Event-based Control for Wirelessly Networked Systems

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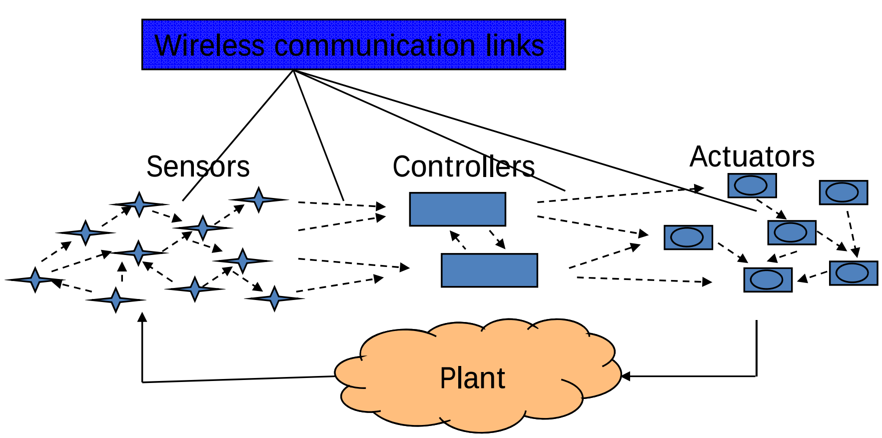
Control over wireless networks

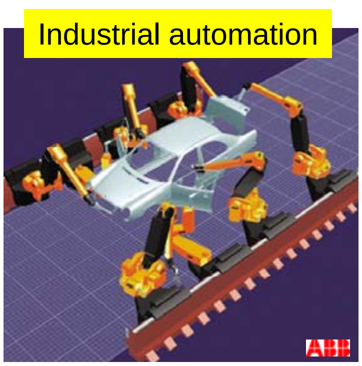
How **efficiently** do closed-loop control when sensor, actuator and controller nodes are wireless network devices?

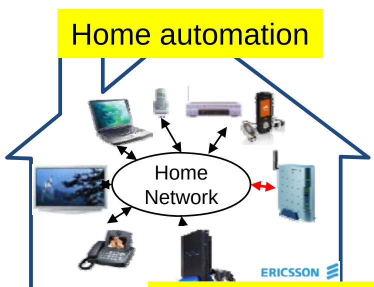


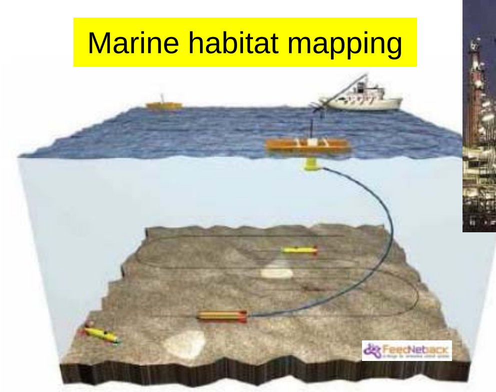
Outline

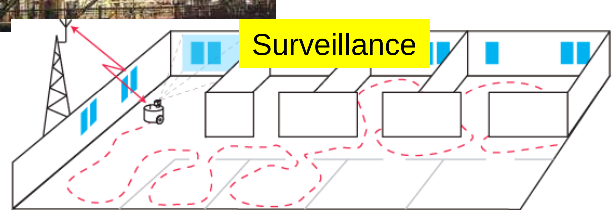
- **Introduction**
- **Motivation**
- **Architecture** for event-based control
- **Design** of event detector
- **Multiple** control **loops** and contention
- **Conclusions**

Today's wireless control systems



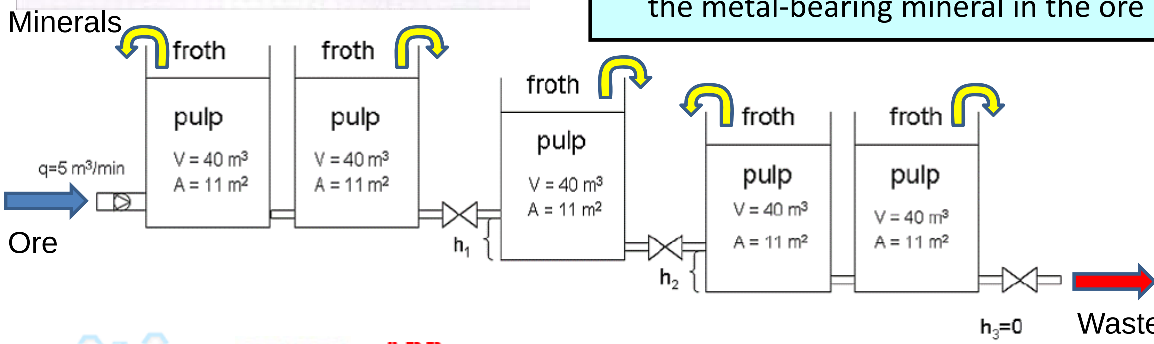






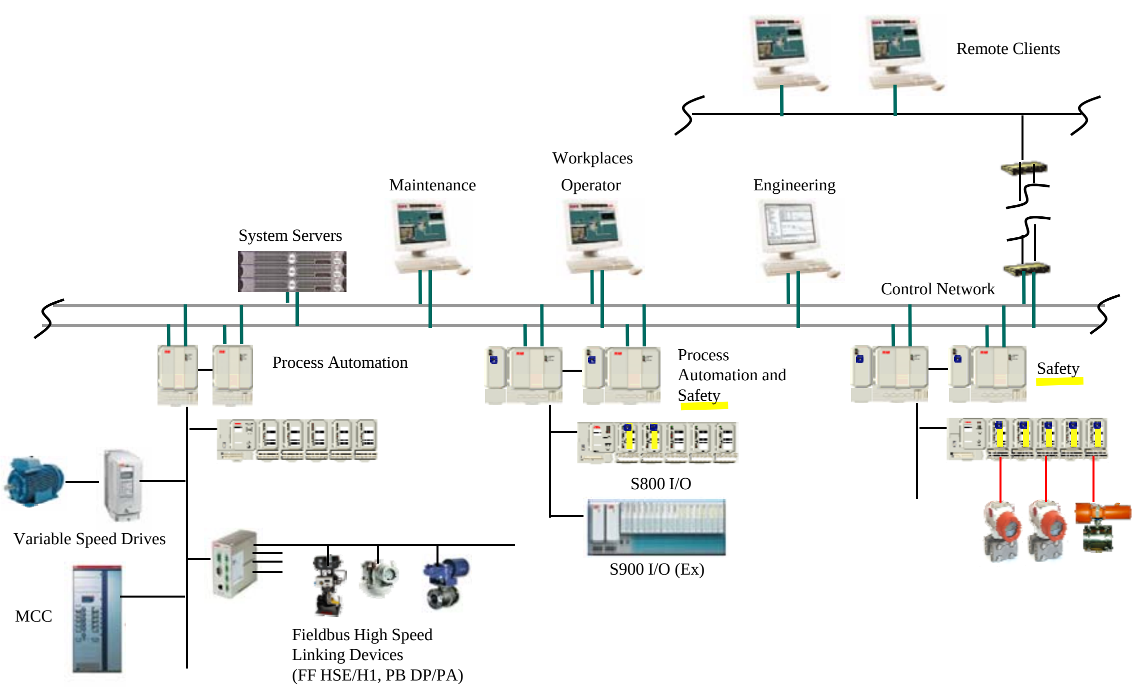
Motivating application:
Froth flotation process

- Froth flotation process concentrates the metal-bearing mineral in the ore



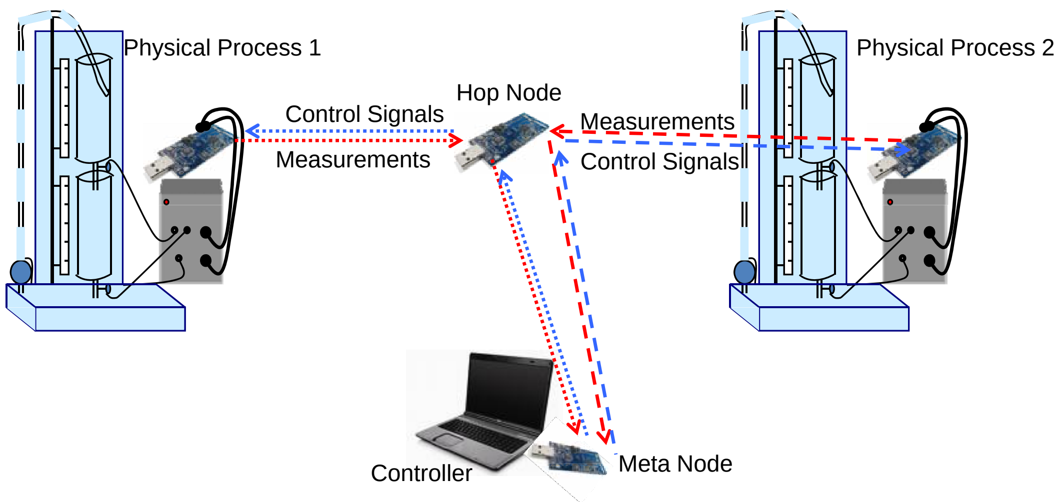
SOCRADES.EU BOLIDEN ABB

A typical communication architecture for industrial automation and control



Fieldbus PROFIBUS HART ABB

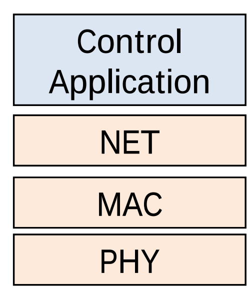
Experimental setup for control over multi-hop network

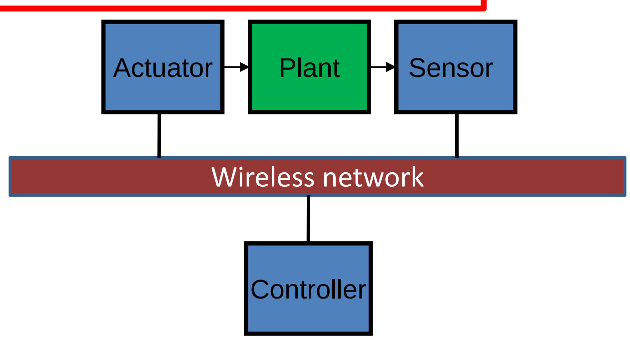


A communication or a control problem?

Approaches to control over wireless networks:

1. Communication protocol suitable for control
2. Controller that compensates for communication imperfections
3. Integrated design of control and communication layers

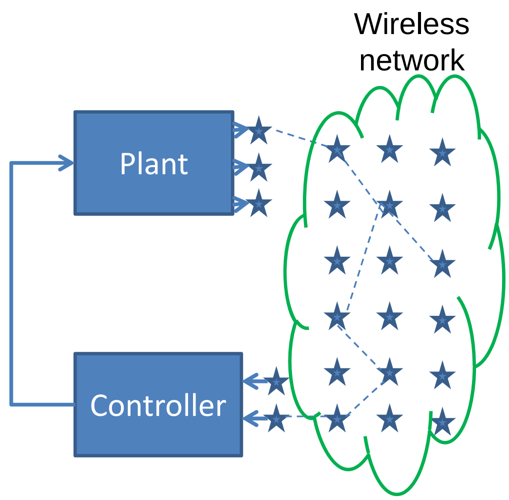




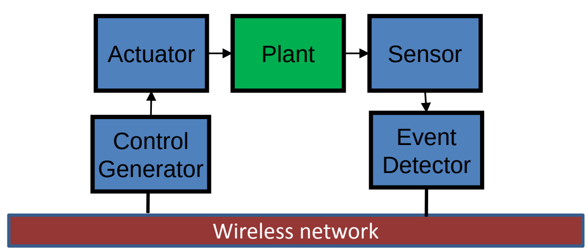
Research challenges on wireless control

To enable wide deployment of wireless control technology, we need to know

- How trade-off network resources and control performance?
- How handle communication imperfections: loss, conflicts, delays?
- How move intelligence from central units to distributed devices?



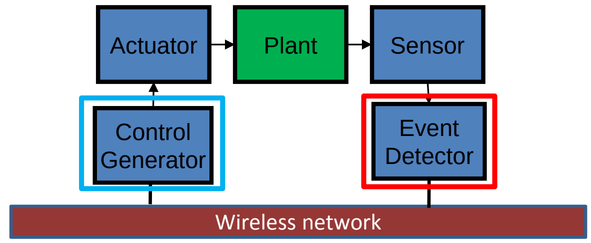
Event-based control architecture



Åström, *2007,* Rabi and J., *WICON, 2008*

When to transmit?

- Medium access control-like mechanism at sensor
  - E.g., fixed threshold crossing, adaptive threshold



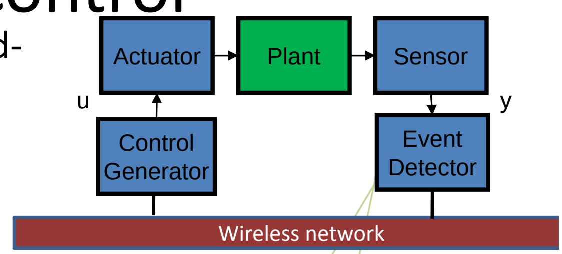
How to control?

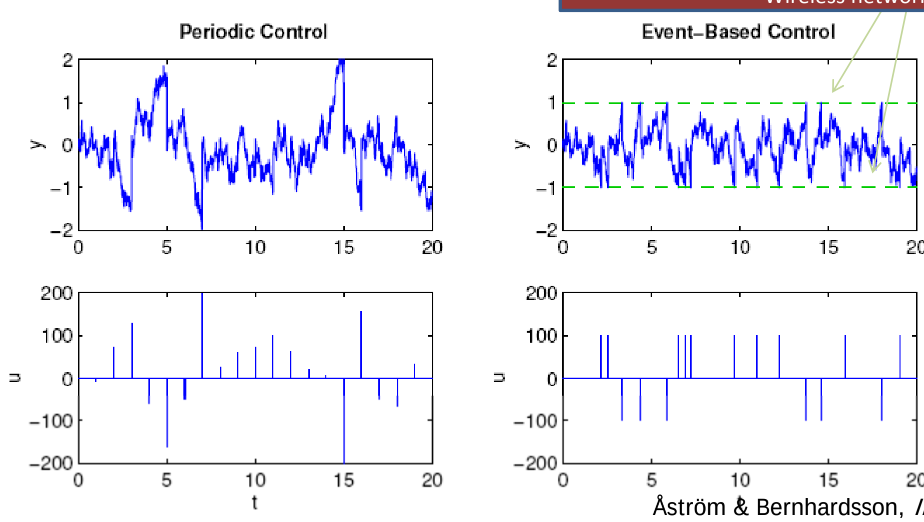
- Execute control law over fixed control alphabet
  - E.g., impulse control, piecewise constant controls

Rabi et al., *2008*

Example: Fixed threshold with impulse control

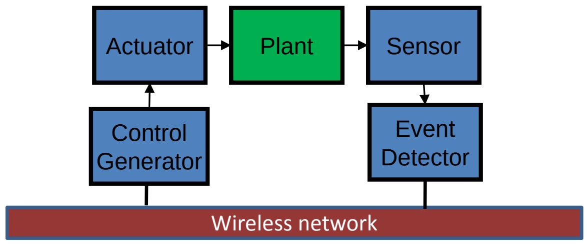
- Event-detector implemented as fixed-level threshold at sensor
- Event-based impulse control better than periodic impulse control

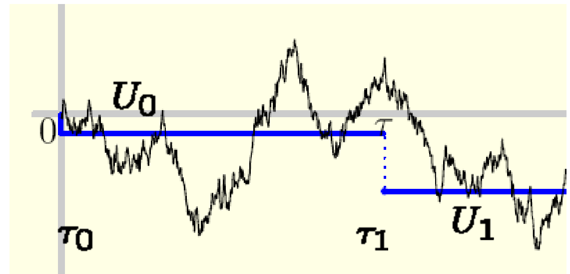




Åström & Bernhardsson, *IFAC*, 1999

Event-based ZoH control with adaptive sampling



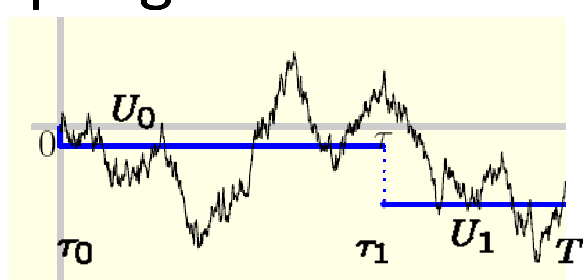


How choose $\{U_i\}$ and $\{\tau_i\}$ to minimize $V = \frac{1}{T} E \int_0^T x^2(t)dt.$

Rabi et al., *2008*

Controlled Brownian motion with one sampling event

$$dx_t = u_t dt + dB_t$$



$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds$$

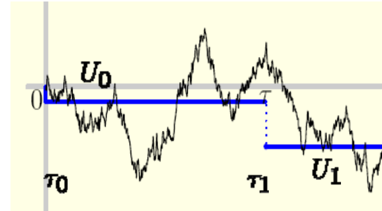
$$= \min_{U_0, U_1, \tau} \left[\mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

A joint optimal control and optimal stopping problem

Rabi et al., *2008*

$$dx_t = u_t dt + dB_t$$

$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$



If τ chosen deterministically (not depending on x_t) and $x_0 = 0$:

$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{T/2}}{T} \qquad \tau^* = T/2$$

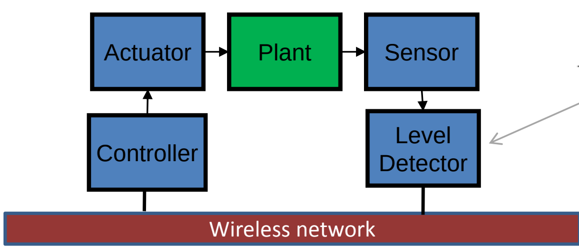
If τ is event-driven (depending on x_t) and $x_0 = 0$:

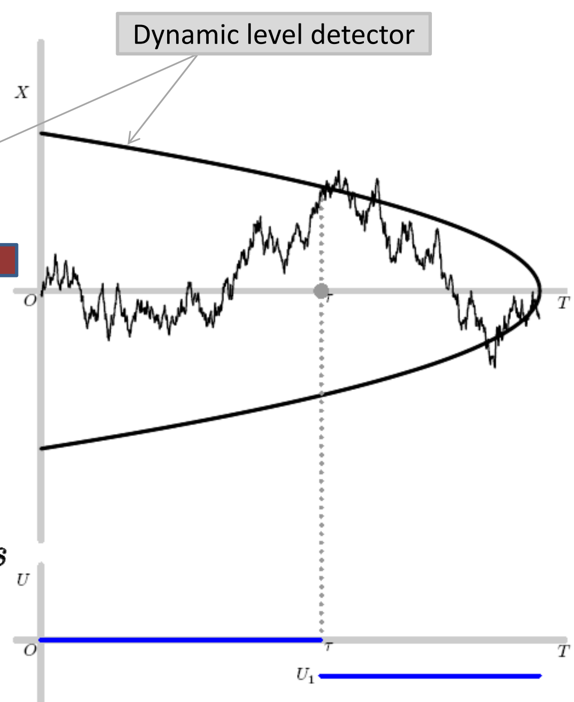
$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{\tau^*}}{2(T-\tau^*)}$$

$$\tau^* = \inf\{t : x_t^2 \geq \sqrt{3}(T-t)\}$$

Envelope defines optimal level detector

Optimal level detector





$$dx_t = u_t dt + dB_t$$

$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$

Policy iteration

For $x_0 \neq 0$ and general dynamics, we have the cost function

$$J_N(x_0, \{U_0, U_1\}, \tau) \triangleq \alpha(x_0, T) - \mathbb{E}[\beta(x_0, U_0, \tau, T)],$$

where

$$\alpha(x_0, U_0, T) = \int_0^T \mathbb{E}\left[\Phi^2_{U_0}(s, 0, x_0)\right] ds$$

$$\beta(x_0, U_0, \tau, T) = \int_\tau^T \mathbb{E}\left[\Phi^2_{U_0}(s, \tau, x_\tau) - \Phi^2_{U_1^*(x_\tau, \tau, T)}(s, \tau, x_\tau)\right]$$

and $\Phi_U(t_2, t_1, x)$ is the solution of the system with constant control

Necessary condition for optimality

$$\begin{cases} \tau^*(x_0) & = \operatorname{ess\,sup}_\tau \ \mathbb{E}\left[\beta\left(x_0, U_0^*(x_0), \tau, T\right)\right], \\ U_0^*(x_0) & = \inf_U \left\{\alpha(x_0, U, T) - \mathbb{E}\left[\beta\left(x_0, U, \tau^*(x_0), T\right)\right]\right\}. \end{cases}$$

suggests iterative search algorithm. Computationally intensive.

Rabi and J., *2009*

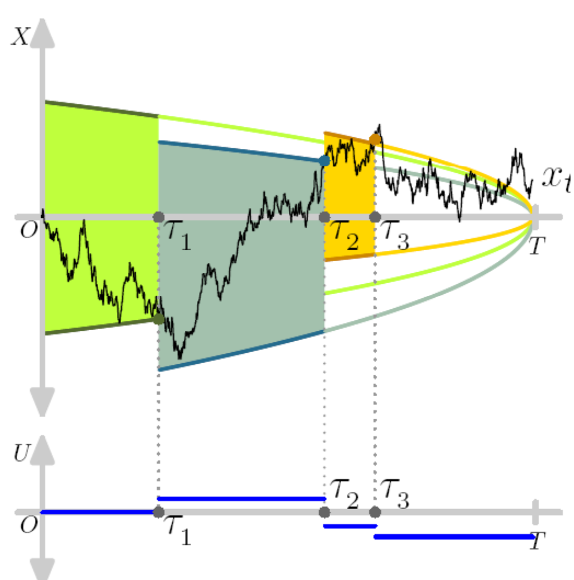
Multiple samples

Extension to N>1 samples

$$J_N\left(x_0, \mathcal{U}, \{\tau\}_{i=1}^N\right) = \mathbb{E}\left[\int_0^T x_s^2 ds \middle| x_0\right]$$

through nested single sample problems

Extension to variable budget sampling, allowing number of samples to depend on x.



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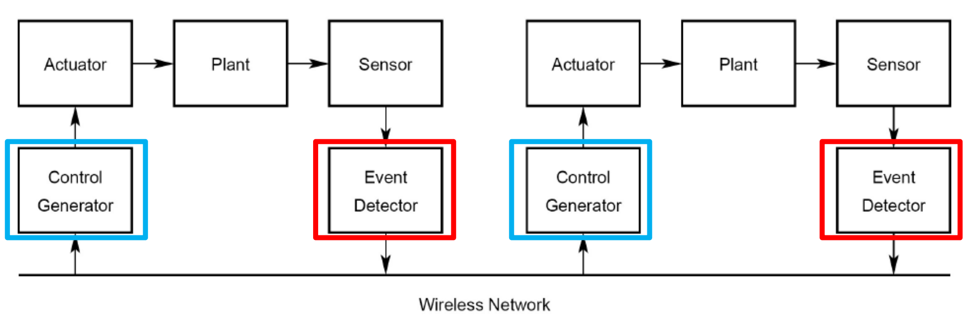
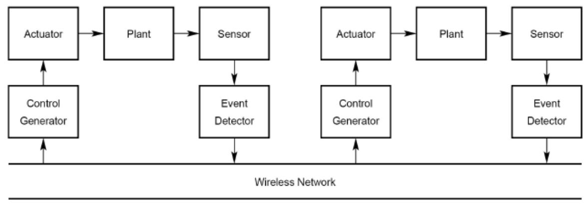
Multiple control loops

- Event-based control often outperforms periodic control for single control loops, e.g., [Åström & Bernhardsson, 1999]
- What if **multiple loops** share a **contention-based medium**?
- What amount of packet losses can the event-based scheme endure and still perform better than TDMA?

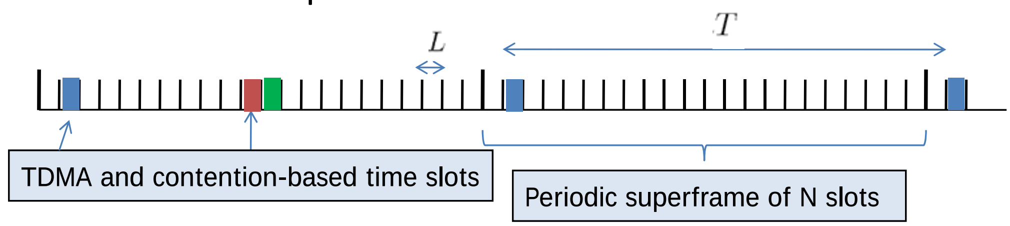


Multiple control loops

- N control loops share the same wireless network



- Time division multiple access vs contention-based medium access

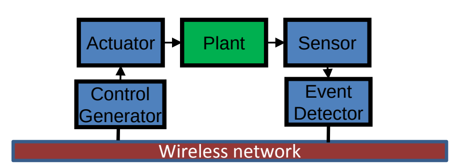


WirelessHART Standard, 2007

System model and performance measures

Plant $$dx_t = dW_t + u_t dt,\ \ x(0) = x_0,$$

Sampling events $$\mathcal{T} = \{\tau_0, \tau_1, \tau_2, \ldots\},$$



Impulse control $$u_t = \sum_{n=0}^{\infty} x_{\tau_n} \delta(\tau_n)$$

Average sampling rate $$R_{\mathcal{T}} = \limsup_{M \to \infty} \frac{1}{M} \mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\tau_n \leq M\}} \delta(s - \tau_n)\, ds\right]$$

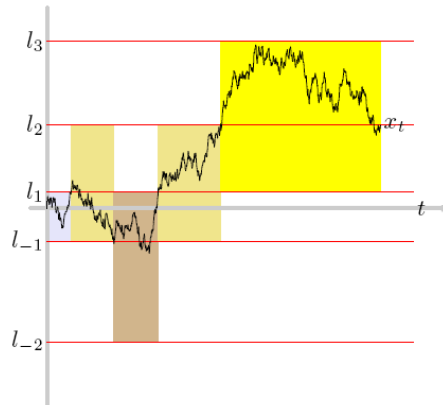
Average cost $$J = \limsup_{M \to \infty} \frac{1}{M} \mathbb{E}\left[\int_0^M x_s^2 ds\right]$$

Level-triggered control

Ordered set of levels $\mathcal{L} = \{\ldots, l_{-2}, l_{-1}, l_0, l_1, l_2, \ldots\}$ $l_0 = 0$

Multiple levels needed because we allow packet loss

Lebesgue sampling $\tau = \inf \{\tau | \tau > \tau_i, x_\tau \in \mathcal{L}, x_\tau \notin x_{\tau_i}\}$



Level-triggered control

For Brownian motion, equidistant sampling is optimal

$$\mathcal{L}^* = \{k\Delta | k \in \mathbb{Z}\}$$

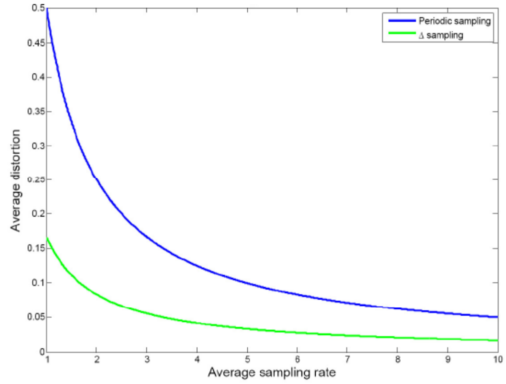
First exit time

$$\tau_\Delta = \inf \{\tau | \tau \geq 0, x_\tau \notin (\xi - \Delta, \xi + \Delta), x_0 = \xi\}$$

Average sampling rate $R_\Delta = \frac{1}{\mathbb{E}[\tau_\Delta]} = \frac{1}{\Delta^2},$

Average cost $J_\Delta = \frac{\mathbb{E}\left[\int_0^{\tau_\Delta} x_s^2 ds\right]}{\mathbb{E}[\tau_\Delta]} = \frac{\Delta^2}{6}.$

Comparison between **periodic** and **event-based** control



$T = \Delta^2$ gives equal average sampling rate for periodic control and event-based control

Event-based impulse control is 3 times better than periodic impulse control

What about the influence of communication losses?
When is event-based sampling better and vice versa?

Influence of communication losses

Times when packets are successfully received $\rho_i \in \{\tau_0 = 0, \tau_1, \tau_2, \ldots\}$,

$$\{\rho_0 = 0, \rho_1, \rho_2, \ldots\}. \qquad \rho_i \geq \tau_i,$$

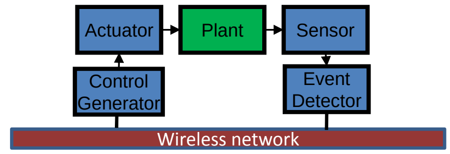
Average rate of packet reception

$$R_\rho = \limsup_{M\to\infty} \frac{1}{M}\mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\rho_n \leq M\}} \delta\left(s - \rho_n\right) ds\right] = p \cdot R_\tau$$

Define the times between successful packet receptions $\rho_{(p,\Delta)}$

Average cost $J_p = \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T x_s^2 ds\right] = \frac{\mathbb{E}\left[\int_0^{\rho_{(p,\Delta)}} x_s^2 ds\right]}{\mathbb{E}\left[\rho_{(p,\Delta)}\right]}$

IID losses



Proposition

If packet losses are IID, then equidistant Lebesque sampling gives

$$J_p = \frac{\Delta^2 (5p+1)}{6(1-p)}$$

Corollary

Event-based control better than periodic control under IID losses if $p < 0.25$

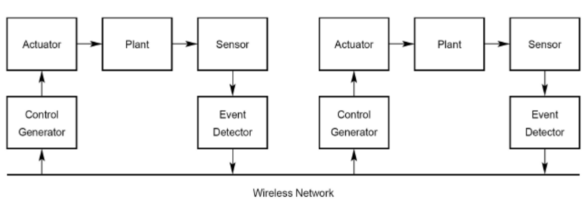
Rabi and J., *2009*

Losses depending on the other loops

Suppose the loss processes across the different loops are independent, so that the sample streams of the other sensors only matter through their average behaviour

The likelihood that a sample generated in one loop faces at least one competing transmission is then

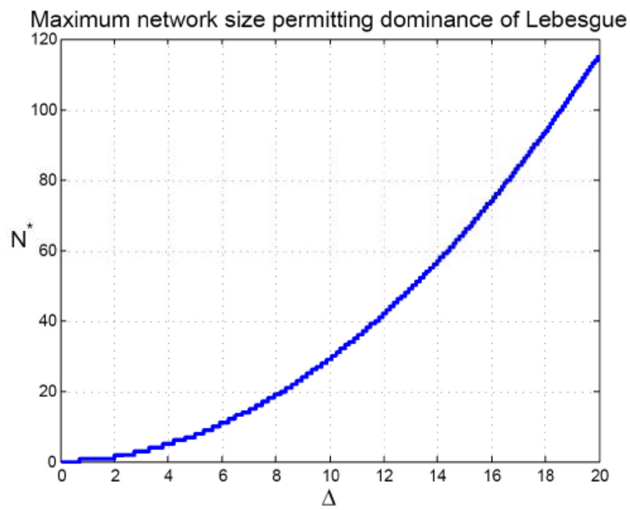
$$p = 1 - \left(1 - \frac{L}{\Delta^2}\right)^{N-1}$$



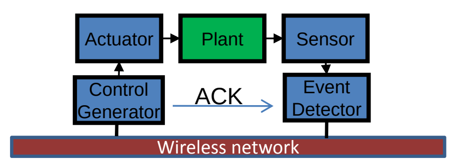
Scalability

Lebesgue sampling better than TDMA sampling for $N < N^*$

$$N^* = 1 + \left\lfloor \frac{\log(0.75)}{\log\left(1 - \frac{L}{\Delta^2}\right)} \right\rfloor.$$



Sensor data ACK's



If controller perfectly acknowledges packets to sensor, event detector can adjust its sampling strategy

Let $$\Delta(l) = \sqrt{l+1}\,\Delta_0$$

where $l \geq 0$ number of samples lost since last successfully transmitted packet

Gives $\mathbb{E}\left[\tau_{i+1}^{\uparrow} - \tau_i^{\uparrow}\right]$ independent of i.

Better performance than fixed $\Delta(l)$ for same sampling rate:

$$J_p^{\uparrow} = \frac{\Delta^2(1+p)}{6(1-p)} \leq \frac{\Delta^2(1+5p)}{6(1-p)} = J_p.$$

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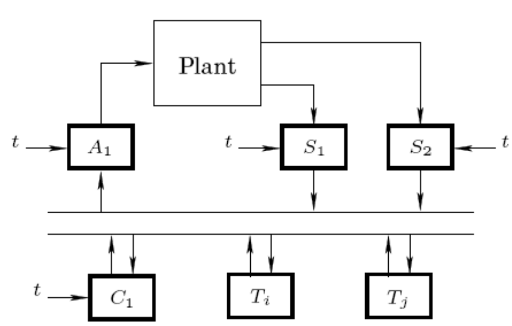
A fundamental challenge in wireless control

A conflict between

- time-driven, synchronous, sampled data **control engineering**

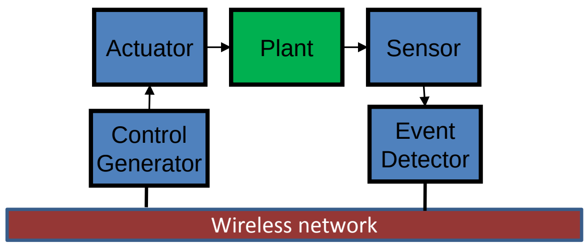
and

- event-driven, asynchronous, ad hoc **wireless networking**



Conclusions

- Event-based control architecture in support of asynchronous wireless network protocols
- Allows network nodes to take local decisions, but still guarantee global system properties
  - Optimal event-detector for LQ criterion
  - Tradeoff between performance and network resources
  - Event-based control under lossy communication



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