Regret-Optimal Cross-Layer Co-Design in Networked Control Systems—Part II: Gauss-Markov Case

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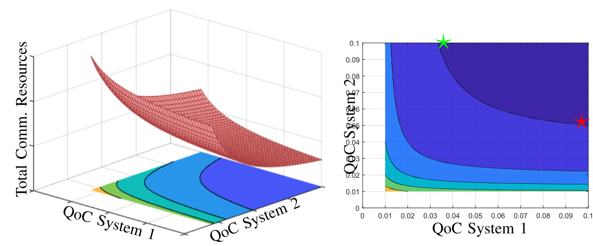
***Abstract*— In the first part of this two-letter series, we proposed a cross-layer framework for joint optimal Quality-of-Control (QoC) and Quality-of-Service (QoS) co-design for networked control systems. In this second part, we employ this framework to perform optimal co-design for networked control systems comprising multiple Gauss-Markov systems. We analytically derive the joint optimal policies based on the information couplings between the physical and the network layers. A numerical case study illustrates the framework.**

***Index Terms*— Gauss-Markov systems, networked control systems, quality-of-control, quality-of-service, regret-optimal co-design.**

I. Introduction

Modern networked control systems (NCSs), involving the exchange of information among physical systems through a common communication network, are evolving rapidly. This evolution includes a shift from separate control and communication designs to a unified control–communication co-design approach. This approach considers the requirements and constraints of both physical and network components, resulting in improved QoC–QoS trade-offs [1]. There's a natural trade-off between available QoS and achievable QoC in NCSs [2]. When multiple systems share the same network and a network manager allocates QoS to each subsystem, a network-wide trade-off is necessary. Fig. 1 illustrates this scenario, with two systems sharing the communication resources. The red surface represents the QoC–QoS trade-off plane. For a given total communication resource, the projection of the red surface onto the horizontal plane shows how different QoCs can be achieved for the two systems by varying their QoS allocations. The objective is to allocate the QoSs to each subsystems in a regret optimal fashion.

In Part I [3], we explored various formulations for the QoC–QoS co-design for generic physical systems and communication networks. In this letter, we focus on physical systems modeled by Gauss-Markov (GM) processes and a communication network supporting multiple services with specific QoS attributes, such as latency and reliability. We develop optimal co-design strategies for an NCS comprising multiple physical systems sharing a common communication network. Additionally, we delve into the fundamental limits on the QoS requirements necessary to achieve desired QoCs for the physical systems. Our work provides analytical expressions for optimal policies, accompanied with an illustrative example.



(a) (b)

Fig. 1. (a) The red surface shows the achievable QoCs for an NCS with two systems. The projection of this red surface on the QoC plane for a given total communication resource is also shown. (b) The projection of the red surface onto the QoC plane. Each black contour line represents a constant total communication resource. The green and red stars represent two of the many possible resource allocations for the top-most contour.

II. Control-Communication Co-Design Problem

Let N controlled physical systems (i.e., end-users) be connected over a shared communication network, where the dynamics of the i-th physical system follows the GM model

$$x_{k+1}^i = A^i x_k^i + B^i u_k^i + w_k^i, \tag{1}$$

where $x_k^i \in \mathbb{R}^{n_x^i}$, $u_k^i \in \mathbb{R}^{n_u^i}$, and $w_k^i \in \mathbb{R}^{n_w^i}$ represent the state vector, the control input, and the uncontrolled stochastic exogenous input, respectively, of system i at time-step k. For different $i \in \{1, \ldots, N\}$ and k, the disturbances w_k^i are assumed to be independent; for the same i and different k they are assumed to be identically distributed with Gaussian distribution $w_k^i \sim \mathcal{N}(0, W^i)$. The initial states $x_0^i \sim \mathcal{N}(0, \Sigma^i)$ are also assumed to be Gaussian and independent of each other for different i and of the disturbances w_k^j for all j and k. Matrix $A^i \in \mathbb{R}^{n_x^i \times n_x^i}$ denotes the *drift* matrix, whereas matrix $B^i \in \mathbb{R}^{n_x^i \times n_u^i}$ denotes the control/actuation matrix. Matrix A^i determines the *stability* of the system whereas the pair (A^i, B^i) determines the *controllability* of the system. Systems of the form (1) are ubiquitous in practice and more complex systems can be *linearized* and approximated by a GM model.

Each system i has sensors to perfectly measure its state x_k^i without any noise. System i also has a controller collocated with an estimator to compute the appropriate input u_k^i. The sensors and controllers are connected via a communication network. At time k, when a new sensor measurement x_k

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is available, it is immediately transmitted to the controller irrespective of whether the previous transmissions were successful or not. The measurements received by controller i at time k is denoted z^i_k. Its freshness and reliability depend on the quality of the communication service. For example, $z^i_k = x^i_{k-d}$ denotes a delay d between the time the measurement is taken and when it arrives at the controller. $z^i_k = \Delta(x^i_k)$ means the state measurement is quantized using a quantization function $\Delta(\cdot)$; $z^i_k = \{x^i_{k-d}, \Delta(x^i_k)\}$ denotes two state measurements simultaneously arriving at the controller at time k, one with a d-step delay and the other non-delayed but quantized. $z^i_k = \varnothing$ implies that no information is received at time k due delay or dropouts. Therefore, the QoS during the entire interval $[0, k]$, and not just at time k, determines the available information $\mathcal{Z}^i_k \triangleq \{z^i_0, \dots, z^i_k\}$ to the control at time k. In this letter, we focus on the latency and reliability aspects and assume that the quantization is perfect, i.e., $\Delta(x^i_k) = x^i_k$. In the case where quantization is not perfect, the quantization error affects the QoC. Higher resolution quantizers results in a better QoC, as expected. A detailed discussion on choosing the best quantization service can be found in [4].

A. Quality-of-Control Metrics

Given a time horizon T, the control objective of system i is to find a control policy $(\mathcal{U}^i)$ to minimize

$$J^{i,\mathrm{QoC}}_T(\mathcal{U}^i) \triangleq \mathsf{E}\Big[\sum_{k=0}^{T-1} c^i_k(x^i_k, u^i_k) + c^i_T(x^i_T)\Big], \quad (2)$$

where, the per-stage costs are

$$c^i_k(x, u) = \|x\|^2_{M^i_k} + \|u\|^2_{N^i_k}, \qquad c^i_T(x) = \|x\|^2_{M_T}, \quad (3)$$

with weight matrices M^i_k and N^i_k being positive semi-definite and positive definite, respectively, and for a vector v and matrix M of compatible dimensions, we define $\|v\|^2_M \triangleq v^\top M v$. The QoC of system i is measured by $\min_{\mathcal{U}^i} J^{i,\mathrm{QoC}}_T$, where a lower value of $\min_{\mathcal{U}^i} J^{i,\mathrm{QoC}}_T$ implies a better QoC.

We additionally consider the corresponding infinite horizon problem to capture the asymptotic performance. The first is the average cost problem where the control objective of the i-th system is to find a *stationary policy* that minimizes

$$J^{i,\mathrm{QoC}}_{\mathrm{avg}}(\mathcal{U}^i) \triangleq \limsup_{T\to\infty} \frac{1}{T}\mathsf{E}\Big[\sum_{k=0}^{T-1} c^i(x^i_k, u^i_k)\Big], \quad (4)$$

where $c^i(\cdot,\cdot)$ represents a stationary per-stage cost of the form (3) with time-invariant weight matrices $M^i_k = M^i$ and $N^i_k = N^i$ for all k. Alongside (4), a *discounted cost* formulation is used extensively in control and reinforcement learning [5] with the objective to minimize

$$J^{i,\mathrm{QoC}}_{\mathrm{dis}}(\mathcal{U}^i) \triangleq \mathsf{E}\Big[\sum_{k=0}^{\infty} \gamma^k c^i(x^i_k, u^i_k)\Big], \quad \gamma \in (0, 1). \quad (5)$$

Whenever it is not ambiguous or the results hold for both $J^{i,\mathrm{QoC}}_{\mathrm{avg}}$ and $J^{i,\mathrm{QoC}}_{\mathrm{dis}}$, we will simply use $J^{i,\mathrm{QoC}}_{\infty}$ for brevity.

Let q^i_k be the desired QoS to transmit the measurement from the sensor to the controller of system i at time k. $\{q^i_0, q^i_1, \dots, q^i_k\}$ affects the available information $\mathcal{Z}^i_k$ which in turn affects the QoC measured by $\min_{\mathcal{U}^i} J^{i,\mathrm{QoC}}_T(\mathcal{U}^i)$ (or $\min_{\mathcal{U}^i} J^{i,\mathrm{QoC}}_\infty(\mathcal{U}^i)$ for infinite horizon). Recent advancements in fast and reconfigurable communication services allow the physical systems to request a certain level of QoS. The network is able to adaptively change its configurations to meet such QoS demands as closely as possible. To receive a requested QoS, the systems are required to pay a corresponding *communication cost*. Let $g^i_k(q^i_k)$ denote the incurred cost of communication services according to the QoS demands of system i. This naturally makes the problem a QoC–QoS trade-off since the joint control–communication cost can now be expressed as

$$\tilde{c}^i_k(x, u, q) = c^i_k(x, u) + g^i_k(q), \qquad \tilde{c}^i_T(x, q) = c^i_T(x) + g^i_T(q),$$

where we similarly define $\tilde{c}^i(x, u)$ for the infinite horizon case. For the finite horizon, we define $J^i_T(\mathcal{U}^i, \mathcal{Q}^i) \triangleq \mathsf{E}\Big[\sum_{k=0}^{T-1} \tilde{c}^i_k(x^i_k, u^i_k) + \tilde{c}^i_T(x^i_T)\Big]$, which is to be minimized by system i by jointly selecting $(\mathcal{U}^i, \mathcal{Q}^i)$. Let $\mathcal{U}^{i,*}$ and $\mathcal{Q}^{i,*}$ be:

$$(\mathcal{U}^{i,*}, \mathcal{Q}^{i,*}) \triangleq \arg\min_{\mathcal{U}^i, \mathcal{Q}^i} \quad J^i_T(\mathcal{U}^i, \mathcal{Q}^i). \quad (6)$$

The infinite horizon co-design problems can be similarly defined using J^i_∞ instead of J^i_T in (6). Most of the following analysis and discussions hold for both finite and infinite horizons as well as for both discounted and averaged costs. For brevity we will use the notation J^i to denote all the three cases J^i_T, J^i_{avg}, and J^i_{dis}.

B. Regret-Optimal Resource Allocation

The local systems solve their respective co-design problems (6) and request QoSs according to the obtained $\mathcal{Q}^{i,*}$. However, the network may not be able to perfectly meet the demands of all the systems due to resource constraints. Let $\mathcal{Q}^{i,r} = \{q^{i,r}_0, q^{i,r}_1, \dots\}$ denote the QoS served by the network to system i, i.e., system i receives service $q^{i,r}_k$ which may differ from q^i_k. Therefore, the QoC of system i becomes

$$J^{i,r} = \min_{\mathcal{U}^i} J^i(\mathcal{U}^i, \mathcal{Q}^{i,r}).$$

The associated local and social regret functions [3] become

$$R^i = J^{i,r} - J^{i,*}, \qquad R^{\mathrm{social}} = \sum_{i=1}^{N} v^i(R^i), \quad (7)$$

where $v^i(\cdot)$ is a function that maps the influence of the individual regrets on the social regret (e.g., $v^i(R^i) = e^{R^i}$).

Notice that the regret is a function of the allocated QoS ($\mathcal{Q}^{i,r}$). The network objective layer is to decide on the service allocations for each system to minimize the social regret, i.e.,

$$\min_{\{\mathcal{Q}^{i,r}\}_{i=1}^N} \quad R^{\mathrm{social}},$$
$$\text{subject to Resource Allocation Constraints}, \quad (8)$$

where the resource constraints are generally in the form of capacity constraints that limit the number of users that can simultaneously access a service; see Section IV for details.

In Part I [3], we discussed different architectures for the network manager to solve (8) under different *awareness structures*. In this Part II, we focus on the regret optimization based on the *passive physical layer* model introduced in Part I. Therefore, the QoS allocation problem needs to be solved only once, and not for each time k.

For GM systems with the per-stage cost function (3), the optimization problem (6) is convex (quadratic program) with respect to $\mathcal{U}^i$ for any fixed $\mathcal{Q}^i$, which yields a closed-form solution for $\mathcal{U}^{i,*}$. Similarly, the computation of the regret functions (7) is tractable since the computation of $J^{i,r}$ is analytical for any given $\mathcal{Q}^{i,r}$. These features make it possible to efficiently quantify the effects of QoS on QoC.

C. Coupling Between QoC and QoS

To quantify the effects of QoS on $J^i(\mathcal{U}^i, \mathcal{Q}^i)$, we find the optimal controller $\mathcal{U}^{i,*}$ that minimizes $J^i(\mathcal{U}^i, \mathcal{Q}^i)$ while keeping $\mathcal{Q}^i$ fixed. As mentioned earlier, the optimization problem has the analytical solution [6, Ch. 5]:

$$u^{i,*}_k = \mathcal{U}^{i,*}_k(\mathcal{Z}^i_k) = -L^{i,*}_k \mathsf{E}[x^i_k | \mathcal{Z}^i_k]. \quad (9)$$

For a finite horizon problem, matrix $L_k^{i,*}$ is given in (11). For the infinite horizon problem, $L_k^{i,*}$ does not depend on k and given in (14a) and (15a) for average and discounted costs, respectively. Under the optimal policy (9), the finite horizon cost reduces to

$$J_T^i = \mathsf{E}\Big[\sum_{k=0}^{T-1} \|e_k^i\|^2_{S_k^i} + g_k^i(q_k^i)\Big] + \mathsf{Tr}(P_0^i\Sigma^i) + \sum_{k=0}^{T-1}\mathsf{Tr}(P_k^i W^i), \tag{10}$$

where the error e_k^i is defined as the difference between the true state x_k^i and the estimated state $\mathsf{E}[x_k^i|\mathcal{Z}_k^i]$.

Optimal Control Parameters [5]:

The control parameters in (9), (10), and (16) have the following structures for the finite and infinite horizon problems:

Case I: Finite Horizon

$$L_k^i = -\left(N_k^i + (B^i)^\top P_{k+1}^i (B^i)\right)^{-1} (B^i)^\top P_{k+1}^i A^i, \tag{11}$$

$$\begin{aligned} P_k^i &= M_k^i + (A^i)^\top P_{k+1}^i A^i \\ &\quad - (A^i)^\top P_{k+1}^i B^i \left(N_k^i + (B^i)^\top P_{k+1}^i (B^i)\right)^{-1} \\ &\quad \times (B^i)^\top P_{k+1}^i A^i, \end{aligned} \tag{12}$$

$$P_T^i = M_T^i, \quad S_k^i = M_k^i + (A^i)^\top P_{k+1}^i A^i - P_k^i. \tag{13}$$

Case II(a): Average Cost Infinite Horizon

$$L_{\text{avg}}^{i,*} = \left(N^i + (B^i)^\top P_{\text{avg}}^i (B^i)\right)^{-1} (B^i)^\top P_{\text{avg}}^i A^i, \tag{14a}$$

$$\begin{aligned} P_{\text{avg}}^i &= M^i + (A^i)^\top P_{\text{avg}}^i A^i - (A^i)^\top P_{\text{avg}}^i B^i \\ &\quad \times \left(N^i + (B^i)^\top P_{\text{avg}}^i (B^i)\right)^{-1} \times (B^i)^\top P_{\text{avg}}^i A^i, \end{aligned} \tag{14b}$$

$$S_{\text{avg}}^i = M^i + (A^i)^\top P_{\text{avg}}^i A^i - P_{\text{avg}}^i. \tag{14c}$$

Case II(a): Discounted Cost Infinite Horizon

$$L_{\text{dis}}^{i,*} = \gamma\left(N^i + (B^i)^\top P_{\text{dis}}^i (B^i)\right)^{-1} (B^i)^\top P_{\text{dis}}^i A^i, \tag{15a}$$

$$\begin{aligned} P_{\text{dis}}^i &= M^i + \gamma (A^i)^\top P_{\text{dis}}^i A^i - \gamma^2 (A^i)^\top P_{\text{dis}}^i B^i \\ &\quad \times \left(N^i + \gamma (B^i)^\top P_{\text{dis}}^i (B^i)\right)^{-1} \times (B^i)^\top P_{\text{dis}}^i A^i, \end{aligned} \tag{15b}$$

$$S_{\text{dis}}^i = M^i + (A^i)^\top P_{\text{dis}}^i A^i - P_{\text{dis}}^i. \tag{15c}$$

Recall that Σ^i and W^i represent the covariances of the initial state x_0^i and disturbances w_k^i, for all k. The matrices P_k^i and S_k^i are defined in (12) and (13), respectively. Similarly, the average and discounted costs J_{avg}^i and J_{dis}^i are

$$\begin{aligned} J_{\text{avg}}^i &= \limsup_{T\to\infty} \frac{1}{T}\mathsf{E}\Big[\sum_{k=0}^{T-1} \|e_k^i\|^2_{S_{\text{avg}}^i} + g^i(q_k^i)\Big] \\ &\quad + \mathsf{Tr}(P_{\text{avg}}^i W^i), \end{aligned} \tag{16a}$$

$$\begin{aligned} J_{\text{dis}}^i &= \mathsf{E}\Big[\sum_{k=0}^{\infty} \gamma^k \big(\|e_k^i\|^2_{S_{\text{dis}}^i} + g^i(q_k^i)\Big] \\ &\quad + \gamma(1-\gamma)^{-1}\mathsf{Tr}(P_{\text{dis}}^i W^i), \end{aligned} \tag{16b}$$

where $P_{\text{avg}}^i, S_{\text{avg}}^i, P_{\text{dis}}^i$ and S_{dis}^i are given in (14) and (15).

The quality of the state estimate depends on the received information $\mathcal{Z}_k^i$ at the controller side, and affects the cost J^i. Recall that $\mathcal{Z}_k^i$ depends on how QoS is allocated according to $\mathcal{Q}^i$, and thus, the estimate is affected by the QoS. Therefore, from (16) it is evident that the QoC (i.e., $J^i(\mathcal{U}^{i,*}, \mathcal{Q}^i)$) depends on $\mathcal{Q}^i$ through the estimation error e_k^i and the communication cost $g_k^i(q_k^i)$.

III. QoS Optimization

In Section II-C, we simplified J^i to the form (16) for any given $\mathcal{Q}^i$. Here, we further analyze the error term e_k^i and find a direct relationship between QoS and e_k^i so that we can optimize J^i in (16) to obtain the optimal QoS ($\mathcal{Q}^{i,*}$) for system i. To this end, let us denote the set of available network QoS by $\{s^1, s^2, \ldots, s^q\}$, where each service $s^i \triangleq (\ell^i, p^i)$ is characterized by a latency ℓ^i and a reliability p^i. $p^i \in [0,1]$ is the packet loss probability. We assume that the transmission is noiseless and the packet drops are i.i.d.

A. Finite Horizon Case

Since each QoS comes with different delays and packet drop probabilities, out-of-order packet delivery may happen regularly. The information at the controller side at time k is given by $\mathcal{Z}_k^i = \{x_{k_1}^i, x_{k_2}^i, \ldots x_{k_m}^i\}$, where $0 \le k_1 < k_2 < \cdots < k_m \le k$ and k_i denotes the time instance for which the transmitted packet is successfully delivered to the controller. Note that the packet originated at k_i may have experienced a delay (depending on the chosen QoS) and arrived at the controller at a delayed time instance.

Due to the Markovian nature of the system dynamics (1), we have

$$\mathsf{E}[x_k^i|\mathcal{Z}_k^i] = \mathsf{E}[x_k^i|x_{k_1}^i, x_{k_2}^i, \ldots x_{k_m}^i] = \mathsf{E}[x_k^i|x_{k_m}^i]. \tag{17}$$

The time index k_m is random due to the stochastic nature of the packet dropout model. Let random variable τ_k^i denote the latest time instance that a measurement has arrived at the controller of system i. We use $\tau_k^i = -1$, to denote that no information has arrived up to time k, i.e., $\mathcal{Z}_k^i = \varnothing$. Given (1) and (17), we obtain

$$e_k^i = \sum_{t=\tau_k^i}^{k-1} (A^i)^{k-t-1} w_t^i, \tag{18}$$

where we define $w_{-1}^i \triangleq x_0^i$; see [2] for details. Our objective is to express $\|e_k^i\|^2_{S_k^i}$ in (10) as an explicit function of the chosen QoS. We next derive the distribution of τ_k^i since it will be essential to simplify $\mathsf{E}[\|e_k^i\|^2_{S_k^i}]$ using (18).

Remark 1: $(k-\tau_k^i)$ represents the *Age-of-Information* (AoI) [7] at the estimator. While minimizing the (area under the) AoI has been widely used in the literature, the effects of AoI on the estimator, as seen in (18), is more complicated and closely coupled with the physical parameter A^i.

Given that the QoS is s^j for time k (i.e., $\mathcal{Q}_k^{i,*}(\mathcal{Z}_k^i) = s^j$), the transmitted packet at time k will either be received at $k + \ell^j$ with probability $1 - p^j$ or will be dropped with probability p^j. Therefore, by denoting the latency experienced by the transmitted packet for system i at time k by δ_k^i, we obtain

$$\mathbb{P}(\delta_k^i = d|\, \mathcal{Q}^{i,*}(\mathcal{Z}_k^i) = s^j) = \begin{cases} (1-p^j)\mathbf{1}_{\{d=\ell^j\}}, & d < \infty, \\ p^j, & d = \infty, \end{cases}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function. Further, we define a binary variable θ_k^{ij} to denote whether the j-th QoS was used by the i-th system at time k, i.e., $\theta_k^{ij} = \mathbf{1}_{\{q_k^i = s^j\}}$ with the service allocation constraint $\sum_{j=1}^q \theta_k^{ij} = 1$ for all i and k. Based on these definitions, one may obtain

$$\mathbb{P}(\delta_k^i = d|\mathcal{Z}_k^i) = \sum_{j=1}^q \theta_k^{ij}\mathbb{P}(\delta_k^i = d|\, \mathcal{Q}^{i,*}(\mathcal{Z}_k^i) = s^j) \triangleq \mu_k^i(d),$$

$$\mathbb{P}(\tau_k^i = k-d) = \left[\prod_{t=0}^{d-1}\left(1 - \sum_{r=0}^{t}\mu_{k-t}^i(r)\right)\right]\sum_{r=0}^{d}\mu_{k-d}^i(r),$$

$$\mathbb{P}(\tau_k^i = -1) = \left[\prod_{t=0}^{k}\left(1 - \sum_{r=0}^{t}\mu_{k-t}^i(r)\right)\right]. \tag{19}$$

Therefore, for any positive definite matrix M, we have

$$\mathsf{E}[\|e_k^i\|^2_M] = \mathsf{E}\left[\sum_{t=\tau_k^i}^{k-1}\mathsf{Tr}\Big(((A^i)^{k-t-1})^\top M (A^i)^{k-t-1}\Phi_k^i\Big)\right], \tag{20}$$

where $\Phi_k^i \triangleq \mathsf{E}[w_k^i {w_k^i}^\top] = W^i$ for $k \ge 0$, and $\Phi_{-1}^i \triangleq \mathsf{E}[x_0^i {x_0^i}^\top] = \Sigma^i$. Note that the expectation on the right hand side of (20) is taken over the randomness of τ_k^i. Since the distribution of τ_k^i depends on the chosen service s^j (as shown in (19)), we simplify (20) and obtain

$$\mathsf{E}[\|e_k^i\|_M^2] = \sum_{t=-1}^{k-1} \mathsf{Tr}\Big(((A^i)^{k-t-1})^\top M (A^i)^{k-t-1} \Phi_k^i\Big) \mathbb{P}(\tau_k \le t).$$

Using the above derivations, we may now rewrite (10) as

$$J_T^i = \mathsf{E}\Big[\sum_{k=0}^{T-1} g_k^i(q_k^i)\Big] + \mathsf{Tr}(P_0^i \Sigma^i) + \sum_{k=0}^{T-1} \mathsf{Tr}(\tilde{P}_k^i \Phi_{k-1}^i), \quad (21)$$

where

$$\tilde{P}_k^i = P_k^i + \Upsilon_k^i, \qquad \Upsilon_T^i = 0,$$

$$\Upsilon_k^i = \sum\nolimits_{t=k}^{T-1} \mathbb{P}(\tau_{t-1} \le k)\big((A^i)^{t-k}\big)^\top S_t^i (A^i)^{t-k}.$$

We refer to [2] for the details of the above derivations. Note that the choice of service θ_k^{ij} affects the QoS cost $\sum_{k=0}^{T-1} g_k^i(q_k^i)$ as well as $\tilde{P}_k^i$ via the distribution of τ_k^i. Therefore, the best choice of θ_k^{ij} can be found by optimizing (21) as a mixed-integer linear program which is illustrated through a numerical example in Sec. IV.

B. Infinite Horizon Case

To analyze the asymptotic behavior, we consider stationary policies. The requested service q_k^i in (16) does not change with time, i.e, for all k, $q_k^i \equiv s^j$ for some j. At each time k, the transmitted data by system i will then be dropped with a probability p^j or will be received at the controller at time $k + \ell^j$ with probability $1 - p^j$. For service s^j, we may write

$$\mathbb{P}(\tau_k^i = (k - \ell^j) - k_0) = (1 - p^j)(p^j)^{k_0}, \forall k_0 \le k - \ell^j,$$
$$\mathbb{P}(\tau_k^i = -1) = (p^j)^{k-\ell^j+1}. \quad (22)$$

Using the expression of e_k^i (18) together with (22), we obtain

$$\mathsf{E}[\|e_k^i\|_M^2] = \mathsf{E}\Big[\sum\nolimits_{t=\tau_k^i}^{k-1} \mathsf{Tr}\Big(((A^i)^{k-t-1})^\top M (A^i)^{k-t-1} \Phi_k^i\Big)\Big], \quad (23)$$

where Φ_k^i was defined after (20). Upon further simplifications,

$$\mathsf{E}[\|e_k^i\|_M^2] = \mathsf{Tr}(M\Psi_k)(p^j)^{k-\ell^j+1} + \sum\nolimits_{t=0}^{\ell^j-1} \mathsf{Tr}(M\Pi_t)$$
$$+ \sum\nolimits_{t=\ell^j}^{k-1} \mathsf{Tr}(M\Pi_t)(p^j)^{t-\ell^j+1}, \quad (24)$$

where $\Psi_t = (A^i)^t \Sigma_0^i ((A^i)^t)^\top$ and $\Pi_t = (A^i)^t W^i (A^i)^{t^\top}$. For $k < \ell^j$, none of the packets has arrived to the controller due to delay. Therefore, $\tau_k^i = -1$, and we obtain

$$\mathsf{E}[\|e_k^i\|_M^2] = \mathsf{Tr}(M\Psi_k) + \sum\nolimits_{t=0}^{k-1} \mathsf{Tr}(M\Pi_t), \quad \forall k < \ell^j.$$

Several remarks can be made from the expression in (24). The reliability p^j affects the first and third terms on the right side of the equality, whereas the latency affects all the three terms. The second term appears in (24), since regardless of the channel reliability, the non-zero latency ℓ^j will result in delayed arrival of measurement at the controller. For a perfectly reliable channel (i.e., $p^j = 0$), this term denotes the effect of latency on J^i. Moreover, we notice that the second term in (24) does not depend on k, whereas the other two terms do. In fact, for certain choices of A^i and p^j, the first and third terms approach to infinity as $k \to \infty$. In order for J_{avg}^i (or J_{dis}^i) to be finite, the service s^j should satisfy the following conditions (see e.g., [8] for a similar result).

Proposition 1: Necessary and sufficient conditions on the probability p^j for J_{avg}^i and J_{dis}^i to be bounded are, respectively,

$$p_{\text{avg}}^j < 1/\|A^i\|^2, \qquad \text{and} \qquad p_{\text{dis}}^j < 1/(\gamma\|A^i\|^2).$$

Remark 2: Reliability plays a crucial role for closed-loop stability. Although latency affects the QoC, it does not make a system unstable as long as it remains bounded. △

TABLE I

SYSTEM PARAMETERS

sys ID.	Physical System Parameters					
	a	b	m	n	P_{avg}	S_{avg}
1	2	1	1	1	4.24	13.71
2	2	1	1	1	4.24	13.71
3	2	1	2	1	5.37	18.12
4	0.5	2	1	2	0.78	0.12
5	0.5	2	1	2	0.78	0.12

	QoS Parameters		
	ℓ^j	p^j	λ^j
s^1	0	0.01	300
s^2	3	0.1	100
s^3	1	0.3	100
s^4	∞	1	0

Assuming that the chosen QoS satisfies Proposition 1, we simplify the expressions in (16) to obtain

$$J_{\text{avg}}^i = \limsup_{T\to\infty} \mathsf{E}\Big[\sum_{k=\ell^j}^{T-1} \mathsf{Tr}(S_{\text{avg}}^i \Pi_k)(p^j)^{k-\ell^j+1} + \frac{1}{T}\sum_{k=0}^{T-1} g^i(q^i)\Big]$$
$$+ \mathsf{Tr}(P_{\text{avg}}^i W^i) + \sum\nolimits_{k=0}^{\ell^j-1} \mathsf{Tr}(S_{\text{avg}}^i \Pi_t), \quad (25a)$$

$$J_{\text{dis}}^i = \sum\nolimits_{k=0}^{\ell^j-1} \gamma^k \,\mathsf{Tr}(S_{\text{dis}}^i(\Psi_k + \bar{\gamma}\Pi_k)) + \sum\nolimits_{k=\ell^j}^{\infty} \gamma^k (p^j)^{k-\ell^j+1}$$
$$\times \mathsf{Tr}(S_{\text{dis}}^i(\Psi_k + \bar{\gamma}\Pi_k)) + \mathsf{Tr}(P_{\text{dis}}^i W^i), \quad (25b)$$

where $\bar{\gamma} = \gamma/(1-\gamma)$. The effects of service s^j on the cost follows from (25), where we explicitly have the latency ℓ^j and reliability p^j incorporated into the costs J_{avg}^i and J_{dis}^i.

By optimizing (25), the physical systems are able to pick the optimal QoS from the given set $\{s^1, \dots s^q\}$. This completes the optimal control and QoS co-design procedure. The preference on the QoS is relayed to the network layer. Upon receiving the preferences from all systems, the network manager is able to allocate the service to each system in a regret-optimal manner by executing the optimization in (8).

Remark 3: If the $g^i(q_k^i)$ terms are excluded from J^i in (16), then J^i is a decreasing function of p^j and ℓ^j, as expected. However, (25) shows that there is a less obvious a trade-off between latency and reliability: the same QoC can be achieved by either a high-latency/high-reliability service or a low-latency/low-reliability service. This provides flexibility for QoS allocation without sacrificing QoC. This is particularly useful if a requested service is unavailable but a performance-equivalent service is available. △

IV. A NUMERICAL CASE STUDY: SCALAR GM SYSTEMS

To numerically illustrate the proposed regret-optimal co-design framework, we consider a heterogeneous NCS consisting of five scalar systems with the following GM dynamics

$$x_{k+1}^i = a^i x_k^i + b^i u_k^i + w_k^i, \; w_k^i \sim \mathcal{N}(0,1), \; x_0^i \sim \mathcal{N}(0,1).$$

We focus on an infinite horizon problem optimizing J_{avg}^i. The quadratic control cost function in (3) is reduced to $m^i(x_k^i)^2 + n^i(u_k^i)^2$. The parameters are chosen arbitrarily and are listed in Table I. For system-i, P_{avg}^i in (14b) becomes

$$P_{\text{avg}}^i = m^i + a_i^2 P_{\text{avg}}^i - a_i^2 b_i^2 (P_{\text{avg}}^i)^2 \big(n^i + b_i^2 P_{\text{avg}}^i\big)^{-1},$$

which is a quadratic in P_{avg}^i having exactly one positive root. The terms L_{avg}^i and S_{avg}^i introduced in (14a) and (14c) become

$$L_{\text{avg}}^i = \frac{a^i b^i P_{\text{avg}}^i}{n^i + (b^i)^2 P_{\text{avg}}^i}, \qquad S_{\text{avg}}^i = m^i + ((a^i)^2 - 1) P_{\text{avg}}^i.$$

For our given systems parameters, numerical values of P_{avg}^i and S_{avg}^i are reported in Table I.

We consider three available services $\{s^1, s^2, s^3\}$, where s^1 is an ultra-low-latency/highly-reliable slice, s^2 provides low-latency/medium-reliable service, and s^3 is a low-latency/low-reliable service. We also add a virtual service s^4 to denote "no service". The systems can select s^4 if they do not wish to communicate. Similarly, the network manager is allowed to allocate this "no service" either when systems

TABLE II
J^i_{avg} UNDER DIFFERENT QoS

	s^1	s^2	s^3	s^4
J^1_{avg}	**318.515**	393.570	∞	∞
J^2_{avg}	**318.515**	393.570	∞	∞
J^3_{avg}	**324.244**	387.759	∞	∞
J^4_{avg}	300.904	100.942	100.914	**0.944**
J^5_{avg}	300.904	100.942	100.914	**0.944**

sys ID.	Allocated service
1	s^1
2	s^2
3	s^2
4	s^4
5	s^4

request it or there is no other available service. Each service s^j comes with an associated service cost λ^j. The service parameters are listed in Table I. Service s^1 is the most costly, whereas s^2 and s^3 incur the same cost. In addition, the services have limited capacities q^j_{total} indicating how many systems can use the service at a time. In this example, $q^j_{\text{total}} = 1, 2, 2, \infty$ for s^1, s^2, s^3 and s^4, respectively.

From Proposition 1 it follows that the first three systems in Table I require $p^j < 0.25$, therefore, s^3 with $p^3 = 0.3$ is not a feasible choice for them. While allocating the services to the systems, the network manager needs to explicitly consider this constraint in the optimization problem (8).[1]

The cost function (25) can be computed for service s^j as

$$J^i_{\text{avg}}(s^j) = S^i_{\text{avg}} \sum_{k=\ell^j}^{\infty} (a^i \sqrt{p^j})^{2k} + \lambda_j + P^i_{\text{avg}} + S^i_{\text{avg}} \sum_{k=0}^{\ell^j - 1} (a^i)^{2k}$$
$$= S^i_{\text{avg}} \left(\frac{1-(a^i)^{2\ell^j}}{1-(a^i)^2} + \frac{(a^i\sqrt{p^j})^{2\ell^j}}{1-(a^i\sqrt{p^j})^2} \right) + \lambda_j + P^i_{\text{avg}}.$$

System i selects service s^j to minimize $J^i_{\text{avg}}(s^j)$. Table II reports J^i_{avg} under different services for all five systems. We notice that service s^1 is requested by systems 1, 2, and 3; s^2 and s^3 are not requested by any system; and s^4 is requested by systems 4 and 5. Systems 4 and 5 are stable and can perform well without its sensor measurements. The improvement in their QoC by using a service s^i is less than the cost λ^i paid for the service and therefore, they chose "no service", i.e., s^4.

Due to the capacity constraints q^j_{total}, the systems will not always be serviced exactly as they requested, and the network manager allocates resources while minimizing the social regret. Let $\theta^{ij} \in \{0, 1\}$ denote whether service s^j is allocated to system i, i.e., $\theta^{ij} = 1$ if and only if $q^{i,r} = s^j$. For this example, the network manager considers the regret function $v^i(R^i) = R^i = J^{i,r} - J^{i,*}$. Since $J^{i,*}$ is constant, optimizing $\sum_{i=1}^{3} R^i$ is equivalent to optimizing $\sum_{i=1}^{3} J^{i,r}$. Therefore, the network manager solves

$$\begin{aligned} \min_{\theta^{ij} \in \{0,1\}} & \sum\nolimits_{i=1}^{5} \sum\nolimits_{j=1}^{3} \theta^{ij} J^i_{\text{avg}}(s^j) \\ \text{s. t. } & \sum\nolimits_{j=1}^{3} \theta^{ij} = 1, \quad \sum\nolimits_{i=1}^{5} \theta^{ij} \le q^j_{\text{total}}, \\ & \sum\nolimits_{j=1}^{3} \theta^{ij} \sqrt{p^j} < (a^i)^{-2}, \forall i = 1, \dots, 5, j = 1, \dots, 4. \end{aligned} \tag{26}$$

The first constraint ensures that each system is assigned exactly one service. The second ensures the number of allocated systems to a service does not exceed the service capacity q^j_{total}. The last one guarantees stability according to Proposition 1.

[1] Recall the parameter ϵ^i_q from Part I which generally accounts for such QoS constraints.

The optimization variables are $\{\theta^{ij}\}$'s as shown under the min operator. The cost function and all the cosntraints are linear in the optimization variables, which makes the overall problem a mixed-integer linear program (MILP). By solving this MILP, the regret-optimal resource allocation is obtained. In our case, systems 4 and 5 are assigned to s^4 (i.e., no service), system 3 is assigned to s^2, and for the remaining two systems (1 and 2), one gets the requested service s^1 and the other gets s^2. The requested services are highlighted by the bold font in Table II.

V. DISCUSSIONS AND CONCLUSION

We addressed the QoC–QoS co-design problem for GM dynamics controlled systems, quantifying communication QoS's impact on control performance. We derived analytical expressions for optimal control and communication service demands, enabling an efficient regret-optimal resource allocation. Our findings showed that reliability cannot be decreased beyond a certain limit without destabilizing the closed-loop systems. Although not discussed in this letter, the data-rate also affects the stability of NCSs, as discussed in [9].

This work primarily focuses on the *passive physical layer* model for regret-optimal resource allocation introduced in Part I. However, a similar approach can be applied to the *reactive physical layer* model presented in Part I. In the reactive model, (26) is solved at each time instance. The network manager allocates QoS at time index k based on this solution, and the physical systems assess the optimal QoS for time $k+1$, sending new requests to the network manager. This iterative process involves continuous interaction between the network manager and physical systems at every time instance.

APPENDIX

Proof of Proposition 1

In order for J^i_{avg} to have a finite value, a necessary and sufficient condition is that $\lim_{k\to\infty} \mathsf{E}[\|e^i_k\|^2_{S^i_{\text{avg}}}]$ is a finite quantity. From (24), $\lim_{k\to\infty} \mathsf{E}[\|e^i_k\|^2_{S^i_{\text{avg}}}]$ is finite if and only if $\lim_{k\to\infty} \mathsf{Tr}(M\Pi_k)(p^j)^k = 0$, which is equivalent to $p^j\|A^i\|^2 < 1$. Similarly, for the discounted cost, $\lim_{k\to\infty} \gamma^k \mathsf{E}[\|e^i_k\|^2_{S^i_{\text{avg}}}] = 0$ needs to hold, which is equivalent to $\gamma p^j \|A^i\|^2 < 1$. ■

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