag{1}$$

with memory of order M_x. The variables x_t, u_t and y_t, in $\mathbb{R}$, are the state, the control, and the measurement, respectively. The process disturbance and the measurement noise are represented by v_t and e_t.

Let $\mathscr{E}_t$ denote the set of "information" available at the encoder at time t, i.e., the set of variables whose values are known to the encoder. In particular, we consider an encoder that *causally* utilizes *full information*, in the sense $\mathscr{E}_t = \{y_0^t, i_0^{t-1}, u_0^{t-1}\}$, where i_t is defined later on. The *encoder* is a mapping from $\mathscr{E}_t$ to a set of integers, $\mathscr{I}_L = \{0, 1, \cdots, L-1\}$, where $L = 2^R$ with R denoting the *rate* of the transmission, in bits per channel use. Formally, the encoder is described by the mapping

$$i_t = f_t(y_0^t, i_0^{t-1}, u_0^{t-1}). \tag{2}$$

The symbol $i_t \in \mathscr{I}_L$ is transmitted over the channel. At time t, the *discrete channel* has input variable i_t and output j_t, with one channel use defined by

$$j_t = k_t(i_{t-M_c}^t), \tag{3}$$

where $k_t : \mathscr{I}_L^{M_c+1} \to \mathscr{I}_L$ is a random mapping, and $M_c \geq 0$ indicates (potential) channel memory.

At the receiver, we denote the information available at the decoder by $\mathscr{D}_t$; in particular, we consider a decoder that causally utilizes full information, i.e., $\mathscr{D}_t = \{j_0^t, u_0^{t-1}\}$. The *decoder* is a deterministic mapping

$$d_t = g_t(j_0^t, u_0^{t-1}), \tag{4}$$

from $\mathscr{D}_t$ to $\mathbb{R}$.

Also, let $\mathscr{C}_t$, in particular $\mathscr{C}_t = \{d_0^t, u_0^{t-1}\}$, denote the (full) information available at the controller. The *controller* is defined by a mapping

$$u_t = z_t(d_0^t, u_0^{t-1}), \tag{5}$$

from $\mathscr{C}_t$ to $\mathbb{R}$. We also define

$$\hat{x}_{s|t} = E\{x_s | j_0^t, u_0^{t-1}\} \tag{6}$$

to be the minimum mean square error estimator of the state x_s, for $s \leq t$, based on j_0^t and u_0^{t-1}.

Finally, we comment that even if all variables are not available at all nodes in the system, we assume that the *functionalities* of the encoder, decoder, controller and state-estimator are known at all nodes.

IV. Problem Statement

Let us focus on a special instance of the system in Fig. 2, namely, a stable scalar linear time-invariant plant

$$x_{t+1} = a x_t + u_t + v_{t+1}, \qquad y_t = x_t, \qquad |a| < 1, \quad (7)$$

and assuming a memoryless noise-free channel, $j_t = i_t$. The disturbance occurs (i.e., v_t is non-zero) at random instances of time. At these time-instants, suppose v_t is drawn from a zero-mean distribution with pdf p_v. Let $t = 0$ denote the occurrence of the first disturbance v_0, and assuming $u_t = x_t = 0,\ \forall\ t < 0$, it is clear that $x_0 = v_0$, drawn according to p_v.

A. Performance Measure

The goal of this paper is to solve an integrated encoder–decoder design and optimal control problem for the system in (7), with a performance measure of the form

$$J = \lim_{K\to\infty} \frac{1}{K} \sum_{t=0}^{K-1} E\{x_t^2 + \rho u_t^2\}, \quad (8)$$

where $\rho \geq 0$ specifies the "weight" assigned to the cost (contribution to J) of closed-loop control; $u_t > 0$. Let $t_i,\ i = 0,1,2,\ldots$, denote the time instance for the occurrence of the ith disturbance, assuming $t_0 = 0$ (corresponding to the initial state $x_0 = v_0$), $t_i > 0,\ \forall\ i > 0$, and $t_i \neq t_j,\ \forall\ i \neq j$. Then we can write

$$J = \lim_{K\to\infty} \frac{1}{K} \sum_{i=0}^{K-1} \left(\frac{1}{t_{i+1} - t_i} \sum_{t=t_i}^{t_{i+1}-1} E\{x_t^2 + \rho u_t^2\} \right) \quad (9)$$

conditioned on a fixed $\{t_i\}$.

Suppose now that a substantial part of the contribution of a disturbance to the performance measure is concentrated to a (small) time-interval of length T time-instants following each disturbance, i.e., the contribution in the interval $t_i + T \leq t < t_{i+1}$ is small compared to the one in $t_i \leq t \leq t_i + T - 1$. This is a reasonable assumption if the feedback control succeeds in attenuating the disturbance within time T, and the intensity of the disturbance is low enough such that, with high probability, there is only one disturbance per interval. Then, J can be approximated by

$$\lim_{K\to\infty} \frac{1}{KT} \sum_{i=0}^{K-1} \left(\sum_{t=t_i}^{t_i+T-1} E\{x_t^2 + \rho u_t^2\} \right). \quad (10)$$

In (10), and from now on, we assume that the plant is not controlled in the interval $t_i + T \leq t < t_{i+1}$, i.e., $u_t = 0$ for these time-instants. We also explicitly assume that $t_{i+1} - t_i > T$.

In (10) the expectation is (implicitly) taken with respect to the pdf p_v, while the random occurrences $\{t_i\}$ are averaged out by the time-average. Alternatively, by treating each disturbance separately and shift time to zero ("re-starting the clock") at the occurrence of each disturbance, we end up with the following criterion

$$J_T = \sum_{t=0}^{T} E\{x_t^2 + \rho u_t^2\}, \quad (11)$$

with the constraint $u_T = 0$.

In (11) the expectation is to be interpreted as averaging over both the values and time-instants of the disturbances, in the sense that the expectation E is taken with respect to a new pdf p_0 for x_0 defined as

$$p_0 = \lim_{K\to\infty} \frac{1}{K} \sum_{i=0}^{K-1} p_i, \quad (12)$$

where p_i is the pdf of x_{t_i}. As per our discussion, it should hold that $J_T \approx J$. In the remaining parts, any reference to optimality will refer to J_T and the implicit averaging over all the different disturbances in the sense of p_0.

B. Disturbance Detector

A detector is employed to mark instances when a disturbance event has taken place. Since no measurement error is present, estimates of the system evolution are readily calculated without error. Hence, a disturbance can be perfectly detected by comparing a predicted value for x_t with the observed value. Even though we assume that there is a perfect disturbance detection mechanism, the quantization effect makes it non-trivial to decide when to act. For example, when the bit rate is low, it may be better to let the disturbance die out by the plant's own stabilizing dynamics, than to apply a control with relatively large estimation errors. We therefore introduce a threshold $\varepsilon > 0$. The control action is triggered only when the magnitude of the observed state is larger than the threshold. The threshold is obviously an additional design parameter. It will be discussed in detail in Sec. V.

C. Encoder–Decoder Structure

In the case of a noise-free connection between the controller and the plant, the encoder is able to predict the future evolution of the state. Encoder–decoder design is therefore essentially equivalent to the problem of constructing an optimal strategy for the encoder to successively inform the decoder about the initial state, x_0, over a discrete noiseless channel. Assuming full information available at the encoder, we notice that $\mathscr{E}_t = \{x_0^t, i_0^{t-1}, u_0^{t-1}\}$ is equivalent to $\mathscr{E}_t = \{x_0, i_0^{t-1}\}$. This since the encoder knows u_0^{t-1} based on i_0^{t-1} (remember, $j_t = i_t$), and since x_1^t can be computed from (x_0, u_0^{t-1}). Hence we assume, from now on, that i_t is generated as

$$i_t = f_t(x_0, i_0^{t-1}), \quad t = 0,\ldots,T-1, \quad (13)$$

based on x_0 and the known previous symbols i_0^{t-1}.

At the receiver, full information is equivalent to $\mathscr{D}_t = \{i_0^t\}$, since $j_t = i_t$ and u_0^t can be computed from i_0^t, so in the following the assumed decoder mapping will be

$$d_t = g_t(i_0^t), \quad t = 0,\ldots,T-1. \quad (14)$$

Similarly, the optimal state estimator, based on $\mathscr{D}_t$, is

$$\hat{x}_{s|t} = E\{x_s | i_0^t\}, \quad t = 0,\ldots,T-1,\ s \leq t. \quad (15)$$

As explained in Sec. V-A, we will specialize the decoder mapping to $d_t = \hat{x}_{t|t}$, so $\hat{x}_{t|t}$ will be utilized by the controller to compute the control output u_t.

V. System Design

This section presents the main results of the paper. After discussing certainty equivalence and encoder–decoder training, we present the overall design algorithm.

A. Certainty Equivalence

In general, optimal performance is achievable only when the encoder–decoder pair and controller are designed jointly. There are examples, however, when the *separation principle* applies. More precisely, the so-called *certainty equivalent* controller remains optimal when the estimation errors are independent of old control commands [3].

To apply the certainty equivalence principle to our problem, we consider the design of optimal sequences of decoders $\{g_t\}$ and controllers $\{z_t\}$, for a fixed sequence of encoders $\{f_t\}$ with $f_t = f_t(x_0, i_0^{t-1})$. We notice that the sufficient condition in [3] corresponds to the requirement that the estimation error $E[(x_0 - \hat{x}_{0|t})^2]$, $\quad t = 0,\ldots,T-1$, must not be a function of u_0^s, for $s = 0,\ldots,t-1$, for any sequence u_0^s of controls. It is straightforward to verify that this condition holds true in our case, since $i_0 = i_0(x_0)$, $i_1 = i_1(x_0, i_0), \ldots, i_t = i_t(x_0, i_0, \ldots, i_{t-1})$. Hence the values taken on by i_0^t, given a fixed set of encoders, depend only on x_0 and not on u_0^s for any $s < t$. Thus, certainty equivalence holds under our assumptions. Consequently, based on a similar argumentation as in [3], the optimal decoder and controller sequences can be shown to be

$$d_t = \hat{x}_{t|t}, \quad \text{and} \quad u_t^* = -\ell_t d_t \qquad (16)$$

(i.e., $z_t(d_0^t) = -\ell_t\, d_t$) with

$$\ell_t = \frac{a\, p_{t+1}}{p_{t+1} + \rho}, \quad p_t = 1 + \frac{a^2 p_{t+1} \rho}{p_{t+1} + \rho}, \qquad (17)$$

for $t = T-1,\ldots,0$ and p_t is initialized with $p_T = 1$. Notice that since certainty equivalence holds, there is no loss in separating the decoder–controller into two separate entities (as done in Fig. 1).

B. Iterative Design Algorithms

Here we propose a framework for designing the encoder, decoder and controller mappings. Similar to traditional iterative algorithms for optimal quantizer design [12], the basic idea is to search for locally optimal encoder–decoder pairs by alternating between updating the decoders for fixed encoders and vice versa, until convergence. Since, for any given set $\{f_t\}_{t=0}^{T-1}$ of encoder mappings, with $f_t = f_t(x_0, i_0^{t-1})$, the optimal decoder and controller mappings are given by (16)–(17), the main issue we need to resolve is the structure of the *optimal encoder mappings*, given $\{g_t\}$ and $\{z_t\}$.

Given the sets of decoder and controller mappings, and assuming in addition that f_t, $t = 0,\ldots,T-2$, are fixed and known, it is straightforward to realize that the optimal $f_{T-1} = f_{T-1}(x_0, i_0^{T-2})$ is described by

$$i_{T-1} = \arg\min_{i \in \mathscr{I}_L} E\left[x_T^2 + \rho u_{T-1}^2 \middle| x_0, i_0^{T-2}, i_{T-1} = i\right]. \qquad (18)$$

Here we note that $x_1,\ldots,x_{T-1}$ and $u_0,\ldots,u_{T-2}$ are known deterministically conditioned on x_0 and i_0^{T-2}, given f_t for $t = 0,\ldots,T-2$ and since the decoder and controller mappings are fixed. Hence, testing different values for i_{T-1} influences only u_{T-1} and $x_T = a x_{T-1} + u_{T-1}$. For $t < T-1$ we need to take the impact of i_t on all future terms into consideration. Hence, for given decoders and controllers, and given the encoder mappings f_t for $t = 0,\ldots,t-1$ and $t+1,\ldots,T-1$, we get the optimal $f_t = f_t(x_0, i_0^{t-1})$ as

$$i_t = \arg\min_{i \in \mathscr{I}_L} E\left[\sum_{k=t}^{T} x_k^2 + \rho u_k^2 \middle| x_0, i_0^{t-1}, i_t = i\right]. \qquad (19)$$

Note that the optimal mapping f_t indeed has the form $i_t = f_t(x_0, i_0^{t-1})$. Although straightforward in principle, the expression in (19) is to our knowledge new.

Based on (16), (17) and (19), we can formulate an encoder–decoder design algorithm:

Encoder–Decoder Design (EDD)

Initialize the encoder and decoder mappings $\{f_t\}$ and $\{g_t\}$. Compute the controller parameters $\{\ell_t\}$ using (17).

1) For each $t = 0,\ldots,T-1$,
   - Update the encoder mapping f_t using (19).
   - Update the decoder mapping d_t using (16).
   - Set $u_t = -\ell_t d_t$.
2) If J_T has not converged, go to 1), otherwise stop.

The algorithm requires that the pdf p_0 is known or described by a training set [12]. Convergence is monitored based on updating the value of J_T in each step. Unfortunately, as mentioned, the above design cannot guarantee global optimality. Still, the algorithm converges to a local minimum, and in this sense produces a "good" solution.

A crucial step in the design is to specify the initial encoder–decoders. Note that given a pdf p_0 for x_0, the values of i_0^{t-1}, and the corresponding u_0^{t-1}, the pdf's $p(x_t|i_0^{t-1})$, $t < T$, can be derived (estimated from a training set). A natural choice to initialize the EDD algorithm above is to use the encoder–decoder of scalar Lloyd–Max quantizers [12] designed for these pdf's. Starting with designing f_0 for p_0, and using the resulting reconstruction points as initial estimates for $\hat{x}_{0|0}$ as well as ℓ_0 from (17), the conditional pdf's $p(x_1|i_0)$, $i_0 \in \mathscr{I}_L$, can be determined. Base on these, L different Lloyd–Max quantizers can be trained to determine $f_1(x_0, i_0)$, and so on. Alternatively, the encoder–decoders can be initialized as properly scaled uniform quantizers [12].

As discussed in Sec. IV, an important parameter with respect to overall performance is the threshold ε. Above, we have discussed the optimal coding and control problem assuming a fixed threshold. The choice of threshold is however taken into account in the overall design algorithm summarized below.

System Design (SD)

1) Initiate the pdf p_0 based on the pdf of the disturbance. Initialize the encoder–decoder $\{f_t\}$ and $\{g_t\}$ using Lloyd–Max or uniform quantizers, as described. Compute $\{\ell_t\}$.

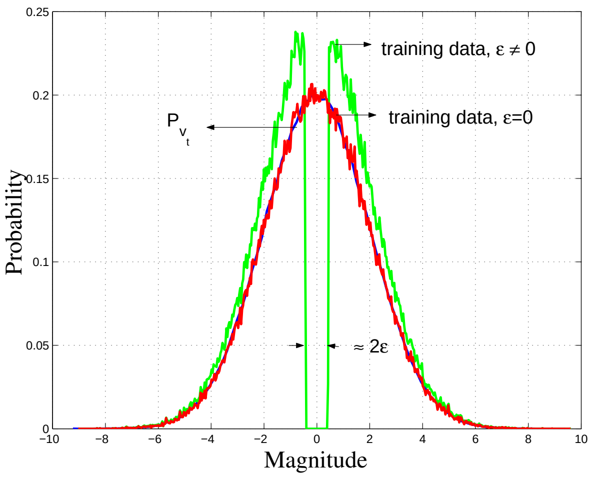


Fig. 3. GGD distribution ('P_v'), empirical distributions for $\varepsilon = 0$ and $\varepsilon \neq 0$.

2) Set $\varepsilon = 0$. Design encoders, decoders and controllers using the EDD algorithm.
3) Use the resulting encoder–decoder pair and simulate the disturbance sequence according to the known statistical properties of $\{v_t\}$. Find the value $\bar{\varepsilon}$ for the threshold ε that minimizes J_T.
4) WHILE J_T has not converged
   a) Apply the trained encoder–decoder and $\bar{\varepsilon}$ to the disturbed control system. Collect a training set that describes p_0.
   b) Train the encoder-decoder using the EDD algorithm.
5) For the final encoder–decoder, find the threshold that minimizes J_T.

In Step 4b) the encoder–decoder pair from the prior iteration is used to initialize the training. Note that selecting a threshold is performed once in Step 3), and for the second time in Step 5) to set a final value for ε.

VI. NUMERICAL RESULTS

We investigate a numerical example with $a = 0.8$, rate $R = 2$ and $T = 3$. The disturbance events $\{t_i\}$ are modeled as a Poisson process, and the pdf p_v of the disturbance magnitude is modeled as a generalized Gaussian distribution (GGD), e.g., [11]. The GGD(α, β) distribution provides a wide coverage from narrow-tailed to broad-tailed pdf's; α describes the exponential rate of decay, and β is the standard deviation. We use $\beta = 2$ throughout.

Fig. 3 illustrates how the distribution of the training data changes when a threshold $\varepsilon \neq 0$ is introduced.

To demonstrate the performance of the proposed iterative algorithm, we compare system performances among several encoder–decoder pairs. As described previously, the iterative algorithm requires a set of initial encoder–decoders. Three initialization methods are studied. The first initialization, referred to as FixU, employs a fixed optimum step-size uniform quantizer for the entire period $0 \leq t \leq T-1$. The second method, referred to as VariU, employs time-varying uniform quantizers with optimized step size. The last initialization, referred to as (LM), applies Llyod-Max quantizers [12], designed for each pdf $p(x_0|i_0^{t-1})$ (Sec. V-B).

Fig. 4 shows the performance improvements after training. The system parameters are still $a = 0.8$, $R = 2$ and $T = 3$. p_v is $GGD(\alpha, 2)$, where α is varied. The performance measure V is defined by

$$V = \frac{E\{\sum_0^T (x_t^2 + \rho u_t^2)\} - E\{\sum_0^T (v_t^2)\}}{E\{\sum_0^T (v_t^2)\}}, \qquad (20)$$

such that the cost function in (11) is normalized by the energy of the disturbance $E\{\sum_{t=0}^T (v_t^2)\}$. Note that a small V indicates an effective control strategy. We observe that the new design algorithm always results in an improvement over the initial encoder–decoder pairs. In addition, all three initializations converge to quite similar final results. However, for disturbances with a peaked distribution, e.g., GGD$(0.5, 2)$, the performance after training is significantly improved. In this case, the optimal quantization should obviously not be uniform. Recalling that the parameter α describes the exponential rate of decay. The pdf is closer to a uniform distribution when α is large.

In Fig. 5, we have investigated the impact of the parameter ρ on the system behavior. Recall that ρ is the penalty on the control signal in the performance measure. The picture illustrates the state evolution and the corresponding control signals over 100 time samples. Note that when ρ is large, the optimal control signal has small magnitude, so consequently the response times to the pulse disturbances are longer. The figure shows also that, due to the effect of coding, the control signal can only be chosen among a few different values.

VII. CONCLUSIONS

Motivated by demands from networked control systems utilizing patches of wireless sensors, an encoder–decoder design problem was proposed and solved. We introduced an approach to jointly optimize an encoder–decoder pair for a closed-loop control system with linear plant and sensor data being communicated over a low-rate channel. The plant was assumed to be affected by rare pulse disturbances. Having argued that the certainty equivalence condition is fulfilled, we fixed the receiver structure to implement separate decoding (estimation) and control. Our main contribution was the introduction of optimized encoders for the given structure on the estimation–control.

We performed various numerical investigations of the proposed system. In particular we have demonstrated that our design is able to outperform a scheme based on fixed or adaptive uniform quantizers.

Even though our results were presented for a scalar system and noiseless transmission, the overall problem was formulated under quite general assumptions, allowing for extensions, e.g., to multiple dimensions and noisy channels. We consider this paper as an important first step toward future work on practical designs in joint control and source–channel coding.

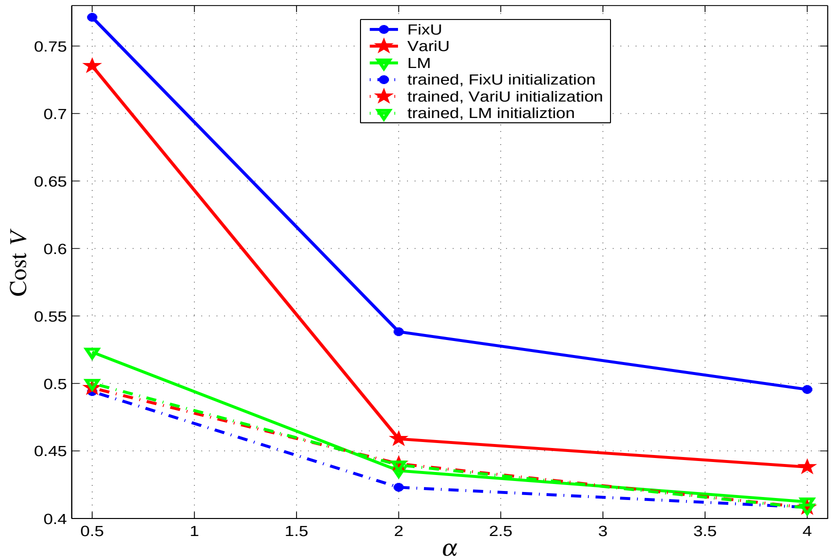
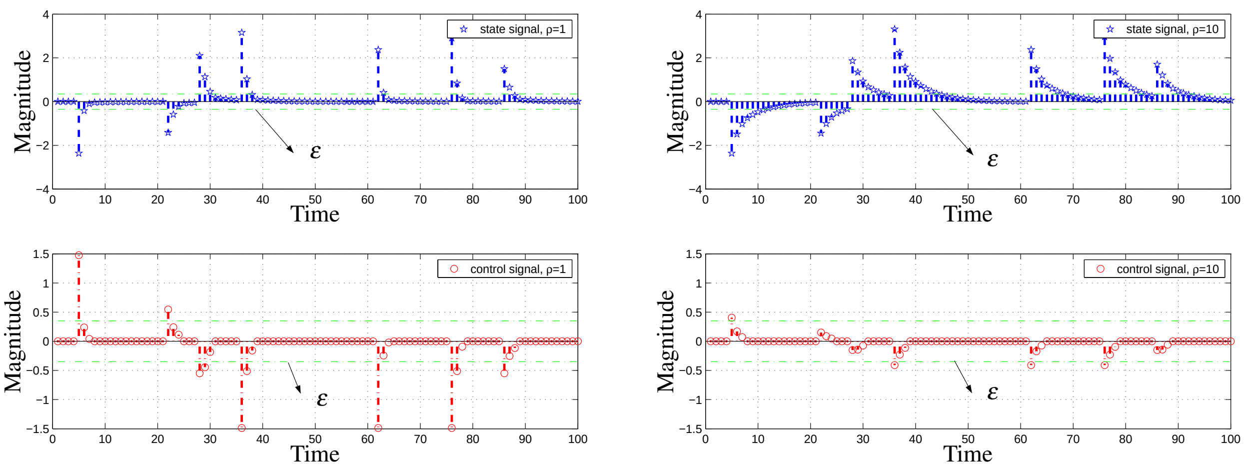


Fig. 4. The improvement of the system performance by applying the proposed encoder–decoder. FixU: time-invariant uniform quantizer. VariU: time-varying uniform quantizer. LM: Lloyd–Max quantizer.



(a) $\rho = 1$ (b) $\rho = 10$

Fig. 5. Optimized encoder–decoder and controls. System behavior for $\rho = 1$ and 10, respectively, and event threshold $\varepsilon = 0.35$.

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