Non-uniform FBMC - A Pragmatic Approach

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Abstract

Outgoing from the known uniform FBMC (Filter-Bank Multi-Carrier) formats with Nyquist spectral shaped sub-channels and its time-frequency dual Time-Limited Orthogonal (TLO) form, we introduce an orthogonal frequency division multiplex of non-uniformly spaced and unequal-width sub-channels. The goal is to attain potential gains in using relatively small number of sub-channels, but still allow for frequency gaps needed for channelization with a small decrease in overall spectral efficiency. The orthogonality conditions are evaluated through simulations using the extended OFDM framework, whereby the corresponding referent filter-bank impulse responses are defined in frequency-domain by straightforward aggregations of the pertaining uniform filter-bank sub-channels spectral shapes (for conventional FBMC with frequency limited sub-channels spectra), and by transforming to frequency-domain of adequately aggregated time-limited referent impulse responses of uniform TLO configuration. An analytical derivation, i.e. confirmation of the orthogonality conditions are also derived with reliance on the uniform FBMC and TLO orthogonality conditions. Non-symmetrical spectral shaped sub-channels in the case of FBMC format are also proposed.

Keywords- Filter-Bank Multi-Carrier (FBMC), uniform filter-bank, non-uniform filter-bank, orthogonal frequency division multiplexing (OFDM), quadrature amplitude modulation (QAM), frequency-limited orthogonal (FLO), time-limited orthogonal (TLO).

I. Introduction

Although the FBMC formats have much better spectral characteristics compared with the traditional CP-OFDM, the longer referent impulse response needed therein has turned into a certain disadvantage regarding the additional latency introduced. To remedy this, the one QAM interval (T-) long impulse response in the form of cosine function in the range of its argument between $-\pi/2$ to $\pi/2$ (the Hanning window) proposed in [1, and its ref. 5], can be used. Since the staggered signaling formats, as essentially real-domain ones, show great advantages in case of the co-channel interference limited scenarios, as shown in for example [2], the TLO format with the zero roll-off factor in time-domain leads to a particular form of Staggered, SCP-OFDM format, while 100% roll off case gives OFDM of MSK signals.

In order to conciliate the advantages of using wider sub-channels in terms of reduction of PAPR and increase of spectral efficiency in situations when predetermined power spectral density (PSD) masks have to be obeyed, while at the same time being able to separate the adjacent channels by relatively narrow frequency guard-intervals, the need arises for a modification which would enable utilization of sub-channels with differing widths within scattered frequency bands (white-zones), and in particular scarcely available frequency-gaps in the targeted Private Mobile Radio (PMR) and Cognitive Radio applications. This in principle can be done by adopting widely explored and quite well studied non-uniform filter-bank configurations known and used for source coding applications, starting from some early proposals, as in[3].

For the time being, we used the framework of extended OFDM [5] for computer simulation based evaluation of orthogonality conditions, which is based on frequency-domain implementation of the conventional overlap-and-add filtering method. We also experimented with orthogonality for the unsymmetrical roll-off factors (in frequency-domain), to be able to possibly better control the latency inherent in the FBMC format.

After highlighting the time-frequency duality between uniformly spaced FBMC and TLO formats in Section II, a pragmatic method of aggregation of uniformly spaced sub-channels and the related derivation of non-uniform filter-bank orthogonality conditions are dealt with in Section III, while in Section IV the asymmetrical sub-channel spectral shaping is introduced. The similar aggregation procedure, but in time-domain, is applied to the TLO format in Section V, followed by the conclusions part in Section VII.

II. Uniformly spaced TLO-MC and FBMC

Difference between the standard FBMC, i.e. OFDM/OQAM formats with frequency-domain shaping (which could be termed as the FLO – Frequency-Limited Orthogonal) and the TLO multi-carrier formats [1] consists in the utilization of time-frequency dual impulse responses, as illustrated in Fig. 1. It may suffice to only note that in the case of zero-percent roll off case TLO, the referent impulse response has rectangular form of length T/2, while in the 100% roll-off case, it has the

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form of the half of the cosine function, and has length of T. Also, since the OFDM requires use of the Cyclic-Prefix (CP), which now (if at all) has to be inserted at the beginning of each of the T/2 long OFDM symbols, in order for it to still be long enough to absorb the delay spreads for particular usage scenarios, more spectral inefficiency has to be allowed for, or the number of sub-channels be doubled, with the consequent increased impact of non-linear distortions and the related spectral broadening caused by the use of High-Power Amplifiers (HPA).

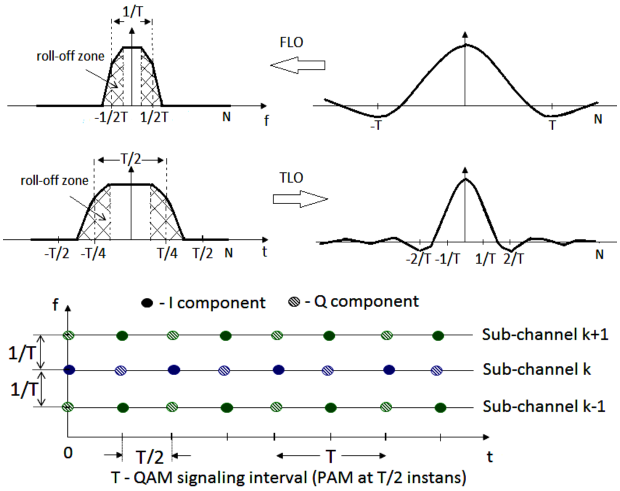

Fig. 1. Time-frequency duality between the TLO and FLO formats.

Implementation of these two forms of filter-banks, both in the uniform and non-uniform configurations is done through the extended OFDM approach [5], essentially a frequency-domain based referent impulse response design method, as described below.

III. Aggregating sub-channels for NU-FBMC

A direct extension of the uniform filter-bank towards the non-uniform filter-bank configuration is to simply aggregate the sub-channels of even, odd or combined uniform filter-banks. In the following we construct certain configurations and study the orthogonality conditions in terms of the Re and j•Im signalling at T/2 spaced instants and their relationship, retaining the roll-off region spectral symmetry from the uniform filter-bank arrangements shown in Fig. 2, with the frequency axis normalized by fs/2π.

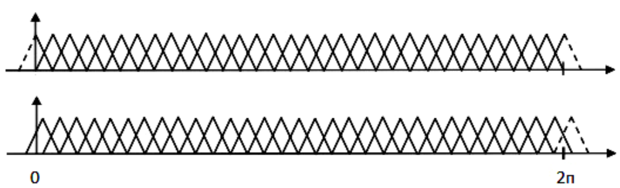

Fig. 2. The even (upper) and odd (lower) uniform filter-banks, M=32.

The separation between the peaks of the stylized 100% roll-off factor (i.e. for their s.r.r.c – square-root raised-cosine Tx and Rx pertinent sub-channels' transfer function) is 1/T, corresponding to QAM symbol rate. The frequency shift between even and odd sub-channels central frequencies is 1/2T. Each of the individual spectral shapes shown here by triangles are represented by 2K+1 equidistant samples. Their overlapping samples are added at the KM-IFFT input, [5, Fig. 10], and the signal samples at the KM-FFT outputs summed-up, [5, Fig. 11], after previously having been weighted by the same set of frequency domain samples.

If the KM-long vectors belonging to equidistantly spaced (real and positive) sub-channels' samples, among which there are only 2K+1 generally non-zero values, are denoted by $\hat{G}_i$, with i = 0 to KM-1, then the wider and generally non-uniformly spaced sub-channels samples can be defined by $\sqrt{\sum_{i=x}^{y}\hat{G}_i^2}$.

For example, from the odd-spaced uniform filter-bank the following arrangement of the non-equidistant sub-channels can be made:

sCh1p=$\sqrt{\sum_{i=0}^{7}\hat{G}_i^2}$, sCh2p=$\sqrt{\sum_{i=8}^{11}\hat{G}_i^2}$, sCh3p=$\sqrt{\sum_{i=12}^{13}\hat{G}_i^2}$, etc.

The sub-channels denoted sCh4p and sCh4m (p and m are used respectively for marking positive and negative frequencies) are thus defined by the original sub-channels with roll-off factor 100%, so that the roll-offs of the wider sub-channels become progressively reduced to 50%, 25% and 12.5% by the effective doubling of sub-channels' bandwidths, along their signaling intervals being correspondingly halved, with central frequencies on the corresponding multiples of 1/2T, as can be inferred from the illustration in Fig. 3a. (In sub-section A. it will be shown that the orthogonality conditions between sub-channels formed in this way reduce to the ones of the uniform FBMC.)

The sum of the pertinent vectors of the non-uniform sub-channels is fed to the input of the KM-size IFFT [5, Fig 10.], and similarly reproduced at the output of demodulator by appropriately weighting the KM-size FFT output [5, Fig 11].

Sequencing of the QAM data samples parts happens at the beginning and the half of the corresponding QAM symbol intervals, and based on simulation experiments it becomes determined by the odd and/or evenness of the filter-bank arrangements produced by equidistant shifting of the particular sub-channels: if particular sub-channel produces an odd filter-bank, on it the signaling is always by Re or j•Im, and if it produces an even filter-bank, then interchangeably Re and j•Im are brought to KM-IFFT block, so that the adjacent sub-channels at the same time instant are purely real and purely imaginary, or the other way around. For the nonuniform filter-bank arrangement of Fig. 3a, the one of the two options for the QAM parts signaling is shown in Fig. 3b. In all the figures Re part is marked by a square, and the j•Im part by a diamond.

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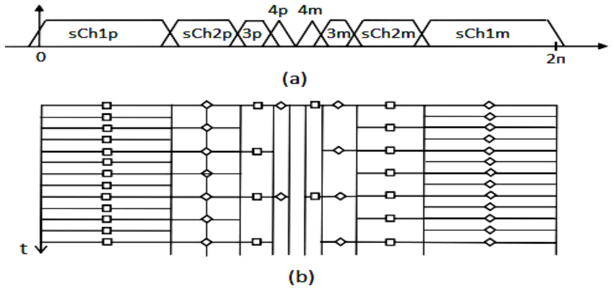

Fig. 3. Non-uniform FBMC arrangement from odd uniform FBMC, (a) and the example of sequencing the QAM I and Q parts in odd-NU-FBMC.

If the sCh1p and sCh1m would be merged (by the square-root of the sum of their squares, as indicated earlier), then the central (DC) sub-channel would correspond to an even-spaced FBMC, illustrated as in Fig. 4.

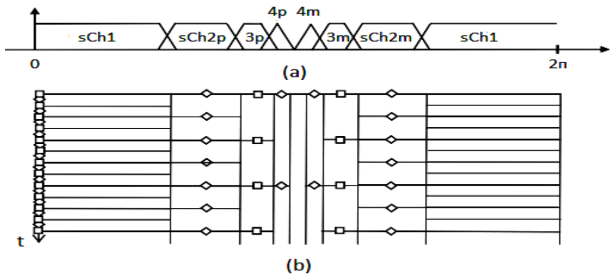

Fig. 4. An example of the I/Q sequencing in NU-FBMC.

For derivation of a non-uniform FBMC configuration by sub-channels aggregation it turns out that only non-integer ratios of the sub-channels bandwidth can be produced. In order to keep the integer ratios, and thus to have every NU-FBMC sub-channel belonging to either the even or the odd extended uniform FBMC 'category', certain sub-channels have to be defined directly, or by some combination of segments of the even and the odd (referent) uniform FBMC arrangements. Fig. 5 illustrates such a situation. Here, the sub-channels 2 and 3 contain 4.5 uniform sub-channels' lengths. Consequently, the roll-off factors are 11.11%, 28.57 (sub-channels 2 and 3). (Although the signaling intervals of sub-channel 4 are not commensurate with the signaling intervals of the adjacent sub-channels, the orthogonality conditions are satisfied with the same QAM parts sequencing as in Fig. 4, indicating that the non-integer signaling speeds are possible, although it might be quite impractical for the symbol synchronization.)

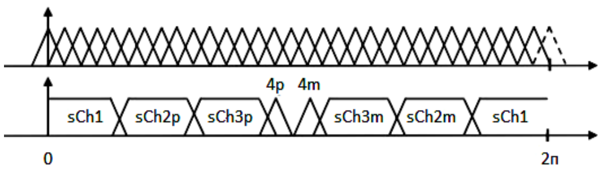

Fig. 5. Non-uniform FBMC arrangement derived from even form FBMC.

A. Analytical derivation of orthogonality conditions

Conditions of intrinsic FLO system orthogonality expressed in time domain provided by [1, i.e. ref. 4 therein] is given by

$$\mathrm{Re}\left\{\int_{-\infty}^{\infty} \hat{g}_{n,k}(t)\hat{g}^{*}_{m,l}(t)dt\right\}=\begin{cases}1 & k=l, n=m\\ 0 & other\end{cases} \quad (1)$$

where the first and second indexes are for sub-channel central frequency and $\frac{T}{2}$ instant, respectively, and $\hat{g}_{n,k}(t)$ is given by

$$\hat{g}_{n,k}(t)=f\left(t-\frac{n\tilde{T}}{2}\right)\exp\left(j\frac{2\pi k}{\tilde{T}}t+j\varphi_{n,k}\right) \quad (2)$$

where

$$\varphi_{n,k}=\begin{cases}\frac{\pi}{2} & n+k\ odd\\ 0 & n+k\ even\end{cases} \quad (3)$$

$\tilde{T}$ is QAM symbol period, and f(t) is impulse response of the square-root Nyquist filter with transfer function F(ω). In [1], for derivation of the uniform FBMC-TLO system it was used the fact that $\{\hat{G}_{n,k}(\omega)\}$, the Fourier transform of $\{\hat{g}_{n,k}(t)\}$ can be used to express the FLO orthogonality conditions in frequency-domain as

$$\mathrm{Re}\left\{\int_{-\infty}^{\infty} \hat{G}_{n,k}(\omega)\hat{G}^{*}_{m,l}(\omega)d\omega\right\}=\begin{cases}1 & k=l, n=m\\ 0 & other\end{cases} \quad (4)$$

where

$$\hat{G}_{n,k}(\omega)=F\left(\omega-\frac{2\pi k}{\tilde{T}}\right)\exp\left(-j\frac{n\tilde{T}}{2}\omega+j\varphi_{n,k}\right) \quad (5)$$

By the straightforward designing of the non-uniformly spaced sub-channels by aggregation of the uniform FBMC sub-channels transfer functions, as shown again in Fig. 6. as an example, it can be shown that the orthogonality conditions of the former can be reduced to conditions of the latter one.

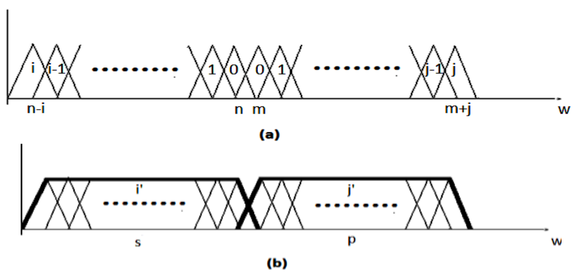

Fig. 6. Spectral aggregation example: (a) uniform configuration (b) corresponding non-uniform configuration

Here, sub-channels i' and j' are described by (6) and (7) respectively

$$\hat{G}_{s,r}(\omega)=\sqrt{\sum_{x=i}^{0}\hat{G}^{2}_{n-x,k-x}(\omega)} \quad (6)$$

$$\hat{G}_{p,q}(\omega)=\sqrt{\sum_{y=0}^{j}\hat{G}^{2}_{m+y,l+y}(\omega)} \quad (7)$$

By placing the newly generated transfer functions into (4), it follows

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$$\mathrm{Re}\left\{\int_{-\infty}^{\infty}\hat{G}_{s,r}(\omega)\hat{G}_{p,q}^{*}(\omega)d\omega\right\}=$$

$$=\mathrm{Re}\left\{\int_{-\infty}^{\infty}\sqrt{\sum_{x=i}^{0}\hat{G}_{n-x,k-x}^{2}(\omega)}\left(\sqrt{\sum_{y=0}^{j}\hat{G}_{m+y,l+y}^{2}(\omega)}\right)^{*}d\omega\right\}$$

Since the complex conjugation (*) can be brought under the square-root, as well as under the squaring operation, and since (c1 + c2)* = c1*+ c2*, the following expression is produced

$$\mathrm{Re}\left\{\int_{-\infty}^{\infty}\sqrt{\hat{G}_{n-i,k-i}^{2}(\omega)+\hat{G}_{n-(i-1),k-(i-1)}^{2}(\omega)+\cdots+\hat{G}_{n-1,k-1}^{2}(\omega)+\hat{G}_{n,k}^{2}(\omega)}\right.$$
$$\left.\sqrt{\hat{G}_{m,l}^{*2}(\omega)+\hat{G}_{m+1,l+1}^{*2}(\omega)+\cdots+\hat{G}_{m+(j-1),l+(j+1)}^{*2}(\omega)+\hat{G}_{m+j,l+j}^{*2}(\omega)d\omega}\right\}$$

Due to spectral confinement to 1/T widths, from all of the products under the root-square operation only $\hat{G}_{n,k}^{2}(\omega)\hat{G}_{m,l}^{*2}(\omega)$ is different from zero, so that the orthogonality conditions are reduced to those of the uniform FLO FBMC format, i.e.

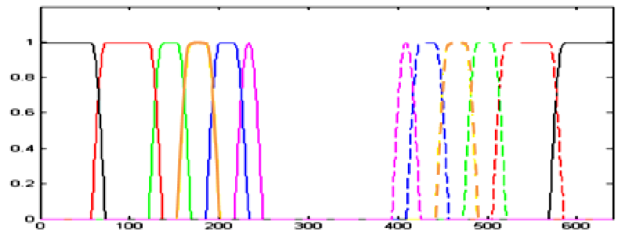
$$\mathrm{Re}\left\{\int_{-\infty}^{\infty}\hat{G}_{s,r}(\omega)\hat{G}_{p,q}^{*}(\omega)d\omega\right\}=\mathrm{Re}\left\{\int_{-\infty}^{\infty}\hat{G}_{n,k}(\omega)\hat{G}_{m,l}^{*}(\omega)d\omega\right\}=$$

$$=\begin{cases}1 & k=l, n=m\\ 0 & other\end{cases} \qquad (8)$$

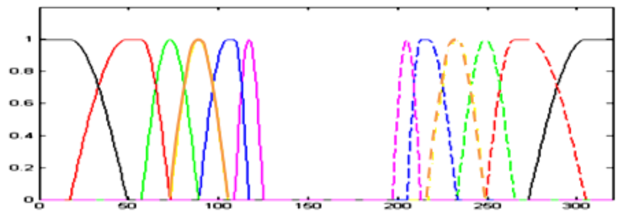
($\widetilde{T}$ is used above for interval T for the reason of generality.)

IV. Using asymmetrical sub-channels for NU-FBMC (NU-FLO)

The relatively low roll-off factors resulting from the uni-form FBMC aggregation might be unfavorable for at least two reasons: increased transmission delay and increased PAPR (Peak-to-Average Power Ratio). To remedy these problems it would be of interest to keep the NU-FBMC sub-channels' roll-off factors as large as possible, while retaining high flexibility in assigning the sub-channel bandwidths. This turns out to be possible by directly defining the sub-channels frequency-domain samples (shapes) for example, as illustrated in Fig. 7b.



(a)

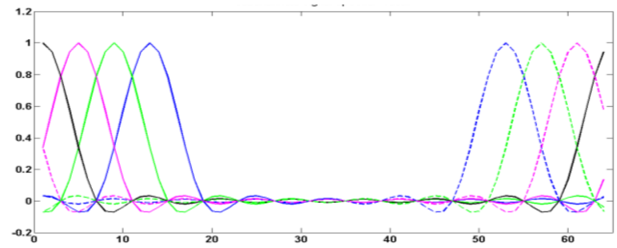


(b)

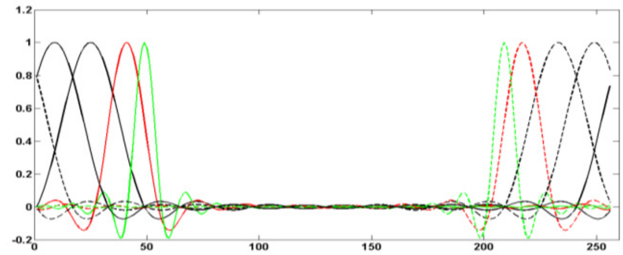
Fig. 7. NU-FBMC arrangement with symmetrical spectra (a), and asymmetrical spectra (b).

V. Aggregating referent IRs for the NU-TLO

Similarly as the frequency domain representations of the sub-channels spectra were produced by aggregation of the uniform frequency-domain description, for the non-uniform TLO formats, the aggregation is performed at corresponding (time-domain) referent impulse responses. Their frequency-domain representations are subsequently produced, and appropriately positioned as in case of the NU-FBMC counterpart. Spectrum of the uniform TLO FBMC format for the even filter banks arrangements is shown in Fig. 8a, and spectrum of the non-uniform TLO FBMC format from the odd filter banks arrangements is shown in Fig. 8b.



(a)



(b)

Fig. 8. TLO FBMC arrangement: (a), even uniform, M = 16, K =4; (b), odd non-uniform, M = 64, K = 4.

As was the case with non-uniform FLO formats, for non-uniform TLO formats the orthogonality conditions can also be shown to reduce to those of the uniformly spaced case. Since in [1] the orthogonality conditions of the uniform TLO FBMC format is derived by applying the time-frequency duality, formally be replacing symbols $\hat{G},\omega,F,\widetilde{T}$ from (5) by g, t, w and 2π/T' respectively, (5) gets converted to

$$g_{n,k}(t)=w(t-kT')\exp(j\frac{n\pi}{T'}t+j\varphi_{n,k}) \qquad (9)$$

As a dual form, $\{g_{n,k}(t)\}$ also satisfies the orthogonality condition given by (10) and thus forms the uniform TLO orthogonal base.

$$\mathrm{Re}\left\{\int_{-\infty}^{\infty}g_{n,k}(t)g_{m,l}^{*}(t)dt\right\}=\begin{cases}1 & k=l, n=m\\ 0 & other\end{cases} \qquad (10)$$

Based on the time-frequency duality between FLO and TLO, expression (8) for non-uniform FLO applies as well to the non-uniform TLO FBMC case, with appropriately exchanged variables. Derivation is given below only for the simplest case, where one sub-channel has the doubly

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longer signaling interval compared to its adjacent one, as it is illustrated in Fig. 9.

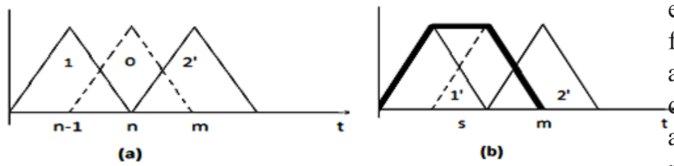

Fig. 9. Uniform configuration (a), and its corresponding non-uniform configuration for two sub-channels (b).

If the referent impulse response of sub-channels marked by 1, 0 and 2′ are $g_{n-1,k-1}(t)$, $g_{n,k}(t)$ and $g_{m,l}(t)$, and the impulse response of the doubly wide sub-channel 1′ is $g_{s,r}(t)$, the process of time-domain aggregation is expressed through

$$g_{s,r}(t) = \sqrt{g^2_{n-1,k-1}(t) + g^2_{n,k}(t)} \qquad (11)$$

Going from (1) and by placing the newly generated impulse responses (11) into the above expression, it follows

$$\mathrm{Re}\left\{\int_{-\infty}^{\infty} g_{s,r}(t) g^*_{m,l}(t)dt\right\} = \mathrm{Re}\left\{\int_{-\infty}^{\infty} \sqrt{g^2_{n-1,k-1}(t)+g^2_{n,k}(t)}\, g^*_{m,l}(t)dt\right\} =$$
$$= \mathrm{Re}\left\{\int_{-\infty}^{\infty} \sqrt{g^2_{n-1,k-1}(t) g^{*2}_{m,l}(t) + g^2_{n,k}(t) g^{*2}_{m,l}(t)}\, dt\right\}$$

Because of absence of overlapping among the uniform TLO sub-channels' impulse responses separated by (at least) one shortest impulse response, the product $g^2_{n-1,k-1}(t) g^{*2}_{m,l}(t)$ is equal to 0, so that the orthogonality criterion reduces to the one corresponding to the uniform FBMC case , i.e.

$$\mathrm{Re}\left\{\int_{-\infty}^{\infty} g_{s,r}(t) g^*_{m,l}(t)dt\right\} = \mathrm{Re}\left\{\int_{-\infty}^{\infty} g_{n,k}(t) g^*_{m,l}(t)dt\right\} =$$
$$= \begin{cases} 1 & k = l, n = m \\ 0 & other \end{cases} \qquad (12)$$

since the same expression is produced as in equation (10), from which we started.

VI. Comparison with Conventional Approaches

The field of the non-uniform filter-banks analysis for the source coding applications has been very well developed, and two basic configurations have emerged – modifications of the uniform filter banks implementation by combination of (I)FFT and Polyphase Networks (PN) by appropriately optimizing the referent low-pass filter impulse response [7][8], and the application of quadrature-mirror filters based branching, or tree architectures [9]. While in the first case the roll-off factors remain relatively high, the tree-branching method keeps the same roll-off factor for all sub-channels, with maximal value inversely proportional to number of non-uniform sub-channels. While these two structures may have similar complexity of implementation, the one considered here appears to be able to provide higher flexibility.

VII. Conclusions

We presented rather pragmatic approach towards extending the uniformly spaced FBMC formats in the forms of FLO and TLO to non-uniform ones. The analysis and elaboration of non-uniform filter-bank, with derivation of their orthogonality conditions, may provide an useful introduction for further elaborations and its practical use as an element of a flexible channelization, and in particular for the realization of the individual user's communication channel. In down-link, the asymmetrical spectral shaping can avoid need for separation of adjacent users' channels realized by uniform FBMC with differing sub-channel widths. For UL directions, by deploying the NU-FBMC individual user's channels can be separated by a negligible loss in spectral efficiency, to accommodate constraints of exploitation, as the PSD mask, unsynchronized symbol timings, and the carrier frequency offset uncertainties.

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