Wireless Control:
Medium Access and Event-based Control

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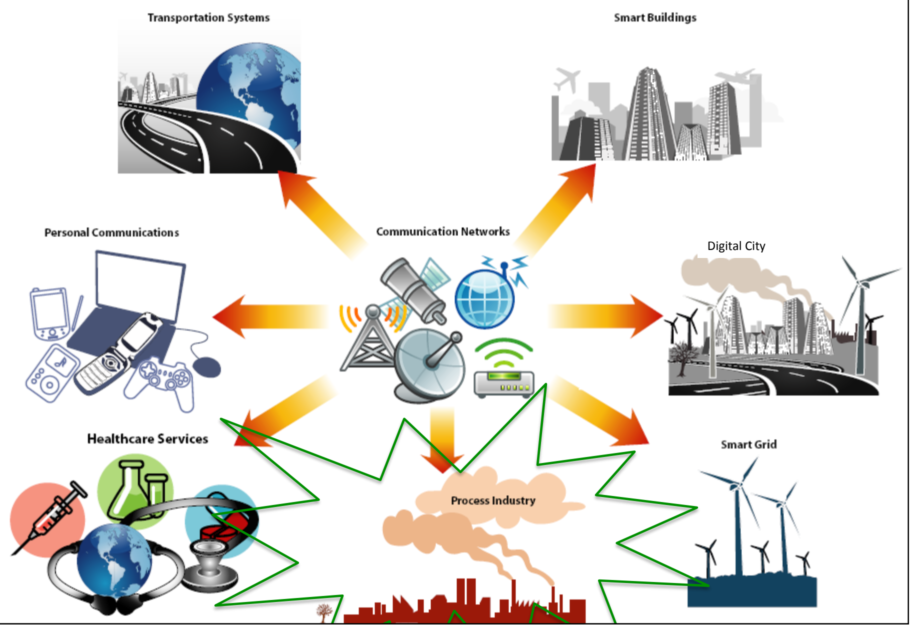
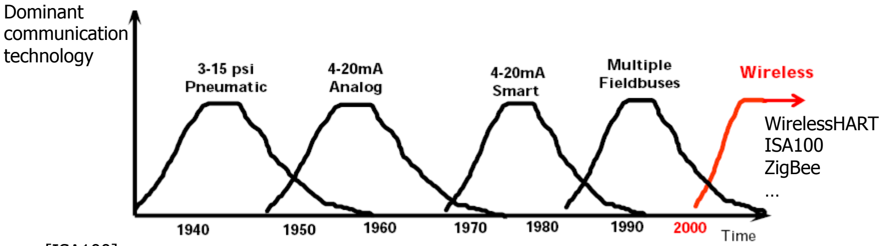
Large-scale networked control systems applications



Outline

- Introduction
- Industrial applications of wireless control
- Medium access
- Event-based control
- Conclusions

Communication in process control



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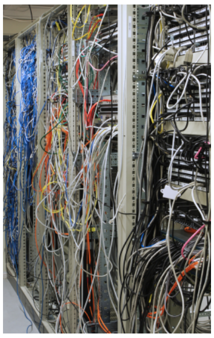


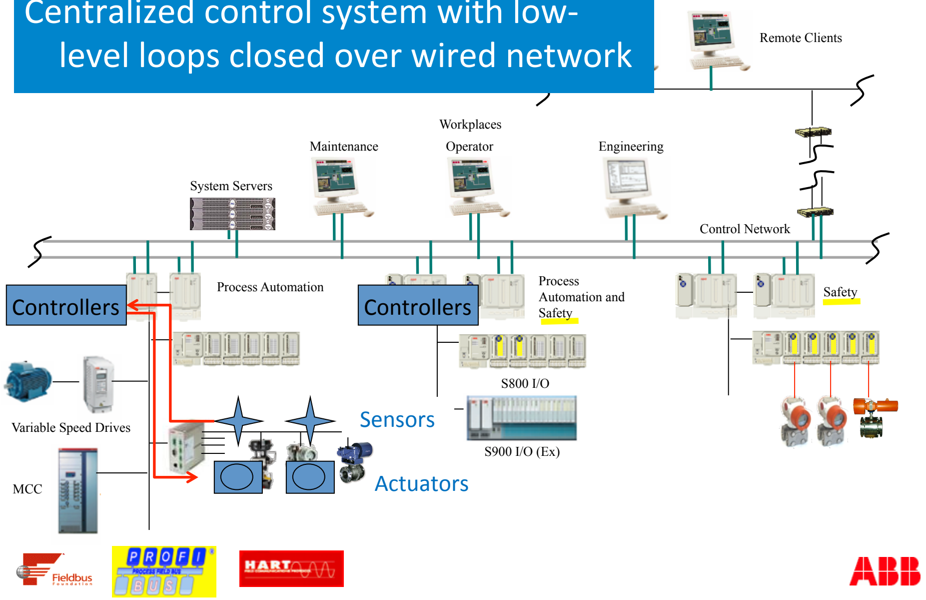
Wireless sensor systems benefit from

- Lower installation and maintenance costs
- Increased sensing capabilities and flexibility

Major consequences for control system architectures

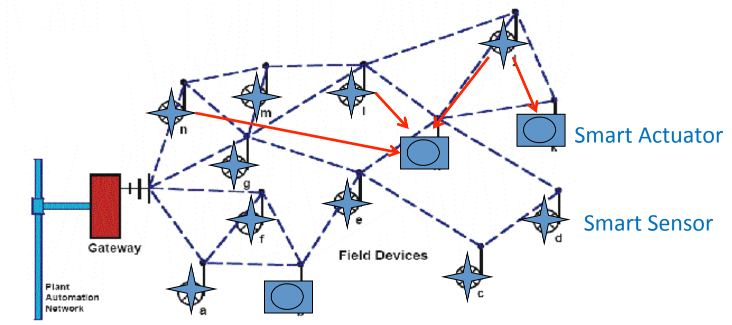
Today's industrial communication architecture

Centralized control system with low-level loops closed over wired network



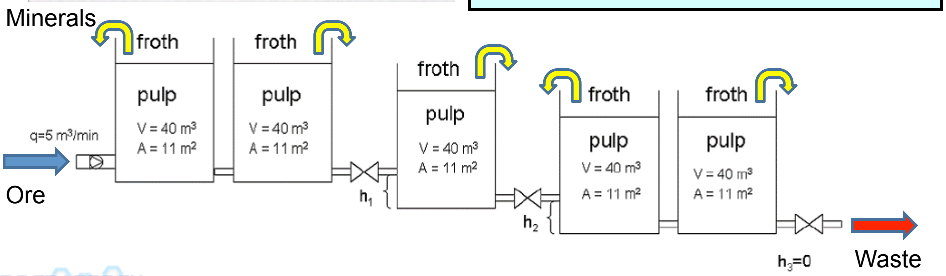
Towards wireless sensor and actuator network architecture

- Local control loops closed over **wireless multi-hop network**
- Potential for a dramatic change:
  - From fixed hierarchical centralized system to flexible distributed
  - Move intelligence from dedicated computers to sensors/actuators



Control of froth flotation process

- Froth flotation process concentrates the metal-bearing mineral in the ore



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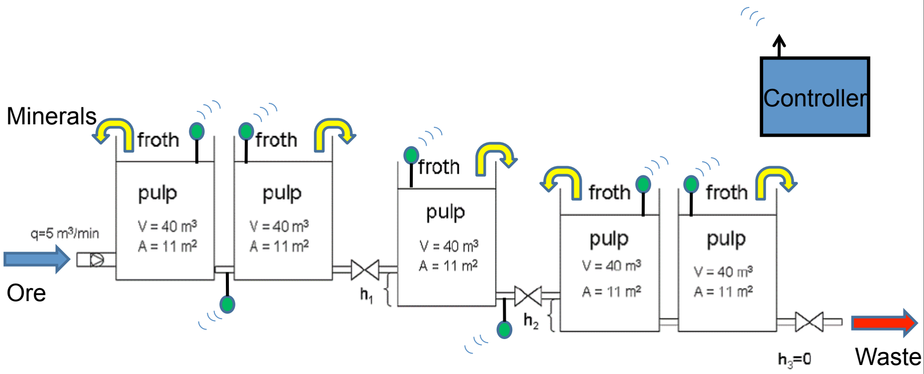
Wireless control of flotation process



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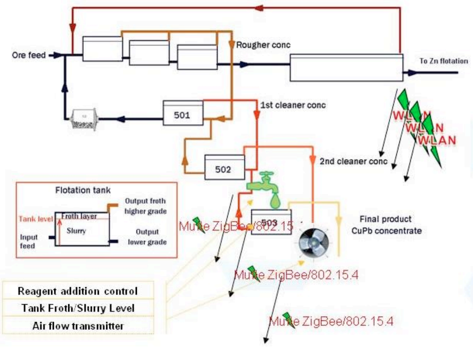
Wireless control of floatation process

The Boliden plant

Existing wired communication system

Wireless communication for tank level control

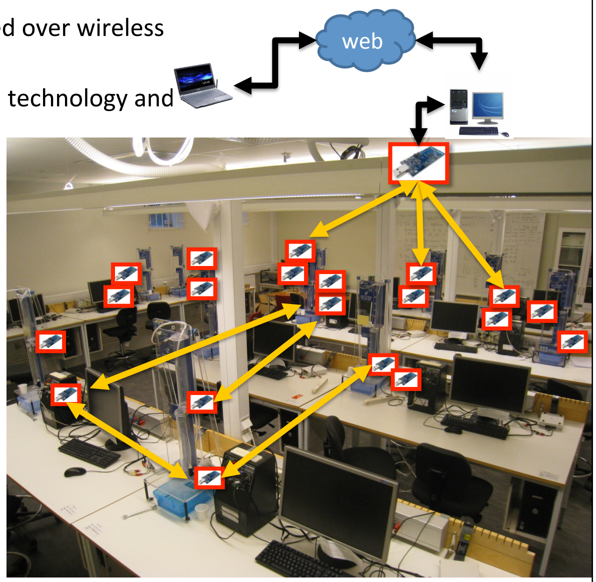


SOCRADES.EU BOLIDEN ABB

Test-bed for control over IEEE 802.15.4

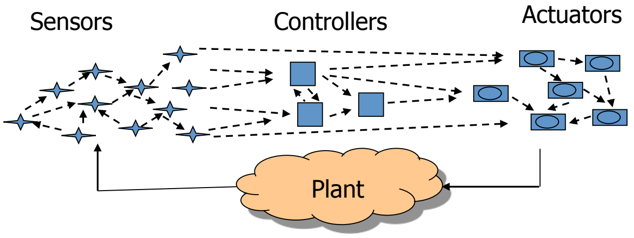
20 coupled water tanks connected over wireless multi-hop network

Test-bed to evaluate new control technology and wireless network protocols



Wireless control system

How share common network resources while maintaining guaranteed closed-loop performance?



- How handle network imperfections: resource constraints, loss, conflicts, delays, outages?
- How move intelligence from a few central units to many distributed devices?

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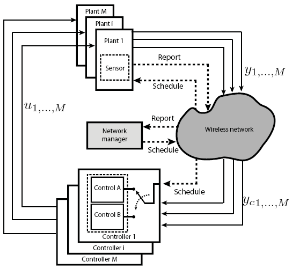
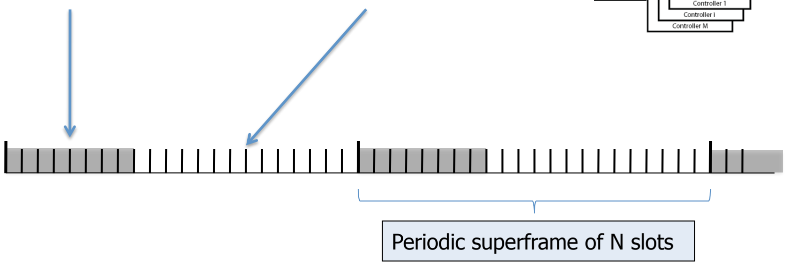
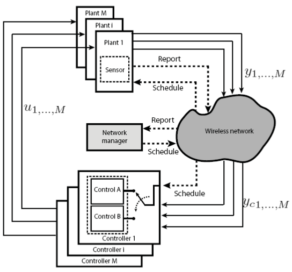
Hybrid MAC protocols



MAC protocols have often both
contention-free and contention access periods

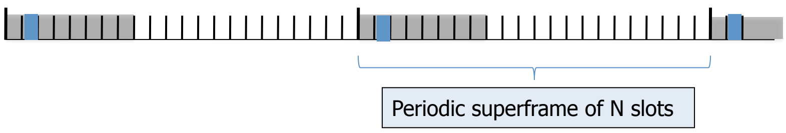


Hybrid MAC protocols

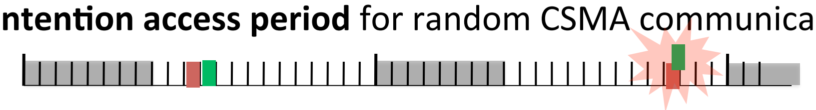


Today's MAC protocol standards have both
contention-free and contention access periods

Contention-free period for TDMA scheduled communication

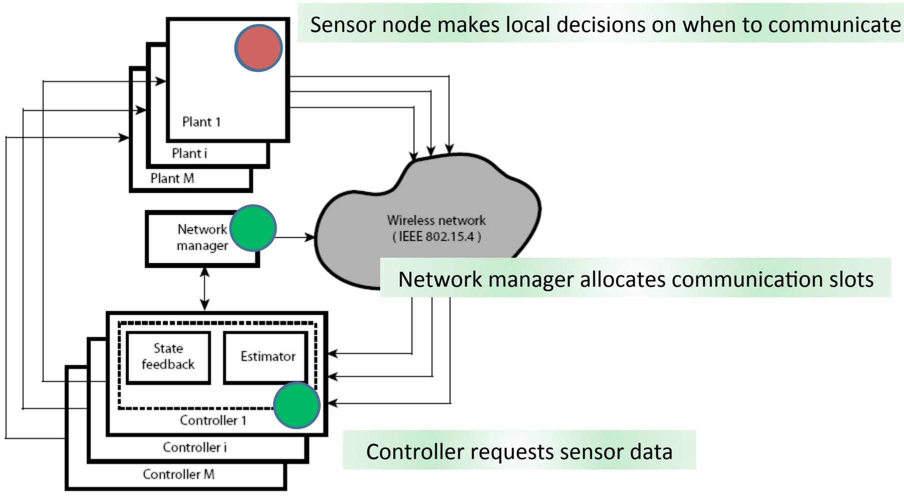


Contention access period for random CSMA communication

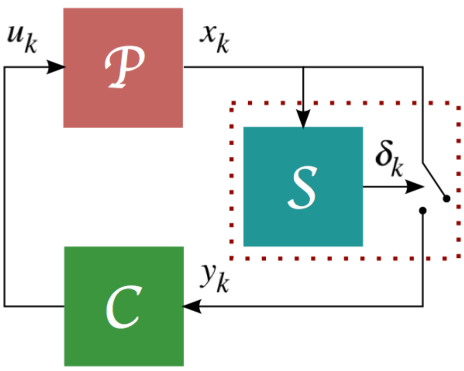


TDMA = Time division multiple access, CSMA/CA = Carrier Sense Multiple Access with Collision Avoidance

Where to take medium access decisions?



Is there a separation principle for scheduling-estimation-control?



Stochastic control formulation

Plant:

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

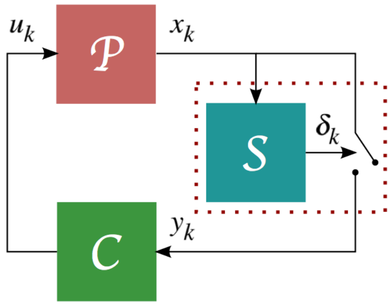
$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$

$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$

$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$

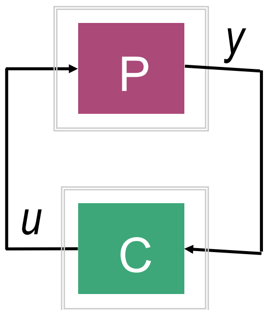


Cost criterion:

$$J(f,g) = \mathrm{E}[x_N^T Q_0 x_N + \sum_{s=0}^{N-1} (x_s^T Q_1 x_s + u_s^T Q_2 u_s)]$$

Certainty equivalence revisited

Definition Certainty equivalence holds if the closed-loop optimal controller has the same form as the deterministic optimal controller with x_k replaced by the estimate $\hat{x}_{k|k} = \mathrm{E}[x_k|\mathbb{I}_k^{\mathbb{C}}]$.



Theorem[Bar-Shalom–Tse] Certainty equivalence holds if and only if $E[(x_k - E[x_k|I_k^c])^2|I_k^c]$ is not a function of past controls $\{u\}_0^{k-1}$ (no dual effect).

Feldbaum, 1965; Åström, 1970; Bar-Shalom and Tse, 1974

State-based scheduler

Plant:

$$x_{k+1} = Ax_k + Bu_k + w_k$$

Scheduler:

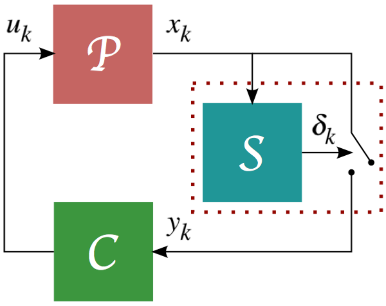
$$\delta_k = f_k(\mathbb{I}_k^{\mathbb{S}}) \in \{0,1\}$$

$$\mathbb{I}_k^{\mathbb{S}} = \left[\{x\}_0^k, \{y\}_0^{k-1}, \{\delta\}_0^{k-1}, \{u\}_0^{k-1}\right]$$

Controller:

$$u_k = g_k(\mathbb{I}_k^{\mathbb{C}})$$

$$\mathbb{I}_k^{\mathbb{C}} = \left[\{y\}_0^k, \{\delta\}_0^k, \{u\}_0^{k-1}\right]$$



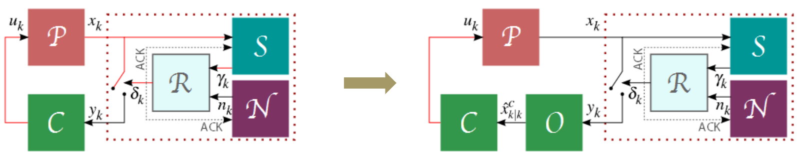
Corollary The control u_k for the optimal closed-loop system has a dual effect.

The separation principle does not hold for the optimal closed-loop system, so the design of the scheduler, estimator, and controller is coupled

Ramesh, Sandberg, Bao, J, 2009, 2010

Conditions for Certainty Equivalence

Corollary: The optimal controller for the system $\{\mathcal{P},\mathcal{S}(f),\mathcal{C}(g)\}$, with respect to the cost J is certainty equivalent if and only if the scheduling decisions are not a function of the applied controls.



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Ramesh, Sandberg, Bao, J, 2011

Observer-Scheduler for Certainty Equivalence

A symmetric scheduling policy results in separation between the estimator and the scheduler, as well as an optimal certainty equivalent

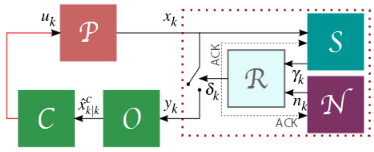
- Observer:

$$\hat{x}_{k|k} = \begin{cases} x_k & \delta_k = 1 \\ A^{k-\tau_k} x_{\tau_k} + \sum_{s=1}^{k-\tau_k} A^{s-1} B u_{k-s} & \\ + \mathbb{E}[\sum_{s=1}^{k-\tau_k} A^{s-1} w_{k-s} | \hat{f}_k, \ldots, \hat{f}_{\tau_k+1} = 0] & \delta_k = 0 \end{cases}$$

- Symmetric scheduler:

$$\gamma_k = f_{|\cdot|}\left(\sum_{s=1}^{k-\tau_{k-1}} A^{s-1} w_{k-s}\right) \qquad \forall k,$$

where, $f_{|\cdot|}(r) = f_{|\cdot|}(-r)$

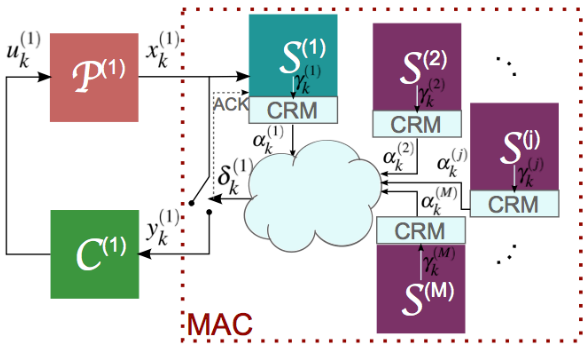


- Certainty equivalence achieved at the cost of optimality

Ramesh, Sandberg, Bao, J, 2011

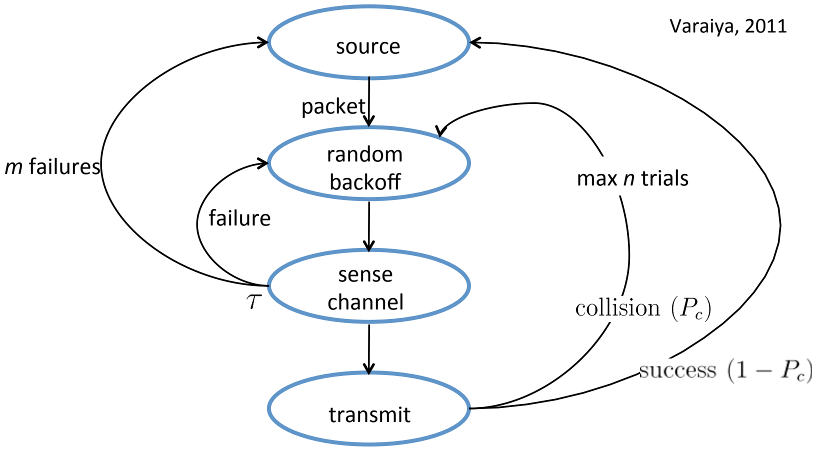
Extensions to multiple loops and contention resolution mechanisms

- Hard problem because of the correlation between the plants imposed by the MAC
- Closed-loop analysis can still be done for a class of event-based schedulers and simple MAC's
- General problem with event-based schedulers and realistic MAC (e.g., CSMA/CA) is open



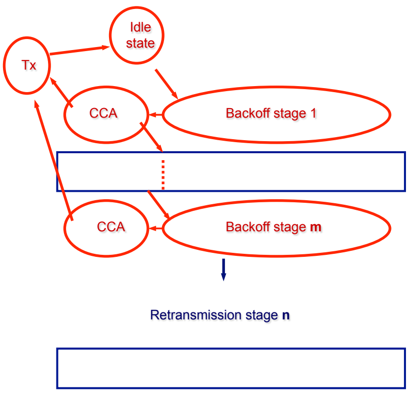
Ramesh et al, 2011

How to model CSMA/CA MAC?



- Every device executes this protocol
- Assume all carrier sense events are independent [Bianchi, 2000]

CSMA/CA mechanism of a node in an IEEE 802.15.4 wireless network

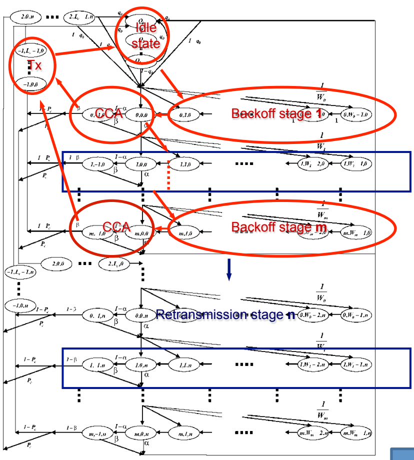


- A transmitting node delays for a random number of backoff periods in [0 , 2^{m_0} - 1], where m_0 is the initial backoff exponent.
- If two consecutive clear channel assessments (CCA) are idle, the node starts the transmission and waits for an ACK
- If the channel is busy, the procedure is repeated increasing the backoff windows until a maximum backoff exponent m_b.
- After a maximum number of backoffs ***m*** the packet is discarded.
- In case of collision the procedure is restarted and repeated until a retry limit ***n***

Park, Di Marco, Soldati, Fischione, J, 2009

Cf., Bianchi, 2000; Pollin et al., 2006

Markov chain model of CSMA/CA



- Markov state (s,c,r)
  - s: backoff stage
  - c: state of backoff counter
  - r: state of retransmission counter
- Model parameters
  - q_0: traffic condition (q_0=0 saturated)
  - m_0, m, m_b, n: MAC parameters
- Computed characteristics
  - α: busy channel probability during CCA1
  - β: busy channel probability during CCA2
  - P_c: collision probability

Park, Di Marco, Soldati, Fischione, J, 2009

Validated in simulation and experiment

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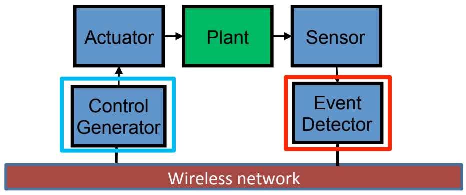
When to transmit?

- Medium access control-like mechanism at sensor
  - E.g., threshold crossing, adaptive sampling



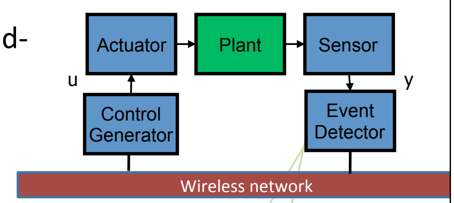
How to control?

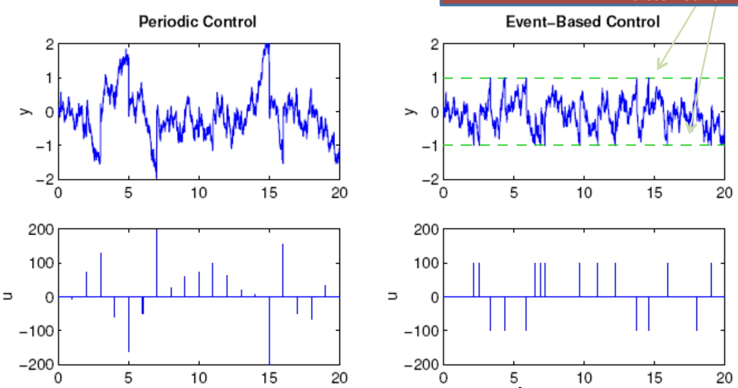
- Execute control law over fixed control alphabet
  - E.g., piecewise constant controls, impulse control

Åström, *2007,* Rabi and J., *WICON, 2008*

Example: Fixed threshold with impulse control

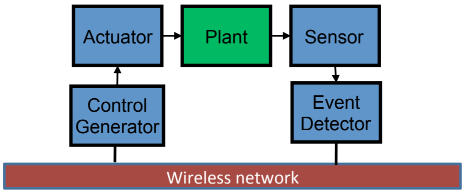
- Event-detector implemented as fixed-level threshold at sensor
- Event-based impulse control better than periodic impulse control

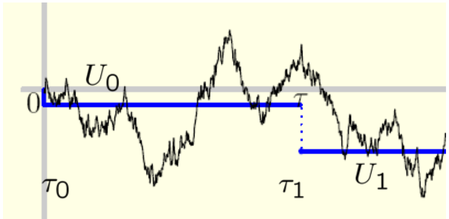




Åström & Bernhardsson, *IFAC,* 1999

Event-based ZoH control with adaptive sampling



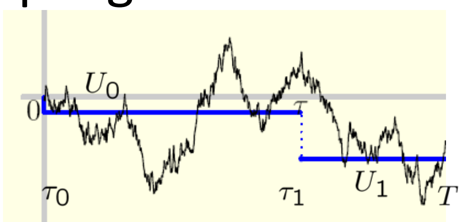


How choose $\{U_i\}$ and $\{\tau_i\}$ to minimize $V = \frac{1}{T} E \int_0^T x^2(t)dt$.

Rabi, J, Johansson, *2008*

Controlled Brownian motion with one sampling event

$$dx_t = u_t dt + dB_t$$



$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$

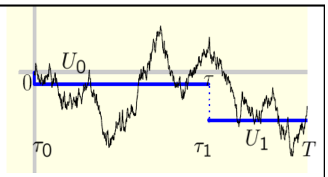
$$= \min_{U_0,U_1,\tau} \left[\mathbf{E} \int_0^\tau x_s^2 ds + \mathbf{E} \int_\tau^T x_s^2 ds \right]$$

A joint optimal control and optimal stopping problem

Rabi, J, Johansson, *2008*

$$dx_t = u_t dt + dB_t$$

$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$



If τ chosen deterministically (not depending on x_t) and $x_0 = 0$:

$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{T/2}}{T} \qquad \tau^* = T/2$$

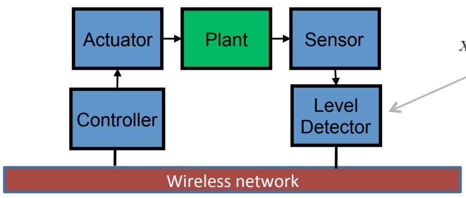
If τ is event-driven (depending on x_t) and $x_0 = 0$:

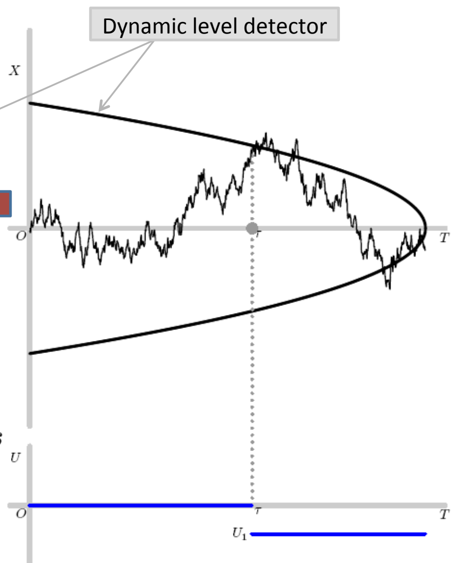
$$U_0^* = 0 \qquad U_1^* = -\frac{3x_{\tau^*}}{2(T-\tau^*)}$$

$$\tau^* = \inf\{t : x_t^2 \geq \sqrt{3}(T-t)\}$$

Envelope defines optimal level detector

Optimal level detector





$$dx_t = u_t dt + dB_t$$

$$\min_{U_0,U_1,\tau} J = \min_{U_0,U_1,\tau} \mathbf{E} \int_0^T x_s^2 ds$$

Policy iteration

For $x_0 \neq 0$ we have in general the cost function

$$J_N\left(x_0, \{U_0, U_1\}, \tau\right) \triangleq \alpha\left(x_0, T\right) - \mathbb{E}\left[\beta\left(x_0, U_0, \tau, T\right)\right],$$

where

$$\alpha\left(x_0, U_0, T\right) = \int_0^T \mathbb{E}\left[\Phi^2_{U_0}(s, 0, x_0)\right] ds$$

$$\beta\left(x_0, U_0, \tau, T\right) = \int_\tau^T \mathbb{E}\left[\Phi^2_{U_0}(s, \tau, x_\tau) - \Phi^2_{U_1^*(x_\tau, \tau, T)}(s, \tau, x_\tau)\right]$$

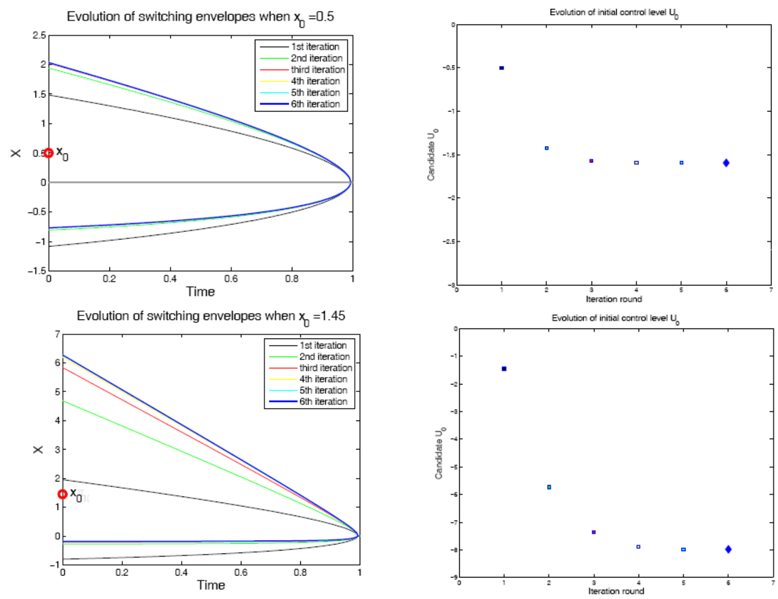
and $\Phi_U(t_2, t_1, x)$ is the solution of the system with constant control

Necessary condition for optimality

$$\begin{cases} \tau^*\left(x_0\right) & = \operatorname{ess\,sup}_\tau \ \mathbb{E}\left[\beta\left(x_0, U_0^*\left(x_0\right), \tau, T\right)\right], \\ U_0^*\left(x_0\right) & = \inf_U \left\{\alpha\left(x_0, U, T\right) - \mathbb{E}\left[\beta\left(x_0, U, \tau^*\left(x_0\right), T\right)\right]\right\}. \end{cases}$$

suggests iterative search algorithm. Computationally intensive.

Example: Non-zero initial conditions



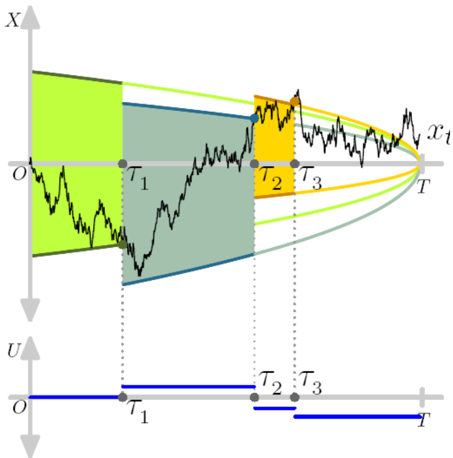
Multiple samples

Extension to N>1 samples

$$J_N\left(x_0, \mathcal{U}, \{\tau\}_{i=1}^N\right) = \mathbb{E}\left[\int_0^T x_s^2 ds \middle| x_0\right]$$

through nested single sample problems

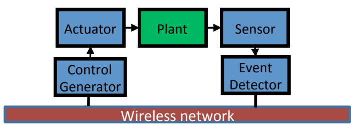
Extensions to variable budget sampling, allowing number of samples to depend on x.



Event-based impulse control

Plant $dx_t = dW_t + u_t dt, \ x(0) = x_0,$

Sampling events $\mathcal{T} = \{\tau_0, \tau_1, \tau_2, \ldots\},$



Impulse control $u_t = \sum_{n=0}^{\infty} x_{\tau_n} \delta(\tau_n)$

Average sampling rate $R_\tau = \limsup_{M\to\infty} \frac{1}{M}\mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\tau_n \le M\}} \delta(s - \tau_n)\, ds\right]$

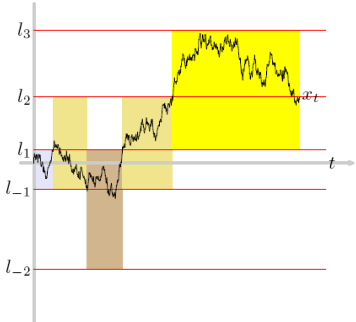
Average cost $J = \limsup_{M\to\infty} \frac{1}{M}\mathbb{E}\left[\int_0^M x_s^2 ds\right]$

Level-triggered control

Ordered set of levels $\mathcal{L} = \{\ldots, l_{-2}, l_{-1}, l_0, l_1, l_2, \ldots\}$ $l_0 = 0$

Multiple levels needed because we allow packet loss

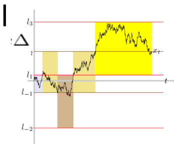
Sampling instances $\tau = \inf \left\{ \tau \middle| \tau > \tau_i, x_\tau \in \mathcal{L}, x_\tau \notin x_{\tau_i} \right\}$



Level-triggered control

For Brownian motion, equidistant sampling is optimal

$$\mathcal{L}^* = \{k\Delta | k \in \mathbb{Z}\}$$



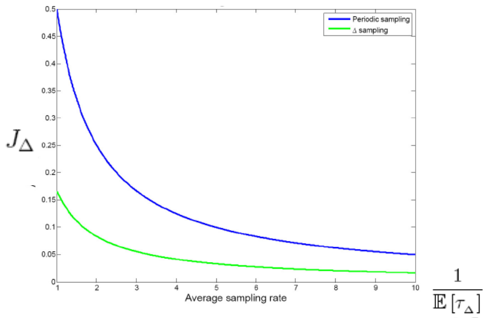
First exit time

$$\tau_\Delta = \inf \{\tau | \tau \geq 0, x_\tau \notin (\xi - \Delta, \xi + \Delta), x_0 = \xi\}$$

Average sampling rate $R_\Delta = \frac{1}{\mathbb{E}[\tau_\Delta]} = \frac{1}{\Delta^2},$

Average cost $J_\Delta = \frac{\mathbb{E}\left[\int_0^{\tau_\Delta} x_s^2 ds\right]}{\mathbb{E}[\tau_\Delta]} = \frac{\Delta^2}{6}.$

Comparison between **time-** and **event-based** control



$T = \Delta^2$ gives equal average sampling rate for periodic control and event-based control

Event-based impulse control is three times better than periodic

Åström & Bernhardsson, 1999

What about the influence of communication losses?
Is event-based sampling still better?

Influence of i.i.d. packet loss

Times when packets are successfully received $\rho_i \in \{\tau_0 = 0, \tau_1, \tau_2, \ldots\}$,

$\{\rho_0 = 0, \rho_1, \rho_2, \ldots\}$. $\quad \rho_i \geq \tau_i$,

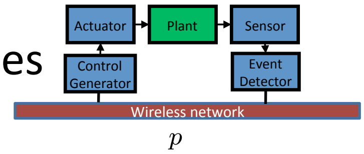
Average rate of packet reception

$$R_\rho = \limsup_{M \to \infty} \frac{1}{M} \mathbb{E}\left[\int_0^M \sum_{n=0}^{\infty} \mathbf{1}_{\{\rho_n \leq M\}} \delta(s - \rho_n)\, ds\right] = p \cdot R_\tau$$

Define the times between successful packet receptions $\rho_{(p,\Delta)}$

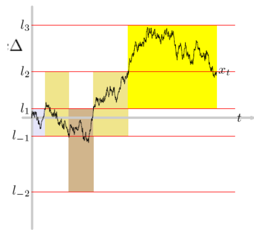
Average cost $J_p = \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}\left[\int_0^T x_s^2 ds\right] = \frac{\mathbb{E}\left[\int_0^{\rho_{(p,\Delta)}} x_s^2 ds\right]}{\mathbb{E}\left[\rho_{(p,\Delta)}\right]}$

Event-based control with losses



Theorem

If packet losses are i.i.d. with probability p, then level-triggered sampling gives



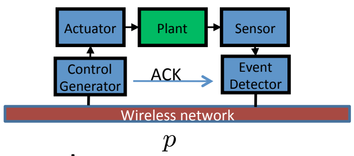
$$J_p = \frac{\Delta^2 (5p+1)}{6(1-p)}$$

Event-based control better than periodic control if loss probability

$$p < 0.25$$

Rabi and J, *2009*

Communication acknowledgements



If controller perfectly acknowledges packets to sensor, event detector can adjust its sampling strategy

Let $\Delta(l) = \sqrt{l+1}\Delta_0$

where $l \geq 0$ number of samples lost since last successfully transmitted packet

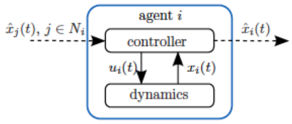
Gives that $\mathbb{E}\left[\tau_{i+1}^{\dagger} - \tau_i^{\dagger}\right]$ becomes independent of i.

Better performance than fixed $\Delta(l)$ for same sampling rate:

$$J_p^{\dagger} = \frac{\Delta^2(1+p)}{6(1-p)} \leq \frac{\Delta^2(1+5p)}{6(1-p)} = J_p.$$

Rabi and J, *2009*

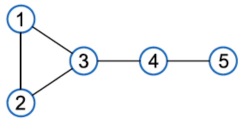
Event-based Control for Multi-agent Systems



$$\dot{x}_i(t) = u_i(t), \quad i \in \mathcal{V},$$

$$u_i(t) = -\sum_{j \in N_i} (\hat{x}_i(t) - \hat{x}_j(t)),$$

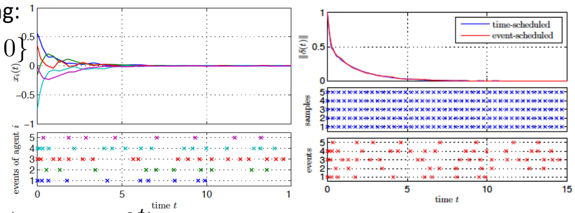
$$\hat{x}_i(t) = x_i(t_k^i),\ t \in [t_k^i, t_{k+1}^i[,$$



Trigger condition for state broadcasting:

$$t_{k+1}^i = \inf\{t > t_k^i :\ f_i(t, e_i(t)) > 0\}$$

$$e_i(t) = \hat{x}_i(t) - x_i(t).$$



Theorem Suppose

$$f_i(t, e_i(t)) = |e_i(t)| - (c_0 + c_1 e^{-\alpha t}), \quad c_0, c_1 \geq 0, c_0 + c_1 > 0$$

and $0 < \alpha < \lambda_2(\mathcal{G})$. Then, the closed-loop system is non-Zeno and $x(t)$ converges to a ball around $(1/N)\mathbf{1}\mathbf{1}^T x(0)$ with radius $\|L\|\sqrt{N} c_0 / \lambda_2(\mathcal{G})$.

Extensions to double-integrator and time-delay systems

[Seyboth et al, 2011; Dimarogonas et al, 2012]

Outline

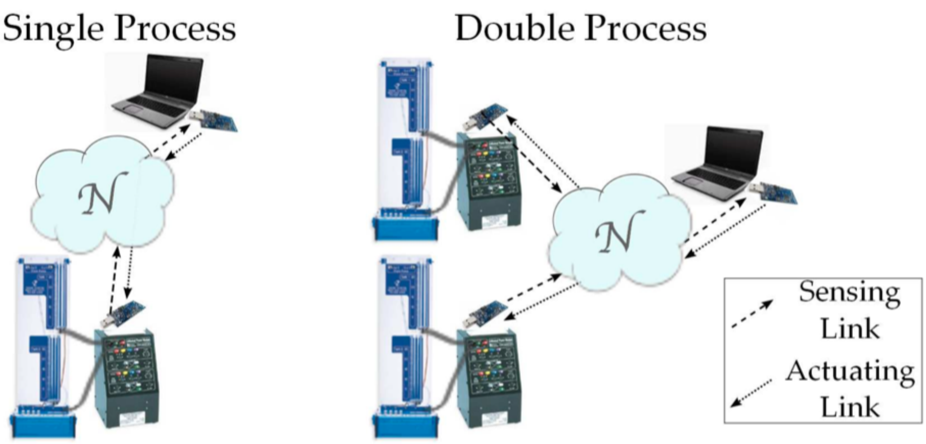
- Introduction
- Industrial applications of wireless control
- Medium access
- Event-based control
- Conclusions

Conclusions

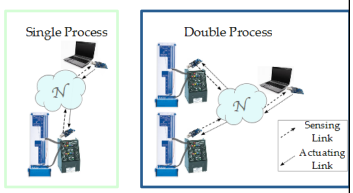
- **Wireless control** is an enabling technology in many emerging industrial applications
- Fundamental challenges related to
  - **time-driven**, synchronous, sampled data control theory, vs
  - **event-driven**, asynchronous, ad hoc wireless networking
- Integrated modeling for medium access and control
- Event-based control provides a natural principle for large-scale wireless control systems

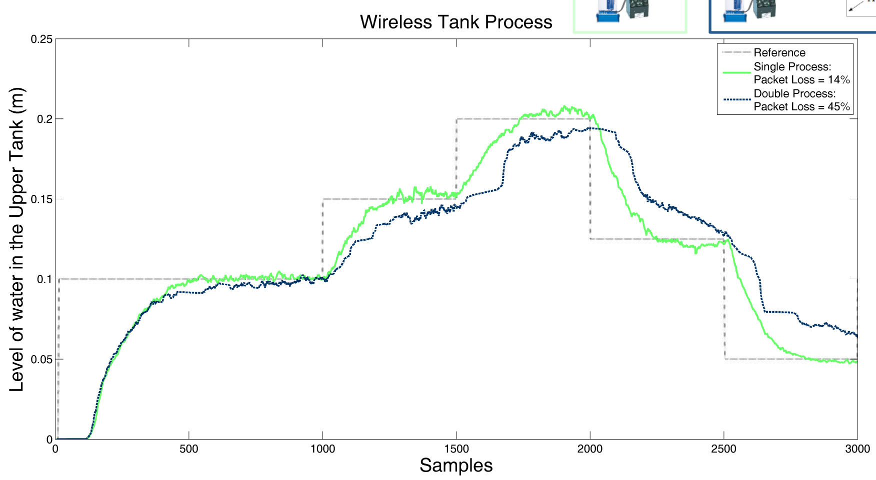
http://www.ee.kth.se/~kallej

Control over wireless network



Packet loss influence on control performance





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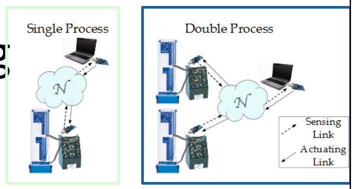
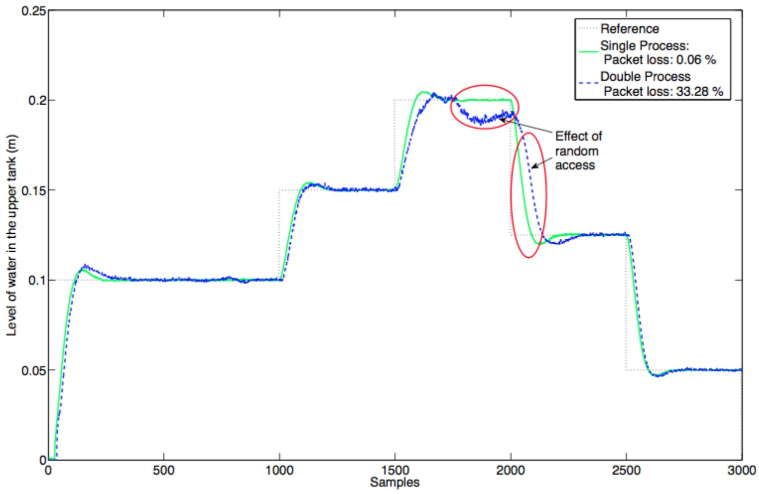
Partial improvement using CSMA/CA medium access





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