Widely Linear Filtering based Kindred Co-Channel Interference Suppression in FBMC Waveforms

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Abstract—This paper evaluates the performance of widely linear filtering based equalization for suppression of kindred cochannel interference. The presented approach is extension of the MMSE method proposed for ICI/ISI compensation for OQAM/FBMC transmission system in [1][2]. The analysis and computer simulation results show the relevance of widely linear filtering based equalization approach when the interfering signal exhibit improper statistics. This indicates the need to consider alternative forms of I/Q staggered multicarrier waveforms for 5G standardization that differs from the conventional one with 100% roll-off subchannels.

Keywords— WL equalization, MMSE, ICI/ISI, co-channel interference suppression; I/Q staggered, OQAM/FBMC, FMT

I. INTRODUCTION

Compensation of the transmission channel induced linear distortions in the subchannels of a FBMC signal with staggered, i.e. by T/2 mutually offset in-phase (I) and quadrature (Q) components of transmitted QAM symbols, was the first challenge towards making it competitive to the traditional CP-OFDM multicarrier waveform. This has been lately very much actualized in various forms of multiple-input multiple-output (MIMO) configurations, where the so-called intrinsic interference [3] has been considered as an obstacle for attaining performance level comparable with CP-OFDM.

With the advent of wireless communications towards more or less heterogeneous nature, where small cells coexist with macro cells, and with the future massive machine-to-machine communications, the problem of presence of co-channel interference gains ever more on importance, the more the reliance on single antenna terminals remains an important limitation for application of complex beam-forming structures.

Yet from the time of second generation wireless standards, notably the GSM, based on (essentially I/Q staggered) GMSK format, there has been an interest in exploiting the degrees of freedom brought by the pulse amplitude modulated (PAM) signals in terms of the co-channel interference originating from signals of the same kind (i.e. kindred). This pertains to the minimum mean square error (MMSE) equalizer which processes both the complex signal and its complex conjugated version tending to align the interfering signal to the imaginary axis, retaining as useful signal the resulting projection on the real axis. As it has been very insightfully demonstrated in [4], this system behaves as having an additional (virtual) antenna.

While for flat fading and rectilinear BPSK or generally PAM modulation just one-tap equalizer suffices, the situation becomes substantially different for MSK (as the linearized model of GMSK [5]) and generally I/Q staggered or Offset QAM (OQAM), which reaches maximal spectral efficiency when applied for orthogonal frequency-division multiplexing. (It should be noted that there is no overlapping of channels in GSM, which - in a linearized form [5]- would correspond to filtered multi-tone - FMT - format [6] with modification for I/Q staggering.) Namely, in this case the presence of intrinsic interference makes the signal "quasi-rectilinearly" modulated, supposedly needing, as suggested in [3], the use of multi-tap equalizer structures. Both rectilinear and quasi-rectilinear modulations induce non-circular second order statisticsallowing for increased degrees of freedom, so that the so-called widely linear filtering (WLF) based MMSE equalizer configuration can potentially cope with co-channel interference. (This term comes as combination of essentially nonlinear two-branch configuration representing the simultaneous processing and summing-up of complex signal and its complex-conjugated version and the linear feature of the second order statistics.). The key indicator for the presence of improper second order statistics is non-zero pseudo-(auto-) correlation matrix [7], the importance of which in the context of linear MMSE equalization and the "optimal" multicarrier waveforms selection is addressed in this paper.

This paper reports on attempts to materialize the WL equalization related expectations by further extending the previous work in [1] and [2] on the per-subchannel MMSE WL equalizer for the single-input single-output (SISO) FBMC waveforms in circular additive white Gaussian noise (AWGN) case, towards presence of the kindred per-subchannel interference. The main focus is on the linear MMSE WL equalization and the analysis of the impact of the type of the FBMC format, regarding subchannels' Nyquist spectral shaping roll-off factors and the amount of their overlapping. As a representative of the non-zero pseudo-correlation cases, every second subchannel with 100% roll-off factor has been studied, at the same time being representative of the singlecarrier, as well as of the filtered-multitone (FMT) formats with I/Q staggering, notwithstanding the spectral efficiency implications, except for its implicit reduction in the latter case.

After presentation of general FBMC transmission system in Section II, equivalence between definitions of WL processing in complex- and real-domain is given in Section III. In Section IV is introduced analytically derivation of equalizer's coefficients in case of presence of co-channel interference signal. Simulation results are given and discussed in Section V for rectilinear (BPSK) and quasi-rectilinear (Offset-QPSK i.e. QPSK with I/Q staggering) modulations, followed by conclusions in section VI.

II. FBMC TRANSMISSION SYSTEM

One of the representations of filter bank multicarrier system (FBMC) is sketched in Fig. 1. As can be see from figure, FBMC data transmission system, comprises of the input QAM data stream de-multiplexing, I/Q staggering, transmit synthesis filter bank (SFB), transmission channel, AWGN source, receiver analysis filter-bank (AFB), equalizer, I/Q de-staggering and de-multiplexing. More detailed block-diagrams of I/Q staggering and I/Q de-staggering with per-subchannel equalizer are shown in Fig. 2.



Fig. 1 FBMC transmission system.



Fig. 2 OQAM staggering and OQAM de-staggering with subcarrier equalizer.

Taking into account that in the "frequency-limited orthogonal" FLO multicarrier [8][9], that is the conventional FBMC case, the energy of the impulse response is mostly localized in a restricted set around the considered symbol time-frequency domain, it can be assumed that significant overlap is present only between immediately adjacent subcarriers. According to this model, input of the *N*-tap equalizer for k-th subchannel, $\mathbf{y}_k[\mathbf{n}] = [\mathbf{r}_{k,\mathbf{n}} \ \mathbf{r}_{k,\mathbf{n}-1} \ \mathbf{r}_{k,\mathbf{n}-2} \ \mathbf{r}_{k,\mathbf{n}-1}]^T$, which consists of the *T*/2 spaced received samples $r_{k,i}$, can be approximately expressed as

$$\mathbf{y}_{k}[n] \approx \mathbf{G}_{k} \cdot \mathbf{x}_{k}[n] + \mathbf{M}_{k} \cdot \mathbf{x}_{k-1}[n] + \mathbf{N}_{k} \cdot \mathbf{x}_{k+1}[n] + \mathbf{\Gamma}_{k} \cdot \mathbf{\eta}[n] \quad (1)$$

The (Toeplitz) convolution matrices $\mathbf{G}_{\mathbf{k}} \in \mathbb{C}^{NxL}$, $\mathbf{M}_{\mathbf{k}} \in \mathbb{C}^{NxL}$ and $\mathbf{N}_{\mathbf{k}} \in \mathbb{C}^{NxL}$ correspond to the down-sampled subchannel impulse response from, respectively, the *k*-th, *(k-1)*-th and *(k+1)*-th subchannel input to the *k*-th subchannel output. $\mathbf{x}_{\mathbf{k}}[\mathbf{n}]$, $\mathbf{x}_{\mathbf{k}-\mathbf{l}}[\mathbf{n}]$ and $\mathbf{x}_{\mathbf{k}+\mathbf{l}}[\mathbf{n}]$ are the vectors of data symbols (formed in accordance with the I/Q staggering of QAM symbols) for the considered subchannels. $\Gamma_{\mathbf{k}} \in \mathbb{C}^{NxR}$ represents the matrix defined by the receiver (AFB) impulse response, and $\boldsymbol{\eta}[\mathbf{n}]$ stands for the vector of the sum of the AWGN and interference signal samples at the AFB input [1].

Down-sampled impulse responses $g_k(n)$, $m_k(n)$ and $n_k(n)$ that define matrices G_k , M_k and N_k respectively are given by:

$$g_{k}(n) = [h_{k}(n) \cdot h_{ch}(n) \cdot h_{k}(n)]_{\downarrow M/2}, m_{k}(n) = [h_{k-1}(n) \cdot h_{ch}(n) \cdot h_{k}(n)]_{\downarrow M/2},$$

$$n_{k}(n) = [h_{k+1}(n) \cdot h_{ch}(n) \cdot h_{k}(n)]_{\downarrow M/2}.$$
(2)

h_k(n), h_{k-1}(n) and h_{k+1}(n) denote prototype filters of length L_g for k-th, (k-1)-th and (k+1)-th subchannels respectively and h_{ch}(n) denotes channel impulse response of length L_{ch} . One of dimensions of convolution matrices, N, represents number of equalizer coefficients, while the other dimension of matrices G_k . M_k and N_k is L=N+Q-1, and $Q=[(2\cdot L_g + L_{ch} - 2)/(M/2)]$ is length of down-sampled impulse responses $g_k(n)$, m_k(n) and n_k(n). The other dimension of down-sampled noise-filtering convolution matrix Γ_k is $R=M/2\cdot N + L_g$. It is important to mention that the matrix Γ_k , in contrast to matrices G_k . M_k and N_k , is not Toeplitz matrix because it contains down-sampling by M/2, where M is the (maximal) number of subchannels.

III. DEFINITION OF WL PROCESSSING IN REAL-DOMAIN AND COMPLEX-DOMAIN NOTATION

As known, simultaneous processing of both the complex signal and its complex-conjugated version is referred as widelinear (WL) processing. The WL formulation can be equivalently given in complex- and real-domain and their relation is described below.

In complex-domain notation, the original observation vector $\mathbf{y}_k[n]$ is extended to include the complex-conjugated version of signal $\mathbf{y}_k^*[n]$ as well:

$$\widetilde{\mathbf{y}}_{k}[n] = \begin{bmatrix} \mathbf{y}_{k}[n] \\ \mathbf{y}_{k}^{*}[n] \end{bmatrix}$$
(3)

For input of equalizer defined by (3), extended correlation matrix of signal is defined by [1]

$$\widetilde{\mathbf{R}}_{\mathbf{y}} = E\{\widetilde{y}_{k}[n]\widetilde{y}_{k}^{H}[n]\} = \begin{bmatrix} \mathbf{R}_{\mathbf{y}} & \mathbf{P}_{\mathbf{y}} \\ \mathbf{P}_{\mathbf{y}}^{*} & \mathbf{R}_{\mathbf{y}}^{*} \end{bmatrix}$$
(4)

where $\mathbf{R}_{\mathbf{y}} = E\{y_k[n]y_k^H[n]\}$ is original correlation matrix and $\mathbf{P}_{\mathbf{y}} = E\{y_k[n]y_k^T[n]\}$ is referred as pseudo-correlation matrix.

In real-domain notation of WL processing input vector is defined through real and imaginary parts of the original observation vector $\mathbf{y}_k[n]$ according to:

$$\overline{\mathbf{y}}_{k}[n] = \begin{bmatrix} \operatorname{Re}\{\mathbf{y}_{k}[n]\} \\ \operatorname{Im}\{\mathbf{y}_{k}[n]\} \end{bmatrix}$$
(5)

Correlation matrix, for input of equalizer defined by (5), is given with

$$\overline{\mathbf{R}}_{\mathbf{y}} = E\{\overline{y}_{k}[n]\overline{y}_{k}^{T}[n]\} = \begin{bmatrix} \mathbf{R}^{rr} & \mathbf{R}^{ri} \\ \mathbf{R}^{ir} & \mathbf{R}^{ii} \end{bmatrix}$$
(6)

T denotes transposition, H is Hilbert transpose, \sim is used to denote complex-domain and to denote real-domain notation.

It can be shown that there is a relation between complex matrix $\mathbf{\tilde{R}}_{v}$ and real matrix $\mathbf{\bar{R}}_{v}$, defined by

$$\overline{\mathbf{R}}_{\mathbf{y}} = \frac{1}{2} \mathbf{E}_R \widetilde{\mathbf{R}}_{\mathbf{y}} \mathbf{E}_C \tag{7}$$

where E_R and E_C are so-called real and complex equivalence matrices, formed from unit matrix I_N and given by

$$\mathbf{E}_{R} = \frac{1}{2} \begin{bmatrix} \mathbf{I}_{N} & \mathbf{I}_{N} \\ -j\mathbf{I}_{N} & j\mathbf{I}_{N} \end{bmatrix}, \ \mathbf{E}_{C} = \begin{bmatrix} \mathbf{I}_{N} & j\mathbf{I}_{N} \\ \mathbf{I}_{N} & -j\mathbf{I}_{N} \end{bmatrix}.$$
(8)

By using the notation of real correlation sub-matrices from (6), the complex correlation and pseudo-correlation matrices \mathbf{R}_{v} and \mathbf{P}_{v} can be expressed as

$$\mathbf{R}_{y} = \mathbf{R}^{rr} + \mathbf{R}^{ii} + j(\mathbf{R}^{ir} - \mathbf{R}^{ri}) , \ \mathbf{P}_{y} = \mathbf{R}^{rr} - \mathbf{R}^{ii} + j(\mathbf{R}^{ir} + \mathbf{R}^{ri})$$
(9)

This equivalence between complex- and real-domain formulations of WL processing is confirmed through simulation results in Section V. The derivation of equalizer coefficients in next Section is given using the real-domain notation (notice that the pseudo-correlation matrix P_y from (9) is equal to zero for $\mathbf{R}^{rr} = \mathbf{R}^{ii}$ and $\mathbf{R}^{ir} = -\mathbf{R}^{ri}$).

IV. ANALYTICAL DERIVATION OF EQUALIZER COEFFICIENTS IN PRESENCE OF KINDRED CO-CHANNEL INTERFERENCE SIGNAL

Starting from expressions for a calculation of equalizer's coefficients in absence of co-channel interference signal [2], we derive expressions for the case when kindred co-channel interference signal is present along the AWGN disturbance. Corresponding expression for *N*-tap equalizers coefficients is then given with

$$\mathbf{u}_{k}^{'} = \left[\mathbf{H}_{k} \cdot \mathbf{R}_{x} \cdot \mathbf{H}_{k}^{T} + \mathbf{F}_{k} \cdot \hat{\mathbf{R}}_{x} \cdot \mathbf{F}_{k}^{T} + \mathbf{R}_{\eta,k}^{'}\right]^{-1} \cdot \mathbf{H}_{k} \cdot \mathbf{R}_{x} \cdot \mathbf{e}_{\nu} \quad (10)$$

where $\mathbf{R}_x = \frac{1}{2}\sigma_d^2 \cdot \mathbf{I}_L$ is correlation matrix of the stationary

OQAM modulated input symbols $d_k[m]$ with the variance σ_d^2 . \mathbf{e}_v is the vector which cuts out the corresponding column of the convolution matrix \mathbf{H}_k according to decision delay v of the equalizer (it is defined as all-zeros vector, except the value 1 at the v-th position). $\hat{\mathbf{R}}_x \in \mathbb{C}^{2L\times 2L}$ is block diagonal correlation matrix formed from correlation matrix \mathbf{R}_x according to

$$\widehat{\mathbf{R}}_{x} = \begin{bmatrix} \mathbf{R}_{x} & \mathbf{0}_{L} \\ \mathbf{0}_{L} & \mathbf{R}_{x} \end{bmatrix}$$
(11)

Matrices $\mathbf{H}_{k} \in \mathbb{C}^{2N_{xL}}$ and $\mathbf{F}_{k} \in \mathbb{C}^{2N_{x2L}}$ are real-valued convolution matrices of desired signal given with the first and the second equation from (12), respectively

$$\mathbf{H}_{k} = \begin{bmatrix} \mathbf{G}_{k}^{(R)} \\ \mathbf{G}_{k}^{(I)} \end{bmatrix}, \ \mathbf{F}_{k} = \begin{bmatrix} \mathbf{M}_{k}^{(R)} \ \mathbf{N}_{k}^{(R)} \\ \mathbf{M}_{k}^{(I)} \ \mathbf{N}_{k}^{(I)} \end{bmatrix}$$
(12)

Influence of noise and co-channel interference signal are included through correlation matrix $R'_{\eta,k}$, which is given by

$$\mathbf{R}'_{\eta,k} = \mathbf{\Gamma}'_{k} \cdot (\mathbf{R}_{\eta} + \mathbf{H}'_{k} \cdot \mathbf{R}_{x} \cdot \mathbf{H}'^{T}_{k} + \mathbf{S}'_{k} \cdot \mathbf{R}_{x} \cdot \mathbf{S}'^{T}_{k} + \mathbf{T}'_{k} \cdot \mathbf{R}_{x} \cdot \mathbf{T}'^{T}_{k}) \cdot \mathbf{\Gamma}'^{T}_{k} (13)$$

where $\mathbf{R}_{\eta} = \sigma_{\eta}^{2} \mathbf{I}_{2R}$ is correlation matrix of uncorrelated stationary noise. $\mathbf{H}'_{\mathbf{k}} \in \mathbb{C}^{2R_{\mathbf{k}}\mathbf{L}'}$ is real-valued convolution matrix of interference signal for *k*-th subcarrier. $\mathbf{S}'_{\mathbf{k}}$ and $\mathbf{T}'_{\mathbf{k}} \in \mathbb{C}^{2R_{\mathbf{k}}\mathbf{L}'}$ are also real-valued convolution matrices of interference signal, which represent interference from *(k-1)*-th and *(k+1)*-th to *k*-th subcarrier, respectively. $\mathbf{\Gamma}'_{\mathbf{k}} \in \mathbb{C}^{2N_{\mathbf{k}}2R_{\mathbf{k}}}$ (15) is interference convolution matrix which includes the impact of only the receiver prototype filter $h_{\mathbf{k}}(\mathbf{n})$, what as result has that $\mathbf{H}'_{\mathbf{k}}$, $\mathbf{S}'_{\mathbf{k}}$ and $\mathbf{T}'_{\mathbf{k}}$ (14) include only the impact of transmitter prototype filters $h_{\mathbf{k}}(\mathbf{n})$, $h_{\mathbf{k}-1}(\mathbf{n})$ and $h_{\mathbf{k}+1}(\mathbf{n})$ (respectively) and channel impulse response $h_{ch}(\mathbf{n})$, unlike matrices $\mathbf{H}_{\mathbf{k}} \mathbf{M}_{\mathbf{k}}$ and $\mathbf{N}_{\mathbf{k}}$.

$$\mathbf{H}_{k}^{'} = \begin{bmatrix} \mathbf{G}_{k}^{'(R)} \\ \mathbf{G}_{k}^{'(I)} \end{bmatrix} \mathbf{S}_{k}^{'} = \begin{bmatrix} \mathbf{M}_{k}^{'(R)} \\ \mathbf{M}_{k}^{'(I)} \end{bmatrix} \mathbf{T}_{k}^{'} = \begin{bmatrix} \mathbf{N}_{k}^{'(R)} \\ \mathbf{N}_{k}^{'(I)} \end{bmatrix}$$
(14)

$$\mathbf{\Gamma}_{k}^{'} = \begin{bmatrix} \mathbf{\Gamma}_{k}^{(R)} & -\mathbf{\Gamma}_{k}^{(I)} \\ \mathbf{\Gamma}_{k}^{(I)} & \mathbf{\Gamma}_{k}^{(R)} \end{bmatrix}$$
(15)

 $\mathbf{G'}_{\mathbf{k}} \in \mathbb{C}^{RxL'}$, $\mathbf{M'}_{\mathbf{k}} \in \mathbb{C}^{RxL'}$ and $\mathbf{N'}_{\mathbf{k}} \in \mathbb{C}^{RxL'}$ are convolution matrices of interference signal defined in complex-domain and formed according to $(17)^1$ from the impulse responses

$$g_{k}(n) = [h_{k}(n) \cdot h_{ch}(n)], m_{k}(n) = [h_{k-1}(n) \cdot h_{ch}(n)],$$

$$n_{k}(n)(n) = [h_{k+1}(n) \cdot h_{ch}(n)].$$
(16)

$$\mathbf{Z}_{k}^{'} = \begin{bmatrix} z_{k}^{'}(n) & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & z_{k}^{'}(n) & & & \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ & & z_{k}^{'}(n) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & z_{k}^{'}(n) \end{bmatrix}_{\downarrow_{M/2}}$$
(17)

(The matrices $\mathbf{G'}_{\mathbf{k}}$, $\mathbf{M'}_{\mathbf{k}}$, $\mathbf{N'}_{\mathbf{k}}$ and $\Gamma_{\mathbf{k}}$ are not Toeplitz, because the down-sampling factor is included in their structures.)

It is important to mention that the every second column of matrices G_k , M_k , N_k , G'_k , M'_k and N'_k is multiplied by *j*, before taking real and imaginary parts (*R* and *I*) in (12), (14). This is done in order to remove *j* from imaginary entries of transmitted sequence (which contains *R* and *I* parts of symbols alternately), so the matrix \mathbf{R}_x becomes purely real-valued.

Above derivation of equalizer's coefficients is given for the case when the adjacent subchannels are overlapped. In [1] it has been commented that for the exponentially modulated subcarriers with identical (100% roll-off) filtering, this results in zero pseudo-correlation matrix $\mathbf{P}_{\mathbf{v}}$ meaning that subcarrier signal is a wide-sense stationary and circular. In that case the one (set) of degrees of freedom is lost, which can be important when kindred co-channel interference signal is present. In other words, WL equalization for zero-valued matrix P_v deteriorates to strictly linear (SL) equalization. For that reason, in this paper the situations with non-zero P_v matrix are emulated through the single-carrier case, that is the FBMC configuration with every second subchannel active - which ensure the non-circular statistics of signal. (Based on the fact that for the 100% roll-off case the adjacent subchannels' related pseudo-correlation terms cancel out the self-pseudocorrelation term, [1], it can be conjectured that the situations from full circularity to full non-circularity can be produced by varying the subchannels' spectral roll-offs from 100% to 0%.)

When every second subchannel is active, all expressions from above are simplified by omission of correlation and convolution matrices which take into account influence of adjacent subcarriers. Equation (10) is then simplified to

$$\mathbf{u}_{k}^{'} = \left[\mathbf{H}_{k} \cdot \mathbf{R}_{x} \cdot \mathbf{H}_{k}^{T} + \mathbf{R}_{\eta,k}^{'}\right]^{-1} \cdot \mathbf{H}_{k} \cdot \mathbf{R}_{x} \cdot \mathbf{e}_{\nu}$$
(18)

where noise-interference correlation matrix is now given by

$$\mathbf{R}_{\eta,k}' = \mathbf{\Gamma}_{k}' \cdot (\mathbf{R}_{\eta} + \mathbf{H}_{k}' \cdot \mathbf{R}_{x} \cdot \mathbf{H}_{k}'^{T}) \cdot \mathbf{\Gamma}_{k}'^{T}$$
(19)

¹**Z**'_k \in {**G**'_k, **M**'_k, **N**'_k } and $z'_k(n) \in$ { $g'_k(n), m'_k(n), n'_k(n)$ }

In Section V are given and discussed results for both of these cases, when subcarriers are overlapped and when there are no overlapped (every second subchannel is active).

The optimization criterion which was used is same as the one defined in [2] and it is minimization of mean square error between the estimated signal $\hat{d}_k[m] = \hat{a}_k[m]+j\hat{b}_k[m]$ and the input signal $d_k[m] = a_k[m]+jb_k[m]$ according to

$$\mathbf{u}_{k,MMSE}^{(R)} = \underset{u_k}{\operatorname{arg min}} E\left[\left|\hat{a}_k[m] - a_k[m-\nu]\right|^2\right]$$

$$\mathbf{u}_{k,MMSE}^{(I)} = \underset{u_k}{\operatorname{arg min}} E\left[\left|\hat{b}_k[m] - b_k[m-\nu]\right|^2\right]$$
(20)

It will be (indirectly) shown that the first and the second expression from (20) lead to same results. It is important to mention that the expressions (10) and (18) offer a solution for real and imaginary parts of coefficients separately, but that the final complex equalizer's coefficients, are calculated concatenating of real and imaginary parts. Relation between $\mathbf{u'}_{\mathbf{k}}$ in (10) and (18) with $\mathbf{u}_{\mathbf{k}}$ in (20) is then given with [2]

$$\mathbf{u}_{k}^{'T} = \left[\mathbf{u}_{k}^{(R),T}, \mathbf{u}_{k}^{(I),T}\right] \in \mathbf{R}^{2N}.$$
 (21)

V. SIMULATION RESULTS

In the following are provided performances of the FBMC system in presence of kindred co-channel interference. Simulation results are given for flat Rayleigh fading case and two type of modulation (quasi-rectilinear Offset-QPSK and rectilinear BPSK). As prototype filter we have used a truncated version of a root raised cosine filter with overlapping factor K=4 and roll-off factor ρ =100%, so the only immediately adjacent subchannels have a significant overlap. The total number of subchannels is M=8 and BER performances are given for Eb/No values in range from 0 dB to 30 dB and three different values of signal-to-interference (SIR) ratio (-10 dB, 0 dB and 10 dB).

The first set of simulation results illustrates equivalence between WL equalization formulated in complex- and realdomain notation. Results are given for QPSK modulation and three different values of SIR. These results are presented in Fig. 3 for case when subchannels are overlapped (7 out of 8 active subchannels) and in Fig. 4 for case when every second subchannel is active (4 out of 8 active subchannels).



Fig. 3 Equivalence between real-domain and complex-domain formulation of WL equalization; overlapped subchannels.

As it was conjectured in previous section, performances are significantly worse for the case of overlapped subchannels (Fig. 3). This comes as a consequence of the loss of additional degrees of freedom, which WL equalization provides for noncircular statistics (e.g. non-overlapped subchannels) i.e. when the pseudo-correlation matrix is different from zero.



Fig. 4 Equivalence between real-domain and complex-domain formulation of WL equalization; every second subchannel is active.

The second set of simulation results gives comparison of "full" WL equalization for BPSK and QPSK modulation, SL equalization (which in complex domain does not use pseudocorrelation matrix for calculating of equalizer coefficients) for QPSK modulation and "incomplete" WL equalization that does not take into account correlation matrix of interferer in equation (19) also for QPSK modulation. Simulation results, for case of non-overlapped subchannels, are given in Fig. 5.



Fig. 5 Comparison of "full" WL equalization for BPSK (magenta) and QPSK (red), SL equalization for QPSK (green) and "incomplete" WL equalization for QPSK (blue); every second subchannel is active.

From Fig. 5 it can be see that the best performances are present in the case of "full" WL equalization, which takes into account correlation matrix of interference signal and the performances are somewhat better for BPSK modulation.

It is important to mention that the above presented results for QPSK modulation correspond to 15-tap equalizer. This is substantially larger number of taps than in the case without cochannel interference, when just one-tap equalizer is sufficient. From that reason, in Fig. 6 are given BER values in function of different number of equalizer's coefficients for SNR = 30 dB, SIR = 0dB. For the case when every second subchannel is active, simulation results are given for QPSK and BPSK modulations and for the case with overlapped subchannels results are given only for QPSK modulation.

As it can be seen from Fig. 6, for QPSK modulation in presence of interference signal and for the case when every second subchannel is active, the one-tap equalizer is not sufficient and BER performance is almost steadily improving with increasing the number of equalizer's coefficients. For the case of (100% roll-off) overlapped subchannels, BER values are very poor and approximately independent from equalizer length. In contrast to OPSK, for BPSK modulation, only onetap equalizer is sufficient even in the case when kindred cochannel interference is present (Fig. 6). This can be explained by comparison of signal constellation for rectilinear (BPSK) and quasi-rectilinear (Offset-QPSK) modulations. In the case of the OQAM format, the signal constellation at the output of AFB, even for the ideal channel, is not composed from only two points, but consists of two lines. The points along them actually represent the interpolated values of data bearing transmitted symbols. For that reason, in corroboration with hint in [4], for OQAM formats more than one tap equalizer will be needed, even in the frequency-flat channel case.



Fig. 6 BER performances for SNR = 30 dB and SIR = 0 dB in function of number of equalizers' coefficients for every second active subchannel-dotted lines and overlapped subchannels-solid lines; (QPSK-red and BPSK-blue).

For simulation results depicted in Fig. 7, parameter \triangle BER is defined as the subtraction between BER value obtained by "full" WL equalization which takes into account correlation matrix of interference signal and BER value obtained by "incomplete" WL equalization which does not take into account correlation matrix of interference signal. Dependence of \triangle BER, WL reduction of BER values from SIR is given for SNR = 20 dB and QPSK and BPSK modulations in Fig. 7.



Fig. 7 BER decrease as function of SIR; BPSK- blue line, QPSK-red line.

From Fig. 7 it can be see that the "full" WL equalization, gives slightly higher improvement compared to "incomplete" WL equalization for BPSK modulation in the case of negative

values of SIR and for QPSK modulation in the case of nonnegative values of SIR. However, this difference in the improvement is small and dependence of \triangle BER from SIR is approximately same for both type of modulation.

VI. CONCLUSION

This rather comprehensive treatment of the co-channel interference resilience of the FBMC format with the use of MMSE WL EQL clearly indicates certain gain in comparison with the SL MMSE EQL approach, attained when the circular statistics was imposed by the avoidance of spectral overlapping among subchannels. It is demonstrated that in the flat fading propagation conditions the multi-tap WL equalizer for quasi-rectilinear modulation attains performance quite close to those that one-tap equalizer has for rectilinear case. Since the difference between SL and WL has been produced by ensuring the non-zero pseudo-correlation matrix, this signals an important alert regarding the likely important inadequacy of the particular FBMC configuration that has been also pursued as a candidate for the 5G standardization effort. Namely, for overlapped adjacent subchannels' spectra exhibiting maximal spectral efficiency one can consider use of smaller roll-off factors in order to improve suppression of cochannel interference by trading it off for somewhat increased transmission latency. On the other side, it hints for possibly even higher SINR gains with reduced spectral efficiency trade-off attainable with FMT format with I/Q staggering. The TLO format [8][9] also gains on importance.

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REFERENCES

- D.S. Waldhauser, "Multicarrier systems based on filter banks," Ph.D. Thesis, Technical University Munich, 2009.
- [2] D.S. Walhauser, et al. "MMSE subcarrier equalization for filter bank based multicarrier systems," Proc. of IEEE Workshop on Signal Processing in Wireless Comms. (ISCAS), May 2005, pp. 524-527.
- [3] M. El Tabach, et al., "Spatial data multiplexing over OFDM/OQAM modulations," ICC 2007.
- [4] P. Chevalier and F. Pipon, "New insights into optimal widely linear array receivers for the demodulation of BPSK, MSK, and GMSK signals corrupted by noncircular interferences – application to SAIC," IEEE Tr. On Signal Processing, Vol. 54, No. 3, March 2006.
- [5] P. A. Laurent, "Exact and approximate construction of digital phase modulations by superposition of amplitude modulated pulses (AMP)," IEEE Trans. Commun. vol. 34, pp. 150-160, Feb. 1986,
- [6] Cherubini et al., "Filter bank modulation techniques for very high speed digital subscriber lines", IEEE Communications Magazine, May 2000.
- [7] B. Picibono and P. Chevalier, "Widely linear estimation with complex data," IEEE Tr. On Sig. Proc., Vol. 43, No. 8, pp. 2030-33, Aug. 1995.
- [8] S. Josilo et al., "Multi-carrier waveforms with I/Q staggering uniform and non-uniform FBMC formats," revised submission to the EURASIP JASP in flexible multi-carrier waveforms for future wireless communications, 2014.
- [9] S. Josilo et al., "Non-uniform FBMC A pragmatic approach", ISWCS'2013, Aug. 2013.