SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

July 2023, Vol. 66 170204:1–170204:15
https://doi.org/10.1007/s11432-022-3791-8

Special Topic: Entanglement of Control and Optimization

Predefined-time distributed multiobjective optimization for network resource allocation

Kunpeng ZHANG[1], Lei XU[1], Xinlei YI[2], Zhengtao DING[3], Karl H. JOHANSSON[2], Tianyou CHAI[1] & Tao YANG[1*]

[1]*State Key Laboratory of Synthetical Automation for Process Industries, Northeastern University, Shenyang* 110819*, China;*
[2]*School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Stockholm* 100 44*, Sweden;*
[3]*School of Electrical and Electronic Engineering, The University of Manchester, Manchester* M13 9PL*, UK*

Received 18 August 2022/Revised 18 March 2023/Accepted 10 May 2023/Published online 27 June 2023

Abstract We consider the multiobjective optimization problem for the resource allocation of the multiagent network, where each agent contains multiple conflicting local objective functions. The goal is to find compromise solutions minimizing all local objective functions subject to resource constraints as much as possible, i.e., the Pareto optimums. To this end, we first reformulate the multiobjective optimization problem into one single-objective distributed optimization problem by using the weighted L_p preference index, where the weighting factors of all local objective functions are obtained from the optimization procedure so that the optimizer of the latter is the desired Pareto optimum of the former. Next, we propose novel predefined-time algorithms to solve the reformulated problem by time-based generators. We show that the reformulated problem is solved within a predefined time if the local objective functions are strongly convex and smooth. Moreover, the settling time can be arbitrarily preset since it does not depend on the initial values and designed parameters. Finally, numerical simulations are presented to illustrate the effectiveness of the proposed algorithms.

Keywords distributed optimization, multiobjective optimization, predefined-time algorithms, time-based generators, weighted L_p preference index

Citation Zhang K P, Xu L, Yi X L, et al. Predefined-time distributed multiobjective optimization for network resource allocation. Sci China Inf Sci, 2023, 66(7): 170204, https://doi.org/10.1007/s11432-022-3791-8

1 Introduction

Recently, resource allocation problems have been substantially studied owing to their wide applications, including economic dispatch [1–3], robot networks [4], and transportation systems [5]. The goal is to minimize the sum of all local objective functions subject to resource constraints. Existing studies on resource allocation problems mainly focus on the single-objective case where each agent contains a single local objective function; see [6–8], recent survey paper [9] and references herein. However, many practical applications tend to contain multiple objective functions. For example, the microgrid in [10] includes three conflicting objective functions, which are economic, environmental and technical objective functions, respectively. The related methods solving such a multiobjective optimization problem are generally summarized into two categories. One is the scalarization approach [11]. For example, in [12–14], the multiple objective functions are weighted and summed into a new objective function; then the multiobjective optimization problems are indirectly solved by single-objective methods. The other one is the evolutionary approach [15], such as ant colony optimization algorithm [16], genetic algorithm [17], particle swarm optimization algorithm [18], nondominated sorting genetic algorithm II [19], and multiobjective evolutionary algorithm based on decomposition [20]. The evolutionary approach tends to be stochastic and lacks theoretical guarantees. On the contrary, owing to the development of single-objective methods, the scalarization approach has a solid theoretical foundation; see [21, 22].

* Corresponding author (email: yangtao@mail.neu.edu.cn)

The aforementioned studies in [11–22] only provide centralized algorithms, where a center exists to collect and compute all objective functions. Noticeably, more objectives tremendously increase the computational burden of the center. Even for the single-objective case, the center struggles with computation and interaction in large-scale networks. Moreover, centralized algorithms are subject to performance limitations, such as a single point of failure, high communication requirement, and limited flexibility and scalability. Therefore, distributed algorithms have been proposed to overcome the limitations where each agent only communicates with its neighboring agents and there is no center; see recent survey studies [9,23] and references herein. Recently, by taking advantage of distributed methods for the single-objective case, Ref. [24] proposed a distributed multiobjective algorithm for the neurodynamic system. Ref. [25] developed a distributed diffusion adaptation strategy for the multiobjective optimization problem. Ref. [26] proposed a distributed framework based on weighted L_p preference index for the constrained multiobjective optimization problem. Moreover, that framework provides a unique robust Pareto optimum, and it does not use any prior knowledge that is often unavailable in many real-world applications; see [27, 28].

The above-mentioned distributed algorithms in [24–26] asymptotically or exponentially converge to the Pareto optimum, which means the Pareto optimum is only obtained as time goes to infinity. However, it is desired that the Pareto optimum or its neighboring solution is obtained within a certain time in time-critical applications, such as distributed energy resource distribution [29]. To the best of our knowledge, such a problem has not been well studied in the literature. For the single-objective case, various finite/fixed/predefined-time distributed algorithms have been proposed in [30–35]. Note that the settling time based on finite/fixed-time convergence theories depends on the initial states or designed parameters. Instead, the settling time in [35] based on predefined-time convergence theory can be arbitrarily preset. Inspired by the aforementioned discussions, this paper considers the predefined-time distributed multiobjective optimization for network resource allocation.

The main contributions are summarized as follows.

(1) We reformulate the multiobjective optimization problem into a single-objective distributed optimization problem by the weighted L_p preference index, where the optimizer of the latter is a Pareto optimum of the former. Compared with [24, 25], which provide the weighting factors directly based on prior knowledge containing global information, the weighting factors in this paper are unknown but can be obtained from the optimization procedure without any prior knowledge, which guarantees that the optimizer of the reformulated single-objective optimization problem is the desired Pareto optimum of the multiobjective optimization problem.

(2) We propose novel predefined-time algorithms based on time-based generators for obtaining the optimizer of the reformulated single-objective optimization problem. Compared with [26], the settling time of the proposed algorithms can be arbitrarily preset since it does not depend on any initial values and designed parameters.

(3) We provide the convergence analysis of the proposed algorithms. Compared with [35], where each agent contains a single objective function, we prove that the proposed algorithms achieve predefine-time convergence, in which each agent contains multiple conflicting objective functions. The considered problem is more challenging because of the trade-off among the conflicting objective functions.

The rest of the paper is organized as follows. Section 2 provides some preliminaries and the multiobjective optimization problem. Section 3 presents the proposed algorithms and convergence analysis. Section 4 utilizes an example to carry out the verification of the proposed algorithms. Finally, Section 5 offers the conclusion.

Notations. $\mathbb{Z}^+$ stands for the set of positive integers. $\mathbb{R}$ and $\mathbb{R}^N$ stand for the set of real numbers and column vectors involving N dimensions, respectively. $\|\cdot\|$ stands for Euclidean norm of vectors. I_N stands for the identity matrix with N dimensions. $\mathfrak{I}_K$ denotes the set $\{1, 2, \ldots, K\}$. $\mathbf{1}_N$ (or $\mathbf{0}_N$) represents the N-dimensional column vector whose component is all 1 (or 0). Given one vector $\boldsymbol{x}$ (or matrix X), $\boldsymbol{x}^{\mathrm{T}}$ (or X^{T}) represents its transposition transform, and $\mathrm{col}\,(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_N) = \left[\boldsymbol{x}_1^{\mathrm{T}}, \ldots, \boldsymbol{x}_N^{\mathrm{T}}\right]^{\mathrm{T}}$. Given one differentiable function C, ∇C represents its gradient.

2 Preliminaries and problem formulation

In this section, some preliminaries and the problem formulation are presented.

2.1 Graph theory

Consider an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with N agents, where $\mathcal{V} = \{1, \ldots, N\}$ stands for the set of agents and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ stands for the set of edges. The weighted adjacency matrix is $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$. If $(j, i) \in \mathcal{E}$, that is, agents j and i can communicate with each other, then a_{ij} is positive and is zero otherwise. $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ is the neighbor set of agent i. A path from agent i_1 to agent i_k is symbolized as $i_1, \ldots, i_k$, where $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \ldots, k-1$. The graph $\mathcal{G}$ is said to be connected if there exists at least one path between any two distinct agents. The Laplacian matrix is defined as $L = [L_{ij}] \in \mathbb{R}^{N \times N}$, where $L_{ii} = \sum_{j=1}^{N} a_{ij}$ and $L_{ij} = -a_{ij}$ for $i \neq j$. For an undirected and connected graph, its Laplacian matrix has an eigenvalue at zero and the other eigenvalues are positive.

2.2 Predefined-time convergence based on time-based generators

Consider a continuous differentiable function $\digamma(t)$ satisfying the following conditions in [35]:

$$\digamma(t) = \begin{cases} 0, & \text{if } t = 0, \\ 1, & \text{if } t \geqslant t_f, \end{cases} \quad \dot{\digamma}(t) = \begin{cases} 0, & \text{if } t = 0 \text{ or } t \geqslant t_f, \\ 1, & \text{if } 0 < t < t_f, \end{cases} \tag{1}$$

where t_f is a predefined time. The function $\digamma(t)$ is referred to as a time-based generator. Next, the following lemma is useful for subsequent analysis.

Lemma 1 ([35])**.** Consider the system as follows:

$$\dot{\boldsymbol{x}}(t) = -\hbar k(t) \boldsymbol{x}(t), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0 \in \mathbb{R}^n, \tag{2}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ with $n \in \mathbb{Z}^+$ is the state, $\hbar$ is a positive constant, $k(t) = \dot{\digamma}(t) / [1 - \digamma(t) + \sigma]$ with $\sigma \in (0, 1)$ being a design parameter, $\boldsymbol{x}(0)$ is the initial state, and $\boldsymbol{x}_0$ is a constant vector which can be arbitrarily chosen. Note that the equilibrium point of the system (2) is $\boldsymbol{x}(t) = \mathbf{0}_n$. The state $\boldsymbol{x}(t)$ converges to the value $[\sigma/(1+\sigma)]^{\hbar} \boldsymbol{x}_0$ at the predefined time t_f if the function $\digamma(t)$ is continuous differentiable and satisfies (1).

The definition of predefined-time convergence is given as follows.

Definition 1 ([35])**.** The system (2) is said to achieve predefined-time convergence when the following conditions are satisfied for arbitrary initial state $\boldsymbol{x}(0)$:

$$\begin{cases} \lim\limits_{t \to t_f} \|\boldsymbol{x}(t)\| \leqslant c, \\ \|\boldsymbol{x}(t)\| \leqslant c, \quad \forall t > t_f, \\ \lim\limits_{t \to \infty} \|\boldsymbol{x}(t)\| = 0, \end{cases} \tag{3}$$

where t_f is independent of the initial state $\boldsymbol{x}(0)$ and can be designed arbitrarily, and c is an arbitrarily small positive constant.

2.3 Pareto optimum and weighted L_p preference index

Consider a constrained multiobjective optimization problem with K conflicting objective functions:

$$\min_{\boldsymbol{x} \in \mathbb{R}^n} \left\{\mathbb{C}^1(\boldsymbol{x}), \ldots, \mathbb{C}^K(\boldsymbol{x})\right\}, \tag{4a}$$

$$\text{subject to } g(\boldsymbol{x}) = 0, \tag{4b}$$

where $\boldsymbol{x} \in \mathbb{R}^n$ with $n \in \mathbb{Z}^+$ is the decision variable, $\mathbb{C}^k : \mathbb{R}^n \to \mathbb{R}$ for $k \in \Im_K$ is the k-th objective function, $g(\boldsymbol{x})$ is the constraint function, and its Pareto optimum is defined as follows.

Definition 2 ([36])**.** A decision $\boldsymbol{x}^* \in \mathbb{R}^n$ is a Pareto optimum of the constrained multiobjective optimization problem (4) if there does not exist any other decision $\boldsymbol{x} \in \mathbb{R}^n$ satisfying the equality constraint (4b) such that $\mathbb{C}^k(\boldsymbol{x}) \leqslant \mathbb{C}^k(\boldsymbol{x}^*)$ for all $k \in \Im_K$ and $\mathbb{C}^j(\boldsymbol{x}) < \mathbb{C}^j(\boldsymbol{x}^*)$ for at least one $j \in \Im_K$.

Definition 3 ([37])**.** The weighted L_p preference index can be written as $\eth = [\sum_{k=1}^{K} w_k (\mathbb{C}^k(\boldsymbol{x}) - \mathbb{C}^{k*}(\boldsymbol{x}))^p]^{\frac{1}{p}}$ with $p \in [1, \infty)$ if $\mathbb{C}^{k*}$ and w_k are the infimum and the weighting factor of $\mathbb{C}^k$ of the constrained multiobjective optimization problem (4), respectively.

2.4 Problem formulation

We consider the following multiobjective optimization problem for the resource allocation of the multiagent network containing N agents:

$$\min_{\boldsymbol{x}\in\mathbb{R}^N} \left\{C_i^1(x_i),\ldots,C_i^K(x_i)\right\}, \text{ for all } i\in\mathcal{V}, \tag{5a}$$

$$\text{subject to } \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} d_i, \tag{5b}$$

where $\boldsymbol{x}=\mathrm{col}(x_1,\ldots,x_N)\in\mathbb{R}^N$ is the global decision variable, $x_i\in\mathbb{R}$ is the local decision variable of agent i, $C_i^k(x_i):\mathbb{R}\to\mathbb{R}$ is the k-th local objective function of agent i, K is the number of the conflicting objective functions of agent i, $d_i\in\mathbb{R}$ is the local constraint of agent i, and $\sum_{i=1}^{N} d_i$ is the global constraint.

Our objective is to find the desired Pareto optimum of the problem (5) in a distributed manner. Specifically, agent i for $i\in\mathcal{V}$ has information only about its local objective functions $\left\{C_i^1(x_i),\ldots,C_i^K(x_i)\right\}$, and minimizes its local objective functions subject to resource constraints while communicating with its neighboring agents via the underlying communication network modeled by an undirected graph $\mathcal{G}$. In the following, we reformulate the multiobjective optimization problem (5) into a single-objective distributed optimization problem subject to resource constraints by using the weighted L_p preference index in [32], and also illustrate that the optimal global decision variable of the reformulated single-objective optimization problem is the desired Pareto optimum of the multiobjective optimization problem (5) by an example.

Before presenting the reformulated single-objective optimization problem, we first define several important symbols: $\tilde{x}_i^{k*}$, ω_i^{k*}, and $\hat{x}_i^{k*}$. Specifically, for any $k\in\Im_K$, $\tilde{\boldsymbol{x}}^{k*}=\mathrm{col}\left(\tilde{x}_1^{k*},\ldots,\tilde{x}_N^{k*}\right)\in\mathbb{R}^N$ is the optimal global decision variable of the following constrained optimization problem:

$$\min_{\tilde{\boldsymbol{x}}^k\in\mathbb{R}^N} \sum_{i=1}^{N} C_i^k\left(\tilde{x}_i^k\right), \tag{6a}$$

$$\text{subject to } \sum_{i=1}^{N} \tilde{x}_i^k = \sum_{i=1}^{N} d_i, \tag{6b}$$

where $\tilde{\boldsymbol{x}}^k=\mathrm{col}\left(\tilde{x}_1^k,\ldots,\tilde{x}_N^k\right)\in\mathbb{R}^N$ is the global decision variable and $\tilde{x}_i^k\in\mathbb{R}$ is corresponding to agent i.

For any $i\in\mathcal{V}$ and $k\in\Im_K$, $\omega_i^{k*}\in\mathbb{R}$ is the weighting factor corresponding to $C_i^k(x_i)$ and is chosen based on the relative importance of objective $C_i^k\left(\tilde{x}_i^k\right)$ compared with the total cost of agent i, which is given by

$$\omega_i^{k*}=\frac{|C_i^k\left(\tilde{x}_i^{k*}\right)|}{\sum_{j=1}^{K}|C_i^j(\tilde{x}_i^{j*})|}. \tag{7}$$

It is straightforward to get that $\sum_{k=1}^{K}\omega_i^{k*}=1$ and $\omega_i^{k*}\geqslant 0$.

For any $i\in\mathcal{V}$ and $k\in\Im_K$, $\hat{x}_i^{k*}\in\mathbb{R}$ is the optimal decision variable of the following optimization problem:

$$\min_{\hat{x}_i^k\in\mathbb{R}} C_i^k\left(\hat{x}_i^k\right), \tag{8}$$

where $\hat{x}_i^k$ is the decision variable and $C_i^k\left(\hat{x}_i^k\right)$ is the k-th local objective function of agent i in the multiobjective optimization problem (5).

To this end, we reformulate the multiobjective optimization problem (5) into the following constrained single-objective distributed optimization problem:

$$\min_{\boldsymbol{x}\in\mathbb{R}^N} U(\boldsymbol{x},\hat{\boldsymbol{x}}^*,\boldsymbol{\omega}^*)=\sum_{i=1}^{N} u_i\left(x_i,\hat{\boldsymbol{x}}_i^*,\boldsymbol{\omega}_i^*\right), \tag{9a}$$

$$\text{subject to } \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} d_i, \tag{9b}$$

where U is the global objective function, $\hat{\boldsymbol{x}}^* = \mathrm{col}\left(\hat{\boldsymbol{x}}_1^*, \ldots, \hat{\boldsymbol{x}}_N^*\right) \in \mathbb{R}^{KN}$ with $\hat{\boldsymbol{x}}_i^* = \mathrm{col}\left(\hat{x}_i^{1*}, \ldots, \hat{x}_i^{K*}\right) \in \mathbb{R}^K$, $\boldsymbol{\omega}^* = \mathrm{col}\left(\boldsymbol{\omega}_1^*, \ldots, \boldsymbol{\omega}_N^*\right) \in \mathbb{R}^{KN}$ with $\boldsymbol{\omega}_i^* = \mathrm{col}\left(\omega_i^{1*}, \ldots, \omega_i^{K*}\right) \in \mathbb{R}^K$, and u_i is the local objective function of agent i, which is given by utilizing the weighted L_p preference index. Specifically,

$$u_i\left(x_i, \hat{\boldsymbol{x}}_i^*, \boldsymbol{\omega}_i^*\right) = \left[\sum_{k=1}^{K} \omega_i^{k*}\left(C_i^k\left(x_i\right) - C_i^k\left(\hat{x}_i^{k*}\right)\right)^p\right]^{\frac{1}{p}}, \tag{10}$$

where $p \in [1, \infty)$.

Throughout this paper, we make the following assumptions, which are commonly adopted in [2,22,26].

Assumption 1. For any $i \in \mathcal{V}$ and $k \in \mathfrak{I}_K$, the local cost function $C_i^k\left(x_i\right)$ is l_i^k-strongly convex with $l_i^k > 0$ and m_i^k-smoothness with $m_i^k > 0$.

Assumption 2. The graph $\mathcal{G}$ is undirected and connected.

Lemma 2 ([38])**.** Supposing Assumption 1 is satisfied, one obtains

$$\left[\nabla C_i^k\left(x\right) - \nabla C_i^k\left(y\right)\right]^{\mathrm{T}}\left(x - y\right) \geqslant l_i^k \|x - y\|^2, \tag{11}$$

$$\left\|\nabla C_i^k\left(x\right) - \nabla C_i^k\left(y\right)\right\| \leqslant m_i^k \left\|x - y\right\|, \text{ for } \forall x, y \in \mathbb{R}. \tag{12}$$

Lemma 3 ([39])**.** Supposing Assumption 2 is satisfied, one obtains

$$\boldsymbol{z}^{\mathrm{T}} L \boldsymbol{z} \geqslant \lambda_2\left(L\right) \boldsymbol{z}^{\mathrm{T}} K_N \boldsymbol{z}, \tag{13}$$

$$\boldsymbol{z}^{\mathrm{T}} L^2 \boldsymbol{z} \leqslant \lambda_N\left(L^2\right) \boldsymbol{z}^{\mathrm{T}} \boldsymbol{z},\ \forall \boldsymbol{z} \in \mathbb{R}^N, \tag{14}$$

where $\lambda_2\left(L\right)$ is the second smallest eigenvalue of L, $\lambda_N(L^2)$ is the largest eigenvalue of L^2, and $K_N = I_N - \frac{1}{N}\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}$.

From [40], we know that the optimal global decision variable of the problem (9) is the desired Pareto optimum of the problem (5). Thus, we aim to solve the problem (9) in a predefined time in a distributed manner.

Next, we illustrate why the optimal global decision variable of the problem (9) is a Pareto optimum of the problem (5). To facilitate analysis, we consider an example of the problem (5) with $K = 2$. For $i \in \mathcal{V}$, the horizontal axis and the vertical axis in Figure 1 represent the decision variables of the local objective functions. Let the irregular ring denote the solution set, which implies the set of cost pairs allowed. The Pareto front, a set of cost pairs of agent i corresponding to all the Pareto optimums of the example, is denoted by the red line. Note that the problem (9) with $K = 2$ and $p = 2$ implies that we minimize the sum of the radiuses of all circles centered on the ideal points of agents when $\omega_i^{1*} = \omega_i^{2*}$, where the ideal point denotes $\left(C_i^{1*}, C_i^{2*}\right)$, and C_i^{1*} and C_i^{2*} denote the infimums of C_i^1 and C_i^2, respectively. When we reformulate the example by the form of the problem (9), it implies that we want to obtain the sum of some values, which are the shortest distances between $\left(C_i^1, C_i^2\right)$ and the ideal point of agent i for $i \in \mathcal{V}$, respectively. The shortest distances are successively denoted by d_i for $i \in \mathcal{V}$. Because of the distribution of the ideal point and the Pareto front, $\left(C_i^1, C_i^2\right)$ corresponding to d_i is at the Pareto front. Since Assumption 1 is satisfied, we know that the Pareto front is $\Lambda^{\geqslant}$-convex, which implies that $\left(C_i^1, C_i^2\right)$ corresponding to d_i is unique for $i \in \mathcal{V}$; see [26].

The weighted L_p preference index has been utilized in [26]. Although its algorithms exponentially converge to the Pareto optimum, they cannot meet the requirements of time-critical applications. Therefore, we propose novel predefined-time algorithms to obtain the desired Pareto optimum of the problem (5). Moreover, the settling time of the proposed algorithms can be arbitrarily preset since it does not depend on any initial values and designed parameters.

3 Main results

In this section, based on time-based generators, we propose algorithms to seek the desired Pareto optimum of the problem (5) and also provide convergence analysis.

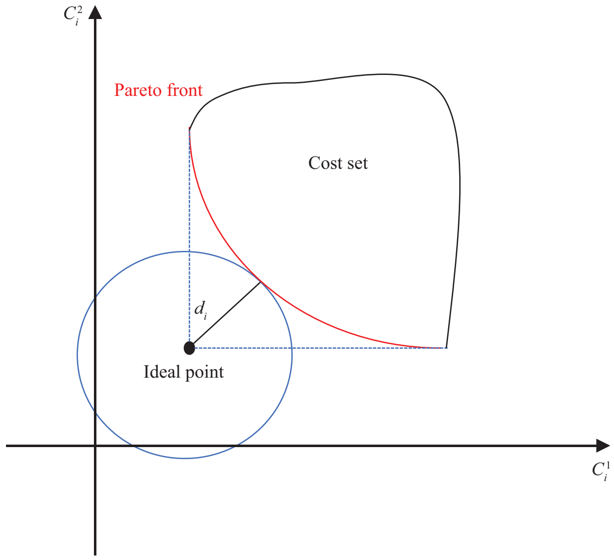

Figure 1 (Color online) Interpretation of problem reconstruction satisfying L_2 preference index.

3.1 Description of the algorithms

We propose new predefined-time algorithms based on time-based generators to solve the problem (9). It is necessary to get $\boldsymbol{\omega}_i^*$ and $\hat{\boldsymbol{x}}_i^*$ of the cost component $u_i\left(x_i, \hat{\boldsymbol{x}}_i^*, \boldsymbol{\omega}_i^*\right)$ for $i \in \mathcal{V}$ by separately solving the problem (6) for $k \in \mathfrak{I}_K$ and the problem (8) for $i \in \mathcal{V}$ and $k \in \mathfrak{I}_K$.

Firstly, to solve the problem (6) for $k \in \mathfrak{I}_K$ in a predefined time, the distributed algorithm is as follows:

$$\dot{\tilde{x}}_i^k(t) = \left[k_1(t) + 1\right]\left[-\nabla C_i^k\left(\tilde{x}_i^k(t)\right) + y_i^k(t)\right], \tag{15a}$$

$$\dot{y}_i^k(t) = \left[k_1(t) + 1\right]\left[-\sum_{j\in\mathcal{V}} a_{ij}\left(y_i^k(t) - y_j^k(t)\right) - \sum_{j\in\mathcal{V}} a_{ij}\left(z_i^k(t) - z_j^k(t)\right) + d_i - \tilde{x}_i^k(t)\right], \tag{15b}$$

$$\dot{z}_i^k(t) = \left[k_1(t) + 1\right]\sum_{j\in\mathcal{V}} a_{ij}\left(y_i^k(t) - y_j^k(t)\right), \tag{15c}$$

$$\omega_i^k(t) = \frac{|C_i^k(\tilde{x}_i^k(t))|}{\sum_{j\in\mathfrak{I}_K}|C_i^j(\tilde{x}_i^j(t))|},\ i \in \mathcal{V}, \tag{15d}$$

where $k_1(t) = \dot{\digamma}_1(t)/[1 - \digamma_1(t) + \sigma_1]$, the function $\digamma_1(t)$ is continuous differentiable and satisfies (1), $\sigma_1 \in (0,1)$, $y_i^k(t), z_i^k(t) \in \mathbb{R}$ for $i \in \mathcal{V}$, $k \in \mathfrak{I}_K$ are auxiliary variables, and $\omega_i^k(t)$ is the estimate of the weighting factor corresponding to $C_i^k(x_i)$.

Next, to solve the problem (8) for $i \in \mathcal{V}$ and $k \in \mathfrak{I}_K$ in a predefined time, the algorithm is as follows:

$$\dot{\hat{x}}_i^k(t) = -\left[k_2(t) + 1\right]\nabla C_i^k\left(\hat{x}_i^k(t)\right), \tag{16}$$

where $k_2(t) = \dot{\digamma}_2(t)/[1 - \digamma_2(t) + \sigma_2]$, the function $\digamma_2(t)$ is continuous differentiable and satisfies (1), and $\sigma_2 \in (0,1)$.

Finally, the optimal global decision variable of the problem (9) is obtained in a predefined time by the following distributed algorithm:

$$\dot{x}_i(t) = \left[k_3(t) + 1\right]\left[-\nabla u_i\left(x_i, \hat{\boldsymbol{x}}_i^*, \boldsymbol{\omega}_i^*\right) + \lambda_i(t)\right], \tag{17a}$$

$$\dot{\lambda}_i(t) = \left[k_3(t) + 1\right]\left[-\sum_{j\in\mathcal{V}} a_{ij}\left(\lambda_i(t) - \lambda_j(t)\right) - \sum_{j\in\mathcal{V}} a_{ij}\left(\rho_i(t) - \rho_j(t)\right) + d_i - x_i(t)\right], \tag{17b}$$

$$\dot{\rho}_i(t) = \left[k_3(t) + 1\right]\left[\sum_{j\in\mathcal{V}} a_{ij}\left(\lambda_i(t) - \lambda_j(t)\right)\right],\ i \in \mathcal{V}, \tag{17c}$$

where $k_3(t) = \dot{\digamma}_3(t) / [1 - \digamma_3(t) + \sigma_3]$, the function $\digamma_3(t)$ is continuous differentiable and satisfies (1), $\sigma_3 \in (0,1)$, and $\lambda_i(t), \rho_i(t) \in \mathbb{R}$ for $i \in \mathcal{V}$ are auxiliary variables.

3.2 Convergence analysis

In this subsection, we show the convergence analysis of the proposed algorithms (15)–(17). Firstly, from (15d), we know that the estimate $\omega_i^k(t)$ of the weighting factor for $i \in \mathcal{V}$ and $k \in \Im_K$ achieves predefined-time convergence if the global decision variable $\tilde{\boldsymbol{x}}^k$ of the constrained distributed optimization problem (6) for $k \in \Im_K$ achieves predefined-time convergence. Therefore, we prove that the global decision variable $\tilde{\boldsymbol{x}}^k$ of the constrained distributed optimization problem (6) for $k \in \Im_K$ achieves predefined-time convergence in the following.

Lemma 4. Supposing Assumptions 1 and 2 are satisfied, the distributed algorithm (15) solves the constrained distributed optimization problem (6) in a predefined time t_f^1, i.e.,

$$\lim_{t \to t_f^1} \left\| \tilde{x}_i^k(t) - \tilde{x}_i^{k*} \right\| \leqslant \sqrt{\frac{2}{\alpha}\left(\frac{\sigma_1}{1+\sigma_1}\right)^{\Phi} V_1^k(0)}, \tag{18a}$$

$$\left\| \tilde{x}_i^k(t) - \tilde{x}_i^{k*} \right\| \leqslant \sqrt{\frac{2}{\alpha}\left(\frac{\sigma_1}{1+\sigma_1}\right)^{\Phi} V_1^k(0)}, \quad \forall\, t > t_f^1, \tag{18b}$$

$$\lim_{t \to \infty} \left\| \tilde{x}_i^k(t) - \tilde{x}_i^{k*} \right\| = 0, \ i \in \mathcal{V}, \tag{18c}$$

where σ_1 is an arbitrarily positive constant, $\Phi = \varepsilon / (\alpha + 3\beta + 2\varepsilon)$ with $\varepsilon > 0$, V_1^k for $k \in \Im_K$ is the Lyapunov candidate function defined later, and

$$\beta \geqslant (3\varepsilon + 1) / [2\lambda_2(L)], \tag{19a}$$

$$\alpha \geqslant \max\left\{[\beta^2 - \varepsilon(l^k - 3/2 - m^{2k}/2 - \lambda_N(L^2))]/l^k, [1 - \varepsilon(2\lambda_2(L) - \lambda_N(L^2) - 1)]/2\lambda_2(L)\right\} \geqslant 0, \tag{19b}$$

with $l^k = \min\{l_i^k\}_{i=1}^N$, $m^k = \max\{m_i^k\}_{i=1}^N$, and $m^{2k} = (m^k)^2$.

Proof. We sometimes drop the dependency t for notational simplification when it is clear from the context.

Firstly, the distributed algorithm (15) is rewritten as follows:

$$\dot{\tilde{\boldsymbol{x}}}^k = (k_1 + 1)\left[-\nabla C^k(\tilde{\boldsymbol{x}}^k) + \boldsymbol{y}^k\right], \tag{20a}$$

$$\dot{\boldsymbol{y}}^k = (k_1 + 1)\left(-L\boldsymbol{y}^k - L\boldsymbol{z}^k + \boldsymbol{d} - \tilde{\boldsymbol{x}}^k\right), \tag{20b}$$

$$\dot{\boldsymbol{z}}^k = (k_1 + 1)L\boldsymbol{y}^k, \tag{20c}$$

where $\tilde{\boldsymbol{x}}^k = \mathrm{col}(\tilde{x}_1^k, \ldots, \tilde{x}_N^k) \in \mathbb{R}^N$, $\nabla C^k = \mathrm{col}(\nabla C_i^k, \ldots, \nabla C_N^k) \in \mathbb{R}^N$, $\boldsymbol{y}^k = \mathrm{col}(y_1^k, \ldots, y_N^k) \in \mathbb{R}^N$, $\boldsymbol{d} = \mathrm{col}(d_1, \ldots, d_N) \in \mathbb{R}^N$, and $\boldsymbol{z}^k = \mathrm{col}(z_1^k, \ldots, z_N^k) \in \mathbb{R}^N$.

Let $(\tilde{\boldsymbol{x}}^{k*}, \boldsymbol{y}^{k*}, \boldsymbol{z}^{k*})$ denote the equilibrium point of (20), which satisfies

$$\mathbf{0}_N = -\nabla C^k(\tilde{x}^{k*}) + \boldsymbol{y}^{k*}, \tag{21a}$$

$$\mathbf{0}_N = -L\boldsymbol{y}^{k*} - L\boldsymbol{z}^{k*} + \boldsymbol{d} - \tilde{\boldsymbol{x}}^{k*}, \tag{21b}$$

$$\mathbf{0}_N = L\boldsymbol{y}^{k*}. \tag{21c}$$

Since Assumption 2 is satisfied, one obtains $y_1^{k*} = y_2^{k*} = \cdots = y_N^{k*}$ from (21c). Then according to (21a), one gets

$$\nabla C_1^k(\tilde{x}_1^{k*}) = \nabla C_2^k(\tilde{x}_2^{k*}) = \cdots = \nabla C_N^k(\tilde{x}_N^{k*}). \tag{22}$$

By premultiplying (21b) with $\mathbf{1}_N^{\mathrm{T}}$, one gets

$$\sum_{i=1}^N \tilde{x}_i^{k*} = \sum_{i=1}^N d_i. \tag{23}$$

From (21)–(23), we conclude easily that the equilibrium point $\left(\tilde{\boldsymbol{x}}^{k*}, \boldsymbol{y}^{k*}, \boldsymbol{z}^{k*}\right)$ satisfies the KKT optimality condition, and thus $\tilde{\boldsymbol{x}}^{k*}$ is the optimal global decision variable of problem (6).

Secondly, the convergence of the global decision variable $\tilde{\boldsymbol{x}}^k$ is analyzed based on Lyapunov theory. For ease of analysis, let $\bar{\boldsymbol{x}}^k = \tilde{\boldsymbol{x}}^k - \tilde{\boldsymbol{x}}^{k*}$, $\bar{\boldsymbol{y}}^k = \boldsymbol{y}^k - \boldsymbol{y}^{k*}$, and $\bar{\boldsymbol{z}}^k = \boldsymbol{z}^k - \boldsymbol{z}^{k*}$. From (20), one obtains

$$\dot{\bar{\boldsymbol{x}}}^k = \left(k_1+1\right)\left(-\boldsymbol{h}^k + \bar{\boldsymbol{y}}^k\right), \tag{24a}$$
$$\dot{\bar{\boldsymbol{y}}}^k = \left(k_1+1\right)\left(-L\bar{\boldsymbol{y}}^k - L\bar{\boldsymbol{z}}^k - \bar{\boldsymbol{x}}^k\right), \tag{24b}$$
$$\dot{\bar{\boldsymbol{z}}}^k = \left(k_1+1\right)L\bar{\boldsymbol{y}}^k, \tag{24c}$$

where $\boldsymbol{h}^k = \nabla C^k\left(\tilde{\boldsymbol{x}}^k\right) - \nabla C^k\left(\tilde{\boldsymbol{x}}^{k*}\right)$.

Consider the following orthogonal transformation:

$$\boldsymbol{\chi}^k = [\boldsymbol{r}, R]^{\mathrm{T}}\bar{\boldsymbol{x}}^k = \operatorname{col}\left(\chi_1^k, \boldsymbol{\chi}_2^k\right), \tag{25a}$$
$$\boldsymbol{\eta}^k = [\boldsymbol{r}, R]^{\mathrm{T}}\bar{\boldsymbol{y}}^k = \operatorname{col}\left(\eta_1^k, \boldsymbol{\eta}_2^k\right), \tag{25b}$$
$$\boldsymbol{\delta}^k = [\boldsymbol{r}, R]^{\mathrm{T}}\bar{\boldsymbol{z}}^k = \operatorname{col}\left(\delta_1^k, \boldsymbol{\delta}_2^k\right), \tag{25c}$$

where $\chi_1^k, \eta_1^k, \delta_1^k \in \mathbb{R}$, $\boldsymbol{\chi}_2^k, \boldsymbol{\eta}_2^k, \boldsymbol{\delta}_2^k \in \mathbb{R}^{N-1}$, $[\boldsymbol{r}, R]$ is an orthogonal matrix, $\boldsymbol{r} = \left(1/\sqrt{N}\right)\mathbf{1}_N$, $\boldsymbol{r}^{\mathrm{T}}R = \mathbf{0}_{N-1}^{\mathrm{T}}$, $R^{\mathrm{T}}R = I_{N-1}$, and $RR^{\mathrm{T}} = I_N - \left(1/N\right)\mathbf{1}_N\mathbf{1}_N^{\mathrm{T}}$.

Since $L\mathbf{1}_N = \mathbf{1}_N^{\mathrm{T}}L = \mathbf{0}_N$ and $RR^{\mathrm{T}}L = LRR^{\mathrm{T}} = L$, it follows from (25) that the system (24) is rewritten into two subsystems.

Subsystem 1:

$$\dot{\chi}_1^k = \left(k_1+1\right)\left(-\boldsymbol{r}^{\mathrm{T}}\boldsymbol{h}^k + \eta_1^k\right), \tag{26a}$$
$$\dot{\eta}_1^k = -\left(k_1+1\right)\chi_1^k, \tag{26b}$$
$$\dot{\delta}_1^k = 0. \tag{26c}$$

Subsystem 2:

$$\dot{\boldsymbol{\chi}}_2^k = \left(k_1+1\right)\left(-R^{\mathrm{T}}\boldsymbol{h}^k + \boldsymbol{\eta}_2^k\right), \tag{27a}$$
$$\dot{\boldsymbol{\eta}}_2^k = \left(k_1+1\right)\left(-R^{\mathrm{T}}LR\boldsymbol{\eta}_2^k - R^{\mathrm{T}}LR\boldsymbol{\delta}_2^k - \boldsymbol{\chi}_2^k\right), \tag{27b}$$
$$\dot{\boldsymbol{\delta}}_2^k = \left(k_1+1\right)R^{\mathrm{T}}LR\boldsymbol{\eta}_2^k. \tag{27c}$$

From (25a) and $\bar{\boldsymbol{x}}^k = \tilde{\boldsymbol{x}}^k - \tilde{\boldsymbol{x}}^{k*}$, we know that the global decision variable $\tilde{\boldsymbol{x}}^k$ of the constrained distributed optimization problem (6) for $k \in \mathfrak{I}_K$ achieves predefined-time convergence if $\boldsymbol{\chi}^k$ for $k \in \mathfrak{I}_K$ achieves predefined-time convergence. Then, we prove the convergence of $\boldsymbol{\chi}^k$ (i.e., χ_1^k and $\boldsymbol{\chi}_2^k$). Consider the following Lyapunov candidate function:

$$\begin{aligned} V_1^k =& \frac{\alpha}{2}\left[\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\boldsymbol{\chi}^k + \left(\boldsymbol{\eta}^k\right)^{\mathrm{T}}\boldsymbol{\eta}^k\right] + \frac{\alpha+\beta}{2}\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\delta}_2^k \\ &+ \frac{\beta}{2}\left(\boldsymbol{\eta}_2^k + \boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\left(\boldsymbol{\eta}_2^k + \boldsymbol{\delta}_2^k\right) + \frac{\varepsilon}{2}\left(\boldsymbol{\chi}^k - \boldsymbol{\eta}^k\right)^{\mathrm{T}}\left(\boldsymbol{\chi}^k - \boldsymbol{\eta}^k\right), \end{aligned} \tag{28}$$

where $\alpha > 0$, $\beta > 0$, and $\varepsilon > 0$, which will be defined later.

Let $\boldsymbol{\varphi}^k = \operatorname{col}\left(\boldsymbol{\chi}^k, \boldsymbol{\eta}^k, \boldsymbol{\delta}_2^k\right)$. One gets

$$\frac{\alpha}{2}\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\boldsymbol{\chi}^k \leqslant V_1^k \leqslant \frac{\alpha+3\beta+2\varepsilon}{2}\left\|\boldsymbol{\varphi}^k\right\|^2. \tag{29}$$

According to (26)–(28), one gets

$$\begin{aligned} \dot{V}_1^k =& \alpha\left[\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\dot{\boldsymbol{\chi}}^k + \left(\boldsymbol{\eta}^k\right)^{\mathrm{T}}\dot{\boldsymbol{\eta}}^k\right] + \left(\alpha+\beta\right)\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\dot{\boldsymbol{\delta}}_2^k \\ &+ \beta\left(\boldsymbol{\eta}_2^k + \boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\left(\dot{\boldsymbol{\eta}}_2^k + \dot{\boldsymbol{\delta}}_2^k\right) + \varepsilon\left(\boldsymbol{\chi}^k - \boldsymbol{\eta}^k\right)^{\mathrm{T}}\left(\dot{\boldsymbol{\chi}}^k - \dot{\boldsymbol{\eta}}^k\right) \\ =& \alpha\left(k_1+1\right)\left[-\left(\bar{\boldsymbol{x}}^k\right)^{\mathrm{T}}\boldsymbol{h}^k - \left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\eta}_2^k\right] \end{aligned}$$

$$
\begin{aligned}
&+\beta\left(k_1+1\right)\left[-\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}\boldsymbol{\chi}_2^k-\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\left(R^{\mathrm{T}}LR\right)\boldsymbol{\delta}_2^k-\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\chi}_2^k\right]\\
&+\varepsilon\left(k_1+1\right)\left[-\left(\bar{\boldsymbol{x}}^k\right)^{\mathrm{T}}\boldsymbol{h}^k+\left(\boldsymbol{\chi}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\eta}_2^k\right.\\
&+\left(\boldsymbol{\chi}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\delta}_2^k+\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\boldsymbol{\chi}^k+\left(\boldsymbol{\eta}^k\right)^{\mathrm{T}}\left[\boldsymbol{r},R\right]^{\mathrm{T}}\boldsymbol{h}^k-\left(\boldsymbol{\eta}^k\right)^{\mathrm{T}}\boldsymbol{\eta}^k\\
&\left.-\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\eta}_2^k-\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\delta}_2^k\right].
\end{aligned}\tag{30}
$$

Since Assumption 1 is satisfied, the term $-\left(\bar{\boldsymbol{x}}^k\right)^{\mathrm{T}}\boldsymbol{h}^k$ in (30) satisfies

$$
-\left(\bar{\boldsymbol{x}}^k\right)^{\mathrm{T}}\boldsymbol{h}^k\leqslant -l^k\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\boldsymbol{\chi}^k.\tag{31}
$$

Since Assumption 2 is satisfied, the terms $-\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\eta}_2^k$ and $-\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\delta}_2^k$ in (30) satisfy

$$
-\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\eta}_2^k\leqslant -\lambda_2(L)\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}\boldsymbol{\eta}_2^k,\tag{32}
$$

$$
-\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\delta}_2^k\leqslant -\lambda_2(L)\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\delta}_2^k.\tag{33}
$$

Moreover, from Young's inequality, one obtains

$$
-\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}\boldsymbol{\chi}_2^k\leqslant\frac{\beta}{2}\left(\boldsymbol{\chi}_2^k\right)^{\mathrm{T}}\boldsymbol{\chi}_2^k+\frac{1}{2\beta}\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}\boldsymbol{\eta}_2^k,\tag{34}
$$

$$
-\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\chi}_2^k\leqslant\frac{\beta}{2}\left(\boldsymbol{\chi}_2^k\right)^{\mathrm{T}}\boldsymbol{\chi}_2^k+\frac{1}{2\beta}\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\delta}_2^k,\tag{35}
$$

$$
\left(\boldsymbol{\eta}^k\right)^{\mathrm{T}}\left[\boldsymbol{r},R\right]^{\mathrm{T}}\boldsymbol{h}^k\leqslant\frac{1}{2}\left(\boldsymbol{\eta}^k\right)^{\mathrm{T}}\boldsymbol{\eta}^k+\frac{m^{2k}}{2}\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\boldsymbol{\chi}^k,\tag{36}
$$

$$
\left(\boldsymbol{\chi}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\eta}_2^k\leqslant\frac{\lambda_N\left(L^2\right)}{2}\left(\boldsymbol{\chi}_2^k\right)^{\mathrm{T}}\boldsymbol{\chi}_2^k+\frac{1}{2}\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}\boldsymbol{\eta}_2^k,\tag{37}
$$

$$
\left(\boldsymbol{\chi}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\delta}_2^k\leqslant\frac{\lambda_N\left(L^2\right)}{2}\left(\boldsymbol{\chi}_2^k\right)^{\mathrm{T}}\boldsymbol{\chi}_2^k+\frac{1}{2}\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\delta}_2^k,\tag{38}
$$

$$
-\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}R^{\mathrm{T}}LR\boldsymbol{\delta}_2^k\leqslant\frac{\lambda_N\left(L^2\right)}{2}\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}\boldsymbol{\eta}_2^k+\frac{1}{2}\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\delta}_2^k.\tag{39}
$$

Substituting (31)–(39) into (30) yields

$$
\begin{aligned}
\dot{V}_1^k\leqslant&-\left(k_1+1\right)\left[\alpha l^k-\beta^2+\varepsilon\Big(l^k-1-\frac{m^{2k}}{2}-\lambda_N\left(L^2\right)\Big)\right]\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\boldsymbol{\chi}^k\\
&-\frac{\varepsilon}{2}\left(k_1+1\right)\left(\boldsymbol{\eta}^k\right)^{\mathrm{T}}\boldsymbol{\eta}^k-\left(k_1+1\right)\left[\beta\lambda_2(L)-\frac{1}{2}-\varepsilon\right]\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\delta}_2^k\\
&-\left(k_1+1\right)\left[\alpha\lambda_2(L)-\frac{1}{2}+\varepsilon\left(\lambda_2(L)-\frac{\lambda_N\left(L^2\right)}{2}-\frac{1}{2}\right)\right]\left(\boldsymbol{\eta}_2^k\right)^{\mathrm{T}}\boldsymbol{\eta}_2^k.
\end{aligned}\tag{40}
$$

From (19a), one gets

$$
\beta\lambda_2\left(L\right)-\frac{1}{2}-\varepsilon\geqslant\frac{\varepsilon}{2}.\tag{41}
$$

From (19b), one gets

$$
\alpha l^k-\beta^2+\varepsilon\left[l^k-\frac{3}{2}-\frac{m^{2k}}{2}-\lambda_N\left(L^2\right)\right]\geqslant\frac{\varepsilon}{2},\tag{42}
$$

$$
\alpha\lambda_2(L)-\frac{1}{2}+\varepsilon\left[\lambda_2(L)-\frac{\lambda_N\left(L^2\right)}{2}-\frac{1}{2}\right]\geqslant 0.\tag{43}
$$

From (40)–(43), one gets

$$
\dot{V}_1^k\leqslant-\frac{\varepsilon}{2}\left(k_1+1\right)\left[\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\boldsymbol{\chi}^k+\left(\boldsymbol{\eta}^k\right)^{\mathrm{T}}\boldsymbol{\eta}^k+\left(\boldsymbol{\delta}_2^k\right)^{\mathrm{T}}\boldsymbol{\delta}_2^k\right]
$$

$$= -\frac{\varepsilon}{2}(k_1+1)\left\|\boldsymbol{\varphi}^k\right\|^2$$
$$\leqslant -\frac{\varepsilon k_1}{2}\left\|\boldsymbol{\varphi}^k\right\|^2. \tag{44}$$

This together with (29) implies that

$$\dot{V}_1^k \leqslant -\frac{\varepsilon k_1}{\alpha+3\beta+2\varepsilon}V_1^k. \tag{45}$$

It then follows from Lemma 1 that

$$V_1^k \to \left(\frac{\sigma_1}{1+\sigma_1}\right)^{\Phi} V_1^k(0), \quad \text{as } t \to t_f^1. \tag{46}$$

Since $V_1^k \geqslant \frac{\alpha}{2}\left(\boldsymbol{\chi}^k\right)^{\mathrm{T}}\boldsymbol{\chi}^k$, one can obtain that

$$\left\|\chi^k\right\| \to \hat{V}_1 = \sqrt{\frac{2}{\alpha}\left(\frac{\sigma_1}{1+\sigma_1}\right)^{\Phi} V_1^k(0)}, \quad \text{as } t \to t_f^1. \tag{47}$$

When $t \geqslant t_f^1$, $k_1 = 0$, one gets

$$\dot{V}_1^k \leqslant -\frac{\varepsilon}{2}\left\|\boldsymbol{\varphi}^k\right\|^2 \leqslant -\Phi V_1^k. \tag{48}$$

Therefore, we conclude that $\boldsymbol{\chi}^k$ for $k \in \Im_K$ converges to $\hat{V}_1$ in a predefined time t_f^1, and continues to converge to the origin as time goes to infinity; i.e., the global decision variable $\tilde{\boldsymbol{x}}^k$ of the constrained distributed optimization problem (6) for $k \in \Im_K$ achieves predefined-time convergence. The proof is completed.

Next, we prove that the decision variable $\hat{x}_i^k$ of the optimization problem (8) for $i \in \mathcal{V}$ and $k \in \Im_K$ achieves predefined-time convergence.

Lemma 5. Supposing Assumption 1 is satisfied, the algorithm (16) solves the optimization problem (8) in a predefined time t_f^2, i.e.,

$$\lim_{t\to t_f^2}\left\|\hat{x}_i^k(t)-\hat{x}_i^{k*}\right\| \leqslant \sqrt{\frac{2}{\varrho}\left(\frac{\sigma_2}{1+\sigma_2}\right)^{2l_i^k}\check{V}_i^k(0)}, \tag{49a}$$

$$\left\|\hat{x}_i^k(t)-\hat{x}_i^{k*}\right\| \leqslant \sqrt{\frac{2}{\varrho}\left(\frac{\sigma_2}{1+\sigma_2}\right)^{2l_i^k}\check{V}_i^k(0)}, \quad \forall\, t > t_f^2, \tag{49b}$$

$$\lim_{t\to\infty}\left\|\hat{x}_i^k(t)-\hat{x}_i^{k*}\right\| = 0, \tag{49c}$$

where $\varrho > 0$, σ_2 is an arbitrarily positive constant, and $\check{V}_i^k$ for $i \in \mathcal{V}$ and $k \in \Im_K$ is the Lyapunov candidate function defined later.

Proof. Let $\check{x}_i^k(t) = \hat{x}_i^k(t) - \hat{x}_i^{k*}(t)$. It then follows from the algorithm (16) that

$$\dot{\check{x}}_i^k(t) = -\left[k_2(t)+1\right]h_{i2}^k(t), \tag{50}$$

where $h_{i2}^k(t) = \nabla C_i^k(\hat{x}_i^k(t)) - \nabla C_i^k(\hat{x}_i^*(t))$.

Consider the following Lyapunov candidate function:

$$\check{V}_i^k(t) = \frac{\varrho}{2}\left[\check{x}_i^k(t)\right]^{\mathrm{T}}\check{x}_i^k(t). \tag{51}$$

Then, one gets that

$$\dot{\check{V}}_i^k(t) = \varrho\left[\check{x}_i^k(t)\right]^{\mathrm{T}}\dot{\check{x}}_i^k(t)$$
$$= -\varrho\left[k_2(t)+1\right]\left[\check{x}_i^k(t)\right]^{\mathrm{T}}h_{i2}^k(t). \tag{52}$$

Since Assumption 1 is satisfied, one gets

$$-\left[\breve{x}_i^k(t)\right]^{\mathrm{T}} h_{i2}^k(t) \leqslant -l_i^k \left[\breve{x}_i^k(t)\right]^{\mathrm{T}} \breve{x}_i^k(t). \tag{53}$$

Next

$$\begin{aligned}\dot{\breve{V}}_i^k(t) &\leqslant -\left[k_2(t)+1\right]\varrho l_i^k \left[\breve{x}_i^k(t)\right]^{\mathrm{T}} \breve{x}_i^k(t)\\ &\leqslant -2\left[k_2(t)+1\right] l_i^k \breve{V}_i^k(t) \qquad (54)\\ &\leqslant -2k_2(t) l_i^k \breve{V}_i^k(t). \qquad (55)\end{aligned}$$

Invoking Lemma 1, one obtains that

$$\breve{V}_i^k(t) \to \left(\frac{\sigma_2}{1+\sigma_2}\right)^{2l_i^k} \breve{V}_i^k(0), \quad \text{as } t \to t_f^2. \tag{56}$$

This together with (51) implies

$$\left\|\breve{x}_i^k(t)\right\| \to \hat{V}_2 = \sqrt{\frac{2}{\varrho}\left(\frac{\sigma_2}{1+\sigma_2}\right)^{2l_i^k} \breve{V}_i^k(0)}, \quad \text{as } t \to t_f^2. \tag{57}$$

From (57), one can obtain

$$\lim_{t\to t_f^2} \left\|\hat{x}_i^k(t) - \hat{x}_i^{k*}\right\| \to \hat{V}_2 = \sqrt{\frac{2}{\varrho}\left(\frac{\sigma_2}{1+\sigma_2}\right)^{2l_i^k} \breve{V}_i^k(0)}. \tag{58}$$

When $t \geqslant t_f^2, k_2(t) = 0$, from (54), one can obtain that

$$\dot{\breve{V}}_i^k(t) \leqslant -2l_i^k \breve{V}_i^k(t). \tag{59}$$

Therefore, we conclude that $\breve{x}_i^k(t)$ for $i \in \mathcal{V}$ and $k \in \mathfrak{I}_K$ converges to $\hat{V}_2$ in a predefined time t_f^2, and continues to converge to the origin as time goes to infinity; i.e., the decision variable $\hat{x}_i^k$ of the optimization problem (8) for $i \in \mathcal{V}$ and $k \in \mathfrak{I}_K$ achieves predefined-time convergence. The proof is completed.

Finally, we prove that the local decision variable x_i for $i \in \mathcal{V}$ of the constrained single-objective distributed optimization problem (9) achieves predefined-time convergence when $\boldsymbol{\omega}_i^*$ and $\hat{\boldsymbol{x}}_i^*$ for $i \in \mathcal{V}$ are obtained.

Theorem 1. Supposing Assumptions 1 and 2 are satisfied, the distributed algorithm (17) solves the constrained single-objective distributed optimization problem (9) in a predefined time t_f^3, i.e.,

$$\lim_{t\to t_f^3} \left\|x_i(t) - x_i^*\right\| \leqslant \sqrt{\frac{2}{\theta}\left(\frac{\sigma_3}{1+\sigma_3}\right)^{\Psi} \tilde{V}(0)}, \tag{60a}$$

$$\left\|x_i(t) - x_i^*\right\| \leqslant \sqrt{\frac{2}{\theta}\left(\frac{\sigma_3}{1+\sigma_3}\right)^{\Psi} \tilde{V}(0)}, \quad \forall\, t > t_f^3, \tag{60b}$$

$$\lim_{t\to\infty} \left\|x_i(t) - x_i^*\right\| = 0,\ i \in \mathcal{V}, \tag{60c}$$

where σ_3 is an arbitrarily positive constant, $\Psi = \vartheta/(\theta + 3\nu + 2\vartheta)$ with $\vartheta > 0$, $\tilde{V}$ is the Lyapunov candidate function defined later, and

$$\nu \geqslant (3\vartheta+1)/[2\lambda_2(L)], \tag{61a}$$

$$\theta \geqslant \max\{[\nu^2 - \vartheta(l - 3/2 - m^2/2 - \lambda_N(L^2))]/l, [1 - \vartheta(2\lambda_2(L) - \lambda_N(L^2) - 1)]/[2\lambda_2(L)]\} \geqslant 0, \tag{61b}$$

with $l = \min\left\{l_i^k\right\}$ and $m = \max\left\{m_i^k\right\}$.

Proof. We sometimes drop the dependency t for notational simplification when it is clear from the context.

Firstly, replacing $\tilde{x}_i^k$, k_1, $C_i^k\big(\tilde{x}_i^k\big)$, y_i^k, and z_i^k by x_i, k_3, $u_i\big(x_i, \hat{x}_i^*, \omega_i^*\big)$, and λ_i, ρ_i, respectively, from (20)–(23), we have

$$\sum_{i=1}^{N} x_i^* = \sum_{i=1}^{N} d_i. \tag{62}$$

It indicates that the equality constraint of the problem (9) is satisfied when the distributed algorithm (17) converges to its equilibrium point $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\rho}^*)$ with $\boldsymbol{x}^* = \mathrm{col}\,(x_1^*, \ldots, x_n^*)$, $\boldsymbol{\lambda}^* = \mathrm{col}\,(\lambda_1^*, \ldots, \lambda_n^*)$, and $\boldsymbol{\rho}^* = \mathrm{col}\,(\rho_1^*, \ldots, \rho_n^*)$. In addition, we conclude easily that the equilibrium point $(\boldsymbol{x}^*, \boldsymbol{\lambda}^*, \boldsymbol{\rho}^*)$ satisfies the KKT optimality condition such that $\boldsymbol{x}^*$ is the optimal global decision variable of problem (9).

Secondly, we analyze the convergence of the global decision variable $\boldsymbol{x}$. Let $\bar{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}^*$, $\bar{\boldsymbol{\lambda}} = \boldsymbol{\lambda} - \boldsymbol{\lambda}^*$, $\bar{\boldsymbol{\rho}} = \boldsymbol{\rho} - \boldsymbol{\rho}^*$, $\boldsymbol{h} = \nabla u\,(\boldsymbol{x}, \hat{\boldsymbol{x}}^*, \boldsymbol{\omega}^*) - \nabla u_i\,(\boldsymbol{x}^*, \hat{\boldsymbol{x}}^*, \boldsymbol{\omega}^*)$, $\chi_1, \eta_1, \delta_1 \in \mathbb{R}$, $\boldsymbol{\chi}_2, \boldsymbol{\eta}_2, \boldsymbol{\delta}_2 \in \mathbb{R}^{N-1}$, $\boldsymbol{\chi} = \big[\tilde{\boldsymbol{r}}, \tilde{R}\big]^{\mathrm{T}} \bar{\boldsymbol{x}} = \mathrm{col}\,(\chi_1, \boldsymbol{\chi}_2)$, $\boldsymbol{\eta} = \big[\tilde{\boldsymbol{r}}, \tilde{R}\big]^{\mathrm{T}} \bar{\boldsymbol{\lambda}} = \mathrm{col}\,(\eta_1, \boldsymbol{\eta}_2)$, and $\boldsymbol{\delta} = \big[\tilde{\boldsymbol{r}}, \tilde{R}\big]^{\mathrm{T}} \bar{\boldsymbol{\rho}} = \mathrm{col}\,(\delta_1, \boldsymbol{\delta}_2)$, where $[\tilde{r}, \tilde{R}]$ is an orthogonal matrix. From (24)–(27), we obtain

$$\dot{\chi}_1 = \big(k_3 + 1\big)\big(-\tilde{\boldsymbol{r}}^{\mathrm{T}} \boldsymbol{h} + \eta_1\big), \tag{63a}$$
$$\dot{\eta}_1 = -\big(k_3 + 1\big)\chi_1, \tag{63b}$$
$$\dot{\delta}_1 = 0, \tag{63c}$$
$$\dot{\boldsymbol{\chi}}_2 = \big(k_3 + 1\big)\big(-\tilde{R}^{\mathrm{T}} \boldsymbol{h} + \boldsymbol{\eta}_2\big), \tag{63d}$$
$$\dot{\boldsymbol{\eta}}_2 = \big(k_3 + 1\big)\big(-\tilde{R}^{\mathrm{T}} L \tilde{R} \boldsymbol{\eta}_2 - \tilde{R}^{\mathrm{T}} L \tilde{R} \boldsymbol{\delta}_2 - \boldsymbol{\chi}_2\big), \tag{63e}$$
$$\dot{\boldsymbol{\delta}}_2 = \big(k_3 + 1\big)\tilde{R}^{\mathrm{T}} L \tilde{R} \boldsymbol{\eta}_2. \tag{63f}$$

Consider the following Lyapunov candidate function:

$$\begin{aligned}\tilde{V} = {} & \frac{\theta}{2}\big[(\boldsymbol{\chi})^{\mathrm{T}} \boldsymbol{\chi} + (\boldsymbol{\eta})^{\mathrm{T}} \boldsymbol{\eta}\big] + \frac{\theta + \nu}{2}(\boldsymbol{\delta}_2)^{\mathrm{T}} \boldsymbol{\delta}_2 \\ & + \frac{\nu}{2}(\boldsymbol{\eta}_2 + \boldsymbol{\delta}_2)^{\mathrm{T}}(\boldsymbol{\eta}_2 + \boldsymbol{\delta}_2) + \frac{\boldsymbol{\vartheta}}{2}(\boldsymbol{\chi} - \boldsymbol{\eta})^{\mathrm{T}}(\boldsymbol{\chi} - \boldsymbol{\eta}),\end{aligned} \tag{64}$$

where $\theta > 0$, $\nu > 0$, $\vartheta > 0$.

From (29)–(45), we have

$$\dot{\tilde{V}} \leqslant -\frac{\vartheta k_3}{\theta + 3\nu + 2\vartheta} \tilde{V}. \tag{65}$$

This together with Lemma 1 and $\tilde{V} \geqslant \frac{\theta}{2}(\boldsymbol{\chi})^{\mathrm{T}} \boldsymbol{\chi}$ implies that

$$\|\boldsymbol{\chi}\| \to \hat{V}_3 = \sqrt{\frac{2}{\theta}\left(\frac{\sigma_3}{1 + \sigma_3}\right)^{\Psi} \tilde{V}\,(0)}, \quad t \to t_f^3. \tag{66}$$

When $t \geqslant t_f^3, k_3 = 0$, one gets

$$\dot{\tilde{V}} \leqslant -\Psi \tilde{V}. \tag{67}$$

Therefore, one concludes that $\boldsymbol{\chi}$ converges to $\hat{V}_3$ in a predefined time t_f^3, and continues to converge to the origin as time goes to infinity; i.e., the local decision variable x_i for $i \in \mathcal{V}$ of the constrained single-objective distributed optimization problem (9) achieves predefined-time convergence. The proof is completed.

Note that the global decision variable of the problem (9) suffers an error caused by the flawed $\boldsymbol{\omega}_i^*$ and $\hat{\boldsymbol{x}}_i^*$ at $t = t_f^3$ as shown in Lemmas 4 and 5 when executing the proposed algorithms (15)–(17) simultaneously at $t = 0$. From (47) and (58), the error decays to a sufficiently small value at $\max\{t_f^1, t_f^2\} + t_f^3$ by choosing sufficiently small σ_1 and σ_2. As time goes to infinity, the error does not exist, and thus the optimal global decision variable of the problem (9) is obtained, which is equivalent to the desired Pareto optimum of the problem (5).

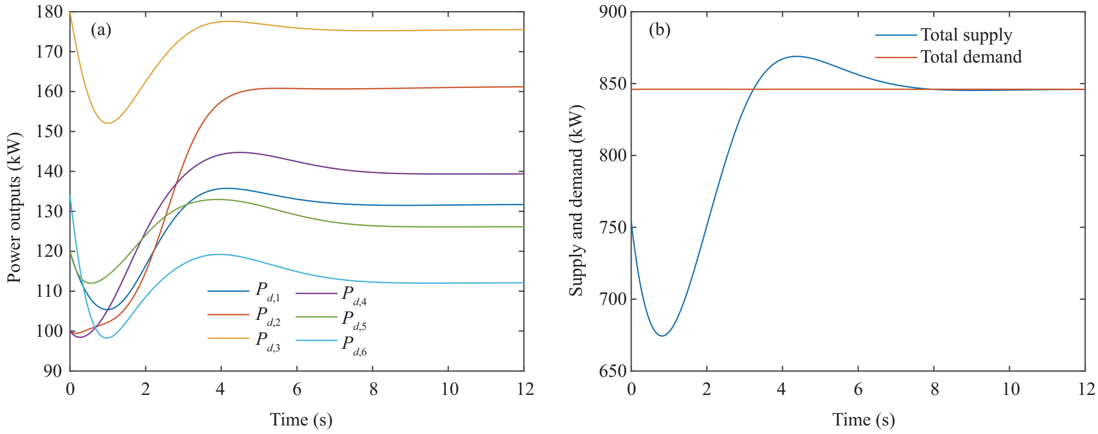

Figure 2 (Color online) Power outputs (a) and supply and demand (b) using the proposed algorithm in [26].

4 Simulation

In this section, one example modeled by an undirected ring graph with six microgrids is provided to verify the fast convergence of the proposed algorithms. Note that the simulation utilizes the weighted L_2 preference index in [26]. The multi-microgrid network is formulated as follows:

$$\min_{P_{d,i}\in\mathbb{R}} \left\{F_{C,i}\left(P_{d,i}\right), F_{N,i}\left(P_{d,i}\right), F_{E,i}\left(P_{d,i}\right)\right\}, \tag{68a}$$

$$\text{subject to } \sum_{i=1}^{N} P_{d,i} = \sum_{i=1}^{N}\left(1+\eta_i\right)P_{l,i},\ \ i\in\mathcal{V}, \tag{68b}$$

where $P_{d,i}$ is the local active power generated from the i-th microgrid, $F_{C,i}(P_{d,i})$, $F_{N,i}(P_{d,i})$, and $F_{E,i}(P_{d,i})$ are the economic, environmental, and technical objective functions, respectively, $\sum_{i=1}^{N}(1+\eta_i)P_{l,i}$ is the mismatch between the supply and demand caused by the intermittent renewable generations and varying load demands, and

$$F_{C,i}\left(P_{d,i}\right) = q_{p,i}P_{d,i}^2 + w_{p,i}P_{d,i} + e_{p,i}, \tag{69}$$

$$F_{N,i}\left(P_{d,i}\right) = m_n\left(s_{l,i}P_{d,i}^2 + g_{l,i}P_{d,i} + h_{l,i}\right), \tag{70}$$

$$F_{E,i}\left(P_{d,i}\right) = v_{e,i}(P_{d,i} - P_{opt,i})^2. \tag{71}$$

For the parameters of the objective functions, see [26].

Next, we make a comparative illustration for demonstrating the effectiveness of the proposed algorithms. We first get $\boldsymbol{\omega}_i^*$ and $\hat{\boldsymbol{x}}_i^*$ of the cost component $u_i\left(x_i, \hat{\boldsymbol{x}}_i^*, \boldsymbol{\omega}_i^*\right)$ for $i\in\mathcal{V}$ by the algorithms (15) and (16), respectively. For the proposed algorithm in [26], Figure 2(a) shows that the power outputs of the microgrids converge to the Pareto optimum after 8 s, and Figure 2(b) shows that the power supply and demand are balanced; that is, the resource allocation satisfies the constraint in the problem (68).

To verify the utility of the time-based generator in the distributed algorithm (17), we introduce the following example with $t_f^3 = 0.7$ s:

$$\digamma_3\left(t\right) = \begin{cases} \frac{10}{0.7^6}t^6 - \frac{24}{0.7^5}t^5 + \frac{15}{0.7^4}t^4, & 0 \leqslant t < t_f^3, \\ 1, & t \geqslant t_f^3. \end{cases} \tag{72}$$

Subsequently, Figure 3(a) shows the power outputs of microgrids converge to the Pareto optimum in the predefined time $t_f^3 = 0.7$ s, and Figure 3(b) shows that the constraint (68b) is satisfied.

Then we choose the following example with $t_f^3 = 0.3$ s:

$$\digamma_3\left(t\right) = \begin{cases} \frac{10}{0.3^6}t^6 - \frac{24}{0.3^5}t^5 + \frac{15}{0.3^4}t^4, & 0 \leqslant t < t_f^3, \\ 1, & t \geqslant t_f^3. \end{cases} \tag{73}$$

As shown in Figures 4(a) and (b), the distributed algorithm (17) still can converge in the predefined time $t_f^3 = 0.3$ s and the constraint (68b) is satisfied.

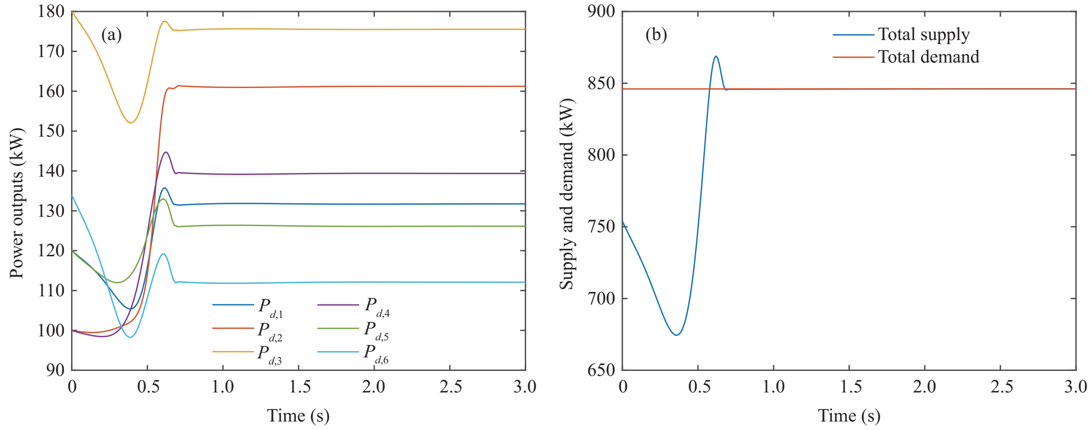

Figure 3 (Color online) Power outputs (a) and supply and demand (b) using our algorithm with $t_f^3 = 0.7$ s.

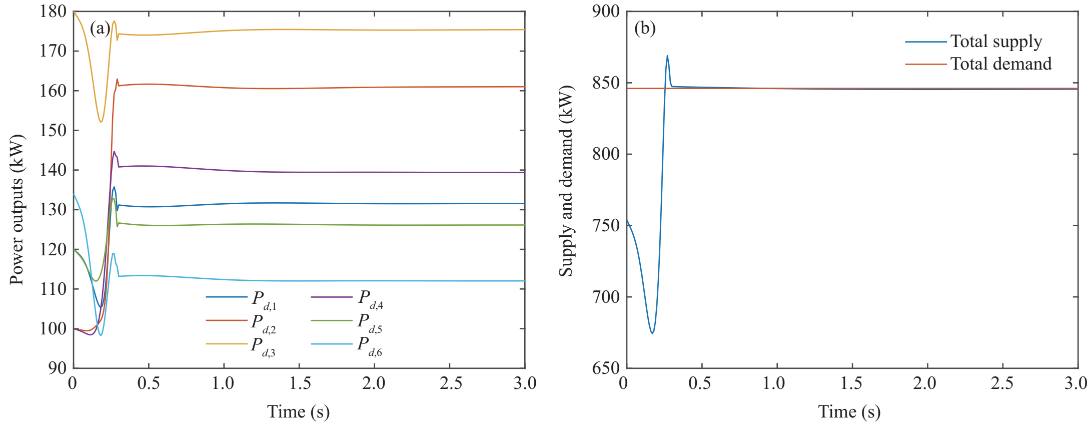

Figure 4 (Color online) Power outputs (a) and supply and demand (b) using our algorithm with $t_f^3 = 0.3$ s.

5 Conclusion

We reformulated the considered multiobjective optimization problem into a single-objective optimization problem by using the weighted L_p preference index. The weighting factors were unknown but can be obtained from optimization procedure without any prior knowledge. In addition, three predefined-time algorithms were proposed based on time-based generators. We proved that the proposed algorithms achieved predefined-time convergence. Finally, the comparative simulation results verified the fast convergence of the proposed algorithms.

The potential drawback of the proposed algorithms is that we only obtain a neighboring solution of the weighting factor $\boldsymbol{\omega}_i^*$ at a predefined time t_f^1, and a neighboring solution of the optimal value $\hat{\boldsymbol{x}}_i^*$ at a predefined time t_f^2, which cause an error for the cost component $u_i\left(x_i, \hat{\boldsymbol{x}}_i^*, \boldsymbol{\omega}_i^*\right)$ of the reformulated constrained single-objective optimization problem (9). Although the error exists until time goes to infinity, it still affects within a certain time. In addition, it is challenging to extend our method to higher-order or mixed-order multiagent systems with box constraints, where the local objective functions are nonsmooth. We will pursue these research directions in the future.

Acknowledgements This work was supported by National Natural Science Foundation of China (Grant Nos. 61991403, 61991404, 62133003), Swedish Research Council Distinguished Professor (Grant No. 2017-01078), Swedish Strategic Research Foundation SUCCESS (Grant No. FUS21-0026), and Knut and Alice Wallenberg Foundation Wallenberg Scholar Grant.

References

1 Yang S, Tan S, Xu J X. Consensus based approach for economic dispatch problem in a smart grid. IEEE Trans Power Syst, 2013, 28: 4416–4426
2 Yang T, Lu J, Wu D, et al. A distributed algorithm for economic dispatch over time-varying directed networks with delays. IEEE Trans Ind Electron, 2016, 64: 5095–5106
3 Li C, Yu X, Yu W, et al. Distributed event-triggered scheme for economic dispatch in smart grids. IEEE Trans Ind Inf, 2016, 12: 1775–1785

4 Jin L, Li S. Distributed task allocation of multiple robots: a control perspective. IEEE Trans Syst Man Cybern Syst, 2018, 48: 693–701
5 Shang R, Dai K, Jiao L, et al. Improved memetic algorithm based on route distance grouping for multiobjective large scale capacitated arc routing problems. IEEE Trans Cybern, 2015, 46: 1000–1013
6 Nedić A, Olshevsky A, Shi W. Improved convergence rates for distributed resource allocation. In: Proceedings of IEEE Conference on Decision and Control (CDC), Miami, 2018. 172–177
7 Deng Z, Liang S, Hong Y. Distributed continuous-time algorithms for resource allocation problems over weight-balanced digraphs. IEEE Trans Cybern, 2017, 48: 3116–3125
8 Zhang J, You K, Cai K. Distributed dual gradient tracking for resource allocation in unbalanced networks. IEEE Trans Signal Process, 2020, 68: 2186–2198
9 Yang T, Yi X, Wu J, et al. A survey of distributed optimization. Annu Rev Control, 2019, 47: 278–305
10 Ren H, Zhou W, Nakagami K, et al. Multiobjective optimization for the operation of distributed energy systems considering economic and environmental aspects. Appl Energy, 2010, 87: 3642–3651
11 Gunantara N. A review of multiobjective optimization: methods and its applications. Cogent Eng, 2018, 5: 1502242
12 Marler R T, Arora J S. The weighted sum method for multiobjective optimization: new insights. Struct Multidisc Optim, 2010, 41: 853–862
13 Yang R, Wang L. Multiobjective optimization for decision-making of energy and comfort management in building automation and control. Sustain Cities Soc, 2012, 2: 1–7
14 Leung M F, Wang J. A collaborative neurodynamic approach to multiobjective optimization. IEEE Trans Neural Netw Learn Syst, 2018, 29: 5738–5748
15 Jin Y, Wang H, Chugh T, et al. Data-driven evolutionary optimization: an overview and case studies. IEEE Trans Evol Computat, 2018, 23: 442–458
16 Dorigo M, Birattari M, Stutzle T. Ant colony optimization. IEEE Comput Intell Mag, 2006, 1: 28–39
17 Fonseca C M, Fleming P J. Multiobjective optimization and multiple constraint handling with evolutionary algorithms. I. A unified formulation. IEEE Trans Syst Man Cybern A, 1998, 28: 26–37
18 Coello C A C, Lechuga M S. MOPSO: a proposal for multiple objective particle swarm optimization. In: Proceedings of IEEE Congress on Evolutionary Computation (CEC), Hawaii, 2002. 1051–1056
19 Deb K, Pratap A, Agarwal S, et al. A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans Evol Computat, 2002, 6: 182–197
20 Zhang Q F, Li H. MOEA/D: a multiobjective evolutionary algorithm based on decomposition. IEEE Trans Evol Computat, 2007, 11: 712–731
21 Eichfelder G. An adaptive scalarization method in multiobjective optimization. SIAM J Optim, 2009, 19: 1694–1718
22 Burachik R S, Kaya C Y, Rizvi M M. A new scalarization technique and new algorithms to generate Pareto fronts. SIAM J Optim, 2017, 27: 1010–1034
23 Nedić A, Liu J. Distributed optimization for control. Annu Rev Control Robot Auton Syst, 2018, 1: 77–103
24 Yang S, Liu Q, Wang J. A collaborative neurodynamic approach to multiple-objective distributed optimization. IEEE Trans Neural Netw Learn Syst, 2017, 29: 981–992
25 Chen J, Sayed A H. Distributed Pareto optimization via diffusion strategies. IEEE J Sel Top Signal Process, 2013, 7: 205–220
26 Li Z, Ding Z. Distributed multiobjective optimization for network resource allocation of multiagent systems. IEEE Trans Cybern, 2021, 51: 5800–5810
27 Lv F R, Liang J, Gong K X, et al. Pareto domain adaptation. In: Proceedings of Conference on Advances in Neural Information Processing Systems (NeurIPS), 2021. 34: 12917–12929
28 Zheng Y, Wang D X. A survey of recommender systems with multiobjective optimization. Neurocomputing, 2022, 474: 141–153
29 Zhao T, Li Z, Ding Z. Consensus-based distributed optimal energy management with less communication in a microgrid. IEEE Trans Ind Inf, 2018, 15: 3356–3367
30 Zuo Z, Han Q L, Ning B, et al. An overview of recent advances in fixed-time cooperative control of multiagent systems. IEEE Trans Ind Inf, 2018, 14: 2322–2334
31 Feng Z, Hu G, Cassandras C G. Finite-time distributed convex optimization for continuous-time multiagent systems with disturbance rejection. IEEE Trans Control Netw Syst, 2019, 7: 686–698
32 Liu H, Zheng W X, Yu W. Continuous-time algorithm based on finite-time consensus for distributed constrained convex optimization. IEEE Trans Automat Contr, 2021, 67: 2552–2559
33 Chen G, Li Z. A fixed-time convergent algorithm for distributed convex optimization in multiagent systems. Automatica, 2018, 95: 539–543
34 Lv Y, Wen G, Huang T. Adaptive protocol design for distributed tracking with relative output information: a distributed fixed-time observer approach. IEEE Trans Control Netw Syst, 2019, 7: 118–128
35 Guo Z, Chen G. Predefined-time distributed optimal allocation of resources: a time-base generator scheme. IEEE Trans Syst Man Cybern Syst, 2022, 52: 438–447
36 Miettinen K. Nonlinear Multiobjective Optimization. New York: Springer, 2012
37 Wang R, Zhang Q, Zhang T. Decomposition-based algorithms using Pareto adaptive scalarizing methods. IEEE Trans Evol Computat, 2016, 20: 821–837
38 Rockafellar R T. Convex Analysis. New Jersey: Princeton University Press, 1970
39 Godsil C, Royle G F. Algebraic Graph Theory. New York: Springer, 2001
40 Zadeh L. Optimality and non-scalar-valued performance criteria. IEEE Trans Automat Contr, 1963, 8: 59–60