Lecture 11: Event-based control over wireless networks

Lecture 11 Outline

• When to schedule transmissions?
• Medium access control
• Predictive and reactive transmissions
Where to take medium access decisions?

Sensor node makes local decisions on when to communicate

Network manager allocates communication slots

Controller requests sensor data

What is the effect on control performance of a shared wireless network?

Single Process

Double Process

Sensing Link
Actuating Link
• A real experiment with water tanks and no MA:
• Let’s compare results of reference tracking

Packet loss influence on control performance

Wireless Tank Process

Level of water in the upper tank (m)

Samples

Packet loss influence on control performance

We need to consider the wireless network in the control system design

Partial improvement using CSMA/CA medium access

We need to consider the wireless network in the control system design
Is there a separation between event-based scheduling-estimation-control?

Stochastic control formulation

**Plant:**
\[ x_{k+1} = Ax_k + Bu_k + w_k \]

**Scheduler:**
\[ \delta_k = f_k(I_k^S) \in \{0, 1\} \]
\[ I_k^S = \{(x)_0^k, (y)_0^{k-1}, (\delta)_0^{k-1}, (u)_0^{k-1}\} \]

**Controller:**
\[ u_k = g_k(I_k^C) \]
\[ I_k^C = \{(y)_0^k, (\delta)_0^k, (u)_0^{k-1}\} \]

**Cost criterion:**
\[ J(f, g) = E[x_N^T Q_0 x_N + \sum_{s=0}^{N-1} \{x_s^T Q_1 x_s + u_s^T Q_2 u_s\}] \]
Control without scheduling = Classical LQG

The controller minimizing

\[ J = \mathbb{E} \left[ x_k^T Q_0 x_N + \sum_{k=0}^{N-1} (x_k^T Q_1 x_k + u_k^T Q_2 u_k) \right] \]

is given by

\[ u_k = -L_k \hat{x}_k, \]

\[ L_k = (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A \]

where

\[ S_k = Q_1 + A^T S_{k+1} A - A^T S_{k+1} B (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A \]

\[ \hat{x}_{k|k} = \mathbb{E}[x_k|y_0^k u_0^{k-1}] \] is the minimum mean-square error (MMSE) estimate

Kalman

Certainty equivalence

**Definition** Certainty equivalence holds if the closed-loop optimal controller has the same form as the deterministic optimal controller with \( x_k \) replaced by the estimate \( \hat{x}_{k|k} = \mathbb{E}[x_k|\mathcal{F}_k] \).

**Theorem** [Bar-Shalom–Tse] Certainty equivalence holds if and only if \( \mathbb{E}[(x_k - \mathbb{E}[x_k|\mathcal{F}_k])^2|\mathcal{F}_k] \) is not a function of past controls \( \{u_j\}_{0}^{k-1} \) (no dual effect).

Feldbaum, 1965; Åström, 1970; Bar-Shalom and Tse, 1974
Event-based scheduler

Plant:
\[ x_{k+1} = Ax_k + Bu_k + w_k \]

Scheduler:
\[ \delta_k = f_k(i_k^S) \in \{0, 1\} \]
\[ I_k^S = \{x_k^k, y_{k-1}^k, \delta_{k-1}^k, u_{k-1}^k\} \]

Controller:
\[ u_k = g_k(i_k^C) \]
\[ I_k^C = \{y_k^k, \delta_k^k, u_{k-1}^k\} \]

Corollary The control \( u_k \) for the optimal closed-loop system has a dual effect.

The separation principle does not hold for the optimal closed-loop system, so the design of the (event-based) scheduler, estimator, and controller is coupled.

Ramesh et al., 2011
Conditions for Certainty Equivalence

Corollary: The optimal controller for the system \( \{ \mathcal{P}, S(f), C(g) \} \), with respect to the cost \( J \) is certainty equivalent if and only if the scheduling decisions are not a function of the applied controls.

Certainty equivalence achieved at the cost of optimality

Event-based control architecture

- Plant \( \mathcal{P} \): 
  \[ x_{k+1} = ax_k + bu_k + w_k \]
- State-based Scheduler \( S \): 
  \[ \gamma_k = \begin{cases} 1, & |x_k - x_{k-1}|^2 > \epsilon, \\ 0, & \text{otherwise}. \end{cases} \]
- Observer \( \mathcal{O} \): 
  \[ y_k^0 = \delta_k^0 x_k \]
- Controller \( C \): 
  \[ u_k = -Lx_k \]

Ramesh et al., CDC, 2012
How to integrate contention resolution mechanisms?

- Hard problem because of correlation between transmissions (and the plant states)
- Closed-loop analysis can still be done for classes of event-based schedulers and MAC’s

Contention resolution through CSMA/CA

- Every transmitting device executes this protocol
- For analysis, assume carrier sense events are independent [Bianchi, 2000]

CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance
Detailed model of CSMA/CA in IEEE 802.15.4

- Markov state \((s,c,r)\)
  - \(s\): backoff stage
  - \(c\): state of backoff counter
  - \(r\): state of retransmission counter

- Model parameters
  - \(q_0\): traffic condition \((q_0=0 \text{ saturated})\)
  - \(m_p, m_r, n\): MAC parameters

- Computed characteristics
  - \(\alpha\): busy channel probability during CCA1
  - \(\beta\): busy channel probability during CCA2
  - \(P_c\): collision probability

- Detailed model for numerical evaluations
- Reduced-order models for control design
- Validated in simulation and experiment

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Slotted medium access

Many medium access protocols have slotted contention-free and contention access periods

- Periodic superframe of \(N\) slots

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Park, Di Marco, Soldati, Fischione, J, 2009

Cf., Bianchi, 2000; Pollin et al., 2006
Hybrid MAC protocols

Exploit the mix of CFP’s and CAP’s for event-based control
E.g., Araujo et al., 2010, Tiberi et al., 2010

**Contention-free period** for TDMA scheduled communication

**Contention access period** for random CSMA communication

TDMA = Time division multiple access, CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance

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**IEEE 802.15.4 MAC**

Device

Network Manager
• 16 slots for CAP and CFP
• Maximum 7 slots for CFP
IEEE 802.15.4 MAC

CSMA/CA

Contention-Access Period (CAP)  
Collision-Free Period (CFP)  
Superframe Duration (S.D.)  
Beacon Interval (B.I.)  
Inactive

Data message
IEEE 802.15.4 MAC

CSMA/CA

Backoff!

Contention-Access Period (CAP)
Collision-Free Period (CFP)
Superframe Duration (S.D.)
Beacon Interval (B.I.)
Inactive
IEEE 802.15.4 MAC

- CFP slot allocation as First-Come First-Served
IEEE 802.15.4 MAC

1. Fixed scheduling of sensing/actuation slots
2. Check triggering condition at every allocated slot
   - One-step ahead triggering condition
3. If triggering condition is true, transmit measurement and perform actuation

- Robust to disturbances
- Unnecessary bandwidth utilization
- Energy spent on checking triggering condition

Event-based sensor communication

- 16 slots for CAP and CFP
- Maximum 7 slots for CFP
- CFP slot allocation as First-Come First-Served

Araujo, 2011
Predictive sensor communication

1. Scheduling of sensing/actuation slots when required, at beacon times

2. If triggering condition is predicted to be true, transmit measurement and perform control action

3. At every transmission, predict and schedule the next triggering time
   - Set node to sleep until next transmission

- Efficient bandwidth utilization
- Low energy consumption
- Less robust to disturbances

Hybrid sensor communication

1. Scheduling of slots as predictive scheme

2. Sensor node checks triggering condition continuously (or during CAP)

3. If triggering condition is true, transmit measurement and perform control action

- Efficient bandwidth utilization
- Robust to disturbances
- Energy spent on checking triggering condition
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