

Regularized Deconvolution-based Approaches for Estimating Room Occupancies

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Abstract—We address the problem of estimating the number of people in a room using information available in standard HVAC systems. We propose an estimation scheme based on two phases. In the first phase, we assume the availability of pilot data and identify a model for the dynamic relations occurring between occupancy levels, CO₂ concentration levels and room temperature signals. In the second phase, we make use of the identified model to formulate the occupancy estimation as a deconvolution problem. In particular, we aim at obtaining an estimated occupancy pattern by trading off between adherence to the current measurements and regularity of the pattern. To achieve this goal we employ a special instance of the so-called *fused lasso* estimator, which promotes piecewise constant estimates by including an ℓ_1 norm-dependent term in the associated cost function. We extend the proposed estimator to include different sources of information, namely the actuation level of the ventilation system and door opening / closing events. We also provide conditions under which the occupancy estimator provides correct estimates within a guaranteed probability. We test the estimator running experiments on a real testbed, in order to both compare it with a few occupancy estimation techniques and assess the value of having additional information sources.

Index Terms—Occupancy estimation, System Identification, Deconvolution, Regularization

Note to Practitioners—Home automation systems benefit from automatic recognition of human presence in built environments. Since dedicated hardware is costly, it may be preferable to have software-based occupancy detection systems which do not require the installation of additional devices. The object of this study is the reconstruction of occupancy patterns in a room using measurements of CO₂ concentration, temperature and fresh air inflow. Additionally, door opening / closing events are considered. All these signals are information sources often available in HVAC systems of modern buildings. We assess the value of such information sources in terms of their relevance in detecting the room occupancy. The proposed estimation scheme is composed of two distinct phases. The first is a training phase where the goal is to derive a mathematical model relating the number of occupants with the CO₂ concentration. In the second phase, we use the derived model to design an online software which collects measurements of the environmental signals and provides

the number of people currently in the room. Other applications of our algorithms include the estimation of occupancy flows in buildings.

I. INTRODUCTION

Estimating occupancy levels in rooms is essential for home automation purposes, e.g., to automate the control of lighting, thermostats, security locks, home entertainment systems, and to improve the energetic performance of Heating, Ventilation and Air Conditioning (HVAC) systems [1], [2], [3]. Estimating occupancy is thus a key enabling factor for improving comfort in smart buildings and energy efficiency.

Direct experience indicates that some standard off-the-shelf dedicated hardware for occupancy estimation (such as cameras and Radio-Frequency Identification (RFID) tags) suffer from several problems. First they may be insufficiently accurate for the employment in HVAC control systems. Second, they may induce large additional deployment and maintenance costs. Last, they may have installation feasibility problems in old buildings. Moreover, hardware-based occupancy detectors may trigger privacy concerns.

Consequently, it is interesting to study how and to what extent hardware-based people counters can be replaced by software-based occupancy estimators that only employ available information in standard HVAC systems (such as CO₂ concentration and temperature), which information sources have to be considered, and what type of statistical processing leads to efficient estimators.

The main objective of this paper is to address the above questions by proposing occupancy estimators that provide with information on the number of occupants using commonly available signals, namely measurements of CO₂ concentration and temperature, HVAC actuation levels (i.e., the amount of fresh air injected in a room), and information on door opening / closing events.

Literature review: the proposed strategies for estimating the occupancy levels in rooms and buildings can be categorized into *hardware-based* and *model-based* approaches.

The first category includes methods working with dedicated hardware such as cameras, RFIDs, etc. [4], [5], [6]. As mentioned before, the applicability of these methods are restricted to certain situations due to their potential drawbacks.

In the second category occupancy levels are inferred indirectly using dynamical models that relate environmental signals with occupancy. These models may be obtained by employing data-driven techniques (i.e., *identification-based*

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methods) or by exploiting knowledge of the underlying physical laws (i.e., *physics-based* methods). The latter techniques comprise strategies based on mass balance equations or first principle considerations to derive dynamical models relating the number of occupants, CO₂ concentration, temperature and humidity [7], [8], [9], [10]. Identification-based approaches aim at estimating input-output models from datasets of past measured data. Other successful approaches exploit machine learning techniques such as Support Vector Machines (SVMs), Neural Networks (NNs) and Hidden Markov Models (HMMs) based on CO₂ features (e.g., averages of the signals in time, first / second-order temporal differences) [11], [12].

Statement of contributions: this paper, extension of [13], describes a two-tier software-based occupancy estimation scheme.

The first tier assumes the availability of both environmental signals and true occupancy levels (as pilot data) for a short and well defined period of time. Black box modeling is then used to model the room under consideration, i.e., no other a priori knowledge on the room properties is assumed.

The second tier formulates the occupancy estimation problem as an inverse problem, i.e., it searches for the occupancy pattern that best explains the measured data given the identified model. In this tier we exploit the fact that the occupancy signal is *piecewise constant* and integer, in order to formulate the estimation problem in a *fused-lasso* framework [14].

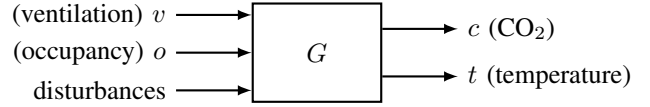
A contribution of the manuscript is to derive different estimators based on the availability of the various information sources. More specifically, we consider the case of adding knowledge of HVAC actuation signals (how much air is injected in the room), and the case of adding a boolean signal accounting for door opening / closing events.

Another contribution is the analysis of the statistical performance of the estimators. We compute bounds on the probability of obtaining incorrect estimates, given the levels of measurement noise, the identified model and the design parameters of the estimators.

Structure of the manuscript: Section II formulates the mathematical problem and the solution methodology. Sections III and IV describe respectively how to identify the model of the room from a training set, and how to exploit this model for estimation purposes. Section V characterizes the performance of the estimator from a statistical perspective. Section VI describes how to modify the original estimation strategy when considering also HVAC actuation levels and information on door opening and closing. Section VII introduces the considered estimation performance indexes, the experimental setup, the results of the estimation processes, and some comparisons with standard tools of Machine Learning. Section VIII then wraps some conclusions, remarks, and ideas for future directions. Proofs are collected in the Appendix.

II. PROBLEM DEFINITION AND METHODOLOGY

We consider the following schematic representation of the dynamics of the concentration of the CO₂ and temperature in a room under well-mixed air assumptions (i.e., these quantities are assumed to be spatially constant).



In the above scheme $c(k)$ represents the concentration of CO₂, $t(k)$ the temperature, $v(k)$ the amount of injected fresh air, $o(k)$ the occupancy, all at time k . G represents an initially unknown dynamic system relating disturbances, events, ventilation and building occupancy levels with temperature and CO₂ concentration signals. In addition, we consider a variable $e(k)$ which is a boolean measurement of door opening and closing events, defined as follows

$$e(k) = \begin{cases} 1 & \text{if the door is open,} \\ 0 & \text{if the door is closed.} \end{cases} \quad (1)$$

The problem we consider in this paper is to find an effective algorithm that transforms measurements of $c(k)$, $c(k-1)$, ... and $t(k)$, $t(k-1)$, ... into estimates of $o(k)$. Our proposal is the following two-tier estimator:

- **Tier 1, training phase:** identify a Linear Time Invariant (LTI) system that captures the dynamics of G from pilot data of $c(k)$, $t(k)$, and $o(k)$, (Section III);
- **Tier 2, test phase:** estimate $o(k)$ from measurements of $c(k)$ and $t(k)$ and the estimated model of the room (Section IV).

The first phase addresses a *system identification* problem, while the second phase addresses a *deconvolution* problem.

A contribution of this paper is the characterization of the proposed estimator in terms of detection error, i.e., probability of obtaining wrong estimates as a function of the parameters of the estimator. We also study extensions of the estimator to include information on venting levels $v(k)$, $v(k-1)$, ..., and door opening / closing events $e(k)$, $e(k-1)$, ... We shall see that, while including venting levels does not change the structure and main properties of the estimator, accounting for door opening and closing requires some modifications of the problem by adding some opportune constraints.

III. IDENTIFICATION OF THE ROOM MODEL

In this section we describe how to obtain a model for G starting from pilot data of $c(k)$, $t(k)$, and $o(k)$.

As in [15], [16], [17], [18], [19], we assume the environmental signals to be stationary, the dynamics of the room to be discrete LTI, measurement devices to be synchronized and operating at the same sample time. We further assume that T_{tr} samples of the aforementioned signals have been collected during an experimental phase.

The dynamics of the room can be expressed as

$$\begin{bmatrix} c(k) \\ t(k) \end{bmatrix} = \begin{bmatrix} G_c(q^{-1}) \\ G_t(q^{-1}) \end{bmatrix} \begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix} + \begin{bmatrix} w_c(k) \\ w_t(k) \end{bmatrix}, \quad (2)$$

where, without loss of generality,

$$\begin{aligned} G_c(q^{-1}) &:= [G_c^c(q^{-1}) \quad G_c^t(q^{-1}) \quad G_c^o(q^{-1})], \\ G_t(q^{-1}) &:= [G_t^c(q^{-1}) \quad G_t^t(q^{-1}) \quad G_t^o(q^{-1})], \end{aligned}$$

are matrix polynomials with all the entries having the same order. The processes $w_c(k)$, $w_t(k)$ are white Gaussian noises, independent of each other, representing the innovation process, i.e., part of $c(k)$ and $t(k)$ that cannot be predicted from past measurements.

To estimate the polynomials $G_c(q^{-1})$ and $G_t(q^{-1})$ we consider a Prediction Error Method (PEM) paradigm. We define the best linear one-step-ahead predictor of the outputs, namely

$$\begin{bmatrix} \hat{c}(k|k-1) \\ \hat{t}(k|k-1) \\ \hat{o}(k|k-1) \end{bmatrix} = \begin{bmatrix} G_c(q^{-1}) \\ G_t(q^{-1}) \end{bmatrix} \begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix}, \quad (3)$$

obtained by simply neglecting the noise processes. Then, using PEM-based techniques we can obtain $\hat{G}_c(q^{-1})$ and $\hat{G}_t(q^{-1})$, such that the variance of the prediction errors $c(k) - \hat{c}(k|k-1)$ and $t(k) - \hat{t}(k|k-1)$ on the data collected during the training phase, is minimized. From (3) it follows that the predictors $\hat{c}(k|k-1)$ and $\hat{t}(k|k-1)$ exploit the same information of the past.

Figure 1 plots the correlation functions defined in (4), and computed using the dataset considered throughout the manuscript. In (4), $\bar{c}(\cdot)$, $\bar{o}(\cdot)$ and $\bar{t}(\cdot)$ represent signals stripped of the mean, m is a time lag, and T_{tr} denotes the size of the dataset:

$$\begin{aligned} r_{c,o}(m) &:= \frac{\sum_{k=0}^{T_{tr}} \bar{c}(k) \bar{o}(k-m)}{\sqrt{\left(\sum_{k=0}^{T_{tr}} \bar{c}(k)^2\right) \left(\sum_{k=0}^{T_{tr}} \bar{o}(k)^2\right)}}, \\ r_{t,o}(m) &:= \frac{\sum_{k=0}^{T_{tr}} \bar{t}(k) \bar{o}(k-m)}{\sqrt{\left(\sum_{k=0}^{T_{tr}} \bar{t}(k)^2\right) \left(\sum_{k=0}^{T_{tr}} \bar{o}(k)^2\right)}}, \end{aligned} \quad (4)$$

The functions $r_{c,o}(m)$ and $r_{t,o}(m)$ indicate the dependency of the occupancy signal, as a function of the time distance, m , on the CO₂ concentration and temperature, respectively. It can be promptly seen that the signal mostly correlated with the occupancy is the CO₂ level. For this reason, in the rest of the paper we shall consider only the predictor $\hat{c}(k|k-1)$ and thus focus on the identification of $\hat{G}_c(q^{-1})$.

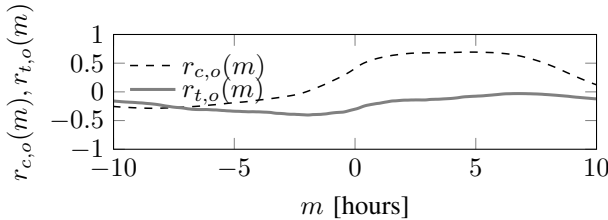


Figure 1. Empirical cross-correlations between occupancy and either temperature ($r_{t,o}(m)$) or CO₂ ($r_{c,o}(m)$), computed using the dataset considered throughout the manuscript (sampling time $t_s = 5$ minutes). To highlight the features of the correlation signals we use a time scale finer than the ones used in the subsequent figures.

A. Nonparametric identification of the CO₂ dynamics

In this paper we adopt a nonparametric approach to the problem of identifying the CO₂ room dynamics. Instead of

directly searching for the coefficients of the polynomials $G_c^o(q^{-1})$, $G_c^t(q^{-1})$ and $G_c^c(q^{-1})$, we aim at estimating the system impulse responses, which are defined in the time domain and which are related to the frequency domain description of the system through the relations

$$\begin{aligned} G_c^o(q^{-1}) &= \sum_{k=1}^{+\infty} g_o(k) q^{-k}, \\ G_c^t(q^{-1}) &= \sum_{k=1}^{+\infty} g_t(k) q^{-k}, \\ G_c^c(q^{-1}) &= \sum_{k=1}^{+\infty} g_c(k) q^{-k}, \end{aligned} \quad (5)$$

where $g_o(k)$, $g_t(k)$, $g_c(k)$ are the impulse responses having the occupancy, temperature and CO₂ as inputs, respectively. We can simplify the problem by truncating the impulse response to a fixed large index p and estimate the first p coefficients of each impulse response. The estimated coefficients can then be used to form the aforementioned polynomials¹. To make the estimation problem well-posed, we define a suitable hypothesis space for the unknown impulse responses. Such a space is a Reproducing Kernel Hilbert Space (RKHS) [20], and its associated kernel is the so-called *stable spline kernel* [21], [22], defined as

$$[K_\beta]_{i,j} = \beta^{\max\{i,j\}}, \quad 0 < \beta < 1, \quad (6)$$

where β is a hyperparameter tuning the decay rate. The choice of this kernel is motivated by the fact that the associated RKHS contains smooth and exponentially decaying functions. These are desirable properties in impulse responses modeling physical systems such as those considered in this problem. We refer to [23] for a thorough description of kernel-based methods in system identification.

Let the training set be indexed by the time instances $0, 1, \dots, T_{tr}$ and \mathbf{g}_c , \mathbf{g}_t , and \mathbf{g}_o be column vectors containing the impulse responses related to $c(k)$, $t(k)$, and $o(k)$, respectively, and

$$\begin{aligned} \mathbf{g} &:= \begin{bmatrix} \mathbf{g}_c^T & \mathbf{g}_t^T & \mathbf{g}_o^T \end{bmatrix}^T, \\ \phi_c(k) &:= \begin{bmatrix} c(k-1) & \dots & c(k-p) \end{bmatrix}, \\ \phi_t(k) &:= \begin{bmatrix} t(k-1) & \dots & t(k-p) \end{bmatrix}, \\ \phi_o(k) &:= \begin{bmatrix} o(k-1) & \dots & o(k-p) \end{bmatrix}, \end{aligned} \quad (7)$$

with $c(k) = t(k) = o(k) = 0$ if $k \leq 0$. Defining

$$\begin{aligned} \Phi &:= \begin{bmatrix} \phi_c(1) & \phi_t(1) & \phi_o(1) \\ \phi_c(2) & \phi_t(2) & \phi_o(2) \\ \vdots & \vdots & \vdots \\ \phi_c(T_{tr}) & \phi_t(T_{tr}) & \phi_o(T_{tr}) \end{bmatrix}, \\ \mathbf{c}_{tr} &:= [c(1) \ c(2) \ \dots \ c(T_{tr})]^T, \end{aligned}$$

we can exploit the theory of function estimation in RKHS [20] and formulate the problem as

$$\hat{\mathbf{g}} = \underset{\mathbf{g} \in \mathbb{R}^{3p}}{\operatorname{argmin}} \|\mathbf{c}_{tr} - \Phi \mathbf{g}\|_2^2 + \gamma (\|\mathbf{g}_c\|_P^2 + \|\mathbf{g}_t\|_P^2 + \|\mathbf{g}_o\|_P^2), \quad (8)$$

i.e., as a regularized Least-Squares (LS) where:

- $\|\mathbf{g}\|_P^2 = \mathbf{g}^T P \mathbf{g}$ with P a positive definite weighting matrix penalizing candidate impulse responses which do not decay to zero for large values of the time index. In this way, P favors outcomes $\hat{\mathbf{g}}$ that well represent impulse

¹In this paper we set $p = 50$.

responses of stable systems. Here we set the matrix P as $P = K_\beta^{-1}$; the choice of the hyperparameter β is discussed below;

- γ is a positive real number representing a trade-off between variance and bias of the estimator, leading to the LS estimate of \mathbf{g} for $\gamma = 0$.

The optimal values of γ and β can be computed via either cross validation-based strategies [24] or empirical Bayes techniques [25], [21]. Once these values have been established, the solution can be computed in closed form [26] as

$$\hat{\mathbf{g}} = (\Phi^T \Phi + \gamma D_P)^{-1} \Phi^T \mathbf{c}_{tr}, \quad (9)$$

where D_P is block diagonal with four blocks all equal to P .

Remark 1 The impulse response estimator (8) may be seen also as a Maximum a Posteriori (MAP) estimator under a Gaussian prior assumption of the unknown impulse responses. Then, the choice of such a prior is well motivated by the underlying theory of the RKHS induced the stable spline kernel.

IV. DECONVOLUTION OF THE OCCUPANCY LEVELS

In this section we derive an estimator $\hat{o}(k)$ of $o(k)$ as a function of the measurements $c(k)$ and $t(k)$ and the estimated room dynamics $\hat{G}_c^c, \hat{G}_c^t, \hat{G}_c^o$. Let

$$\hat{c}(k|k-1) = \hat{G}_c(q^{-1}) \begin{bmatrix} c(k-1) \\ t(k-1) \\ o(k-1) \end{bmatrix}, \quad (10)$$

and consider the CO₂ levels prediction error

$$\varepsilon(k) := c(k) - \hat{c}(k|k-1). \quad (11)$$

Under the stated assumptions $\varepsilon(k)$ is a zero-mean Gaussian white noise [27]. Substituting (10) into (11) and rearranging properly, we obtain

$$\begin{aligned} & \hat{G}_c^o(q^{-1}) o(k-1) \\ &= c(k) - \left[\hat{G}_c^c(q^{-1}) \quad \hat{G}_c^t(q^{-1}) \right] \begin{bmatrix} c(k-1) \\ t(k-1) \end{bmatrix} - \varepsilon(k), \end{aligned} \quad (12)$$

where the unknowns are only $o(k-1)$ and $\varepsilon(k)$, since

$$\tilde{c}(k) := c(k) - \left[\hat{G}_c^c(q^{-1}) \quad \hat{G}_c^t(q^{-1}) \right] \begin{bmatrix} c(k-1) \\ t(k-1) \end{bmatrix},$$

can be computed given the available information. Thus (12) becomes

$$\tilde{c}(k) = \hat{G}_c^o(q^{-1}) o(k-1) + \varepsilon(k), \quad (13)$$

which shows that the problem of estimating the unknown $o(\cdot)$ is a deconvolution problem, i.e., the unknown occupancy signal $\hat{o}(k)$ can be estimated as the signal best describing the observed output $\tilde{c}(k)$, given the knowledge of the transfer function \hat{G}_c^o . Since $\varepsilon(k)$ is assumed white and Gaussian, the natural approach to this problem would be to employ a LS estimator of $o(\cdot)$, because this would minimize the residual error [24, Ch. 7]. More specifically, let $\hat{G}_c^o(q^{-1}) =$

$g_1 q^{-1} + \dots + g_p q^{-p}$; we consider two variants of the occupancy estimation problem:

- 1) online monitoring;
- 2) offline estimation.

We begin by dealing with the first case. At each time instant we consider a window in the past of N data samples of each signals, from $k - N + 1$ to k , with $N \geq p$. Considering the auxiliary notation

$$\hat{G} := \begin{bmatrix} g_1 & 0 & \dots & & 0 \\ g_2 & g_1 & & & \\ \vdots & \ddots & \ddots & & \vdots \\ g_p & \dots & g_2 & g_1 & \\ & \ddots & & \ddots & \ddots \\ 0 & & g_p & \dots & g_2 & g_1 \end{bmatrix} \quad \tilde{\mathbf{o}} := \begin{bmatrix} o(k-N) \\ \vdots \\ o(k-1) \\ \tilde{c}(k-N+1) \\ \vdots \\ \tilde{c}(k) \end{bmatrix} \quad (14)$$

a basic occupancy estimator can be formulated as the LS-type problem

$$\hat{\mathbf{o}} = \arg \min_{\tilde{\mathbf{o}} \in \mathbb{R}_+^N} \|\tilde{\mathbf{c}} - \hat{G} \tilde{\mathbf{o}}\|_2^2. \quad (15)$$

The performance of this estimator is usually unsatisfactory, since the estimates are noisy, due to the high variance, and they do not reflect suitable room occupancy patterns. To overcome this issue we account for the prior information that $o(k)$ is *non-negative, integer, and piecewise constant* and we formulate the deconvolution problem as the problem of finding the least-changing positive piecewise constant input signal giving a prescribed mismatch between the estimated and measured outputs of the system. Let us define

$$\Delta o(i) := o(i) - o(i-1), \quad \Delta \mathbf{o} := \begin{bmatrix} \Delta o(k-N+1) \\ \vdots \\ \Delta o(k-1) \end{bmatrix}. \quad (16)$$

The estimation problem then becomes

$$\begin{aligned} \hat{\mathbf{o}}(k-1) &= \arg \min_{\tilde{\mathbf{o}} \in \mathbb{N}_+^N} \|\Delta \tilde{\mathbf{o}}\|_0 \\ \text{s.t.} \quad & \|\tilde{\mathbf{c}} - \hat{G} \tilde{\mathbf{o}}\|_2^2 \leq \rho, \end{aligned} \quad (17)$$

where:

- $\hat{\mathbf{o}}(k-1)$ is a N -dimensional vector with the estimated values of occupancy at the time instants $k-1, \dots, k-N$ (for online estimation purposes one might consider to use just its first entry $\hat{o}(k-1)$);
- the cost function $\|\cdot\|_0$, the ℓ_0 norm, counts the number of variations of the candidate inputs, thus penalizing candidate inputs with frequent variations;
- the LS-type term accounts for adherence to data and tries to match the estimated and measured outputs of the system, up to a precision given by the user-choice parameter ρ .

Problem (17) can be reformulated as follows [28]

$$\hat{\mathbf{o}}(k-1) = \arg \min_{\tilde{\mathbf{o}} \in \mathbb{N}_+^N} \|\tilde{\mathbf{c}} - \hat{G} \tilde{\mathbf{o}}\|_2^2 + \lambda \|\Delta \tilde{\mathbf{o}}\|_0, \quad (18)$$

where λ is a regularization parameter (strictly related to ρ) that trades off the two previous terms, how to choose λ is discussed

in details in Section IV-A. Unfortunately, Problem (18) is a non-convex non-linear integer program; to solve it directly one must search through all possible combinations of non-zero element in $\Delta\tilde{\mathbf{o}}$. Hence, the search space is increasing exponentially in the number of parameters and the problem cannot be solved efficiently [29]. To circumvent this computational drawback we adopt two relaxations. First, we substitute the ℓ_0 norm with the ℓ_1 norm [30, Ch. 3.4], which represents its best convex relaxation. Second, extend the domain of the plausible inputs to \mathbb{R}_+^N instead of \mathbb{N}_+^N , so that the estimation problem becomes

$$\hat{\mathbf{o}}(k-1) = \left[\arg \min_{\tilde{\mathbf{o}} \in \mathbb{R}_+^N} \left\| \tilde{\mathbf{c}} - \hat{G} \tilde{\mathbf{o}} \right\|_2^2 + \lambda \|\Delta\tilde{\mathbf{o}}\|_1 \right], \quad (19)$$

with $[\cdot]$ the vector-wise rounding operator. Problem (19) is a particular case of *fused-lasso* estimator, where the solution is searched among *sparse* regressor vectors where less frequent jumps (i.e., non-zero impulses in the derivatives) are preferred, and the strength of this preference is dictated by the regularization parameter λ .

Remark 2 The estimator in (19) can be also seen as a MAP estimator of the occupancy (when the rounding operator is removed). In this case, the prior distribution on the unknown process is Laplacian with independent components (see e.g., [31]), that is

$$p(\Delta\mathbf{o}) = \frac{1}{\alpha} \prod_{i=1}^{N-1} \exp\left(-\frac{|\Delta\tilde{o}(i)|}{\alpha}\right), \quad (20)$$

where α is a user parameter that tunes the sharpness (and so the sparsity) of the pdf. Since $\varepsilon(k)$ is white and Gaussian, then the distribution of the vector $\tilde{\mathbf{c}}$ given $\Delta\tilde{\mathbf{o}}$ is Gaussian, with mean $\hat{G} \tilde{\mathbf{o}}$ and variance $\sigma^2 I$, where σ^2 is the prediction error variance and I is the identity matrix. The MAP estimation of $\tilde{\mathbf{o}}$ can thus be formulated as

$$\begin{aligned} \hat{\mathbf{o}}(k-1) &= \arg \max_{\tilde{\mathbf{o}} \in \mathbb{R}_+^N} \log(p(\tilde{\mathbf{c}}|\Delta\tilde{\mathbf{o}})p(\Delta\tilde{\mathbf{o}})) \\ &= \arg \min_{\tilde{\mathbf{o}} \in \mathbb{R}_+^N} \frac{1}{\sigma^2} \left\| \tilde{\mathbf{c}} - \hat{G} \tilde{\mathbf{o}} \right\|_2^2 + \frac{2}{\alpha} \|\Delta\tilde{\mathbf{o}}\|_1. \end{aligned} \quad (21)$$

The above expression reveals that the regularization parameter $\lambda = \frac{2\sigma^2}{\alpha}$ is the ratio of the noise variance and the user parameter α regulating the (prior) sparseness of the derivative of the occupancy.

The parameter N plays an important role in (19), since it defines the amount of data employed for estimating $\hat{\mathbf{o}}(k-1)$ (and in particular $\hat{o}(k-1)$) at each time instant. Clearly, a large value of N yields more accurate estimates, since more information is used. However, a large value of N brings computational issues which could make the computation of (19) too slow for online operations. Thus, as will be discussed in Section VII-B, a good choice of N should consider both these aspects.

The derivation of the offline estimator is straightforward. Let the test set be indexed by the time instants $1, \dots, T_{ts}$.

Redefining the vectors introduced in (14) and (16), so that they include all the T_{ts} measurement of the test set, we can reuse the estimator defined in (19). In this case, its output will be a vector $\hat{\mathbf{o}}$ containing the estimated occupancy pattern for the time instants $0, \dots, T_{ts} - 1$.

A. Finding the optimal regularization parameter λ

The regularization parameter λ establishes the typical variability of the room occupancy signal. Large values of λ penalize changes in the value of estimated occupancy, leading to estimates that are constant for long periods. Small values of λ , instead, yield occupancy signals with high frequency components, thus behaving similarly to the outcomes of the LS estimator (which is obtained by setting $\lambda = 0$).

A reasonable value of λ is computed by finding the value of such a parameter giving the best estimation performance during the training phase. This optimal value can then be computed by the following algorithm:

- 1) define a grid Λ of candidate values of λ ;
- 2) for each $\lambda \in \Lambda$ solve Problem (19) using the $c(k)$, $t(k)$ and $v(k)$ collected *during the training phase*, obtaining $\hat{\mathbf{o}}(\lambda)$, i.e., an occupancy estimate as function of λ ;
- 3) compute the optimal regularization parameter as

$$\hat{\lambda} = \arg \min_{\lambda \in \Lambda} \|\hat{\mathbf{o}}(\lambda) - \mathbf{o}\|_2^2, \quad (22)$$

with \mathbf{o} being the occupancy signal collected during the training phase.

Remark 3 To find the set Λ one can start by first finding an opportune λ_{max} for which the problem (19) leads to constant occupancy estimates. Consider moreover the cardinality of Λ be given; then one can define the set Λ between 0 and the obtained λ_{max} exploiting a logarithmic grid. The main advantage of logarithmic gridding is that the grid will be finer for smaller values of λ , where the sensitivity is usually higher (see Figure 4).

V. CHARACTERIZATION OF THE OCCUPANCY ESTIMATOR

In this section we derive relations between the probability of obtaining wrong occupancy estimates and the quantities parameterizing the estimator, namely the identified linear models, the noise level of the measurements, and the regularization parameter λ .

Our first result regards the performance of the estimator when the occupancy is constant in a window of N past values.

Proposition 4 Let σ_ε be the variance of the noise in (13), N the window length in the estimator, and λ the regularization parameter. Assume that $o(k)$ is a constant signal. Define

$$\Delta := \begin{bmatrix} -1 & 1 & & \\ & \ddots & \ddots & \\ & & -1 & 1 \end{bmatrix} \in \mathbb{R}^{N-1 \times N} \quad (23)$$

and $V^T := (\Delta \widehat{G}^{-1})^\dagger$, where $(X)^\dagger$ denotes the Moore-Penrose pseudoinverse of X . Then $\widehat{o}(k)$ is detected as constant with probability of at least α if

$$\lambda^2 > \sigma_\varepsilon^2 \chi_\alpha^{-1}(N) \|V_m\|^2 \quad (24)$$

where $\chi_\alpha^{-1}(N)$ is the inverse of the chi-square Cumulative Distribution Function (CDF) with N degrees of freedom for the corresponding probability α and $\|V_m\|^2 := \max_i \|V_i\|^2$, with V_i the i -th row of V .

The following result studies the case where $o(k)$ has a variation.

Proposition 5 Let σ_ε be the variance of the noise in (13), N the window length in the estimator, λ the regularization parameter. Define $\bar{\Delta} \in \mathbb{R}^{N-1 \times N-1}$, obtained removing the first column of Δ and $\bar{V}^T := (\bar{\Delta} \bar{H}^{-1})^{-1}$. Assume that the first value of the estimated occupancy is set to the true one, i.e., $\widehat{o}(k-N) = o(k-N)$, and that $o(k)$ has a unique discontinuity given by a variation of one unit. Then, $\widehat{o}(k)$ is detected as constant, i.e., there is a missed change with probability of at least α if

$$\lambda^2 > \sigma_\varepsilon^2 \chi_\alpha^{-1}(N) \|\bar{V}_1\|^2 + (1 + o(k-N))^2 \|\bar{V}_1\|^4, \quad (25)$$

where \bar{V}_1 is the first row of \bar{V} .

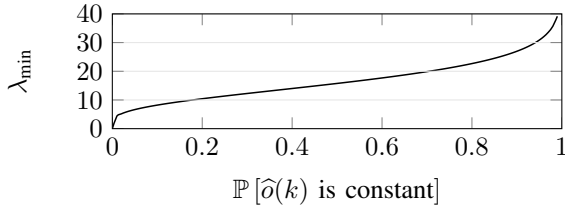


Figure 2. Graphical representation of bound (24) as functions of the probability α for a given σ_ε^2 .

Figure 2 shows the behavior of the bound derived in Proposition 4 as a function of the probability that the detected occupancy pattern is constant.

The previous results can easily be extended to the more general case where the true occupancy is piecewise constant with ρ discontinuities of +1 units. The sufficient condition to estimate a constant signal in this case is

$$\lambda^2 > \sigma_\varepsilon^2 \chi_\alpha^{-1}(N) \|V_1\|^2 + (\rho + o(k-N))^2 \|V_1\|^4.$$

VI. ACCOUNTING FOR ADDITIONAL INFORMATION

In this section we address the cases where the available information contains the additional signals $v(k)$ (venting levels) and $e(k)$ (door events, defined in (1)).

A. Accounting for venting levels

When the signal $v(k)$ is available, a straightforward generalization of (10) yields

$$\widehat{c}(k|k-1) = \widehat{G}_c(q^{-1}) \begin{bmatrix} c(k-1) \\ t(k-1) \\ v(k-1) \\ o(k-1) \end{bmatrix}, \quad (26)$$

$$\widehat{G}_c(q^{-1}) = \begin{bmatrix} \widehat{G}_c^c(q^{-1}) & \widehat{G}_c^t(q^{-1}) & \widehat{G}_c^v(q^{-1}) & \widehat{G}_c^o(q^{-1}) \end{bmatrix}.$$

Consequently, we can extend the system identification procedure of Section III, with $\mathbf{g} := [\mathbf{g}_c^T \quad \mathbf{g}_t^T \quad \mathbf{g}_v^T \quad \mathbf{g}_o^T]^T$, and

$$\widehat{\mathbf{g}} = \underset{\mathbf{g} \in \mathbb{R}^{4p}}{\operatorname{argmin}} \|\mathbf{c}_{tr} - \Phi \mathbf{g}\|_2^2 + \gamma (\|\mathbf{g}_c\|_P^2 + \|\mathbf{g}_t\|_P^2 + \|\mathbf{g}_v\|_P^2 + \|\mathbf{g}_o\|_P^2).$$

The same extension applies to the deconvolution step: the estimator (13) remains structurally the same as soon as $\widetilde{c}(k)$ is redefined as

$$\widetilde{c}(k) := c(k) - \begin{bmatrix} \widehat{G}_c^c(q^{-1}) & \widehat{G}_c^t(q^{-1}) & \widehat{G}_c^v(q^{-1}) \end{bmatrix} \begin{bmatrix} c(k-1) \\ t(k-1) \\ v(k-1) \end{bmatrix}.$$

B. Accounting for door opening and closing events

Assume now the knowledge of $e(k)$, i.e., a boolean signal measuring door opening and closing events. Using the definition in (1) we infer that $e(k) = 0$ implies $o(k) = o(k-1)$, while no deduction on the behavior of $o(k)$ can be made when $e(k) \neq 0$.

As for the system identification problem, information on $e(k)$ is non-influential, i.e., it does not modify the derivations in Section III, since during the identification the occupancy levels are assumed known. In other words, $o(k)$ contains already the information in $e(k)$.

As for the deconvolution problem, knowing $e(k)$ changes the structure of the estimator, since $e(k)$ naturally constrains the estimand occupancy levels to be identical when $e(k) = 0$. More precisely, knowing $e(k)$ corresponds to knowing the sparsity pattern of the to-be-reconstructed signal. This implies that the regularization term $\|\Delta \widetilde{o}\|_0$ in (18) is a constant factor that does not depend on the decision variables; thus (18) is equivalent to the Integer Quadratic Program (IQP)

$$\begin{aligned} \widehat{o}(k-1) = \operatorname{arg} \min_{\substack{\widetilde{o} \in \mathbb{N}_+^N \\ \widetilde{o} \in \mathbb{N}_+^N}} & \|\widetilde{c} - \widehat{G} \widetilde{o}\|_2^2 \\ \text{s.t.} & \Delta \widetilde{o}(k) = 0 \text{ for all } e(k) = 0. \end{aligned} \quad (27)$$

Following the motivations that brought from (18) to (19), (27) can be relaxed with

$$\begin{aligned} \widehat{o} = \operatorname{arg} \min_{\widetilde{o} \in \mathbb{R}_+^{Ts}} & \left[\|\widetilde{c} - \widehat{G} \widetilde{o}\|_2^2 \right] \\ \text{s.t.} & \Delta \widetilde{o}(k) = 0 \text{ for all } e(k) = 0. \end{aligned} \quad (28)$$

Due to the lack of the regularization term, (28) does not require tunings of regularization parameters.

Estimator (28) is based on the hypothesis that the noise process $\varepsilon(k)$ is white and Gaussian. Such an assumption may be unrealistic, since the identification phase is likely to yield non-exact models (due to disturbances and unmodeled dynamics). One way to address this issue and robustify the estimator is

to further modify (28) adding back the ℓ_1 regularization term $\lambda \|\Delta\tilde{o}\|_1$ to obtain

$$\hat{o} = \arg \min_{\tilde{o} \in \mathbb{R}_+^{T_{ts}}} \left[\|\tilde{c} - \hat{G} \tilde{o}\|_2^2 + \lambda \|\Delta\tilde{o}\|_1 \right] \quad (29)$$

s.t. $\Delta\tilde{o}(k) = 0$ for all $e(k) = 0$.

As noticed before, this regularization term corresponds thus to favor, in the occupancy signal, small changes to big ones, with the strength of this preference dictated by the regularization parameter λ . Obviously, implementing estimator (29) requires to find the optimal λ , as described in Section IV-A.

VII. EXPERIMENTS

We have tested the proposed estimator on one of the rooms of the KTH ACL-HVAC testbed, see <http://hvac.ee.kth.se/> for more information. The collected information, available at <http://hvac.ee.kth.se/datasets.html>, comprises two weeks of measurements of CO₂ and temperature levels from HDH sensors, and of venting, cooling, and heating actuation levels from the central HVAC system. Occupancy levels were manually registered for the whole period. To uniform the sampling times of the various signals (5 minutes), or in case of missing measurements, the information was resampled using linear interpolation schemes. The first week was used as a training set, while the second week was used as a test set.

A. Definition of the performance indexes

We consider four performance indexes: *i*) the *Mean Squared Error (MSE)* (30), characterizing the relative estimation errors; *ii*) the *accuracy* (32), reporting how many times the estimator returns the correct value; *iii*) the *false positive / false negative occupancy detection rates* (35), describing the ability of discriminating the presence / absence of occupants in terms of false positives (when the room is estimated to be occupied while it is not) and false negatives (when the room is estimated to be empty while it is not).

The MSE associated with o and \hat{o} is

$$\text{MSE}(\hat{o}) := \frac{\|\hat{o} - o\|_2^2}{\|o\|_2^2}. \quad (30)$$

To define the other performance indexes we then transform the signals o , \hat{o} with codomain \mathbb{N}_+ (number of occupants) to signals with codomain $\{0, 1\}$ (room is not occupied, room is occupied) through indicator function

$$\mathbb{1}(o(k)) := \begin{cases} 1 & \text{if } o(k) > 0 \\ 0 & \text{otherwise} \end{cases}, \quad \mathbb{1}(o) := \begin{bmatrix} \mathbb{1}(o(1)) \\ \vdots \\ \mathbb{1}(o(N)) \end{bmatrix}. \quad (31)$$

Given (31), the accuracy of the estimate \hat{o} is

$$\text{Acc}(\hat{o}) := \frac{N - \sum_{k=1}^N \mathbb{1}(o(k) - \hat{o}(k))}{N}. \quad (32)$$

To define the false positive / negative rates we introduce

$$\mathcal{N}_\theta := \{t \text{ s.t. } \mathbb{1}(o(k)) = \theta\}, \quad (33)$$

dividing the time indexes in two sets: \mathcal{N}_0 , for the time indexes k for which the room was not occupied, and \mathcal{N}_1 , for the k 's for

which the room was occupied. Using this definition we may capture the mistakes “the room is estimated to be occupied while it is empty”, “the room is considered empty while it is occupied” with

$$\hat{\beta}(\theta) := \frac{1}{|\mathcal{N}_\theta|} \sum_{k \in \mathcal{N}_\theta} \mathbb{1}(\hat{o}(k)), \quad (34)$$

where we remark that the summation is performed over the set \mathcal{N}_θ . With (34) the false positive and false negative rates become

$$\text{FP}(\hat{o}) := \hat{\beta}(0), \quad \text{FN}(\hat{o}) := 1 - \hat{\beta}(1). \quad (35)$$

B. Summary of the results

1) *Evaluation of the importance of additional information:* We assume the parameters λ and N are optimally scaled (we discuss tuning of these parameters in the following subsections). Table I then numerically assesses the value of knowing the ventilation levels $v(k)$ and the door openings / closing flags $e(k)$, while Figure 3 depicts graphically the realizations of the results. From Table I, it can be seen that adding the information regarding ventilation levels can improve the accuracy of the estimator. Moreover, the estimator (29) has the best performance in terms of MSE. It is worth mentioning that due to the constraint on $\Delta\tilde{o}$ in (29), the ℓ_1 regularization term does not impose (further) sparsity, however, it shrinks the estimates of the occupancy. It is well known that shrinking may improve the MSE [30]; we can get similar results with other shrinkage methods such as the ridge regression [30].

Estimator	MSE	Accuracy	FP	FN
(19)	0.208	0.822	0.039	0.028
(19)+ v	0.124	0.888	0.007	0.018
(28)	0.217	0.884	0.001	0.028
(29)	0.109	0.886	0.006	0.008

Table I
COMPARISON OF THE PERFORMANCE OF ESTIMATORS (19), (19) WITH KNOWLEDGE OF VENTILATION LEVELS $v(k)$, (28) AND (29).

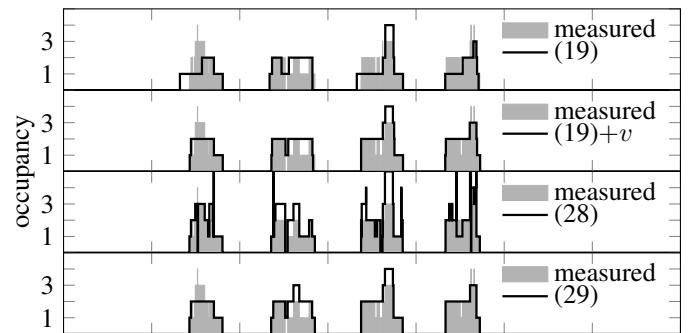


Figure 3. Realizations of the estimates for the test set considered in our experiments for the various estimators proposed in this manuscript.

2) *Evaluation of the sensitivity to the regularization parameter λ* : We evaluate the effectiveness of selection strategy for the parameter λ described in Section IV-A. Since the best value of such a parameter for the test set may be different from its best value in the training set, it is important to evaluate the effects of this unavoidable mismatch.

Figure 4 plots the MSE for different λ 's for estimator (19)+ v for both the training and test sets. The dependency on λ appears relatively weak in the test set, and the MSE of the training and test sets attain their minima at approximately the same point. This suggests that the proposed estimation strategy for λ is reliable and effective.

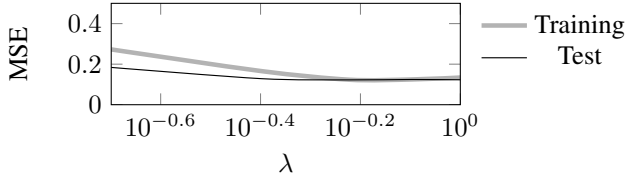


Figure 4. Sensitivity of the performance of estimator (19)+ v w.r.t. the choice of λ .

3) *Evaluation of the sensitivity to the optimization horizon N* : N trades-off computational requirements with information: the larger the optimization horizon, the more information the estimators have about the dynamics of the system. Intuition suggests that, after a certain horizon length, adding more information does not improve the estimation performance, i.e., after this horizon the old dynamics do not influence the current estimates. The results shown in Figure 5 indicate that this length is, in our experiments, of about 5 days.

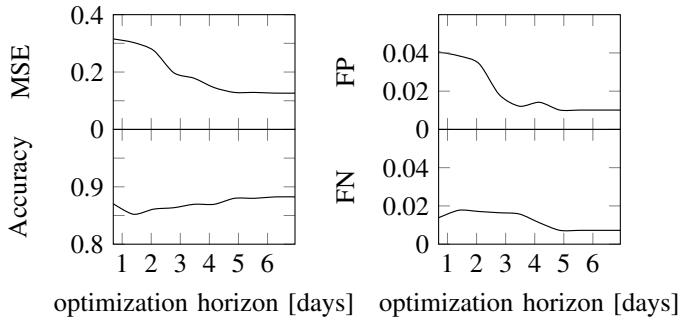


Figure 5. Dependency of the performance of estimator (29) w.r.t. the choice of N .

C. Alternative occupancy estimation methods

We here consider two classical Machine Learning strategies and compare them against estimator (19) with knowledge of ventilation levels $v(k)$.

1) *Estimation using Support Vector Machine (SVM)*: In their basic form, SVMs perform classification tasks as follows: given a dataset \mathcal{D} of samples (\mathbf{x}_k, y_k) for $k = 0, \dots, N$ with $\mathbf{x}_k \in \mathbb{R}^n$ and $y_k \in \{-1, +1\}$, try to find a separating hyperplane in \mathbb{R}^{n+1} that: (i) separates the points of the form $(\mathbf{x}_k, +1)$ from those of the form $(\mathbf{x}_k, -1)$; (ii) maximizes

its minimum distance from the \mathbf{x}_i 's. This concept can then be extended to cope with non-linear and imperfect separation rules, and with multi-classes classification tasks [32, Part III].

SVMs have already been exploited for building occupancy estimation tasks, e.g., in [11], [12]. The most common approach is to let \mathbf{x}_k contain functions of the current and past CO_2 , temperature and ventilation levels (e.g., the average of $c(k), \dots, c(k-n)$). y_k instead usually represents the building occupancy level $o(k)$. With these definitions it is possible to train a general multi-class SVM on the couples (\mathbf{x}_k, y_k) that form the training set. After this step one can then estimate the unknown building occupancy by applying the trained SVM on the \mathbf{x}_k that form the test set.

The SVM implemented in our tests that led to the best estimation error performance is a C -SVM exploiting a polynomial kernel of order 3. As features, it considers current and past values of the temperature, CO_2 , and ventilation levels up to 1 hour in the past, and their first and second derivatives in time.

2) *Estimation using Neural Network (NN)*: The Neural Networks (NNs) maps considered in this manuscript are of the form [33, Sec. 44]

$$y_k = \Psi'' \left(\sum_i \omega_i'' h_i(\mathbf{x}_k) + \theta'' \right)$$

$$h_i(\mathbf{x}_k) = \Psi_i' \left(\sum_j \omega_j' x_{k,j} + \theta_i' \right)$$

with y_k and \mathbf{x}_k having the same meanings of Section VII-C1. The structures of the functions Ψ'' , Ψ_i' , h_i are design parameters, that usually remind how biological neurons electrically react to external stimuli. Once the design parameters have been chosen, training the network corresponds to search that particular set of weights \mathbf{w} for which the corresponding NN best fits the training examples. Once this function has been learned, it can be used for prediction purposes as did in the SVM case.

The NN implemented in our tests that led to the best estimation error performance is a complete feed-forward network with Sigmoid activation rules and one hidden layer composed by 8 neurons. It considers the same features exploited to train the SVM based estimator.

3) *Results of comparisons*: We compare the performance of estimator (19) with knowledge of ventilation levels against the performance of the NN and Support Vector Classification (SVC) strategies described above. All these estimators are comparable in the sense that they use the same amount of information. It can then be noticed that the estimation strategy proposed in this manuscript outperforms the more classic Machine Learning strategies on the considered dataset.

Estimator	MSE	Accuracy	FP	FN
(19)+v	0.124	0.888	0.007	0.018
SVM	0.342	0.826	0.018	0.277
NN	0.268	0.811	0.067	0.095

Table II
COMPARISON OF THE PERFORMANCE OF ESTIMATOR (19) WITH
KNOWLEDGE OF VENTILATION LEVELS AGAINST THE PERFORMANCE OF
EQUIVALENT NN AND SVC STRATEGIES.

VIII. CONCLUSIONS

We have proposed methods for estimating occupancy levels in closed environments. These methods exploit different sources of information, and are aimed at understanding which of such sources are mostly meaningful in the addressed task of estimating how occupancy levels change in time. The main standing assumption is that the estimator can, for system identification purposes, access to direct measurements of the true occupancy levels for a limited period.

The proposed estimation scheme first obtains Linear Time Invariant (LTI) model by a suitable identification method. Then, it formulates the occupancy estimation problem as a regularized deconvolution problem (where the regularization exploits prior information on the features of the searched signal). The obtained results show that adding information on ventilation and door opening / closing events can significantly improve the performance of the estimator.

We also analyzed the theoretical statistical performance of the estimators, and showed that the probability of obtaining wrong estimates can be suitably bounded when we know specific design parameters and the measurement noise variance.

The idea considered in this paper can be extended towards the construction of occupancy estimators for whole buildings, and towards the identification of building occupancy pattern models. Moreover it may be possible to adapt the models identified in a single room to other rooms of the same building, by an opportune rescaling of the identified impulse responses accounting variations in the structural properties of rooms.

APPENDIX

A. Proof of Proposition 4

The proof is divided in 3 main parts: *i*) rewrite (19), derive the dual of the new problem and the structure of its solution. *ii*) find some analytical relations between the estimated and the true occupancy levels. *iii*) exploit these relations to derive bounds that characterize the statistical performance of the estimator.

i): Introduce the variable $z := \Delta \tilde{\mathbf{o}}$ and rewrite (19) as

$$\arg \min_{\substack{\tilde{\mathbf{o}} \in \mathbb{R}_+^N \\ \mathbf{z} \in \mathbb{R}^{N-1}}} \frac{1}{2} \left\| \tilde{\mathbf{c}} - \widehat{G} \tilde{\mathbf{o}} \right\|_2^2 + \lambda \|\mathbf{z}\|_1 \quad (36)$$

s.t. $\mathbf{z} = \Delta \tilde{\mathbf{o}}$,

where, for the purposes of the proof, the function $[\cdot]$ (the vector-wise rounding operator) is omitted. The Lagrangian

of (36) is then

$$\mathcal{L}(\tilde{\mathbf{o}}, \mathbf{z}, \mathbf{u}) = \frac{1}{2} \left\| \tilde{\mathbf{c}} - \widehat{G} \tilde{\mathbf{o}} \right\|_2^2 + \lambda \|\mathbf{z}\|_1 + \mathbf{u}^T (\Delta \tilde{\mathbf{o}} - \mathbf{z}), \quad (37)$$

where \mathbf{u} is the Lagrange multiplier. The dual problem, obtained minimizing \mathcal{L} w.r.t. $\tilde{\mathbf{o}}$ and \mathbf{z} , is [34]

$$\arg \min_{\mathbf{u} \in \mathbb{R}^N} \frac{1}{2} \left\| \tilde{\mathbf{c}} - \left(\Delta \widehat{G}^{-1} \right)^T \mathbf{u} \right\|_2^2 \quad (38)$$

s.t. $|\mathbf{u}|_\infty \leq \lambda$.

We notice that, since \widehat{G} is a lower triangular matrix, \widehat{G} admits inverse as soon as $g_1 \neq 0$. This is then satisfied as soon as there is (only) one delay in the effects of the occupancy on the CO₂ levels of the room.

To obtain the structure of the dual solution, consider again the derivative of the Lagrangian with respect to \mathbf{z}

$$\min_{\mathbf{z}} \mathcal{L}(\tilde{\mathbf{o}}, \mathbf{z}, \mathbf{u}) = \min_{\mathbf{z}} (\lambda \|\mathbf{z}\|_1 - \mathbf{u}^T \mathbf{z}) = \begin{cases} 0 & \text{if } |\mathbf{u}|_\infty \leq \lambda, \\ -\infty & \text{otherwise} \end{cases} \quad (39)$$

Let then $\widehat{\mathbf{u}}_\lambda$ be the dual solution and $\widehat{\mathbf{z}} = \Delta \widehat{\mathbf{o}}_\lambda$ be the primal solution of (36) for a specific λ . Given the computations above, it satisfies

$$\widehat{\mathbf{u}}_{\lambda,i} \in \begin{cases} \{+\lambda\} & \text{if } (\Delta \widehat{\mathbf{o}})_i > 0, \\ \{-\lambda\} & \text{if } (\Delta \widehat{\mathbf{o}})_i < 0, \\ [-\lambda, \lambda] & \text{if } (\Delta \widehat{\mathbf{o}})_i = 0. \end{cases} \quad (40)$$

In other words, to maximize (39), the i^{th} element of the dual solution, i.e., $\widehat{\mathbf{u}}_{\lambda,i}$, should be $+\lambda$ if the corresponding element in the primal solution is positive and it should be $-\lambda$ if the corresponding element in the primal solution is negative, see [34]. For those elements of the primal solution with zero values we can only say that the dual problem must satisfy the condition $|\mathbf{u}|_\infty \leq \lambda$.

From (40), one can conclude that $|\widehat{\mathbf{u}}_{\lambda,i}| \neq \lambda$, only if $(\Delta \widehat{\mathbf{o}})_i = 0$.

ii): relax problem (38) by removing the ∞ -norm constraint. The resulting problem is a unconstrained Least-Squares (LS) problem, with solution

$$\mathbf{u}_{\text{LS}} = \left(\Delta \widehat{G}^{-1} \right)^T \tilde{\mathbf{c}}. \quad (41)$$

If $\|\mathbf{u}_{\text{LS}}\|_\infty < \lambda$ holds, then two facts hold:

- 1) \mathbf{u}_{LS} is also the solution of problem (38);
- 2) due to the last implication described in *i*), $\Delta \widehat{\mathbf{o}} = 0$, i.e., the estimated occupancy is a constant signal.

These two facts connect variations in the estimate $\Delta \widehat{\mathbf{o}}$ with the measured signal $\tilde{\mathbf{c}}$, considering $V^T := \left(\Delta \widehat{G}^{-1} \right)^T$, since they read as

$$\|V \tilde{\mathbf{c}}\|_\infty < \lambda \Rightarrow \Delta \widehat{\mathbf{o}} = 0. \quad (42)$$

To explicit $\tilde{\mathbf{c}}$, consider that the vectorized version of (13) reads as

$$\tilde{\mathbf{c}} = \widehat{G} \mathbf{o} + \varepsilon, \quad (43)$$

with $\varepsilon \in \mathbb{R}^{T_{ts}}$ white and Gaussian innovation, and \mathbf{o} the true occupancy signal. Rewriting \mathbf{V} as

$$\mathbf{V} = \left(\Delta \hat{\mathbf{G}}^{-1} \hat{\mathbf{G}}^{-T} \Delta^T \right)^{-1} \Delta \hat{\mathbf{G}}^{-1} \quad (44)$$

and eventually, substituting (44) into (42), we rewrite the latter as

$$\left\| \left(\Delta \hat{\mathbf{G}}^{-1} \hat{\mathbf{G}}^{-T} \Delta^T \right)^{-1} \Delta \hat{\mathbf{G}}^{-1} (\hat{\mathbf{G}} \mathbf{o} + \varepsilon) \right\|_{\infty} < \lambda, \quad (45)$$

which in turn implies $\Delta \hat{\mathbf{o}} = 0$. As can be seen, (45) relates conditions on the true occupancy \mathbf{o} and the innovation process ε with conditions on the final estimate $\hat{\mathbf{o}}$.

iii): we now analyze the case when the true occupancy is constant ($\Delta \mathbf{o} = \mathbf{0}$). In this case condition (45) reads as

$$\|V\varepsilon\|_{\infty} < \lambda \Rightarrow \Delta \hat{\mathbf{o}} = 0, \quad (46)$$

that is equivalent to

$$\left\{ |\langle V_i, \varepsilon \rangle|^2 < \lambda^2 \right\}_{i=1, \dots, N} \Rightarrow \Delta \hat{\mathbf{o}} = 0. \quad (47)$$

The Cauchy-Schwarz inequality yields $\langle V_i, \varepsilon \rangle^2 \leq \|V_i\|^2 \|\varepsilon\|^2$. Letting $\|V_m\|^2 := \max_i \|V_i\|^2$, the sufficient condition for (46) becomes

$$\|V_m\|^2 \|\varepsilon\|^2 < \lambda^2 \Rightarrow \Delta \hat{\mathbf{o}} = 0. \quad (48)$$

In (48) V_m is known, while ε is white Gaussian noise: thanks to the Prediction Error Method (PEM) paradigm, $\varepsilon_i \sim \mathcal{N}(0, \sigma_{\varepsilon}^2)$, with σ_{ε}^2 estimated during the system identification phase. It thus follows that

$$\left\| \frac{\varepsilon}{\sigma_{\varepsilon}} \right\|^2 = \sum_{i=1}^{T_{ts}} \left(\frac{\varepsilon_i}{\sigma_{\varepsilon}} \right)^2 \sim \chi^2(N), \quad (49)$$

where $\chi^2(N)$ is a Chi-squared distribution with N degrees of freedom. Thus, with the probability of at least α , $\|\varepsilon\|^2$ will have the following upper bound

$$\|\varepsilon\|^2 \leq (\sigma_{\varepsilon})^2 \chi_{\alpha}^{-1}(N), \quad (50)$$

where $\chi_{\alpha}^{-1}(N)$ is the inverse of the chi-square cdf with N degrees of freedom for the corresponding probability α . Substituting (50) into (48), we get the statement of the proposition.

B. Proof of Proposition 5

In this case, we impose another constraint on the optimization problem (19) by setting the first element in the occupancy signal to its true value. Using the same approach as in the proof of the proposition 1, we will have (36) subject to $\bar{\mathbf{o}}(1) = o(k-N)$, where $o(k-N)$ is the true value of the occupancy signal at time $k-N$. Substituting the new constraint $\bar{\mathbf{o}}(1) = o(k-N)$ into the cost function, one can rewrite (36) as

$$\begin{aligned} \arg \min_{\substack{\bar{\mathbf{o}} \in \mathbb{R}_+^N \\ \mathbf{z} \in \mathbb{R}^{N-1}}} & \frac{1}{2} \|\bar{\mathbf{c}} - \bar{\mathbf{H}} \bar{\mathbf{o}}\|_2^2 + \lambda \|\mathbf{z} - \bar{\mathbf{o}}^*\|_1 \\ \text{s.t.} & \quad \mathbf{z} = \bar{\Delta} \bar{\mathbf{o}}, \end{aligned} \quad (51)$$

where

$$\bar{\mathbf{o}}^* := [o(k-N) \ 0 \ \dots \ 0]^T \in \mathbb{R}^{N-1 \times N-1}.$$

Using the same approach as before the dual problem for (51) will be

$$\begin{aligned} \arg \min_{\mathbf{u} \in \mathbb{R}^N} & \frac{1}{2} \|\bar{\mathbf{c}} - (\bar{\Delta} \bar{\mathbf{H}}^{-1})^T \mathbf{u}\|_2^2, \\ \text{s.t.} & \quad \|\mathbf{u}\|_{\infty} \leq \lambda \end{aligned} \quad (52)$$

where the Lagrange multipliers satisfy

$$\hat{u}_{\lambda, i} \in \begin{cases} \{+\lambda\} & \text{if } (\bar{\Delta} \bar{\mathbf{o}} - \bar{\mathbf{o}}^*)_i > 0, \\ \{-\lambda\} & \text{if } (\bar{\Delta} \bar{\mathbf{o}} - \bar{\mathbf{o}}^*)_i < 0, \\ [-\lambda, \lambda] & \text{if } (\bar{\Delta} \bar{\mathbf{o}} - \bar{\mathbf{o}}^*)_i = 0. \end{cases} \quad (53)$$

Notice that $\bar{\Delta}$ is invertible and thus the condition (45) for this case reads as

$$\left\| (\bar{\mathbf{V}} \bar{\mathbf{V}}^T)^1 \mathbf{o}^* + (\bar{\mathbf{V}} \bar{\mathbf{V}}^T)^k \pm \bar{\mathbf{V}} \varepsilon \right\|_{\infty} < \lambda \Rightarrow \bar{\Delta} \bar{\mathbf{o}} - \bar{\mathbf{o}}^* = 0, \quad (54)$$

where $(\bar{\mathbf{V}} \bar{\mathbf{V}}^T)^k$ is the k -th column of $\bar{\mathbf{V}} \bar{\mathbf{V}}^T$ and $\bar{\mathbf{V}} = (\bar{\mathbf{H}} \bar{\Delta}^{-1})^T$. Notice that this is a upper triangular Toeplitz matrix, satisfying (letting \bar{V}_j be the j -th row of $\bar{\mathbf{V}}$)

$$\|\bar{V}_1\|^2 \geq \|\bar{V}_2\|^2 \geq \dots \geq \|\bar{V}_N\|^2. \quad (55)$$

This implication refers to the case where the estimator makes the error of not finding the change in the occupancy signal at time k .

The ∞ -norm above can be expanded, as before, to obtain the component-wise equivalent condition

$$\left\{ |\langle \bar{V}_i, \bar{V}_1 \rangle \mathbf{o}^* + \langle \bar{V}_i, \bar{V}_k \rangle \pm \langle \bar{V}_i, \varepsilon \rangle| < \lambda \right\}_{i=1, \dots, N} \Rightarrow \bar{\Delta} \bar{\mathbf{o}} = \bar{\mathbf{o}}^* \quad (56)$$

or, using the bilinearity of inner products,

$$\left\{ |\langle \bar{V}_i, \bar{V}_1 \mathbf{o}(k-N) + \bar{V}_k \pm \varepsilon \rangle| < \lambda \right\}_{i=1, \dots, N} \Rightarrow \bar{\Delta} \bar{\mathbf{o}} = \bar{\mathbf{o}}^*. \quad (57)$$

Cascading now Cauchy-Schwarz and triangular inequalities with (55) and (57) it is possible to derive the sufficient condition

$$\begin{aligned} \|\bar{V}_1\|^2 \left(\|\bar{V}_1\|^2 \mathbf{o}(k-N)^2 + \|\bar{V}_1\|^2 + \|\varepsilon\|^2 \right) & < \lambda^2 \\ \Rightarrow \bar{\Delta} \bar{\mathbf{o}} = \bar{\mathbf{o}}^* \end{aligned} \quad (58)$$

or, equivalently,

$$\left\| \frac{\varepsilon}{\sigma_{\varepsilon}} \right\|^2 < \frac{\lambda^2 - (1 + \mathbf{o}(k-N)^2) \|\bar{V}_1\|^4}{\sigma_{\varepsilon}^2 \|\bar{V}_1\|^2} \Rightarrow \bar{\Delta} \bar{\mathbf{o}} = \bar{\mathbf{o}}^*. \quad (59)$$

Same considerations as in the previous case thus follow and (59) can be read as

$$\lambda^2 > \sigma_{\varepsilon}^2 \chi_{\alpha}^{-1}(N) \|\bar{V}_1\|^2 + (1 + \mathbf{o}(k-N)^2) \|\bar{V}_1\|^4. \quad (60)$$

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