M17: Event-triggered and Self-triggered Control Lectures 10-14

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Karl H. Johansson
KTH Royal Institute of Technology
Stockholm, Sweden

Me

Karl H. Johansson, KTH
www.ee.kth.se/~kallej

- MS, PhD, Lund U, EE
- Postdoc, UC Berkeley

Research interests

- Networked control systems
- Hybrid systems
- Applications in smart mobility, automation, and energy systems

Acknowledgements

- Maben Rabi
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Outline

Lecture 10: Stochastic event-based control
Lecture 11: Event-based control over wireless networks
Lecture 12: Distributed and saturated event-based control
Lecture 13: Applications
Lecture 14: Summary and outlook

Outline

Lecture 10: Stochastic event-based control
- Introduction to Lectures 10-14
- Stochastic control (Maben)
- When to transmit? (KJ, Maben)

Lecture 11: Event-based control over wireless networks
- Models of wireless networks (Chithrupa)
- Certainty equivalence (Chithrupa)
- Event-triggered control over MAC (Chithrupa, Rainer)
- Time-triggered and event-triggered communication (Jose)

Lecture 12: Distributed and saturated event-based control
- Event-based multi-agent control (George, Dimos, Guodong, Maria)
- Anti-windup (Daniel), Event-based PID control (Maben)

Lecture 13: Applications
- Smart mobility: real-time management for heavy-duty vehicle platooning
- Industrial wireless control
- Smart buildings: hvac and building automation, demand response

Lecture 14: Summary and outlook
- Summary of course
- What was not covered? (sequential detection, optimal stopping, multi-rate sampling etc)
- Outlook (cyber-physical systems, cyber-security, ncs lecture)

Biography

Questions to Answer

Lecture 10: Stochastic event-based control
- **What if there are stochastic uncertainties in the control loop?**

Lecture 11: Event-based control over wireless networks
- **How to model wireless networks in control loops?**

Lecture 12: Distributed and saturated event-based control
- **Can event-based control be implemented as a distributed system?**

Lecture 13: Applications
- **Are event-based control systems used in practice?**

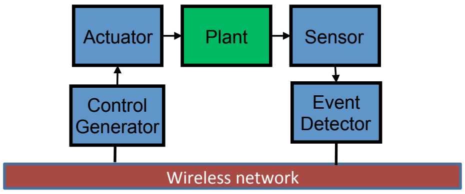
Lecture 14: Summary and outlook
- **Are there any exciting open research problems to work on?**

Lecture 10: Stochastic event-based control

Lecture 10 Outline

- Stochastic control
- Optimal event-based control
- Event-based control with packet losses

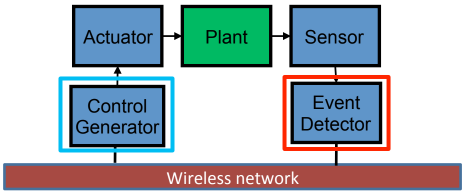
Event-based control loop



Åström, *2007,* Rabi and J., *WICON, 2008*

When to transmit?

- Event detector mechanism on sensor side
  - E.g., threshold crossing



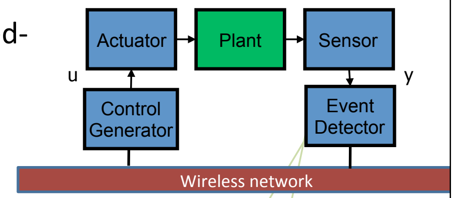
How to control?

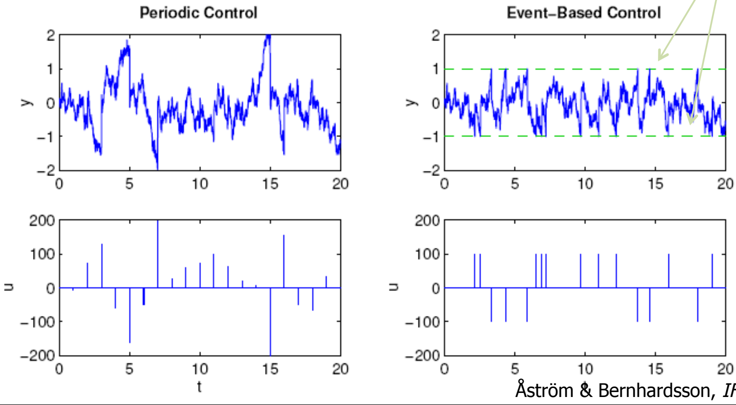
- Execute control law at actuator side
  - E.g., piecewise constant controls, impulse control

Rabi et al., *2008*

Example: Fixed threshold with impulse control

- Event-detector implemented as fixed-level threshold at sensor
- Event-based impulse control better than periodic impulse control



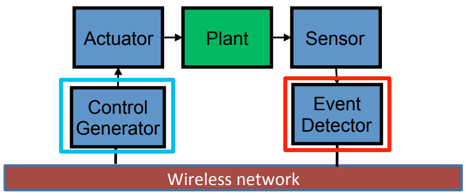


Åström & Bernhardsson, *IFAC,* 1999

Control generators and event detectors

1. Impulse
2. Zero order hold
3. Higher order hold

1. Fixed threshold
2. Time-varying
3. Adaptive



Plant model

Plant $dx = udt + dv,$

Stochastic differential equation, interpreted as

$$x(s+\tau) - x(\tau) = \int_{\tau}^{s+\tau} u(t)dt + \int_{\tau}^{s+\tau} dv(t)$$

with one ordinary (Lebesgue) integral and one stochastic (Ito) integral.

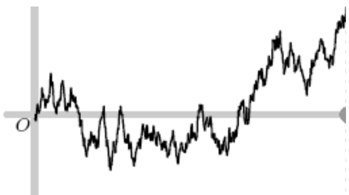
v is a Wiener process (or Brownian motion)

See bibliography incl Øksendal (2003) for an introduction to stochastic differential equations

Wiener process

A Wiener process v(t) fulfills

1. v(0)=0
2. v(t) is almost surely continuous
3. v(t) has independent increments with v(t)-v(s) ~ N(0,t-s) for t>s≥0



Remark The variance of a Wiener process is growing like

$$E(v(t+s) - v(t))^2 = |s|$$

Plant model

Plant $dx = udt + dv,$

Stochastic differential equation, interpreted as

$$x(s+\tau) - x(\tau) = \int_{\tau}^{s+\tau} u(t)dt + \int_{\tau}^{s+\tau} dv(t)$$

with one ordinary (Lebesgue) integral and one stochastic (Ito) integral.

When $s > 0$ is a small, the change of $x(\tau)$ is
normally distributed with mean $su(\tau)$ and variance s.

Plant model and control cost

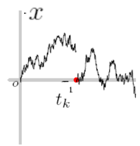
Plant $dx = udt + dv,$

v is a Wiener process: $E(v(t+s) - v(t))^2 = |s|$

Cost function $V = \frac{1}{T} E \int_0^T x^2(t)dt.$

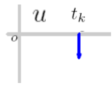
Periodic impulse control

Impulse applied at events t_k



$$u(t) = -x(t_k)\delta(t - t_k),$$

Periodic reset of state every event.



State grows linearly as

$$E(v(t+s) - v(t))^2 = |s|$$

between sample instances, because $dx = udt + dv,$

Average variance over sampling period h is $\frac{1}{2}h$ so the cost is $V_{PIH} = \frac{1}{2}h.$

Åström, 2007

Periodic ZoH control

Traditional sampled-data control theory gives that $V = \frac{1}{h}\int_0^h Ex^2(t)\,dt$ is minimized for the sampled system

$$x(t+h) = x(t) + hu(t) + e(t),$$

with

$$u = -Lx = \frac{1}{h}\frac{3+\sqrt{3}}{2+\sqrt{3}}x$$

derived from

$$S = \Phi^T S\Phi + Q_1 - L^T RL, \quad L = R^{-1}(\Gamma^T S\Phi + Q_{12}^T), \quad R = Q_2 + \Gamma^T S\Gamma,$$

The minimum gives the cost

$$V_{PZOH} = \frac{3+\sqrt{3}}{6}h$$

Åström, 2007

Event-based impulse control with fixed threshold

Suppose an event is generated whenever

$$|x(t_k)| = a$$

generating impulse control

$$u(t) = -x(t_k)\delta(t - t_k),$$

One can show that the average time between two events is

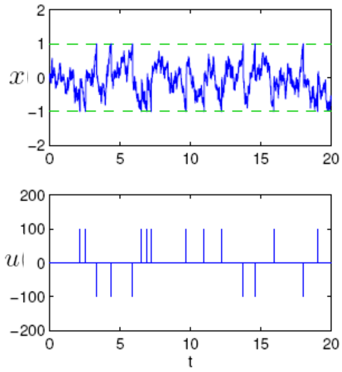
$$h_E := E(T_{\pm d}) = E(x^2_{T_{\pm d}}) = a^2$$

and that the pdf of x is triangular:

$$f(x) = (a - |x|)/a^2$$

The cost is

$$V_{EIH} = \frac{a^2}{6} = \frac{h_E}{6}$$

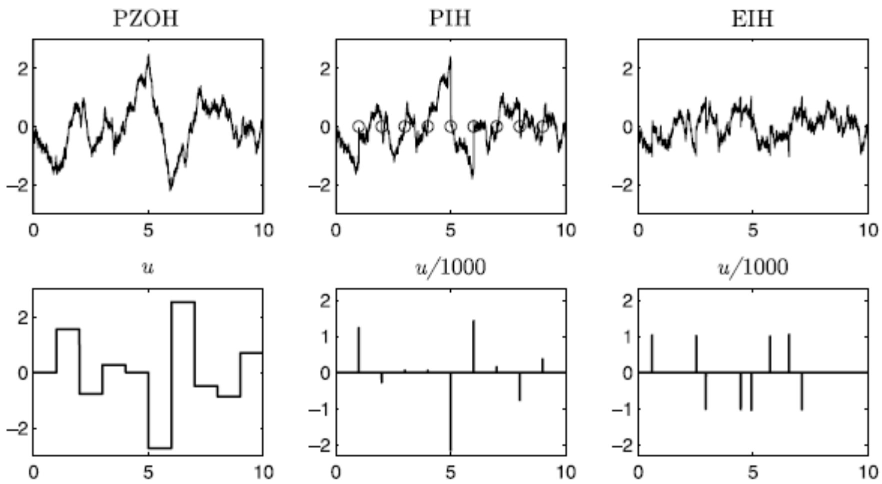


Åström, 2007

Pdf $f(x) = (a - |x|)/a^2$ is the solution to the forward Kolmogorov forward equation (or Fokker–Planck equation)

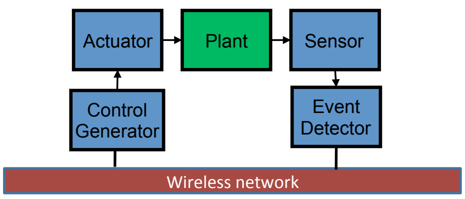
$$\frac{\partial f}{\partial t} = \frac{1}{2}\frac{\partial^2 f}{\partial x^2}(x) - \frac{1}{2}\frac{\partial f}{\partial x}(d)\delta_x + \frac{1}{2}\frac{\partial f}{\partial x}(-d)\delta_x, \qquad f(-a) = f(a) = 0,$$

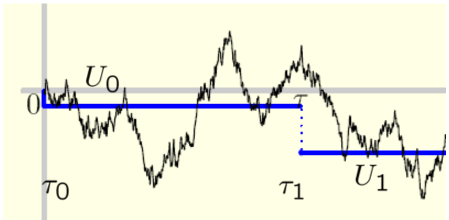
Comparison



Åström, 2007

Event-based ZoH control with adaptive sampling



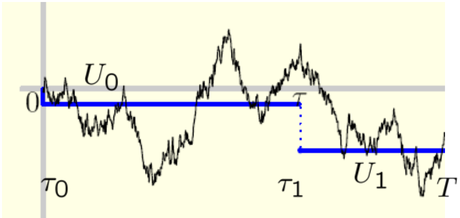


How choose $\{U_i\}$ and $\{\tau_i\}$ to minimize $V = \frac{1}{T} E \int_0^T x^2(t)dt.$

Rabi et al., *2008*

Optimal control with one sampling event

$$dx_t = u_t dt + dB_t$$



$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$

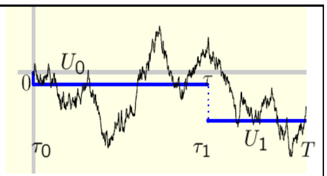
$$= \min_{U_0,U_1,\tau} \left[\mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

A joint optimal control and optimal stopping problem

Rabi et al., *2008*

$$dx_t = u_t dt + dB_t$$

$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$



If τ chosen deterministically (not depending on x_t) and $x_0 = 0$:

$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{T/2}}{T} \qquad \tau^* = T/2$$

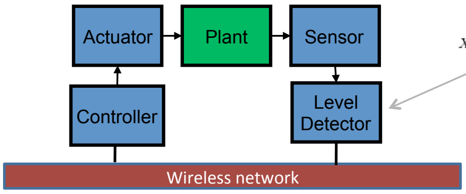
If τ is event-driven (depending on x_t) and $x_0 = 0$:

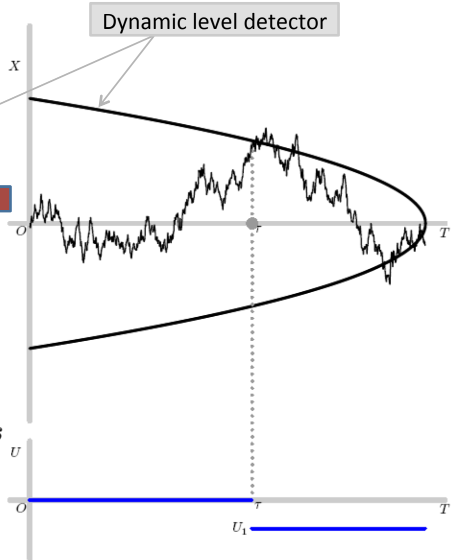
$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{\tau^*}}{2(T-\tau^*)}$$

$$\tau^* = \inf\{t : \underbrace{x_t^2 \geq \sqrt{3}(T-t)}\}$$

Envelope defines optimal level detector

Optimal level detector





$$dx_t = u_t dt + dB_t$$

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds$$

Proof

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds = \min_{U_0, U_1, \tau} \left[\mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

$$\mathbf{E} \left\{ \int_\tau^T x_s^2 ds \middle| \tau, x_\tau, U_1 \right\} = \left[x_t = x_\tau + \int_\tau^t U_1 ds + \int_\tau^t dB_s \right]$$

$$= \int_\tau^T \mathbf{E} \{ [x_\tau^2 + U_1^2 (t-\tau)^2 + (B_t - B_\tau)^2 + 2 x_\tau U_1 (t - \tau)$$

$$+ 2 x_\tau (B_t - B_\tau) + 2 U_1 (t - \tau)(B_t - B_\tau)] \} dt$$

$$= \left[\mathbf{E} B_t = 0,\ \mathbf{E} B_t^2 = t,\ \delta := T - \tau \right] = \delta x_\tau^2 + \frac{\delta^3}{3} U_1^2 + \frac{\delta^2}{2} + \delta^2 x_\tau U_1$$

$$= \frac{\delta}{4} x_\tau^2 + \delta \left(\frac{x_\tau \sqrt{3}}{2} + \frac{\delta U_1}{\sqrt{3}} \right)^2 + \frac{\delta^2}{2}$$

Hence, optimal control $U_1^* = U_1^*(x_\tau, T - \tau) = -\frac{3 x_\tau}{2(T - \tau)}$

$$J(U_0,U_1^*,\tau) = \mathbf{E}\int_0^\tau x_s^2 ds + \mathbf{E}\left\{\frac{T-\tau}{4}x_\tau^2 + \frac{(T-\tau)^2}{2}\right\}$$

If τ chosen deterministically (not depending on x_t) and $x_0 = 0$:

$$J(U_0,U_1^*,\theta) = \frac{\theta^3}{3}U_0^2 + \frac{\theta^2}{2} + \frac{T-\theta}{4}(U_0^2\theta^2+\theta) + \frac{(T-\theta)^2}{2}$$

Hence,

$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{T/2}}{T} \qquad \tau^* = T/2$$

which gives

$$J(U_0^*,U_1^*,\tau^*) = \frac{5T^2}{16}$$

If τ is event-driven (depending on x_t) and $x_0 = 0$:

$$J(U_0,U_1^*,\tau) = \mathbf{E}\int_0^\tau x_s^2 ds + \mathbf{E}\left\{\frac{T-\tau}{4}x_\tau^2 + \frac{(T-\tau)^2}{2}\right\} = \cdots$$

$$= \frac{T^2}{2} + \frac{U_0^2T^3}{3} - \mathbf{E}\left\{\left(\frac{x_\tau\sqrt{3}}{2} + \frac{(T-\tau)U_0}{\sqrt{3}}\right)^2 (T-\tau)\right\}$$

$$= \frac{T^2}{2} - \frac{3}{4}\mathbf{E}\left\{x_\tau^2(T-\tau)\right\}$$

because from symmetry $U^* = 0$.

Find τ that maximizes $f(x_\tau,\tau) = \mathbf{E}\left\{x_\tau^2(T-\tau)\right\}$

Find τ that maximizes $f(x_\tau, \tau) = \mathbf{E}\left\{x_\tau^2(T-\tau)\right\}$

Suppose there exists smooth $g(x,t)$ such that

$$g(x,t) \geq x^2(T-t)$$
$$\frac{1}{2}g_{xx}(x,t) + g_t(x,t) = 0$$

Then, for $0 \leq t \leq \tau \leq T$,

$$f(x_\tau,\tau) = \mathbf{E}\left\{x_\tau^2(T-\tau)\right\} \leq \mathbf{E}\left\{g(x_\tau,\tau)\right\} = g(x_t,t) + \mathbf{E}\int_t^\tau dg(x_\tau,\tau)$$
$$= \text{[Ito formula]} = g(x_t,t) + \mathbf{E}\int_t^\tau \left(\frac{1}{2}g_{xx} + g_t\right)dt$$
$$= g(x_t,t)$$

Hence, g is an upper bound for the expected reward.

We next show that equality can be achieved.

$$g(x_t,t) = \frac{\sqrt{3}}{1+\sqrt{3}}\left(\frac{x_t^4}{6} + x_t(T-t)^2 + \frac{(T-t)^2}{2}\right)$$

is a solution to

$$\frac{1}{2}g_{xx}(x,t) + g_t(x,t) = 0$$

Moreover,

$$g(x_t,t) - x_t^2(T-t) = \frac{1}{2(1+\sqrt{3})}\left(\frac{x_t^4}{3} - \frac{2}{\sqrt{3}}x_t^2(T-t) + (T-t)^2\right)$$
$$= \frac{1}{2(1+\sqrt{3})}\left(\frac{x_t^4}{\sqrt{3}} - (T-t)^2\right) = 0$$

if $x_t^2 = \sqrt{3}(T-t)$.

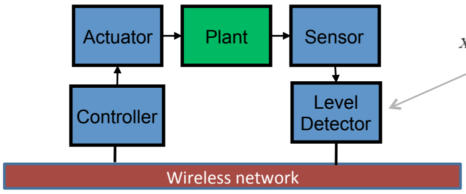
Hence, the optimal sampling time is

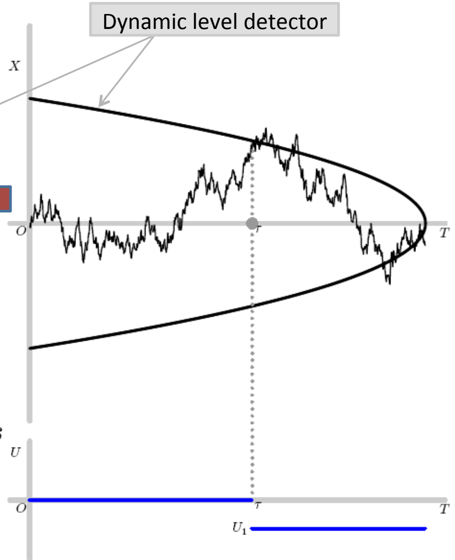
$$\tau^* = \inf\{t: \ x_t^2 \geq \sqrt{3}(T-t)\}$$

which gives

$$J(U_0^*, U_1^*, \tau^*) = \frac{T^2}{8}$$

Optimal level detector





$$dx_t = u_t dt + dB_t$$

$$\min_{U_0, U_1, \tau} J = \min_{U_0, U_1, \tau} \mathbf{E} \int_0^T x_s^2 ds$$

Policy iteration

For $x_0 \neq 0$ we have in general the cost function

$$J_N(x_0, \{U_0, U_1\}, \tau) \triangleq \alpha(x_0, T) - \mathbb{E}[\beta(x_0, U_0, \tau, T)],$$

where

$$\alpha(x_0, U_0, T) = \int_0^T \mathbb{E}\left[\Phi^2_{U_0}(s, 0, x_0)\right] ds$$

$$\beta(x_0, U_0, \tau, T) = \int_\tau^T \mathbb{E}\left[\Phi^2_{U_0}(s, \tau, x_\tau) - \Phi^2_{U_1^*(x_\tau, \tau, T)}(s, \tau, x_\tau)\right]$$

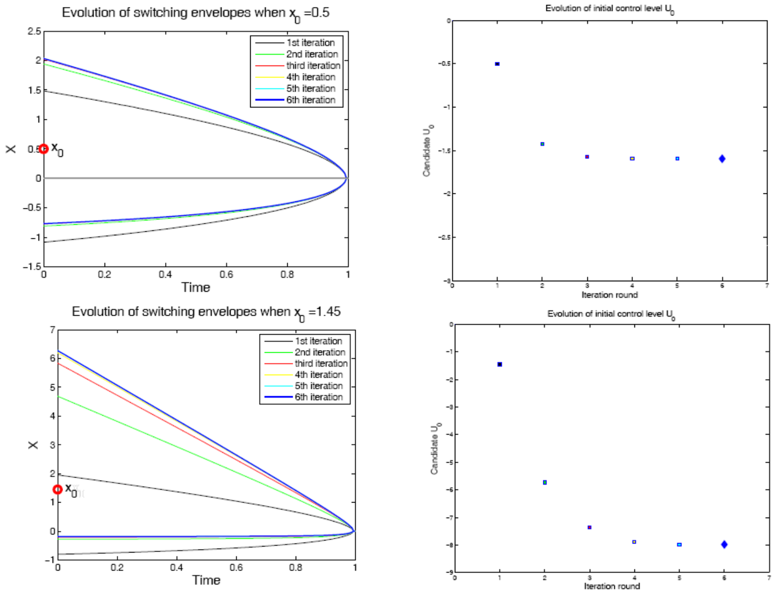
and $\Phi_U(t_2, t_1, x)$ is the solution of the system with constant control

Necessary condition for optimality

$$\begin{cases} \tau^*(x_0) & = \operatorname{ess\,sup}_\tau \mathbb{E}[\beta(x_0, U_0^*(x_0), \tau, T)], \\ U_0^*(x_0) & = \inf_U \left\{ \alpha(x_0, U, T) - \mathbb{E}[\beta(x_0, U, \tau^*(x_0), T)] \right\}. \end{cases}$$

suggests iterative search algorithm. Computationally intensive.

Example: Non-zero initial conditions



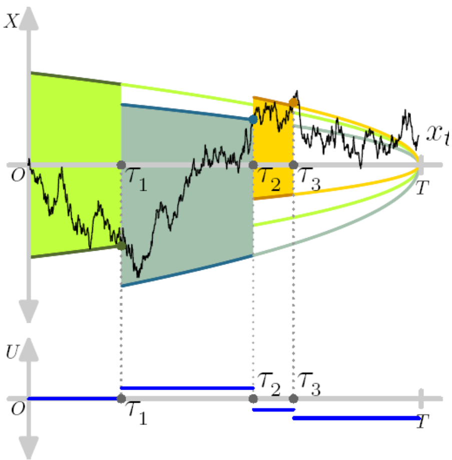
Multiple samples

Extension to N>1 samples

$$J_N\left(x_0, \mathcal{U}, \{\tau\}_{i=1}^N\right) = \mathbb{E}\left[\int_0^T x_s^2 ds \middle| x_0\right]$$

through nested single sample problems

Extension to variable budget sampling, allowing number of samples to depend on x.



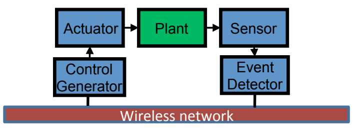
Lecture 10 Outline

- Stochastic control
- Optimal event-based control
- **Event-based control with packet losses**

Event-based impulse control over wireless network with communication losses

Plant $dx_t = dW_t + u_t dt, \;\; x(0) = x_0,$

Sampling events $\mathcal{T} = \{\tau_0, \tau_1, \tau_2, \ldots\},$



Impulse control $u_t = \sum_{n=0}^{\infty} x_{\tau_n} \delta(\tau_n)$

Average sampling rate $R_\tau = \limsup_{M\to\infty} \frac{1}{M} \mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\tau_n \le M\}} \delta(s-\tau_n)\, ds\right]$

Average cost $J = \limsup_{M\to\infty} \frac{1}{M} \mathbb{E}\left[\int_0^M x_s^2 ds\right]$

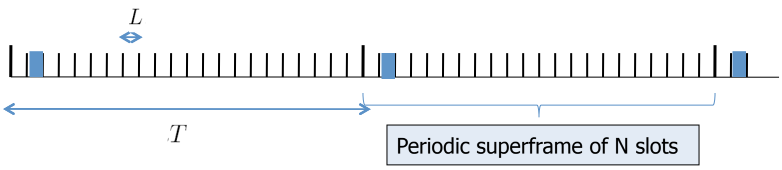
Periodic impulse control

Sampling events $\tau_n = nT \quad \text{for} \quad n \geq 0$

Slot length L gives $T = NL$

Average sampling rate $R_{\text{Periodic}} = \dfrac{1}{T}$

Average cost $J_{\text{Periodic}} = \dfrac{T}{2}$

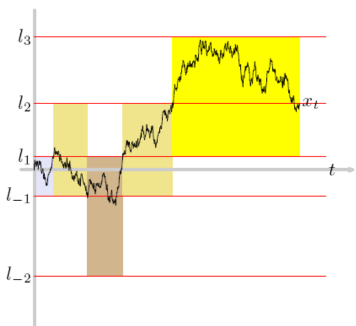


Level-triggered event-based control

Ordered set of levels $\mathcal{L} = \{\ldots, l_{-2}, l_{-1}, l_0, l_1, l_2, \ldots\} \qquad l_0 = 0$

Multiple levels needed because we allow packet loss

Lebesgue sampling $\tau = \inf\left\{\tau \middle| \tau > \tau_i, x_\tau \in \mathcal{L}, x_\tau \notin x_{\tau_i}\right\}$



Level-triggered control

For Brownian motion, equidistant sampling is optimal

$$\mathcal{L}^* = \{k\Delta | k \in \mathbb{Z}\}$$

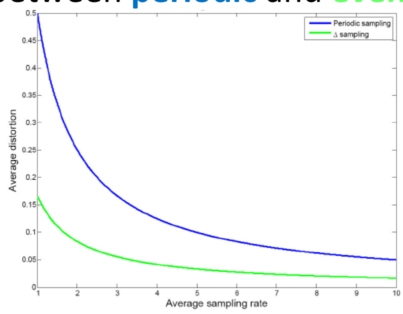
First exit time

$$\tau_\Delta = \inf\{\tau | \tau \geq 0, x_\tau \notin (\xi - \Delta, \xi + \Delta), x_0 = \xi\}$$

Average sampling rate $R_\Delta = \frac{1}{\mathbb{E}[\tau_\Delta]} = \frac{1}{\Delta^2},$

Average cost $J_\Delta = \frac{\mathbb{E}\left[\int_0^{\tau_\Delta} x_s^2 ds\right]}{\mathbb{E}[\tau_\Delta]} = \frac{\Delta^2}{6}.$

Comparison between **periodic** and **event-based** control



$T = \Delta^2$ gives equal average sampling rate for periodic control and event-based control

Event-based impulse control is 3 times better than periodic impulse control

What about the influence of communication losses?
When is event-based sampling better and vice versa?

Influence of communication losses

Times when packets are successfully received $\rho_i \in \{\tau_0 = 0, \tau_1, \tau_2, \ldots\}$,

$$\{\rho_0 = 0, \rho_1, \rho_2, \ldots\}. \qquad \rho_i \geq \tau_i,$$

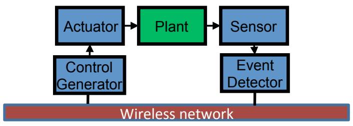
Average rate of packet reception

$$R_\rho = \limsup_{M\to\infty} \frac{1}{M} \mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\rho_n \leq M\}} \delta(s-\rho_n)\, ds\right] = p \cdot R_\tau$$

Define the times between successful packet receptions $\rho_{(p,\Delta)}$

Average cost $J_p = \limsup_{T\to\infty} \frac{1}{T} \mathbb{E}\left[\int_0^T x_s^2 ds\right] = \frac{\mathbb{E}\left[\int_0^{\rho_{(p,\Delta)}} x_s^2 ds\right]}{\mathbb{E}\left[\rho_{(p,\Delta)}\right]}$

IID losses



Proposition

If packet losses are IID with prob p, then equidistant Lebesque sampling gives

$$J_p = \frac{\Delta^2(5p+1)}{6(1-p)}$$

Remark

Event-based control better than periodic control under IID losses if

$$\frac{(1+5p)}{3(1-p)} \geq 1$$

So if the loss probability

$$p \geq 0.25$$

then TDMA do better than event-based sampling.

Rabi and J., *2009*

Proof

$$J_p = \limsup_{T\to\infty} \frac{1}{T}\mathbb{E}\left[\int_0^T x_s^2 ds\right] = \frac{\mathbb{E}\left[\int_0^{\rho_{(p,\Delta)}} x_s^2 ds\right]}{\mathbb{E}\left[\rho_{(p,\Delta)}\right]}$$

$$\begin{aligned}
\mathbb{E}\left[\rho_{p,\Delta}\right] &= \sum_{i=1}^{\infty} \mathbb{E}\left[\tau_i\right]\mathbb{P}\left[\rho=\tau_i\right],\\
&= \sum_{i=1}^{\infty} i\mathbb{E}\left[\tau_\Delta\right]\mathbb{P}\left[\rho=\tau_i\right],\\
&= (1-p)\mathbb{E}\left[\tau_\Delta\right]\sum_{i=1}^{\infty} ip^{i-1},\\
&= \frac{\Delta^2}{1-p}.
\end{aligned}$$

$$\begin{aligned}
\mathbb{E}\left[\int_0^{\rho_{p,\Delta}} x_s^2 ds\right] &= \sum_{i=1}^{\infty}\mathbb{E}\left[\int_0^{\tau_i} x_s^2 ds\right]\mathbb{P}\left[\rho=\tau_i\right],\\
&= (1-p)\sum_{i=1}^{\infty} p^{i-1}\sum_{n=1}^{i}\mathbb{E}\left[\int_{\tau_{n-1}}^{\tau_n} x_s^2 ds\right].
\end{aligned}$$

$$\begin{aligned}
\nu_n &= \mathbb{E}\left[\int_{\tau_{n-1}}^{\tau_n} x_s^2 ds\right],\\
&= \mathbb{E}\left[x_{\tau_{n-1}}^2\int_{\tau_{n-1}}^{\tau_n} ds + \int_{\tau_{n-1}}^{\tau_n}\left(x_s - x_{\tau_{n-1}}\right)^2 ds\right].
\end{aligned}$$

Let $\{\theta_i\}$ be an infinite sequence of binary IID variables. Let θ take vales in $\{-1,+1\}$ with equal probabilities. Then, we can say that the following random variables are equal in probability law:

$$x_{\tau_n} \overset{\mathrm{d}}{=} \sum_{m=1}^{n}\theta_m\Delta \qquad \forall n \in \mathbb{N}.$$

$$\begin{aligned}
\nu_n &= \mathbb{E}\left[\left(\sum_{m=1}^{n}\theta_m\Delta\right)^2\right]\mathbb{E}\left[\tau_\Delta\right] + \mathbb{E}\left[\int_0^{\tau_\Delta} x_s^2 ds \,\middle|\, x_0 = 0\right]\\
&= (n-1)\,\Delta^4 + \frac{\Delta^4}{6}.
\end{aligned}$$

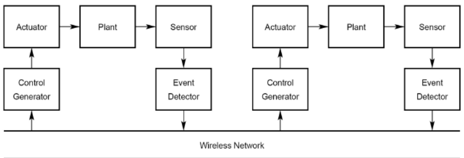
$$\begin{aligned}
\mathbb{E}\left[\int_0^{\rho_{p,\Delta}} x_s^2 ds\right] &= \sum_{i=1}^{\infty}(1-p)p^{i-1}\sum_{n=1}^{i}\frac{\Delta^4}{6} + (n-1)\Delta^4,\\
&= (1-p)\Delta^4\sum_{i=1}^{\infty}p^{i-1}\left(\frac{i}{6}+\frac{i(i-1)}{2}\right),\\
&= (1-p)\frac{\Delta^4}{6}\sum_{i=1}^{\infty}p^{i-1}\left(3i^2-2i\right),\\
&= \frac{\Delta^4\,(5p+1)}{6(1-p)^2}.
\end{aligned}$$

Losses depending on the other loops

Suppose the loss processes across the loops are independent, so that the sample streams of the other sensors only matter through their average behavior

The likelihood that a sample generated in one loop faces at least one competing transmission is then

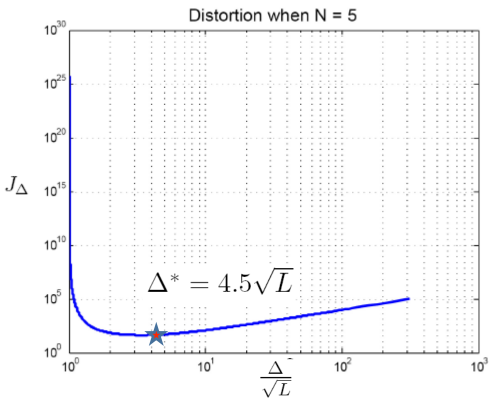
$$p = 1 - \left(1 - \frac{L}{\Delta^2}\right)^{N-1}$$



Losses depending on the other loops

Average cost $J_\Delta = \frac{L\left(6 - 5\beta^{N-1}\right)}{6\beta^{N-1}\left(1-\beta\right)}$ $\beta = 1 - \frac{L}{\Delta^2}$

Trade-off between control performance and network resources

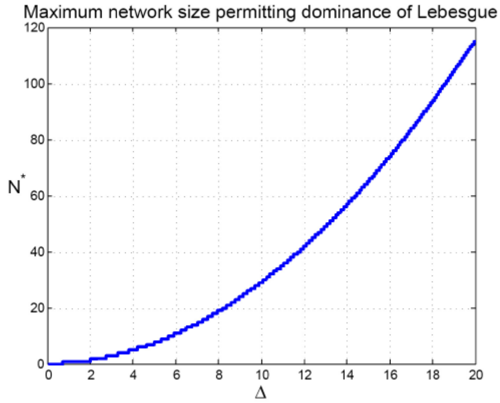


Rabi and J., *2009*

Event-based vs periodic control

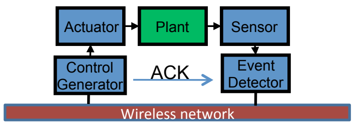
Event-based sampling better than periodic when $N < N^*$

$$N^* = 1 + \left\lfloor \frac{\log(0.75)}{\log\left(1 - \frac{L}{\Delta^2}\right)} \right\rfloor .$$



Rabi and J., *2009*

Sensor data ACK's



If controller perfectly acknowledges packets to sensor, event detector can adjust its sampling strategy

Let $\Delta(l) = \sqrt{l+1}\Delta_0$

where $l \geq 0$ number of samples lost since last successfully transmitted packet

Gives $\mathbb{E}\left[\tau^{\uparrow}_{i+1} - \tau^{\uparrow}_{i}\right]$ independent of i.

Better performance than fixed $\Delta(l)$ for same sampling rate:

$$J^{\uparrow}_{p} = \frac{\Delta^2(1+p)}{6(1-p)} \leq \frac{\Delta^2(1+5p)}{6(1-p)} = J_p.$$

Lecture 10 Outline

- Stochastic control
- Optimal event-based control
- Event-based control with packet losses