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A Hierarchical Distributed MPC for HVAC Systems

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Abstract— **In this paper a hierarchical distributed model predictive control scheme is proposed for heating, ventilation and air-conditioning (HVAC) systems in buildings. The building consists in multiple connected rooms and zones. The control objective is to keep the temperature of each room and zone at a given comfortable level with minimal energy consumption. This control scheme is divided into two levels. The upper level controller collects temperature and predictive information of all rooms and zones to generate reference trajectories while a lower level controller only uses local information to track the reference and optimize energy efficiency and thermal comfort. By using a contraction property of building's dynamics, recursive feasibility of the proposed algorithm is guaranteed. Simulation results are given to show the performance of our proposed control strategy.**

I. Introduction

From the United Nations environment programme report [10], buildings account for 40 percent of energy consumption and resources and one third of greenhouse gas emissions. In Singapore, 30 percent of energy is consumed by buildings while air-conditioning systems are responsible for more than half of the total energy consumed by buildings [2]. Therefore it is attractive to reduce the energy cost of buildings and one of the most promising directions is to optimize the energy efficiency of heating, ventilation and air conditioning systems.

Model Predictive Control (MPC) is one of the most popular methods used to optimize the energy efficiency of HVAC systems. The main idea of this method is to obtain a control sequence at each time step by solving a finite horizon optimal control problem, which is formulated with system dynamics and current measurement. Only the first part of the control sequence will be implemented and the optimization problem will be formulated and solved again when the next measurement comes. Compared with traditional control strategies, MPC is superior due to its optimal nature and the ability to handle input and output constraints. Therefore, in recent years, a lot of researchers have proposed MPC algorithms to increase energy efficiency of HVAC systems, see, e.g. [7], [15], [21], [20] and so on.

Though MPC usually has a better performance than other control strategies, the size of optimization problem grows rapidly when the dimension of system becomes large. Especially, when MPC algorithm is applied to control HVAC systems in buildings, due to the large numbers of rooms and zones, it will become impractical to implement a centralized MPC algorithm since the optimal control problem may not be solved in reasonable time and the control system may become difficult to maintain.

Motivated by the above issues, several distributed algorithms were proposed recently to attenuate the online computational burden. In [16], a nonlinear optimal control problem is formulated and solved through tailed sequential quadratic programming. Then the subproblem is decomposed further by subgradient method. Local controller approaches the optimal solution by repeatedly negotiating with its neighbours in every sample period. Another distributed MPC algorithm is proposed in [18]. Compared with the iterative approach in [16], the algorithm in [18] only requires controller to exchange the predicted output with its neighbours for once in each sample period. However, this algorithm can obtain only Nash equilibrium, which may not be the optimal solution. Unlike aforementioned two works where the optimization problems are formulated and solved online, in [9], explicit solutions of the optimal control problems are provided in an off-line manner by using Karush-Kuhn-Tucker conditions so that the online computational burden is reduced significantly. However, the prediction horizon is only one step in this algorithm and it may lead to inefficient operation. More recently, inspired by algorithms in [22] and so on, authors of [8] proposed a consensus-based strategy to coordinate and control heating energy distribution with guaranteed convergence. However, efficiency of this algorithm is not considered.

In this paper, a hierarchical distributed MPC algorithm is proposed to regulate the temperature of buildings. This control scheme is divided into two levels. The upper level controller collects global measurement information, formulates and solves a centralized optimization problem to generate reference temperature trajectories for lower level controllers. The lower level controllers only use local information to track the reference trajectories given by the upper level controller by solving local optimal control problems. Since the centralized optimization problem cannot be solved efficiently, the upper level controller works with a long sampling period while the lower level controllers update more fre-

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quently because local problems can be solved fast. Compared with [16], this algorithm does not require exchanging information among subsystems in each sampling period so that communication requirements are reduced. Besides, compared with [8], the performance index is explicitly considered in the problem formulation. Furthermore, based on contraction property [14] and some other mild assumptions, the recursive feasibility of this algorithm, which is usually missed in the literature, is guaranteed.

The paper is organized as follows. Section II introduces a control-oriented model of HVAC systems considered in this paper. Section III outlines the proposed hierarchical distributed MPC control algorithm as well as the recursive feasibility analysis. Numerical examples and some discussions are given in Section IV. Finally, conclusions are drawn in Section V.

II. System model

In this section, a simplified HVAC system structure will be introduced and a control-oriented model will be established.

Due to the complex nature of air flow and heat transfer process, HVAC systems are usually modelled as time-varying nonlinear partial differential equations, which are not suitable for control and optimization. Therefore similar to [6], [15] and [25], the following assumptions are made to simplify the modelling.

- The air in each room and outdoor environment is well mixed immediately so that the temperature distributions are uniform.
- The heat capacity of air is assumed to be constant.

An undirected graph is used to describe the communication topology among rooms. An undirected graph is defined as a set $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V}$ is the set of all nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of all edges. We treat room i as the node i in graph $\mathcal{G}$. If room i and j are adjacent, edge (i,j) is in $\mathcal{E}$. The set of all neighbors of room i is defined as $\mathcal{N}_i = \{j \in \mathcal{V} | (i,j) \in \mathcal{E}\}$.

The indoor temperature dynamics for one single room is considered as a Resistive-Capacitive (RC) system [5] and temperature of the whole building is considered as a network of RC systems. Each node in this network is a room and its state represents temperature. The thermal dynamic model of room i is given by the following equation:

$$C_i\dot{T}_i = u_i c_p(T_s - T_i) + \sum_{j\in\mathcal{N}_i} \frac{T_j - T_i}{R_{ij}} + \frac{T_{oa} - T_i}{R^i_{oa}} + P^i_d, \quad (1)$$

where C_i is the lumped mass of room i, T_i is the temperature of air in room i. u_i represents the mass flow rate of air entering room i, which is the control input. $c_p = 1012(J/kg \times K)$ is the heat capacity of room air, T_s is the temperature of the supply air. $R_{ij} = R_{ji}$ denotes thermal resistances between room i and its neighbour j while R^i_{oa} models thermal resistance between room i and the outside environment. T_{oa} is environment temperature and unmeasured disturbances in room i such as occupancy, usage of electronic devices and so on are collected in P^i_d. The prediction of T_{oa}, which is denoted as $\tilde{T}_{oa}$, can be obtained from weather forecast station and the prediction of P^i_d, which is denoted as $\tilde{P}^i_d$ can be learned from historical data and schedule. In the following of this paper, we assume that prediction errors are zero since the main purpose of this paper is to introduce the hierarchical distributed structure and the compensation of bounded disturbances has been discussed in many other works [17].

The system states and control inputs are also subject to the following constraints: 1) $T_i \in [\underline{T}, \bar{T}] = [20, 24]°C$. 2) $u_i \in [\underline{\dot{m}}, \bar{\dot{m}}] = [0.005, 5]kg/s$. The maximal mass flow rate is limited by the power of fans and the size of VAV boxes while the minimal mass flow rate is used to guarantee ventilation requirement.

The objective of this paper is to design a distributed control law to maintain temperature T_i within a comfortable region while minimizing energy consumption.

III. Controller design

In this section, the proposed control algorithm will be stated. Firstly, contraction property of nonlinear systems will be introduced and some lemmas based on this property will be given. After that, the optimization problem will be formulated and feasibility issue of the algorithm will be analysed.

A. Contraction property

In some existing works [16] [18], the recursive feasibility issue is not studied while in some other works [4] [11], the conditions to guarantee recursive feasibility are rather conservative and not practical because they are derived by using Lipschitz continuity. In this paper, instead of using Lipschitz condition, a novel distributed MPC algorithm is designed based on contraction property proposed in [13]. It is noted that contraction theory has been extended to process control [14], distributed systems [12] and more recently, contraction theory has also been developed for the study of networked systems [23], observer and Kalman filter design [1] [3] and so on.

Contraction property is given in the following definition.

Definition 3.1: Consider a subsystem $\mathcal{S}_i : \dot{x}_i = f_i(x_i, x_{-i}, u_i, d_i)$ where x_i is the state, $-i$ denotes $\{j | j \in \mathcal{N}_i\}$, u_i is the control input and d_i is the disturbance. If there exist positive matrix M_i and positive number β_i such that

$$\frac{\partial f_i}{\partial x_i}^T M_i + M_i \frac{\partial f_i}{\partial x_i} \leq -\beta_i M_i$$

then subsystem $\mathcal{S}_i$ is contractive.

Then we have the following proposition.

Proposition 3.1: Subsystem (1) is contractive.

Proof: By choosing $M_i = 1$ and noticing that $u_i \geq \underline{\dot{m}}$, it can be calculated that

$$\begin{aligned}\frac{\partial f_i}{\partial x_i}^T M_i + M_i \frac{\partial f_i}{\partial x_i} &= -2(u_i c_p + \sum_{j\in\mathcal{N}_i} \frac{1}{R_{ij}} + \frac{1}{R^i_{oa}})/C_i \\ &\leq -2(\underline{\dot{m}} c_p + \sum_{j\in\mathcal{N}_i} \frac{1}{R_{ij}} + \frac{1}{R^i_{oa}})/C_i\end{aligned}$$

which completes the proof. ■

Lipschitz continuity is also required in the design procedure.

Assumption 3.1: f_i satisfies that

$$\|f_i(x_i, x_{-i}, u_i) - f_i(x_i, x'_{-i}, u_i)\| \le \sum_{j\in\mathcal{N}_i} L_{ij}\|x_j - x'_j\|$$

Clearly, subsystem (1) satisfies the above assumption with $L_{ij} = \frac{1}{R_{ij}}$.

When MPC algorithm is used, the predicted trajectory and the real one are usually not the same due to uncertainties. Based on the above two properties, we introduce the following lemma to estimate the discrepancy. This lemma is an extension of Theorem 5 in [24] to the case with disturbance.

Lemma 3.1: Consider two systems $\dot{x}_0 = f(x_0, \mathbf{0}, t)$ and $\dot{x}_1 = f(x_1, w, t)$, where w is a bounded disturbance. Suppose that $\frac{\partial f}{\partial x}^T M + M\frac{\partial f}{\partial x} \le -\beta M$ holds for some positive number β and positive definite matrix M in some region C where the two systems evolve. Then $V(t) \le \frac{2Ld\lambda_{\max}(M^{\frac{1}{2}})}{\beta\lambda_{\min}(M^{\frac{1}{2}})} + (V(0) - \frac{2Ld\lambda_{\max}(M^{\frac{1}{2}})}{\beta\lambda_{\min}(M^{\frac{1}{2}})})e^{-\frac{1}{2}\beta t}$ where $V(t) = \|x_0(t) - x_1(t)\|_M$, L is a positive constant satisfies that $\|f(x,y,z) - f(x,y',z)\| \le L\|y - y'\|$, $d = \sup_w \|w\|_M$.

Proof: Similar to the proof of Theorem 5 in [24], pick $\gamma(r) = x_0(0) + r(x_1(0) - x_0(0))$. Consider an auxiliary trajectory $\dot{x}_r = f(x_r, rw, t)$ which starts from $x_r(0) = \gamma(r),\ r \in [0,1]$. Define $p(t,r) = \frac{\partial x_r}{\partial r}(t,r)$. Then it follows that $\frac{\partial p}{\partial t}(t,r) = \frac{\partial}{\partial t}(\frac{\partial x_r}{\partial r}) = \frac{\partial}{\partial r}(\frac{\partial x_r}{\partial t}) = \frac{\partial}{\partial r} f(x_r, rw, t) = \frac{\partial f}{\partial x_r}p + \frac{\partial f}{\partial rw}w$.

Consider $\Phi(t,r) = p(t,r)^T M p(t,r)$. Then by the contraction property we have $\frac{\partial}{\partial t}\Phi(t,r) \le -\beta\Phi(t,r) + 2\sqrt{\Phi(t,r)}L\frac{d\lambda_{\max}(M^{\frac{1}{2}})}{\lambda_{\min}(M^{\frac{1}{2}})}$. Note that $\sqrt{\Phi(0,r)} = V(0)$ and it follows from comparison principle that

$$\sqrt{\Phi(t,r)} \le \mu + (V(0) - \mu)e^{-\frac{1}{2}\beta t}, \qquad (2)$$

where $\mu = \frac{2Ld\lambda_{\max}(M^{\frac{1}{2}})}{\beta\lambda_{\min}(M^{\frac{1}{2}})}$, $\forall x \in C,\ t \ge 0$, and $\forall r \in [0,1]$.

According to the fundamental theorem of calculus one can write that $x_1(t) - x_0(t) = \int_0^1 p(t,r)dr$ and by inequality (2), $V(t) \le \int_0^1 \|p(t,r)\|_M dr = \frac{2Ld\lambda_{\max}(M^{\frac{1}{2}})}{\beta\lambda_{\min}(M^{\frac{1}{2}})} + (V(0) - \frac{2Ld\lambda_{\max}(M^{\frac{1}{2}})}{\beta\lambda_{\min}(M^{\frac{1}{2}})})e^{-\frac{1}{2}\beta t}$. ■

B. Hierarchical distributed MPC

Because of the high computational complexity of the centralized MPC, it is not practical to apply it in real application when the dimension of the system becomes high. Distributed MPC, which decomposes the centralized optimization problem to a group of small ones, makes it possible to be used for large scale dynamically coupled systems. However, purely distributed MPC algorithms are usually difficult to have convergence and feasibility guarantees [18] or those conditions are very conservative for application [4] [11] especially when the system is nonlinear. In this work we combine the distributed and centralized structure together to design a hierarchical control scheme to balance the computational complexity and the conservativeness.

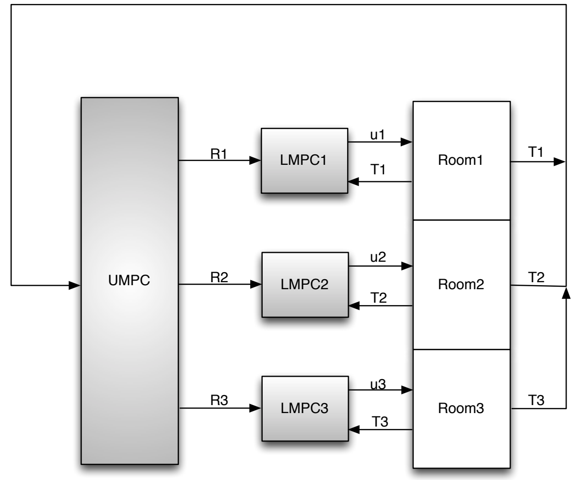

Fig. 1. Control scheme

The control structure is divided into two layers. The upper layer solves a global optimal control problem with long prediction horizon by collecting information from all subsystems to generate reference temperature trajectories for lower layer controllers. The lower layer controllers solve local optimal control problems with short prediction horizon by only using its local information. Both layers work in a receding horizon manner but the sampling period of the upper layer is much larger than that of the lower layer. Therefore, though the upper layer controller solves a global optimization problem, the computational burden is acceptable.

To facilitate the description of the proposed algorithm, the notation $\cdot^u$ will be used for upper layer MPC while $\cdot^l$ will be used for lower layer MPC. We shall abbreviate upper layer MPC controller to UMPC while the lower layer one of room i will be called LMPCi for short. t_k^u denotes the sampling time instant of the UMPC while t_k^l denotes that of the lower layer MPCs; define $c(k,p) = k(M+1) + p$ where M is a positive integer corresponding to the number of sampling instants of LMPC between two sampling instants of UMPC; $t_k^u = t_{c(k,0)}^l$; $\delta^u \triangleq t_{k+1}^u - t_k^u$ and $\delta^l \triangleq t_{k+1}^l - t_k^l$ represent sampling period of the UMPC and LMPC respectively; $\delta^u = (M+1)\delta^l$; T^u denotes the prediction horizon of the UMPC and T^l denotes that of LMPC which satisfies that $T^u \ge T^l + \delta^u$. The control scheme is illustrated in Fig. 1.

The hierarchical distributed MPC algorithm is summarized below:

1) At sampling time instant t_k^u, UMPC receives temperature measurement $T(t_k^u) = [T_1(t_k^u), \ldots, T_N(t_k^u)]^T$.

2) UMPC computes the optimal control input $u^0(s;t_k^u) = [u_1^0(s;t_k^u), \ldots, u_N^0(s;t_k^u)]^T,\ s \in [t_k^u, t_k^u + T^u]$ and corresponding temperature trajectory $T^0(s;t_k^u) = [T_1^0(s;t_k^u), \ldots, T_N^0(s;t_k^u)]^T,\ s \in [t_k^u, t_k^u + T^u]$. $T_i^0(s;t_k^u)$ is transmitted to LMPCi and its neighbours.

3) At sampling time instant $t_{c(k,p)}^l$, LMPCi receives local temperature measurement $T_i(t_{c(k,p)}^l)$ and computes the optimal control input $u_i^*(s;t_{c(k,p)}^l),\ s \in [t_{c(k,p)}^l, t_{c(k,p)}^l + T^l]$.

4) Apply $u_i^*(s;t^l_{c(k,p)}),\ s\in[t^l_{c(k,p)}, t^l_{c(k,p)}+\delta^l]$.

5) If $0\le p<M$, $p=p+1$ and go to 3); else $k=k+1$, $p=0$ and go to 1).

$u^0(s;t_k^u),\ s\in[t_k^u, t_k^u+T^u]$ is the optimal solution of the following optimization problem:

$$\min_{\hat{u}(\cdot)} J^u(T^u(t_k^u), \hat{u}(\cdot))$$

subject to

$$\begin{aligned} C_i\dot{\hat{T}}_i^u(s) &= \hat{u}_i(s)c_p(T_s-\hat{T}_i^u(s))+\frac{\sum_{j\in\mathcal{N}_i}(\hat{T}_j^u(s)-\hat{T}_i^u(s))}{R_{ij}} \\ &\quad +(\tilde{T}_{oa}(s)-\hat{T}_i^u(s))/R^i_{oa}+\tilde{P}^i_d(s), \\ &\quad \hat{u}_i(s)\in[\underline{\dot{m}}, \bar{\dot{m}}], \\ &\quad \hat{T}_i^u(s)\in[\underline{T}, \bar{T}], \\ &\quad \hat{T}^u(t_k^u)=T(t_k^u), \\ &\quad s\in[t_k^u, t_k^u+T^u],\ i=1,\ldots,N \end{aligned}$$

where J^u represents performance index which will be optimized by UMPC.

Denote $r(s;t_k)=\frac{c_i}{\beta_i}(1-e^{-\frac{1}{2}\beta_i(s-t_k)})$. After obtaining temperature trajectory $T^0(s;t_k^u)=[T_1^0(s;t_k^u),\ldots,T_N^0(s;t_k^u)]^T$, $s\in[t_k^u, t_k^u+T^u]$, $u_i^*(s;t^l_{c(k,p)}),\ s\in[t^l_{c(k,p)}, t^l_{c(k,p)}+T^l]$ is computed by solving the following optimization problem:

$$\min_{\hat{u}_i(\cdot)} J_i^l(T_i^l(t^l_{c(k,p)}), \hat{u}_i(\cdot))$$

subject to

$$\begin{aligned} C_i\dot{\hat{T}}_i^l(s) &= \hat{u}_i(s)c_p(T_s-\hat{T}_i^l(s)) \\ &\quad +\sum_{j\in\mathcal{N}_i}(T_j^0(s;t_k)-\hat{T}_i^l(s))/R_{ij} \\ &\quad +\frac{(\tilde{T}_{oa}(s)-\hat{T}_i^l(s))}{R^i_{oa}}+\tilde{P}^i_d(s), \quad (3) \\ |\hat{T}_i^l(s)-T_i^0(s;t_k)| &\le l_i-r(s;t^l_{c(k,p)}), \quad (4) \\ \hat{u}_i(s) &\in [\underline{\dot{m}}, \bar{\dot{m}}], \\ \hat{T}_i^l(t^l_{c(k,p)}) &= T_i(t^l_{c(k,p)}), \\ s &\in [t^l_{c(k,p)}, t^l_{c(k,p)}+T^l] \end{aligned}$$

where J_i^l is the performance index which will be optimized by LMPCi, $\beta_i=\frac{2}{C_i}(\underline{\dot{m}}c_p+\sum_{j\in\mathcal{N}_i}\frac{1}{R_{ij}}+\frac{1}{R^i_{oa}})$, $c_i=2\sum_{j\in\mathcal{N}_i}\frac{l_j}{C_iR_{ij}}$ and l_i is a parameter to be chosen by designer. Denote the optimal control input of the above problem as $u_i^*(s;t^l_{c(k,p)})$ and the corresponding optimal temperature trajectory as $T_i^*(s;t^l_{c(k,p)})$.

The purpose of constraint (4) is to guarantee that the real temperature trajectory will not be far away from the reference. Therefore (3) is a reasonable approximate model of subsystem i.

Now we give the following proposition to ensure the recursive feasibility of the proposed hierarchical distributed MPC algorithm.

Proposition 3.2: Consider a group of interconnected rooms with thermal dynamics described by (1). If the optimization problems of the UMPC formulated at t_k^u is feasible and $\frac{c_i}{\beta_i}\le l_i$, then problems for LMPC$i$ formulated at $t_{k(M+1)+p},\ 0\le p<M$ are feasible.

Proof: Since UMPC is feasible, for any t_k^u, $T^0(s;t_k^u)$ and $u^0(s;t_k^u)$ are available. Clearly, when $p=0$, subprolems formulated at $t^l_{c(k,0)}$ are feasible since $u_i^0(s;t_k),\ s\in[t_k, t_k+T^l]$ is a feasible solution for each LMPCi. Now suppose that subproblem of LMPCi formulated at $p\ge 0$ has optimal solution $u_i^*(s;t^l_{c(k,p)}),\ i=1,\ldots,N$. We will show that subproblem formulated at $p+1$ has a feasible solution given by

$$\tilde{u}_i(s;t^l_{c(k,p+1)})=\begin{cases} u_i^*(s;t^l_{c(k,p)}), \\ s\in[t^l_{c(k,p+1)}, t^l_{c(k,p)}+T^l] \\ u_i^0(s;t_k), \\ s\in[t^l_{c(k,p)}+T^l, t^l_{c(k,p+1)}+T^l] \end{cases}$$

According to the definition of u_i^* and u_i^0, it is obvious that $\tilde{u}_i\in[\underline{\dot{m}}, \bar{\dot{m}}]$. Then we will show that $\tilde{x}(s;t^l_{c(k,p+1)})$ given by

$$\begin{aligned} C_i\dot{\tilde{T}}_i(s;t^l_{c(k,p+1)}) &= \tilde{u}_i(s;t^l_{c(k,p+1)})c_p(T_s-\tilde{T}_i(s;t^l_{c(k,p+1)})) \\ &\quad +\sum_{j\in\mathcal{N}_i}\frac{T_j^0(s;t_k)-\tilde{T}_i(s;t^l_{c(k,p+1)})}{R_{ij}} \\ &\quad +\frac{\tilde{T}_{oa}(s)-\tilde{T}_i(s;t^l_{c(k,p+1)})}{R^i_{oa}}+\tilde{P}^i_d(s) \end{aligned}$$

with initial condtion $\tilde{T}_i(t^l_{c(k,p+1)};t^l_{c(k,p+1)})=T_i(t^l_{c(k,p+1)})$ satisfies constraints (4).

Based on Lemma 3.1 and (4), it can be proved that $|T_i(s)-T_i^0(s;t_k)|\le l_i$. The proof will be omitted here due to limited space. Then by considering $w(s)=T_{-i}(s)-T^0_{-i}(s;t_k)$ and some simple calculations we get $|\tilde{T}_i(t^l_{c(k,p+1)};t^l_{c(k,p+1)})-T_i^*(t^l_{c(k,p+1)};t^l_{c(k,p)})|\le\frac{c_i}{\beta_i}(1-e^{-\frac{1}{2}}\beta_i\delta^l)$ and it leads to

$$\begin{aligned} &|\tilde{T}_i(s;t^l_{c(k,p+1)})-T_i^*(s;t^l_{c(k,p)})| \\ \le\ &\frac{c_i}{\beta_i}(1-e^{-\frac{1}{2}\beta_i\delta^l})e^{-\frac{1}{2}\beta_i(s-t^l_{c(k,p+1)})}, \quad (5) \end{aligned}$$

where $s\in[t^l_{c(k,p+1)}, t^l_{c(k,p)}+T^l]$. On the other hand, by constraint (4), $|T_i^*(s;t^l_{c(k,p)})-T_i^0(s;t_k)|\le l_i-r(s;t^l_{c(k,p)})$. Combining the above inequalities results in $|\tilde{T}_i(s;t^l_{c(k,p+1)})-T_i^0(s;t_k)|\le l_i-r(s;t^l_{c(k,p+1)}),\ s\in[t^l_{c(k,p+1)}, t^l_{c(k,p)}+T^l]$. When $s\in[t^l_{c(k,p)}+T^l, t^l_{c(k,p+1)}+T^l]$, we check $|\tilde{T}_i(s;t^l_{c(k,p+1)})-T_i^0(s;t_k)|$ directly. Note that $|\tilde{T}_i(t^l_{c(k,p)}+T^l;t^l_{c(k,p+1)})-T_i^0(t^l_{c(k,p)}+T^l;t_k)|\le l_i-r(t^l_{c(k,p)}+T^l;t^l_{c(k,p)})$. Then by Lemma 3.1 $|\tilde{T}_i(s;t^l_{c(k,p+1)})-T_i^0(s;t_k)|\le(l_i-r(t^l_{c(k,p)}+T^l;t^l_{c(k,p)}))e^{-\frac{1}{2}\beta_i(s-t^l_{c(k,p)}-T^l)}\le l_i-r(s;t^l_{c(k,p+1)})$. This proves that constraint (4) is satisfied.

■

IV. Simulation and Discussion

In this section, a six-room model is used to simulate the temperature response under the proposed hierarchical distributed MPC algorithm. The six

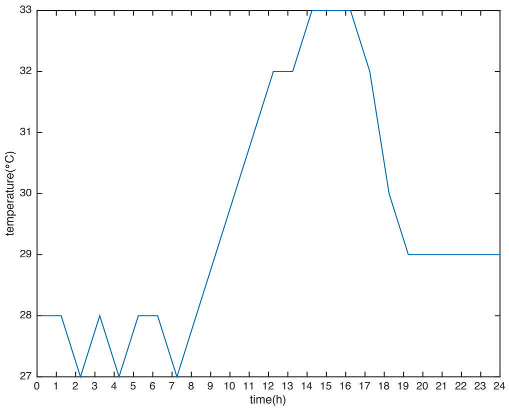

Fig. 2. Environment temperature

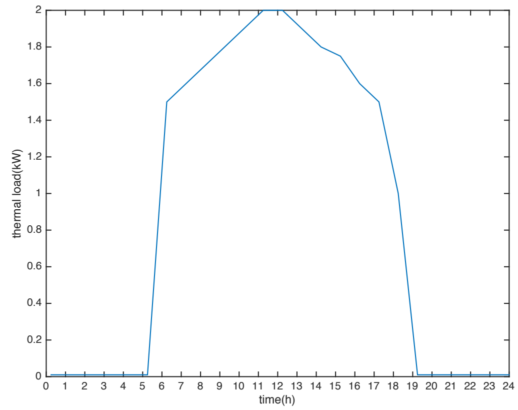

Fig. 3. Thermal load

rooms are connected and the undirected graph is $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V} = \{1, 2, 3, 4, 5, 6\}$ and $\mathcal{E} = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 1)\}$. System parameters are given: $C_i = 9.163 \times 10^3$kg/K, $T_s = 16°C$, $R_{ij} = 4$K/kW, $R^i_{oa} = 57$K/kW. The design parameter l_i is chosen as 2. Prediction horizon of UMPC is 24 hours while that of LMPCi is 2 hours. Sampling periods of UMPC and LMPCi are 1 hour and 15 minutes respectively. The objective function of UMPC is defined as

$$\begin{aligned} & J^u(T(t^u_k), \hat{u}(\cdot; t^u_k)) \\ = & \int_{t^u_k}^{t^u_k+T^u} (0.01\|\hat{T}^u(s; t^u_k) - T_d\|^2 + 100\|\hat{u}(s; t_k)\|^2) ds \end{aligned}$$

and the objective function of LMPCi is defined as

$$\begin{aligned} & J^l_i(T_i(t^l_{c(k,p)}), \hat{u}_i(\cdot; t^l_{c(k,p)})) \\ = & \int_{t^u_k}^{t^u_k+T^l} (c_1\|\hat{T}^l_i(s; t^l_{c(k,p)}) - T_d\|^2 + \|\hat{u}(s; t_k)\|^2 \\ & +c_2\|\hat{T}^l_i(s; t^l_{c(k,p)}) - T^0_i(s; t^u_k)\|^2) ds \end{aligned}$$

where T_d is the desired temperature which is set as $21°C$. Note that different types of objective functions are used in two layers. In particular, the upper layer mainly considers energy efficiency since it has a long prediction horizon while the lower layer can balance energy efficiency and thermal comfort by tuning c_1 and c_2.

The simulation starts from 0:00 am to 23:59 pm. The environment temperature information in two days is downloaded from a local weather station and the temperature profile is plotted in Fig. 2

In Fig. 3 thermal load profile P_d in two days is depicted. Here we assume that from 6:00 to 19:00 rooms are occupied so the thermal load is relatively higher while the rest time of the day rooms are empty thus the thermal load is set to the minimum caused by electronic devices. The thermal load for each room i is set as $P^i_d = (1 + (i - 1)/10)P_d$. The initial room temperature is set as $[21\ 21\ 21\ 21\ 21\ 21]°C$. We also set $c_1 = c_2 = 0.1$.

In the simulation, we compare the temperature responses under the proposed hierarchical distributed MPC and a decentralized PI control scheme with $u_i = (T_i - T_d) + 10^{-4}\int(T_i - T_d)$. Note that the temperature constraint $T_i \in [20, 24]°C$ is satisfied in all examples. The proposed algorithm are coded in Matlab® and run on a PC with Intel Core Duo i5-2400 CPU 3.10GHz. The optimization problems are described by using ICLOCS [19] and solved by IPOPT [26]. The average time to solve a subproblem is 6.8 seconds.

In Fig. 4, temperature response and control input under the proposed algorithm are depicted. Temperature response and manipulated input under a decentralized PI control are shown in Fig. 5. The proposed algorithm starts to cool down room temperature before 4:00 while the decentralized PI keeps almost inactive. Between 5:00 and 6:00, the proposed algorithm exhibits a precooling effect which saves energy because of lower environment temperature and thermal load which leads to cheaper cooling. After 18:00, due to the predictive information, the proposed algorithm can avoid overcooling such that the room temperature will not be too low because of the lower environment temperature and thermal load. On the other hand, the decentralized PI controller continues to cool down room temperature and leads to some negative overshoot. It can also be observed that the proposed algorithm balances the tracking performance and energy saving. It does not track the desired temperature as well as the decentralized PI controller. Therefore the proposed algorithm saves more energy with some loss of tracking performance. However, it should be noticed that the temperature is still in the comfort zone so that the thermal comfort is not affected.

We also compare the energy cost defined as $J_{energy} = \int_0^{T^l} \|u(s)\|^2 ds$ for the two examples. In the two examples, the value of J_{energy} are 3.24×10^4 and 5.98×10^4 respectively. Clearly, in the first example, controllers consume less energy because of the optimal nature of MPC. Strategy in the second example consumes more energy though it has better tracking performance.

V. Conclusion

In this paper, a hierarchical distributed MPC strategy is proposed for building temperature regulation problem. This control structure is divided into two levels. The upper

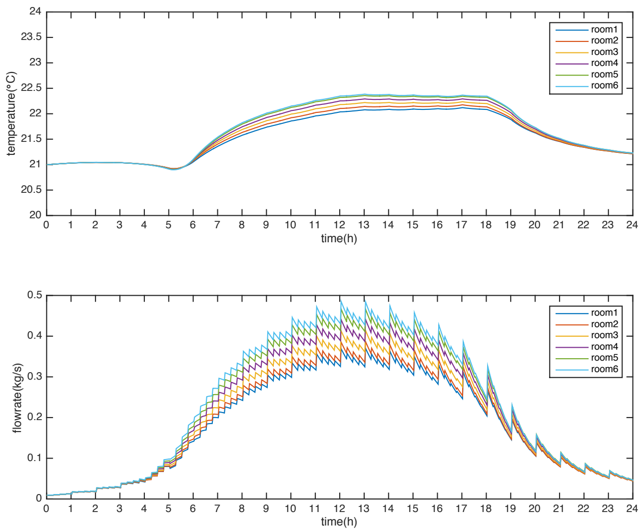

Fig. 4. Temperature response and control input under the proposed algorithm

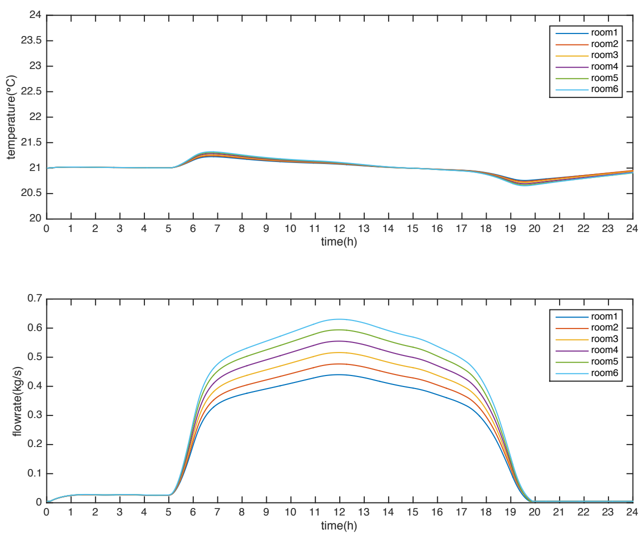

Fig. 5. Temperature response and control input under decentralized PI algorithm

level is a centralized coordinator, which can collect global information and solve an optimal control problem to generate reference signal, which will be tracked by lower layer controllers. The reference signal is also used to decouple the interconnected system. The lower level controllers only use local information to formulate and solve local optimization problems to track the reference trajectories while optimize energy efficiency. Different objective functions can be used in the two levels. Numerical examples are given to illustrate the performance of the proposed control scheme.

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