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Adaptive Field Gradient Estimation Based Extremum Circumnavigation

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Abstract— **This paper studies adaptive steering control of a sensory vehicle toward a planar circumnavigation orbit around a signal field source using the signal intensity measurements of the vehicle along its motion path. The signal field is approximated by a quadratic function of location, and has its extremum (maximum) at the signal source location. The proposed adaptive motion control design is based on on-line estimation of the gradient parameters of the signal field. Stability analysis is provided for the proposed adaptive estimation and motion control schemes, establishing asymptotic convergence of the gradient parameter estimates to their true values and settlement of the vehicle on an orbital trajectory on which the signal intensity is equal to a predefined constant value. Simulation test results verify the established properties of the proposed scheme as well as robustness to signal measurement noise.**

I. Introduction

A particular motion task considered in recent research studies on surveillance and target localization and monitoring via autonomous vehicle or robot agents is circumnavigation over a specific orbit around the target [1]. The applications include rescue, environmental, biomedical, transportation related ones [2], [3], where the target or signal source of interest emits electromagnetic signals or is a source of pollution or natural phenomena [4]–[6]. For example, in oceanic monitoring, traditional techniques involving ships and buoy arrays provide limited coverage compared to distributed or adaptive methods developed for fleets of autonomous underwater vehicles (AUVs) [5] for various monitoring purposes, e.g., to map and track marine blooms.

In [1], the circumnavigation control scheme is designed around a motion control law which guides an autonomous vehicular agent to orbit a target source with unknown location x, which can be stationary or drifting, in a circular path. The proposed adaptive control scheme utilizes estimation of the source location x through a linear parametric model based on filtered target distance and self-location measurements of the autonomous agent, following the adaptive localization scheme designed in [7]. A similar adaptive control scheme is developed in [8], [9] for target pursuit, i.e. reaching x rather than circumnavigating around it, utilizing the same certainty equivalence based indirect adaptive control approach and a similar adaptive localization system. In [1], [8], [9], stability analysis is performed for both static and drifting target cases via Lyapunov analysis, and the proposed designs are proven to be exponentially stable and robust to drifting of the target location.

Vehicle network approaches to distance and relative position based target reconnaissance and circumnavigation of targets, in formation, can be seen, e.g., in [10]–[12]. The design in [10] follows a geometric and time-trajectory analysis based procedure and achieves the reconnaissance control goal in finite time. The work in [11] utilizes a non-cooperative game-theoretic formulation and reformulates the circumnavigation problem as a Nash equilibrium seeking problem. In [12], the notion of circumnavigation is generalized to be an act of traveling around an encompassed region.

The ideal problem settings of [1], [8], [10]–[12] assume availability of geometric measurements, such as distance, bearing, distance difference, directly related to relative position of the signal source target. However, in many real-world scenarios, the vehicular agent can only sense the signal field intensity related to this source target. Cooperative signal source localization by formations of such agents have been studied in [13]–[17]. All these works utilize the gradient of the cooperatively measured signal field intensity to go towards the signal source, either assuming that the gradient is perfectly measurable or approximating it via signal intensity measurement differences and variations.

An adaptive localization based signal field extremum seeking scheme (ES) for the latter practical setting, where only the signal field intensity is measurable, is developed in [18]. In [18], the distribution of the signal field is approximated as a quadratic function, accommodating adaptive estimation of both source target location and the field Hessian matrix. In this paper, aligned with the framework of [18] but utilizing estimation of the field gradient parameters without explicitly estimating the source location and the Hessian parameters, an adaptive control scheme is designed for steering a sensory vehicle toward a planar circumnavigation orbit around the source of a signal field. The steering control scheme utilizes only the signal intensity measurements of the vehicle along its motion path, in addition to self location of the vehicle. Stability analysis is provided for for the proposed field gradient estimation based adaptive steering and circumnavigation control scheme, establishing asymptotic convergence of the field gradient parameter estimates to their true values and settlement of the vehicle on an orbital trajectory on which the signal intensity is equal to a predefined constant value. Simulation test results verify the established properties of the proposed scheme as well as robustness to signal measurement noise.

Beyond the existing literature, including [1], [8], [9], [14]–[17], the paper aims the following contributions: (1)

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Design of a circumnavigation control law that uses the signal intensity directly instead of relative position or distance information. (2) A new adaptive control scheme based on estimation of the gradient parameters of the signal field F only and bypassing localization of the extremum. (3) More accurate and faster adaptation via use of RLS algorithms.

II. Problem Definition and Proposed Approach

Consider a sensory vehicle A, moving over a planar region of interest for A, and a planar signal field effective in this region, represented by an unknown function $F(\cdot) : \mathbb{R}^2 \to [0, \infty)$, which satisfies the following assumptions.

Assumption 1:

i The entries of F are twice continuously differentiable.

ii The Hessian $\nabla^2 F(y)$ of F is Lipschitz continuous and bounded from above and below for all $y \in \mathbb{R}^2$.

iii The function F has a single maximum $F_{\max}$ at point $x \in \mathbb{R}^2$, and $\nabla^2 F(y)\big|_{y=x}$ is negative definite at x.

iv There exists a scalar $\gamma_H > 0$ such that, for all $\xi_1, \xi_2 \in \mathbb{R}^2$, $||\nabla^2 F(\xi_1) - \nabla^2 F(\xi_2)|| \leq \gamma_H$.

By Taylor expansion around x [19] utilizing Assumption 1, the intensity of the planar signal field at any point $y \in \mathbb{R}^2$ within the region of interest is represented by

$$F(y) = F_{\max} - \frac{1}{2}(y - x)^\top H(y - x) + h(y) \qquad (1)$$

where $H = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix} := -\nabla^2 F(y)\big|_{y=x}$ is symmetric positive definite and $h(y)$ represents the higher order terms in the Taylor series expansion. Neglecting $h(y)$ in the model (1), the gradient of the field F is obtained as

$$\nabla F(y) = -H(y - x). \qquad (2)$$

Above, the maximizer point $x \in \mathbb{R}^2$ represents the unknown location of the signal field source. The main task of this paper is to design an adaptive motion control law for the sensory agent A to steer it to and circumnavigate over an orbit of specified signal strength level F^* around x.

We consider the velocity integrator kinematic model

$$\dot{y}(t) = v(t) \qquad (3)$$

for the motion of the sensory agent A, where $y(t) = [y_1(t), y_2(t)]^\top, v(t) = [v_1(t), v_2(t)]^\top \in \mathbb{R}^2$ denote the lateral position and velocity vectors of A, respectively. Our control design will focus on producing the required velocity v for achieving the circumnavigation task. The circumnavigation control problem is formulated as follows.

Problem 1: Consider a sensory vehicle A with lateral motion kinematics (3) and an unknown planar signal field $F(\cdot)$ in the form (1). Let the agent A continuously be able to measure its current location $y(t)$, the signal strength $F(y(t))$ at $y(t)$, and the signal strength variation $z(t) = \frac{dF(y(\tau))}{d\tau}\Big|_{\tau=t}$. Given a desired level of signal F^*, design a motion control scheme to have the agent A asymptotically circumnavigate around the target location x such that $F(y(t))$ asymptotically converges to a pre-defined value $F^* > 0$.

Remark 1: In actual implementation, our control design can be used as a velocity trajectory generator and integrated with a velocity trajectory tracking control system and low level dynamic controllers that take the detailed motion dynamics of A into account.

Our approach to Problem 1 is similar to that of [18], using parameter identifier and certainty equivalence based adaptive control techniques [20]. The base motion control design is a geometric one, where the control law is defined based on the gradient ∇F of the field. The details of this base control design are given in Section III. The unknown field gradient to feed this motion control law is estimated using the parametric quadratic model (1) and a parameter identification scheme developed in Section IV. The structure of the proposed adaptive extremum circumnavigation control scheme is shown in Figure 1.

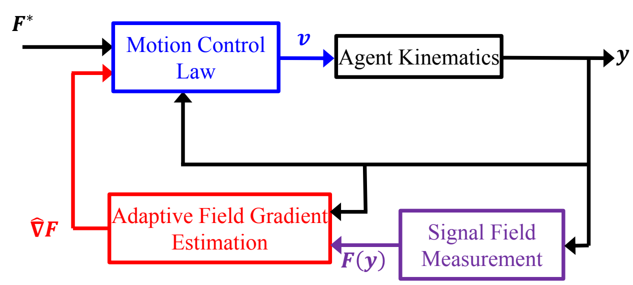

Fig. 1: Structure of the proposed adaptive extremum circumnavigation control scheme.

III. Control Design for Known Field Parameters

In this section, a motion control law is designed to move the agent A towards an orbit with a pre-defined desired signal level $F^* > 0$ and circumnavigate over this orbit around the target. It will be assumed that the field $F(\cdot)$ is in the form (1). Further, to solely focus on the motion control design, in this section, we will fictitiously assume that the target location x and the parameters of the field $F(\cdot)$ are known and the field gradient ∇F can be perfectly measured by A. Later, for application to the actual case where ∇F is not available for measurement, we will replace this base control with its adaptive version, which involves a field gradient estimator.

Consider the unit vector

$$u_\nabla(y) := \frac{1}{\|\nabla F(y)\|} \nabla F(y)$$

in the direction of $\nabla F(y)$ and let $u_\perp(y)$ be one of the two unit vectors perpendicular to $u_\nabla(y)$, defined by

$$u_\perp(y) := S u_\nabla(y),$$

for the skew-symmetric matrix $S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. By definition, we have $\|u_\perp\| = 1$ and $u_\perp^\top \nabla F(y) = 0$.

At any time instant t, if $F(y(t)) < F^*$, the agent A should move in the direction of $u_\nabla(y)$ to obtain maximum increase in $F(y)$, and otherwise if $F(y(t)) > F^*$, it should move in the direction of $-u_\nabla(y)$ to obtain maximum decrease in $F(y)$. Once A reaches a location where $F(y) = F^*$, to move on a closed path maintaining $F(y) = F^*$, its velocity should be aligned with either $u_\perp(y)$ or $-u_\perp$. A generic smooth motion control law satisfying the above requirements is

$$\begin{aligned} v &= -k_v \tanh(\delta_F(y))u_\nabla + k_v \text{sech}(\delta_F(y)u_\perp \quad (4) \\ \delta_F(y) &= F(y) - F^*, \end{aligned}$$

where k_v is a preset speed parameter. Since u_∇ and $u_\perp$ are mutually orthogonal unit vectors and $\tanh^2(\delta_F) + \text{sech}^2(\delta_F) = 1$ for any δ_F, we indeed have $\|v\| = k_v$. Closed loop stability and convergence properties guaranteed by control law (4) are stated in the following proposition.

Proposition 1: Consider the field model (1). Let $\|y(0) - x\| \geq \bar{d}_0$ for some lower bound $\bar{d}_0 > 0$ and $h(y)$ be negligible compared to $\delta_F(y)$, i.e., let $\delta_F(y) \approx \delta_F(y) - h(y)$. Then, the closed loop system (3), (4) is stable. Further, it is guaranteed that $F(y(t))$ asymptotically converges to F^* as $t \to \infty$.

Proof: Consider the Lyapunov function

$$V_F = \frac{1}{2}\delta_F^2(y). \quad (5)$$

Applying (1) and (2), time-derivative of (5) is obtained as

$$\begin{aligned} \dot{V}_F &= \delta_F(y)\dot{\delta}_F(y) \\ &= -\delta_F(y)(y - x)^\top H v \quad (6) \\ &= \delta_F(y)\nabla F^\top(y)v. \end{aligned}$$

Substituting (4) in (6), we obtain

$$\dot{V}_F = -k_v \tanh(\delta_F(y))\delta_F(y)\|\nabla F(y)\|. \quad (7)$$

Since $\text{sgn}(\tanh(\delta_F)) = \text{sgn}(\delta_F)$ for any δ_F, $\dot{V}_F < 0$ if $\delta_F \neq 0$ and $\dot{V}_F = 0$ if $\delta_F = 0$. Hence, for all $t \geq 0$, $0 \leq V_F(t) \leq V_F(0)$ and $F(t)$ approaches to F^* as $t \to \infty$. This further implies, by (1) and $\delta_F(y) \approx \delta_F(y) - h(y)$, that

$$(y(t) - x)^\top H(y(t) - x) \geq 2\bar{F}_{\min} \quad (8)$$

for $\bar{F}_{\min} := \min\{F_{\max} - F(0), F_{\max} - F^*\}$ and all $t \geq 0$.

Since H is symmetric positive definite matrix, there exists a symmetric positive definite matrix H_1, whose eigenvalues are the square roots of the eigenvalues $0 \leq \lambda_{H1} \leq \lambda_{H2}$ of H and $H = H_1^2$ [20], [21]. Hence, (8) can be rewritten as

$$\|H_1(y(t) - x)\|^2 \geq 2\bar{F}_{\min},$$

which implies that

$$\|\nabla F(t)\|^2 = \|H(y(t) - x)\|^2 \geq 2\lambda_{H1}\bar{F}_{\min}. \quad (9)$$

Since for any $0 < \gamma_{\tanh} < 1$, there exists $r_{\tanh} > 0$ such that $\tanh(x_{\tanh})/x_{\tanh} \geq \gamma_{\tanh}$ for any $x_{\tanh} \in (-r_{\tanh}, r_{\tanh})$, by (7) and (9), there exists $t_1 \geq 0$ and a constant k_δ, which depends on k_v, $\bar{F}$, λ_{H1} and t_1, such that

$$\dot{V}_F \leq -k_\delta V_F$$

for all $t \geq t_1$. Therefore, we establish that V_F converges to zero as $t \to \infty$, completing the proof. ■

Remark 2: The control law (4) allows the vehicle speed $\|v\|$ to be assigned as k_v.

IV. Field Gradient Estimation

The proposed motion control law (4) relies on the knowledge of the field gradient (2), which depends on the source location x and the positive definite matrix H. Since these values are unknown to the agent, we design a parameter identifier to estimate (2), utilizing the estimator design in [18]. As different from [18], estimation of x is not required, estimation of only Hx and H is sufficient.

Assuming that the source is stationary and $dh(y)/dt$ is negligible compared to $z(t) = \left.\frac{dF(y(\tau))}{d\tau}\right|_{\tau=t}$, i.e., $z \approx z - \frac{\mathrm{d}h(y)}{\mathrm{d}t}$, the time-derivative of (1) is derived as

$$\begin{aligned} z &= -\dot{y}^\top H(y - x) \\ &= -\dot{y}^\top H y + \dot{y}^\top H x \\ &= -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(y^\top H y\right) + \dot{y}^\top H x \\ &= -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(H_{11}y_1^2 + 2H_{12}y_1y_2 + H_{22}y_2^2\right) + \dot{y}^\top H x \\ &= -\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\begin{bmatrix} y_1^2 & 2y_1y_2 & y_2^2 \end{bmatrix}\right)\theta_H^* + \dot{y}^\top \theta_x^* \\ &= \theta^{*\top}\frac{\mathrm{d}\Psi(y)}{\mathrm{d}t} \end{aligned}$$

where $\theta_H^* = \begin{bmatrix} H_{11} & H_{12} & H_{22} \end{bmatrix}^\top$, $\theta_x^* = Hx$,

$$\theta^* = [\theta_H^{*\top},\ \theta_x^{*\top}]^\top \in \mathbb{R}^5,$$
$$\Psi(y) = \begin{bmatrix} -\frac{1}{2}y_1^2 & -y_1y_2 & -\frac{1}{2}y_2^2 & y^\top \end{bmatrix}^\top \in \mathbb{R}^5.$$

This results in the linear parametric model

$$z = \theta^{*\top}\phi, \quad (10)$$

$$\phi = \frac{\mathrm{d}\Psi(y)}{\mathrm{d}t} = [-y_1v_1,\ -y_1v_2 - v_1y_2,\ -y_2v_2,\ v^\top]^\top. \quad (11)$$

Next, two alternative parameter identifiers, similar to those in [18], are designed based on (10) to produce the estimate $\hat{\theta} = [\hat{\theta}_H^\top,\ \hat{\theta}_x^\top]^\top$ of $\theta^* = [\theta_H^{*\top},\ \theta_x^{*\top}]^\top$, which is used to generate the gradient estimate

$$\hat{\nabla}F = -\hat{H}(\hat{\theta}_H)y + \hat{\theta}_x = -\begin{bmatrix} \hat{H}_{11} & \hat{H}_{12} \\ \hat{H}_{12} & \hat{H}_{22} \end{bmatrix} y + \hat{\theta}_x. \quad (12)$$

In order to guarantee that $\hat{H}(t)$ is positive definite, parameter projection will be applied in the parameter identifier designs, utilizing the following assumption on H [18]:

Assumption 2: In (1), H satisfies:

(i) $H_{ii} > 0$ for all $i = 1, 2$.
(ii) $H_{ii} > |H_{ij}|$ for all $i, j = 1, 2$ and $i \neq j$.

Lemma 1: [18] If H satisfies Assumption 2, then it is positive definite.

As the first alternative parameter identifier based on (10), the following gradient based adaptive estimator [18] is considered:

$$\dot{\hat{\theta}} = \gamma\phi(z - \hat{\theta}^\top\phi), \quad (13)$$

where $\gamma > 0$ is a scalar design constant. The gradient based parameter identifier with projection is formulated as follows:

$$\dot{\hat{\theta}} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{\gamma\phi(z - \hat{\theta}^\top \phi)\}, \tag{14}$$

where S_H is the convex compact set of all vectors $\hat{\theta}_H = [\hat{H}_{11}, \hat{H}_{12}, \hat{H}_{22}]^\top$ such that the corresponding 2×2 matrix $\hat{H}$ satisfies Assumption 2, and $\underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{\cdot\}$ is the parameter projection operator [20], [21] defined to maintain $\hat{\theta}_H$ in S_H.

For constant θ^*, the base adaptive law (13) and the adaptive law (14) with parameter projection, respectively, lead to dynamic equations

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = -\gamma\phi\phi^\top \tilde{\theta} \tag{15}$$

and

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{-\gamma\phi\phi^\top \tilde{\theta}\}, \tag{16}$$

of the parameter estimation error vector

$$\tilde{\theta} = \begin{bmatrix} \tilde{\theta}_H \\ \tilde{\theta}_x \end{bmatrix} = \begin{bmatrix} \hat{\theta}_H - \theta_H^* \\ \hat{\theta}_x - \theta_x^* \end{bmatrix} = \hat{\theta} - \theta^*. \tag{17}$$

The second alternative parameter identifier is based on the following RLS adaptive estimation algorithm [18]:

$$\dot{\hat{\theta}} = P\phi(z - \hat{\theta}^\top \phi), \tag{18}$$

$$\dot{P} = \begin{cases} \beta P - P\phi\phi^\top P, & \text{if } \lambda_{\max}(P) < \rho_{\max} \\ 0, & \text{otherwise} \end{cases} \tag{19}$$

where the initial gain matrix $P(0)$ and hence the gain matrix $P(t)$ for any $t \geq 0$ are symmetric positive definite, $\beta > 0$ is the fixed forgetting factor, and $\rho_{\max}$ is a preset upper limit for the maximum eigenvalue $\lambda_{\max}(P)$. The RLS based parameter identifier with projection is formulated as follows:

$$\dot{\hat{\theta}} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{P\phi(z - \hat{\theta}^\top \phi)\}. \tag{20}$$

For constant θ^*, the base adaptive law (18) and the adaptive law (20) with parameter projection, respectively, lead to parameter estimation error dynamic equations

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = -P\phi\phi^\top \tilde{\theta} \tag{21}$$

and

$$\dot{\tilde{\theta}} = \dot{\hat{\theta}} = \underset{\hat{\theta}_H \in S_H}{\text{Proj}} \{-P\phi\phi^\top \tilde{\theta}\}, \tag{22}$$

The stability and asymptotic convergence properties of the gradient and RLS based adaptive scheme above are similar [18], as summarized in the following proposition. Nevertheless, the RLS based alternative facilitates fine-tuning for faster settling and robustness to measurement noises [3], [18], and hence provides better performance for the adaptive motion control scheme introduced in the next section.

Proposition 2: [18] Suppose $\theta^* \in \mathbb{R}^5$ is a constant. Consider the parametric model (10). For each of the base adaptive laws (13) and (18) and the adaptive laws (14) and (20) with parameter projection , there exist $\rho_1, \rho_2, \lambda > 0$ such that for all $t \geq 0$ and $||\theta^*(0)||$

$$||\tilde{\theta}(t)|| \leq (\rho_1||\theta^*(0)|| + \rho_2)e^{-\lambda t} \tag{23}$$

if ϕ satisfies the persistence of excitation (PE) condition

$$\alpha_1 I \leq \int_t^{t+T} \phi(\tau)\phi(\tau)^\top d\tau \leq \alpha_2 I \tag{24}$$

for some $\alpha_1 > 0$, $\alpha_2 > 0$, $T > 0$ and for all $t \geq 0$.

Remark 3: [9], [18] Symmetric positive definitess of $P(0)$ and the update law (19), together with boundedness of ϕ, guarantee that $P(t)$ and $Q(t) := P^{-1}(t)$ are bounded and symmetric positive definite for all t. Further (19) can be rewritten in terms of $Q(t)$ as

$$\dot{Q} = \begin{cases} -\beta Q + \phi\phi^\top, & \text{if } \lambda_{\min}(Q) > 1/\rho_{\max} \\ 0, & \text{otherwise} \end{cases} \tag{25}$$

Further, (24) guarantees that

$$\alpha_{Q1} I \leq Q(t) \leq \alpha_{Q2} I \tag{26}$$

for all $t \geq 0$, where $\alpha_{Q1} = \min\{\alpha_1, \lambda_{\min}(Q(0))\}$ and $\alpha_{Q2} = \alpha_2 + \lambda_{\max}(Q(0))$, with $\lambda_{\min}$ and $\lambda_{\max}$ indicating minimum and maximum eigenvalues [9], [18].

V. Adaptive Motion Control Design

Our adaptive motion control design directly utilizes the base motion control law (4), where the required field gradient ∇F is replaced with its estimate (12) generated by one of the gradient estimation schemes presented in Section IV:

$$\begin{aligned} v &= -k_v \tanh(\delta_F(y))\hat{u}_\nabla + k_v \text{sech}(\delta_F(y))\hat{u}_\perp \\ &\quad + \tanh(\delta_F(y))v_a, \\ \delta_F(y) &= F(y) - F^*, \\ \hat{u}_\nabla &= \frac{1}{\|\hat{\nabla} F(y)\|}\hat{\nabla} F(y), \quad \hat{u}_\perp(y) = S\hat{u}_\nabla, \end{aligned} \tag{27}$$

where v_a is an auxiliary control signal, with magnitude $\|v_a\| << k_v$, which is selected to satisfy that the regressor vector ϕ in (11) satisfies the PE condition (24) of Proposition 2. Such selection in turn guarantees that $\|\tilde{\theta}(t)\|$ and hence $\|\hat{\nabla} F(y(t)) - \nabla F(y(t))\|$ asymptotically converge to zero. Hence, with such selection of v_a, (27) asymptotically converges to

$$-k_v \tanh(\delta_F(y))u_\nabla + k_v \text{sech}(\delta_F(y))u_\perp + \tanh(\delta_F(y))v_a,$$

and the analysis in the proof of Proposition 1 still applies, leading to the following proposition, which states the closed loop stability and convergence properties for Problem 1 guaranteed by the adaptive control law (4).

Proposition 3: Consider the field model (1). Let $\|y(0) - x\| \geq \bar{d}_0$ for some lower bound $\bar{d}_0 > 0$ and $h(y)$ be negligible compared to $\delta_F(y)$, i.e., let $\delta_F(y) \approx \delta_F(y) - h(y)$. Further let the PE condition (24) be satisfied. Then, the closed loop system (3), (12), (27) is stable and guarantees that $F(y(t))$ asymptotically converges to F^* as $t \to \infty$.

Having three distinct frequencies, i,e., being sufficiently rich of order 6 [20], which is larger than the dimension of the parameter vector θ^*, one intuitive selection for the auxiliary control signal to satisfy the PE condition (24) is as follows:

$$v_a(t) = k_a \tanh(\delta_F(y(t)))(\sigma_1(t) + \sigma_2(t) + \sigma_3(t)), \quad (28)$$

$$\dot{\sigma}_i(t) = A_i\sigma_i(t), \;\; A_i = c_i \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ for } i = 1, 2, 3, \quad (29)$$

$\|\sigma_i(t)\| \leq 1, \forall t$ for $i = 1, 2, 3$, c_1, c_2, c_3 are distinct positive numbers defining three distinct frequencies of $v_a(t)$, and $0 < k_a << k_v$.

VI. Simulations

Here, we present results of MATLAB/Simulink simulations of the proposed adaptive motion control scheme (27),(28) together with the RLS based gradient estimator (12),(19),(20) with parameter projection, which has been observed to provide faster convergence and more accurate results than the gradient based estimator (12),(14). The presented results are for four different scenarios with stationary and drifting source location x and with perfect and noisy measurement. In all the four scenarios, the source location x is assumed to be unknown, an approximate scaled quadratic signal field model of the form (1) is considered with $F_{\max} = 10.0$ and $H = \begin{bmatrix} 5.0 & -1.0 \\ -1.0 & 3.0 \end{bmatrix}$ based on the algal bloom imagery approximation presented in [4], the desired circumnavigation level is set as $F^* = 8.0$, the control gain is chosen as $k_v = 1.0$, and the initial vehicle position is $y(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top$. In all the scenarios, the field gradient estimator parameters are chosen as $\beta = 0.5$, $\rho_{\max} = 10.0$, $\hat{\theta}(0) = \begin{bmatrix} 1 & 0.1 & 1 & -2 & -6 \end{bmatrix}^\top$, and $P(0) = \mathbb{I}_5$.

A. Scenario 1: Stationary Source and Perfect Measurement

The source is stationary, located at $x = \begin{bmatrix} 5.0 & 3.0 \end{bmatrix}^\top$. As seen in Figures 2 and 3, the trajectory of the vehicle is significantly affected by the instantaneous field gradient estimate; nevertheless, it converges to the orbit with desired field intensity level.

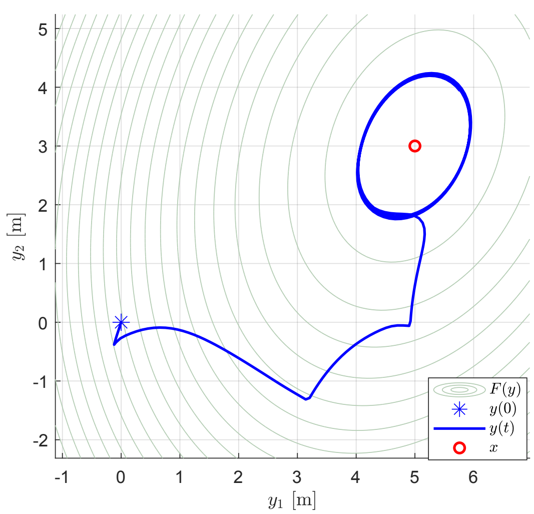

Fig. 2: Vehicle trajectory for Scenario 1.

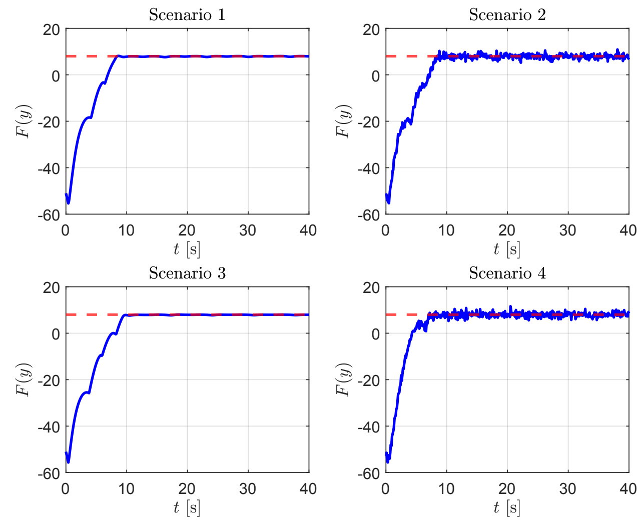

Fig. 3: Field intensity measurements for the four scenarios.

B. Scenario 2: Stationary Source and Noisy Measurement

The setting is the same as Scenario 1, except that the signal measurement from the agent $F(y)$ is assumed to exhibit a zero-mean white noise with variance of 1.00. The results are shown in 3 and Figures 4, which demonstrate that the control task is successfully achieved. Note that the measured field intensity, although noisy, stays close to the desired level-set.

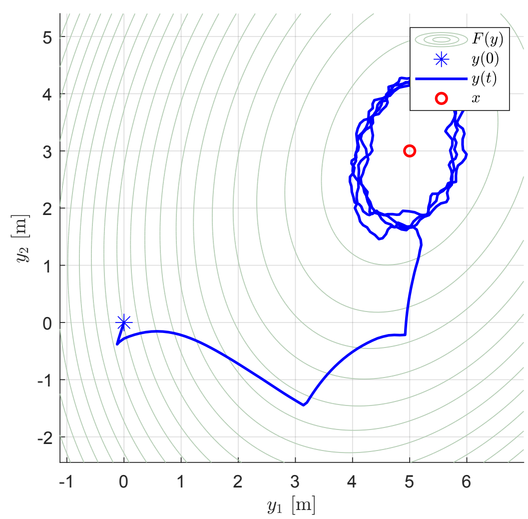

Fig. 4: Vehicle trajectory for Scenario 2.

C. Scenario 3: Drifting Source and Perfect Measurement

A slow drift in source location is introduced, leading to

$$x(t) = \begin{bmatrix} 5 + 2\sin\frac{\pi}{250}t, & 3 + 2\sin\frac{\pi}{150}t \end{bmatrix}^\top.$$

The estimation and motion control results are shown in Figures 3 and 5. Although the source drifts from the original location, the proposed adaptive control scheme keeps the vehicle on the orbit with the desired field intensity value.

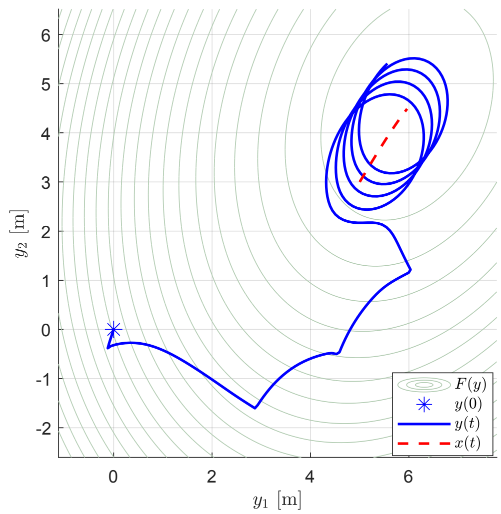

Fig. 5: Vehicle trajectory for Scenario 3.

D. Scenario 4: Drifting Source and Noisy Measurement

This scenario is a combination of Scenarios 2 and 3, where the unknown source location drifts and the field measurement is noisy with the same zero-mean Gaussian noise. The results are shown in Figures 3 and 6. The proposed adaptive control scheme keeps the vehicle on the desired orbit, under source drift and noisy measurement as well.

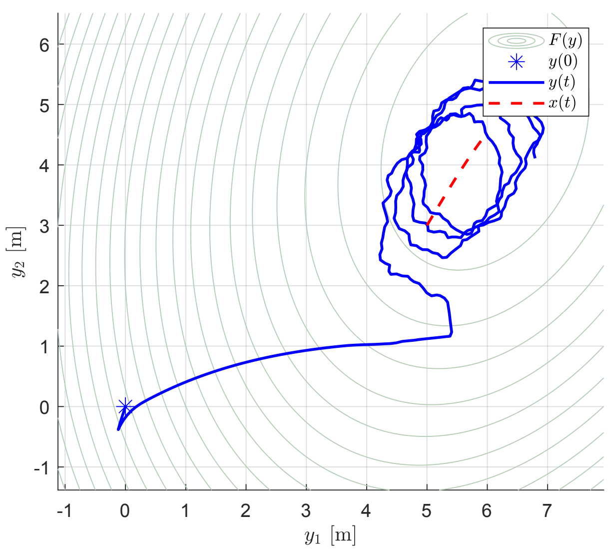

Fig. 6: Vehicle trajectory for Scenario 4.

VII. Conclusion

In this paper, an adaptive motion control scheme has been designed for navigating a sensory vehicle toward a planar circumnavigation orbit, with a specified field intensity level F^*, around the a signal source using the signal intensity measurements of the vehicle along its motion path. The proposed adaptive motion control scheme is effective in accurate and fast navigation of the vehicle to the orbit with the specified field intensity. The stability of the proposed adaptive gradient estimation and motion control schemes has been formally established. Simulation results demonstrate accuracy and robustness to source drift and field measurement noises. Follow-up research directions include extension of the proposed scheme for formations of multiple sensory vehicles and alternative adaptive motion control designs.

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