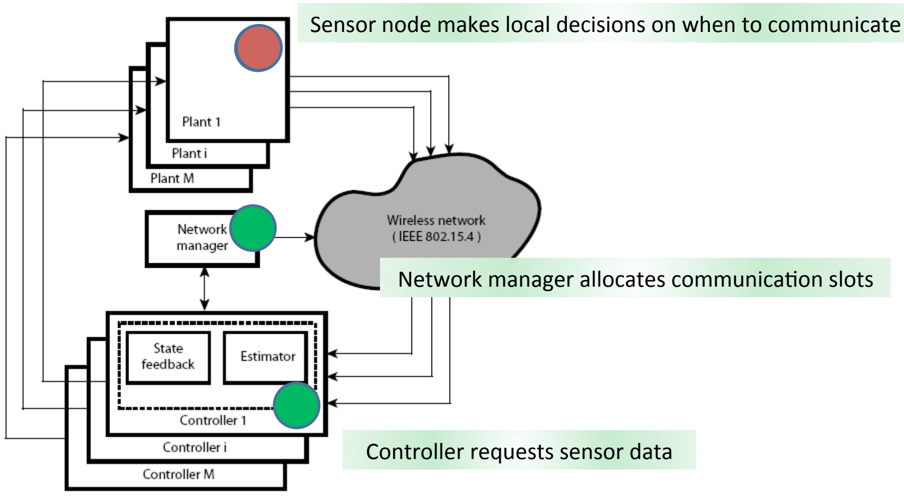
Lecture 11: Event-based control over wireless networks

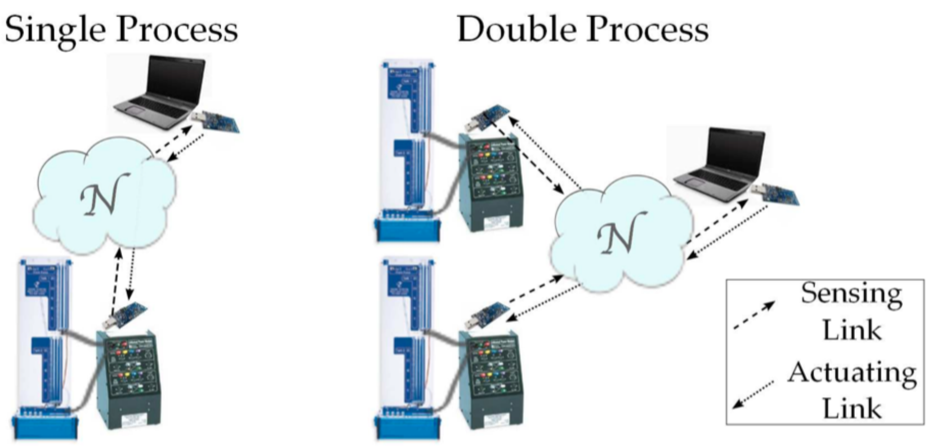
Lecture 11 Outline

- When to schedule transmissions?
- Medium access control
- Predictive and reactive transmissions

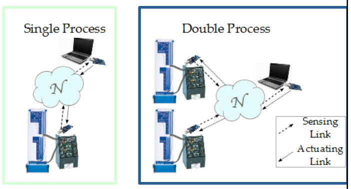
Where to take medium access decisions?

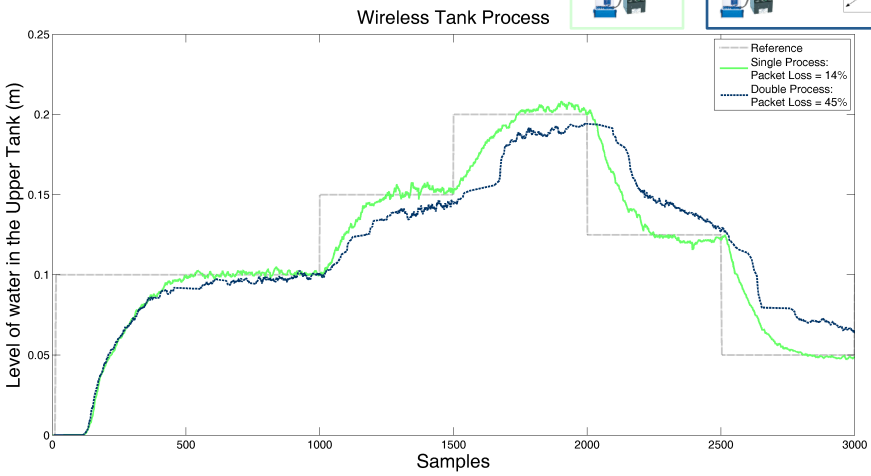


What is the effect on control performance of a shared wireless network?

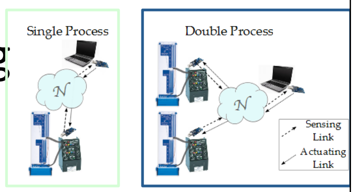


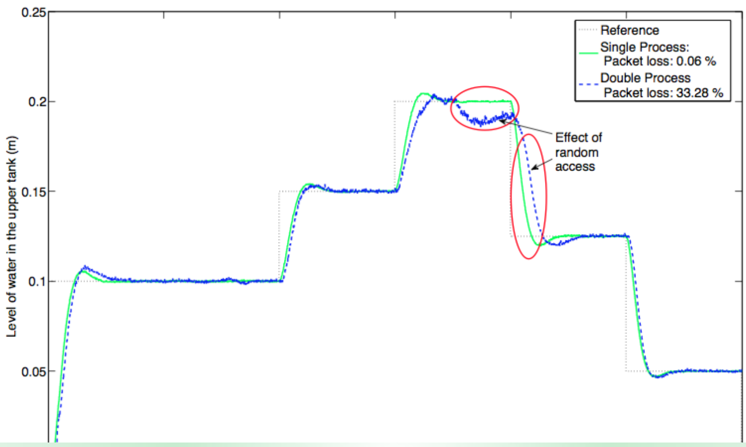
Packet loss influence on control performance





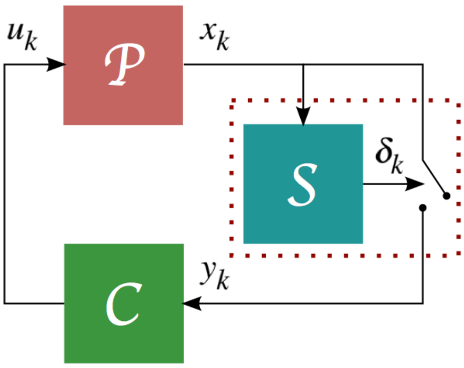
Partial improvement using CSMA/CA medium access





We need to consider the wireless network in the control system design

Is there a separation between event-based scheduling-estimation-control?



Stochastic control formulation

Plant:

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

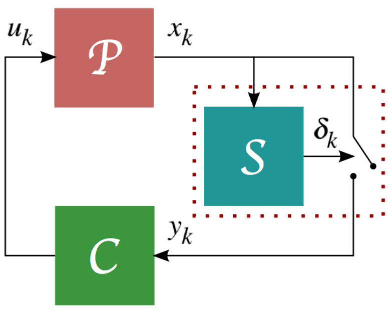
$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$

$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$

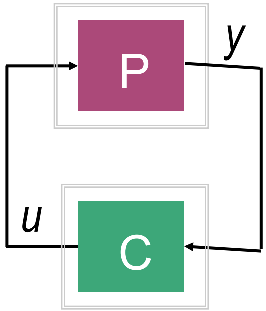
$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$



Cost criterion:

$$J(f,g) = \mathrm{E}[x_N^T Q_0 x_N + \sum_{s=0}^{N-1} (x_s^T Q_1 x_s + u_s^T Q_2 u_s)]$$

Control without scheduling = Classical LQG



The controller minimizing

$$J = \mathbb{E}\left[x_N^T Q_0 x_N + \sum_{s=0}^{N-1}(x_s^T Q_1 x_s + u_s^T Q_2 u_s)\right]$$

is given by

$$u_k = -L_k \hat{x}_{k|k} \ ,$$
$$L_k = (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A$$

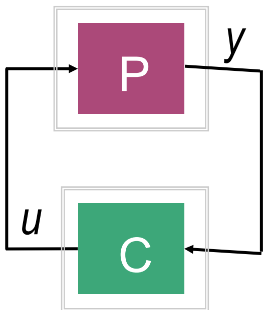
where

$$S_k = Q_1 + A^T S_{k+1} A - A^T S_{k+1} B (Q_2 + B^T S_{k+1} B)^{-1} B^T S_{k+1} A$$

$\hat{x}_{k|k} = \mathbb{E}[x_k | \{y\}_0^k u_0^{k-1}]$ is the minimum mean-square error (MMSE) estimate

Kalman

Certainty equivalence



Definition Certainty equivalence holds if the closed-loop optimal controller has the same form as the deterministic optimal controller with x_k replaced by the estimate $\hat{x}_{k|k} = \mathrm{E}[x_k | \mathbb{I}_k^{\mathbb{C}}]$.

Theorem[Bar-Shalom–Tse] Certainty equivalence holds if and only if $E[(x_k - E[x_k|I_k^c])^2|I_k^c]$ is not a function of past controls $\{u\}_0^{k-1}$ (no dual effect).

Feldbaum, 1965; Åström, 1970; Bar-Shalom and Tse, 1974

Event-based scheduler

Plant:

$x_{k+1} = Ax_k + Bu_k + w_k$

Scheduler:

$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$

$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$

Controller:

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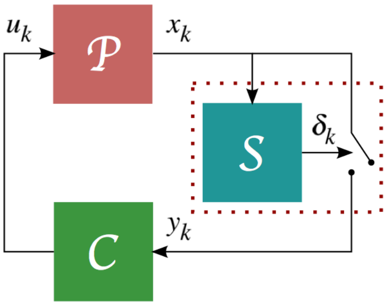

Corollary The control u_k for the optimal closed-loop system has a dual effect.

The separation principle does not hold for the optimal closed-loop system, so the design of the (event-based) scheduler, estimator, and controller is coupled

Ramesh et al., 2011

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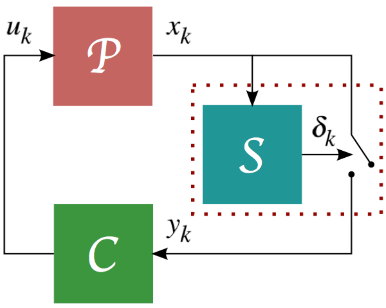
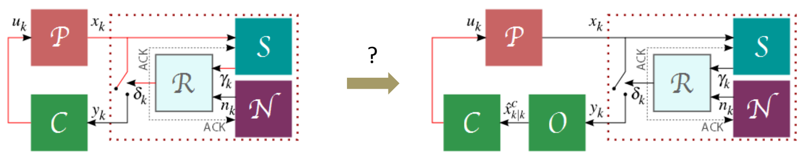


The separation principle does not hold for the optimal closed-loop system, so the design of the (event-based) scheduler, estimator, and controller is coupled

Ramesh et al., 2011

Conditions for Certainty Equivalence

Corollary: The optimal controller for the system $\{\mathcal{P},\mathcal{S}(f),\mathcal{C}(g)\}$, with respect to the cost J is certainty equivalent if and only if the scheduling decisions are not a function of the applied controls.



Certainty equivalence achieved at the cost of optimality

Ramesh et al., 2011

Event-based control architecture

- Plant $\mathcal{P}$:

  $x_{k+1} = ax_k + bu_k + w_k$

- State-based Scheduler $\mathcal{S}$:

$$\gamma_k = \begin{cases} 1, & |x_k - \hat{x}^s_{k|\tau_{k-1}}|^2 > \epsilon_d, \\ 0, & \text{otherwise.} \end{cases}$$

$$\hat{x}^s_{k|\tau_{k-1}} = a\hat{x}^c_{k-1|k-1} + bu_{k-1}$$

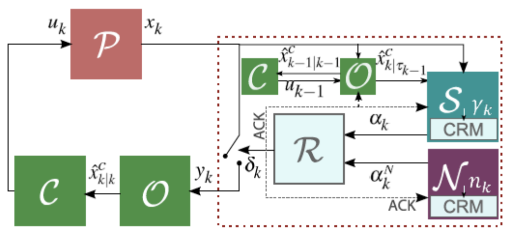
- CRM: $\mathbb{P}(\alpha_k{=}1|\gamma_k{=}1) = \mathbb{P}(\alpha_k^N{=}1|n_k{=}1) = p_\alpha$

$$\delta_k = \alpha_k(1-\alpha_k^N)$$

- Observer $\mathcal{O}$: $y_k^{(j)} = \delta_k^{(j)} x_k^{(j)}$

$$\hat{x}^c_{k|k} = \bar{\delta}_k(a\hat{x}^c_{k-1|k-1} + bu_{k-1}) + \delta_k x_k$$

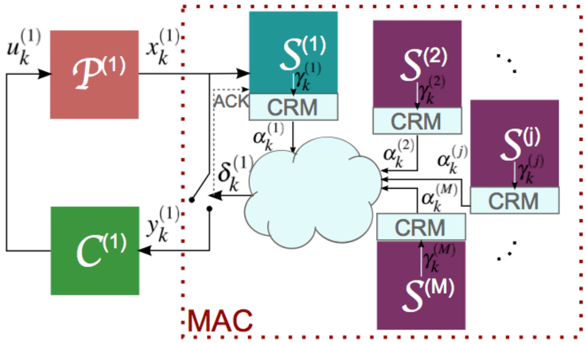
- Controller $\mathcal{C}$: $u_k = -L\hat{x}^c_{k|k}$



Ramesh et al., CDC, 2012

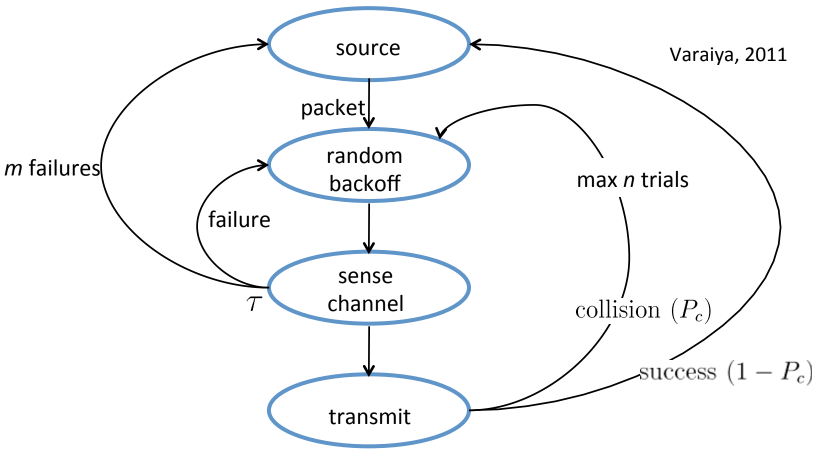
How to integrate contention resolution mechanisms?

- Hard problem because of correlation between transmissions (and the plant states)
- Closed-loop analysis can still be done for classes of event-based schedulers and MAC's



Ramesh et al., CDC 2011

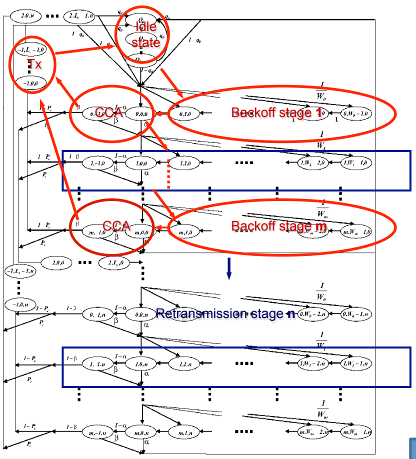
Contention resolution through CSMA/CA



- Every transmitting device executes this protocol
- For analysis, assume carrier sense events are independent [Bianchi, 2000]

CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance

Detailed model of CSMA/CA in IEEE 802.15.4



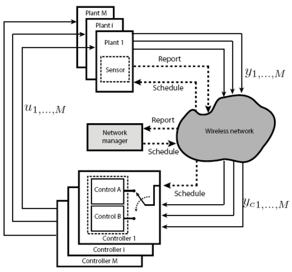
- Markov state (*s*,*c*,*r*)
  - s: backoff stage
  - c: state of backoff counter
  - r: state of retransmission counter
- Model parameters
  - q_0: traffic condition (q_0=0 saturated)
  - m_0, m, m_b, n: MAC parameters
- Computed characteristics
  - α: busy channel probability during CCA1
  - β: busy channel probability during CCA2
  - P_c: collision probability

- Detailed model for numerial evaluations
- Reduced-order models for control design
- Validated in simulation and experiment

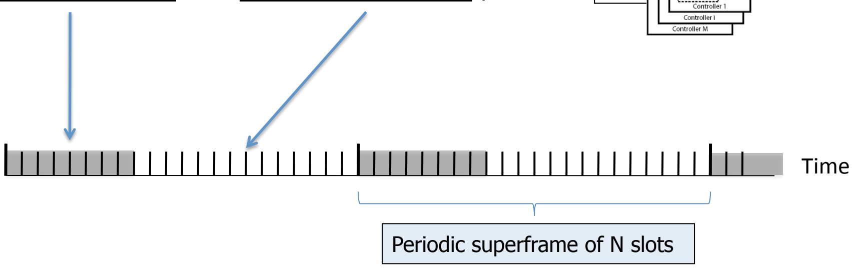
Park, Di Marco, Soldati, Fischione, J, 2009

Cf., Bianchi, 2000; Pollin et al., 2006

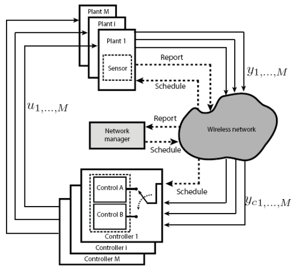
Slotted medium access



Many medium access protocols have slotted
contention-free and contention access periods



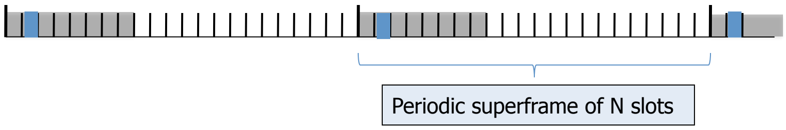
Hybrid MAC protocols



Exploit the mix of CFP's and CAP's for event-based control

E.g., Araujo et al., 2010, Tiberi et al., 2010

Contention-free period for TDMA scheduled communication



Contention access period for random CSMA communication

TDMA = Time division multiple access, CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance

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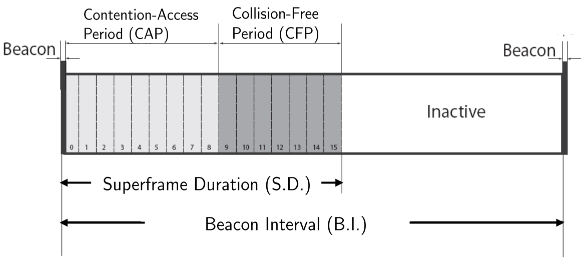
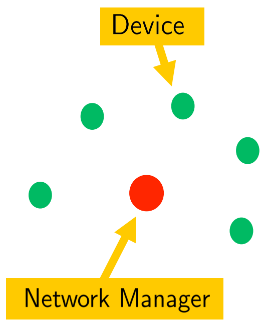
IEEE 802.15.4 MAC





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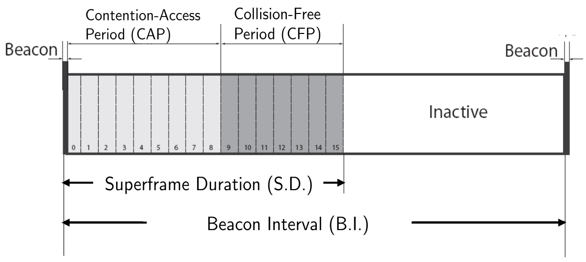
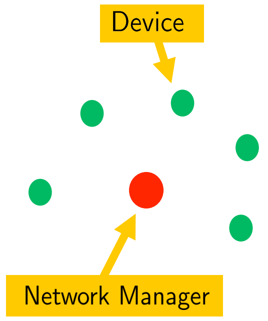
IEEE 802.15.4 MAC





- 16 slots for CAP and CFP
- Maximum 7 slots for CFP

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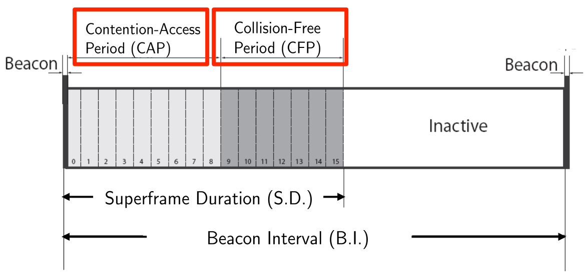
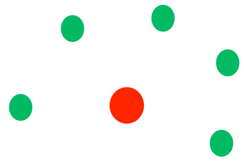
IEEE 802.15.4 MAC





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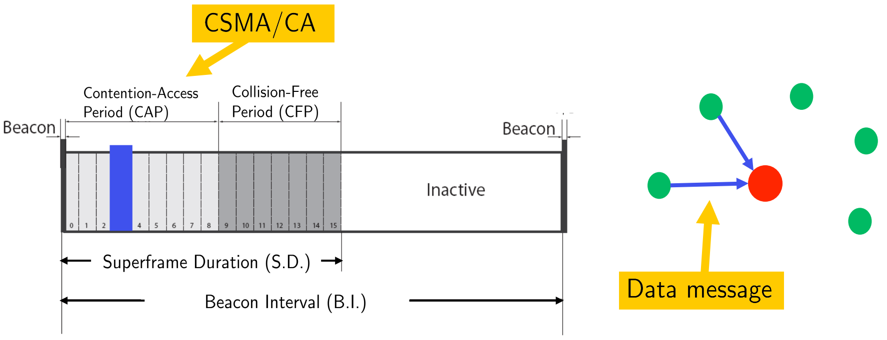
IEEE 802.15.4 MAC



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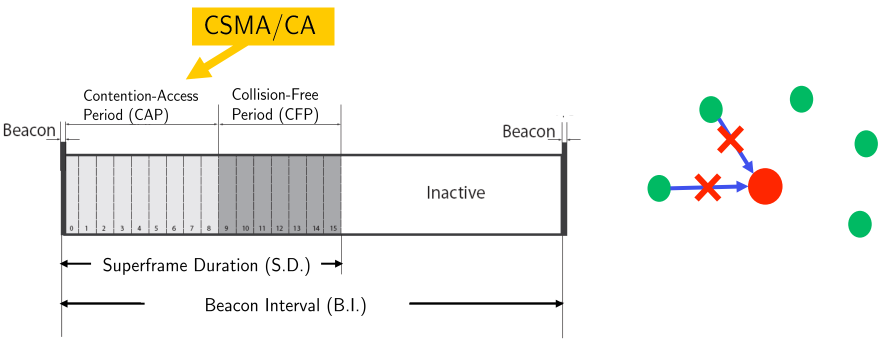
IEEE 802.15.4 MAC



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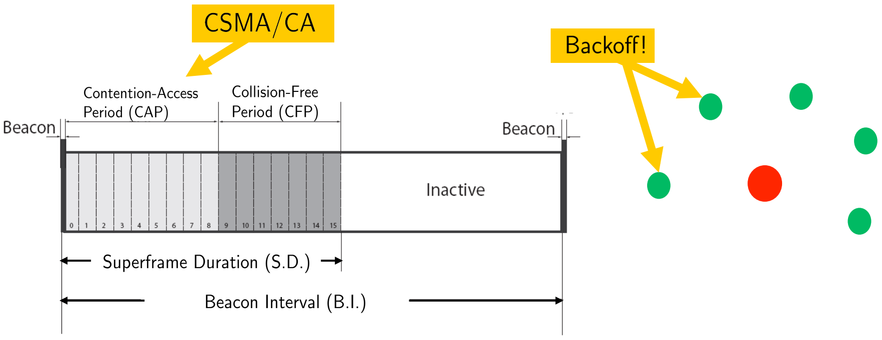
IEEE 802.15.4 MAC



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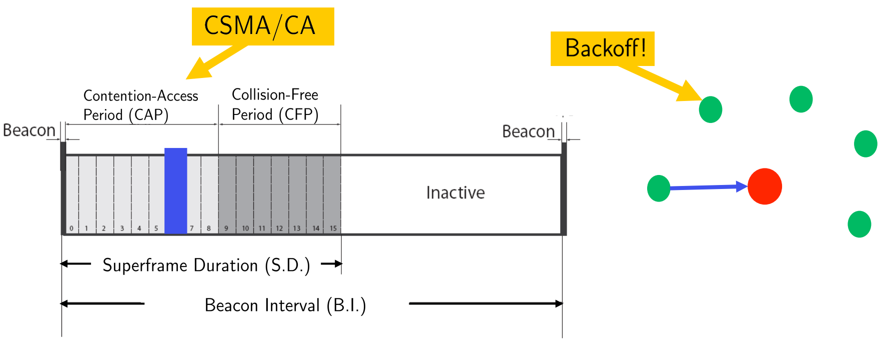
IEEE 802.15.4 MAC



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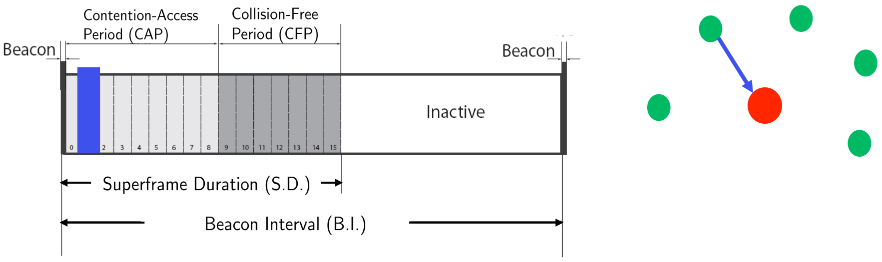
IEEE 802.15.4 MAC



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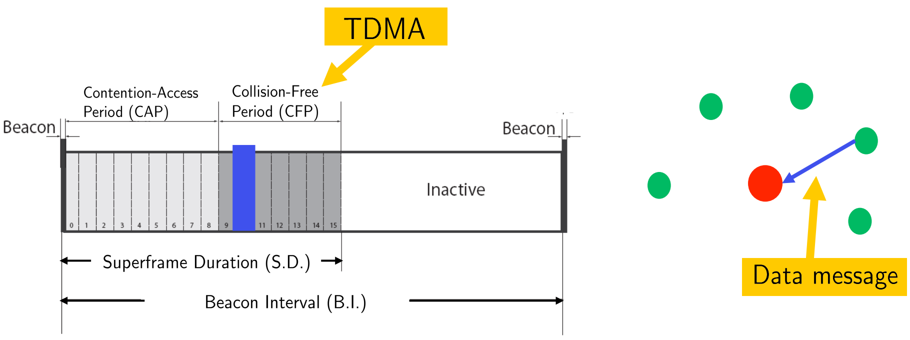
IEEE 802.15.4 MAC



- CFP slot allocation as First-Come First-Served

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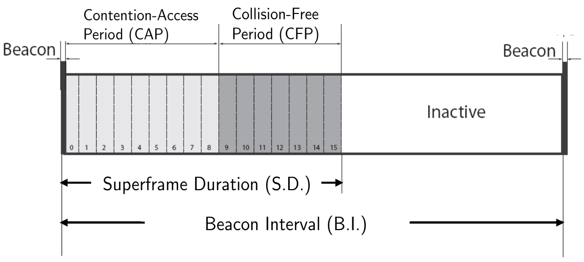
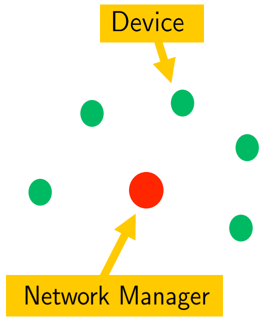
IEEE 802.15.4 MAC





- 16 slots for CAP and CFP
- Maximum 7 slots for CFP
- CFP slot allocation as First-Come First-Served

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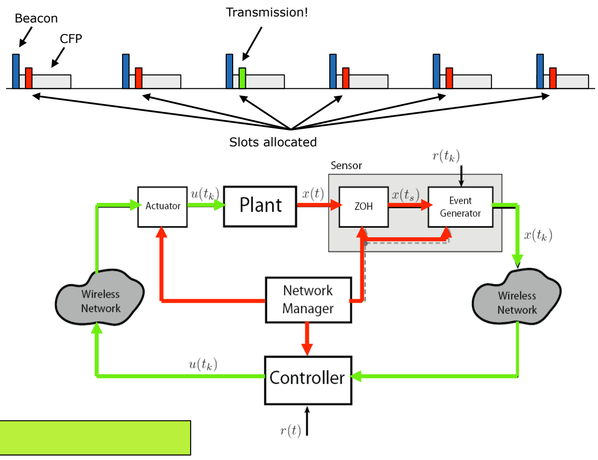
Event-based sensor communication

1. Fixed scheduling of sensing/actuation slots
2. Check triggering condition at every allocated slot
   - One-step ahead triggering condition
3. If triggering condition is true, transmit measurement and perform actuation



- **Robust to disturbances**

- **Unnecessary bandwidth utilization**
- **Energy spent on checking triggering condition**

Araujo, 2011 30

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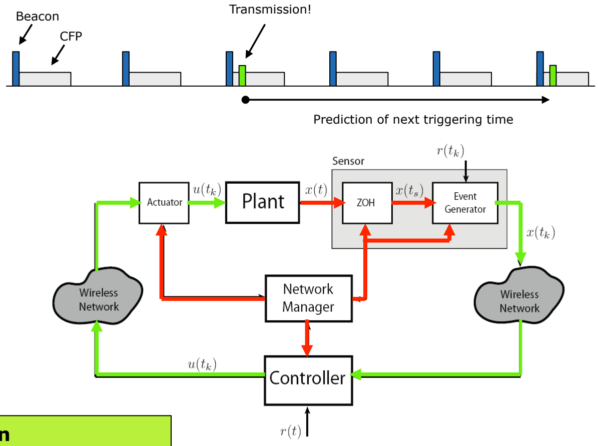
Predictive sensor communication

1. Scheduling of sensing/ actuation slots when required, at beacon times
2. If triggering condition is predicted to be true, transmit measurement and perform control action
3. At every transmission, predict and schedule the next triggering time
   - Set node to sleep until next transmission



- **Efficient bandwidth utilization**
- **Low energy consumption**

- **Less robust to disturbances**

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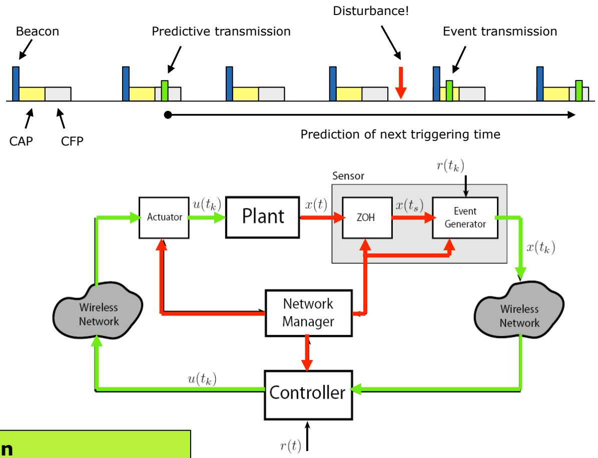
Hybrid sensor communication

1. Scheduling of slots as predictive scheme
2. Sensor node checks triggering condition continuously (or during CAP)
3. If triggering condition is true, transmit measurement and perform control action



- **Efficient bandwidth utilization**
- **Robust to disturbances**

- **Energy spent on checking triggering condition**

Araujo, 2011 32

Lecture 11 Outline

- When to schedule transmissions?
- Medium access control
- Predictive and reactive transmissions