Wireless and Computing Resource Allocation for Selfish Computation Offloading in Edge Computing

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Mobile Edge Computing

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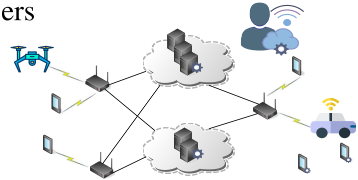
Enabler of 5G that provides

- computing resources close to the end users
- support for autonomous IoT devices
- bandwidth and computing resource orchestration on demand



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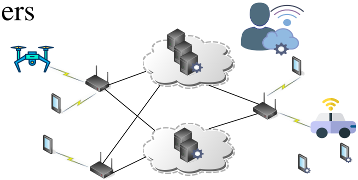
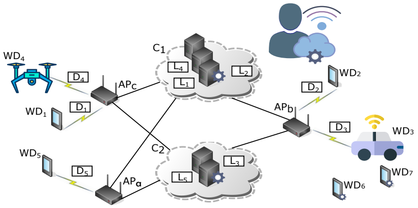


- How do device autonomy and infrastructure resource management interact?

Mobile Edge Computing System



- Wireless devices (WDs) $\mathcal{N}$

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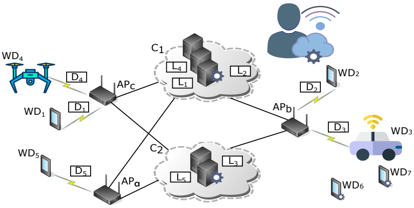

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- Operator manages
  - Edge clouds (ECs) $\mathcal{C}$
  - APs $\mathcal{A}$

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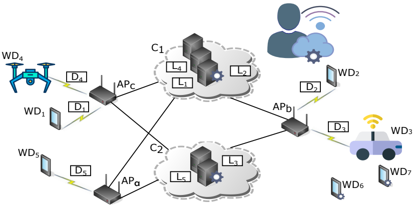

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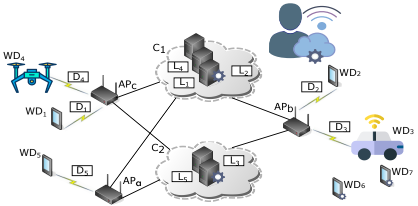

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Computation offloading

- Task of WD i, $< D_i, L_i >$
  - size of the input data D_i
  - computational complexity L_i

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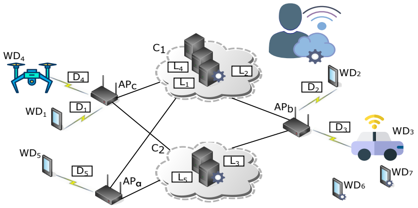

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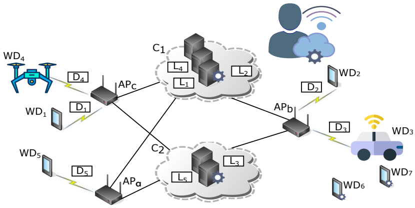

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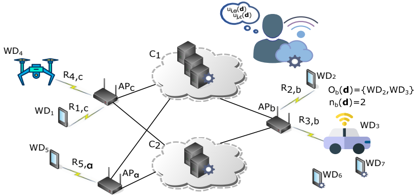
- Set of decisions for all WDs is a *strategy profile* **d**

Communication Model



- $R_{i,a}$: PHY rate of WD i on AP a
- $u_{i,a}$: uplink access provisioning coefficient

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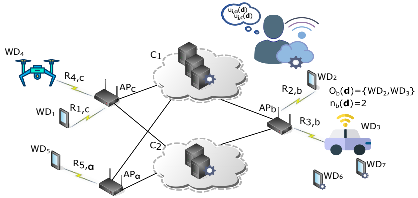

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Transmission time

$O_a(\mathbf{d})$: set of offloaders via AP a in strategy profile $\mathbf{d}$

- Uplink rate of WD i via AP a

$$\omega_{i,a}(\mathbf{d}, \mathbf{u}_a) = R_{i,a} \frac{u_{i,a}}{\sum_{j \in O_a(\mathbf{d})} u_{j,a}}$$

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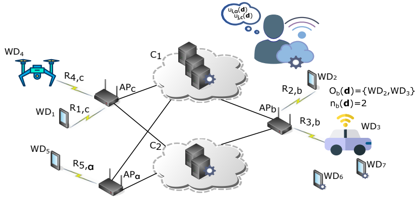

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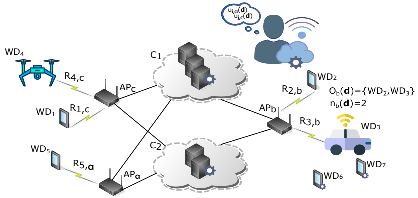

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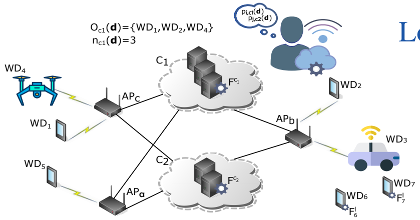
- Transmission time of WD i for offloading via AP a

$$T^{off}_{i,a}(\mathbf{d}, \mathbf{u}_a) = \frac{D_i}{\omega_{i,a}(\mathbf{d},\mathbf{u}_a)}$$

Computation Model

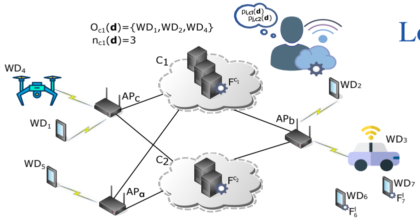


Local computing

- F_i^l computing capability of WD i
- Local execution time of WD i's task

$$T_{i,l}^{exe} = \frac{L_i}{F_i^l}$$

Computation Model



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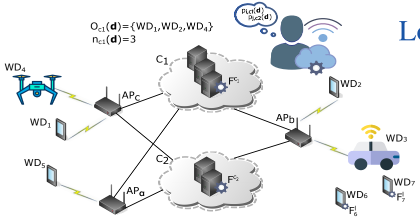
$O_c(\mathbf{d})$: set offloaders to EC c in strategy profile $\mathbf{d}$

- Computing capability allocated to WD i by EC c

$$F_i^c(\mathbf{d}, \mathbf{p}_c) = F^c \frac{p_{i,c}}{\sum_{j\in O_c(\mathbf{d})} p_{j,c}}$$

Computation Model



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- Execution time of WD i's task in EC c

$$T_{i,c}^{exe}(\mathbf{d}, \mathbf{p}_c) = \frac{L_i}{F_i^c(\mathbf{d}, \mathbf{p}_c)}$$

Cost - Task Completion Time

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Computation offloading cost

$$C^{c}_{i,a}(\mathbf{d}, \mathbf{u}_a, \mathbf{p}_c) = T^{off}_{i,a}(\mathbf{d}, \mathbf{u}_a) + T^{exe}_{i,c}(\mathbf{d}, \mathbf{p}_c)$$

Local computing cost

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Local computing cost

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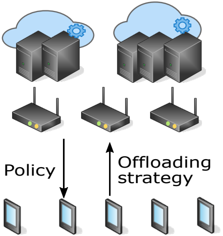
System cost

$$C(\mathbf{d},\mathbf{u},\mathbf{p}) = \underbrace{\sum_{i\in\mathcal{N}} \sum_{(a,c)\in\mathcal{A}_i\times\mathcal{C}} I_{d_i,(a,c)} C^c_{i,a}(\mathbf{d},\mathbf{u}_a,\mathbf{p}_c)}_{\text{offloading}} + \underbrace{\sum_{i\in\mathcal{N}} I_{d_i,i} C^l_i}_{\text{local execution}},$$

$$I_{d_i,r} = \begin{cases} 1, & \text{if } d_i = r \\ 0, & \text{otherwise} \end{cases}$$

Mobile Edge Computation Offloading Game (MEC-OG)



- Multi-leader common-follower Stackelberg game

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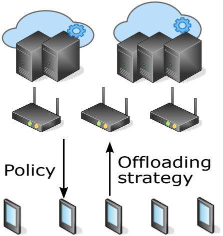

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Objective of the operator

- Minimization of total cost

$$\min_{\mathbf{u},\mathbf{p}\succeq 0} C(\mathbf{d},\mathbf{u},\mathbf{p})$$

Objective of WDs

- Minimization of own cost

$$\min_{d_i\in\mathfrak{D}_i} C_i(d_i, d_{-i}, \mathbf{u}_a^*, \mathbf{p}_c^*)$$

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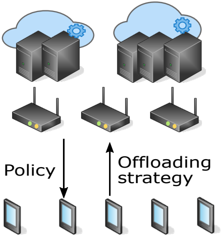

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Strategic game played by WDs

- For any allocation policy of the operator, game $\Gamma = < \mathcal{N}, (\mathfrak{D}_i)_i, (C_i)_i >$ played by WDs is a player-specific weighted congestion game

Questions Addressed

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1. Does the MEC-OG have a subgame perfect equilibrium (SPE)?
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2. Can it be computed using a decentralized algorithm?
3. How good is the system performance?

Results

Optimal resource allocation policy of the operator

- Best response of the operator to strategy profile **d** chosen by WDs

$$u^*_{i,a}(\mathbf{d}) = \frac{\sqrt{D_i/R_{i,a}}}{\sum_{j\in O_a(\mathbf{d})}\sqrt{D_j/R_{j,a}}}, \forall i \in O_a(\mathbf{d}), \forall a \in \mathcal{A}$$

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Game Γ under the optimal operator policy

- We transform Γ into a congestion game Γ^* with resource dependent weights

Offloading cost: $C^c_{i,a}(\mathbf{d}) = \omega_{i,a}\sum_{j\in O_a(\mathbf{d})}\omega_{j,a} + \omega_{i,c}\sum_{j\in O_c(\mathbf{d})}\omega_{j,c}$

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- Does strategic game Γ^* have a Nash equilibrium (NE)?

Results

NE existence

- Game Γ^* has a NE $\mathbf{d}^*$
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Improve Local Computing (ILC) algorithm

- Starts from a strategy profile in which all WDs perform computation locally
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SPE existence

- The MEC-OG has a SPE $(\mathbf{d}^*, \mathbf{u}^*, \mathbf{p}^*)$
  - Optimal provisioning coefficients $\mathbf{u}^*$ and $\mathbf{p}^*$ have closed form expressions
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Price of Anarchy (PoA) bound

- Upper bound on the PoA for the MEC-OG is $\frac{3+\sqrt{5}}{2}$

User Focused Performance Analysis

Evaluation scenario

- $A = 5$ APs, heterogeneous ECs $F^{c,tot} = 192$ *Gcycles*
- Tasks: $D_i \sim \mathcal{U}(0.2, 4)$ *Mb* , $L_i = D_i X$ *Gcycles* , $X \sim \Gamma(0.5, 1.6)$ *Gcycles/b*

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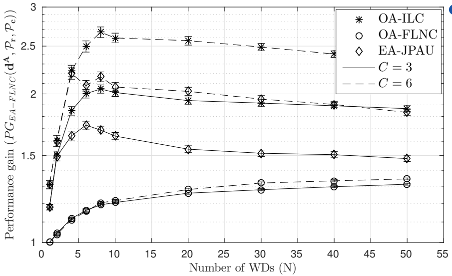
Performance gain

Defined w.r.t. *equal alocation* (EA) policy and the *fastest-link nearest-cloud* (FLNC) algorithm



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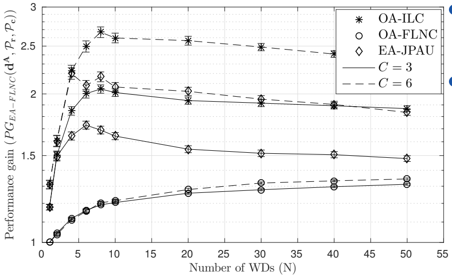

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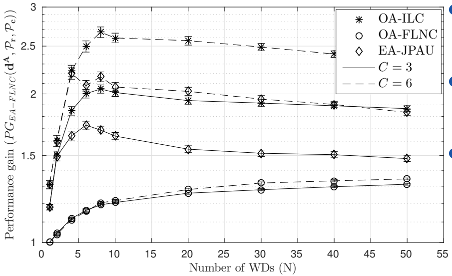

- *Performance gain* increases with decreasing marginal gain in N
- *Performance gain* increases with the number of edge ECs
- Largest *performance gain*
  - Operator implements OA policy
  - WDs compute offloading decisions using the ILC algorithm

Cloud Focused Performance Analysis

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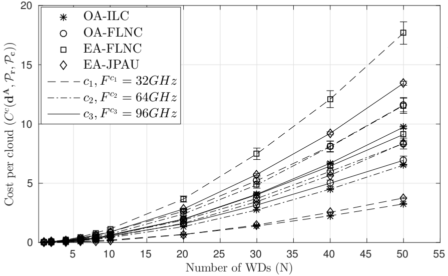
Cost per cloud

Defined as $C^c(\mathbf{d}) = \sum_{i \in O_c(\mathbf{d})} C_i(\mathbf{d})$



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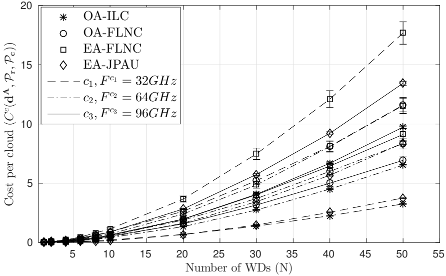

- *Cost per cloud* increases with the number N of WDs
- *Cost per cloud* is proportional to the EC's computing capability in case of equilibria under OA and EA policies

Computational Complexity

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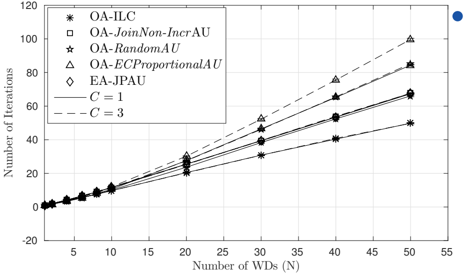
Number of iterations

- Randomly chosen strategy profile
- All WDs offload-congestion per EC proportional to its computing capability
- Empty system-WDs added in non-increasing order of their task complexities



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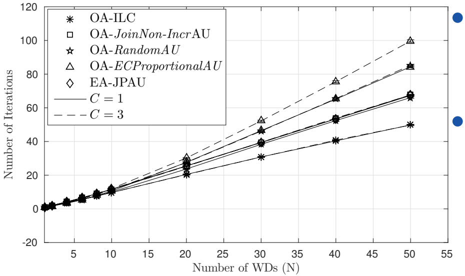

- Number of iterations scales approximately linearly with the number N of WDs
- Number of iterations is sensitive to the starting strategy profile
  - Smallest in the case of the ILC algorithm

Summary and Future Work

- Provided game theoretical analysis of the interaction between
  - an edge operator that jointly manages wireless and computing resources
  - autonomous WDs that aim at minimizing their own tasks completion times

Summary and Future Work

- Provided game theoretical analysis of the interaction between
  - an edge operator that jointly manages wireless and computing resources
  - autonomous WDs that aim at minimizing their own tasks completion times

- Interesting extensions
  - energy consumption minimization problem
  - stochastic model of task arrivals
  - learning in congestion games

Wireless and Computing Resource Allocation for Selfish Computation Offloading in Edge Computing

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