Complexity

Johan Montelius

KTH

VT23

run-time complexity of sum

Calculating the sum of all elements in a list:

sum/1

```
def sum([]) do 0 end
def sum([h|t]) do
   s = sum(t)
   h + s
end
```

sum/2

```
def sum([], s) do s end
def sum([h|t], s) do
   s1 = h+s
   sum(t, s1)
end
```

What are the run-time complexities of sum/1 and sum/2?

run-time complexity of foo

foo/1

```
def foo([]) do [] end
def foo([h|t]) do
   z = foo(t)
   bar(z, h)
end
```

foo/2

```
def foo([], y) do y  end
def foo([h|t], y) do
   z = zot(h, y)
   foo(t, z)
end
```

What are the run-time complexities of foo/1 and foo/2?

run-time complexity of reverse

nreverse/1

```
def nreverse([]) do [] end
def nreverse([h|t]) do
   z = nreverse(t)
   append(z, [h])
end
```

reverse/2

```
def reverse([], y) do y end
def reverse([h|t], y) do
   z = [h | y]
   reverse(t, z)
end
```

What are the run-time complexities of nreverse/1 and reverse/2?

the recurence relation

nreverse/1

```
def nreverse([]) do [] end
def nreverse([h|t]) do
   z = nreverse(t)
   append(z, [h])
end
```

Assume that append/2 takes kn ms to execute, where k is some constant time and n is the length of the list.

Describe the time T_n it takes to execute nreverse/1 of a list of length n:

$T_0 = a$ ms
$T_n = T_{n-1} + k(n-1) + b$ ms

the recurence relation

$$
\begin{aligned}
T_n &= T_{n-1} + k(n-1) + b \\
&= T_{n-2} + k(n-2) + k(n-1) + 2b \\
&= T_{n-3} + k(n-3) + k(n-2) + k(n-1) + 3b \\
&\vdots \\
&= T_{n-n} + k(n-n) + ...k(n-1) + nb \\
&= a + 0 + k + 2k + 3k......(n-1)k + nb \\
&= n\frac{(n-1)}{2}k + nb + a \\
&= (\frac{k}{2})n^2 - \frac{k}{2}n + bn + a \\
&= (\frac{k}{2})n^2 + (b - \frac{k}{2})n + a
\end{aligned} \tag{1}
$$

the big-O notation

We know:

$$T_n = (\frac{k}{2})n^2 + (b - \frac{k}{2})n + a$$

$$T_n \in O(n^2)$$

Ordo calculations

Do ordo calculations in your head without specifying the full T_n relation.

If we know that append/2 is in $O(n)$ then:

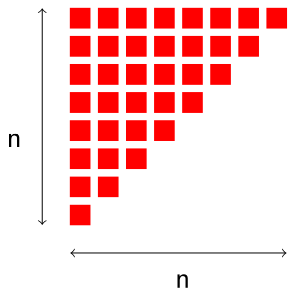
$$T_n \in n * O(n) + bn + a$$

Which means that:

$$T_n \in O(n^2)$$

run-time complexity of reverse/1

`nreverse/1`

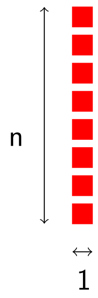


nreverse/1

```
def nreverse([]) do  end
def nreverse([h|t]) do
   z = nreverse(t)
   append(z, [h])
end
```


run-time complexity of reverse/2

`reverse/2`



reverse/2

```
def reverse([], y) do y end
def reverse([h|t], y) do
   z = [h | y]
   reverse(t, z)
end
```


complexity of quick-sort

```
def qsort([]) do [] end
def qsort([h]) do [h] end
def qsort(all) do
   {low, high} = partition(all)
   lowS = qsort(low)
   highS = qsort(high)
   append(lowS, highS)
end
```

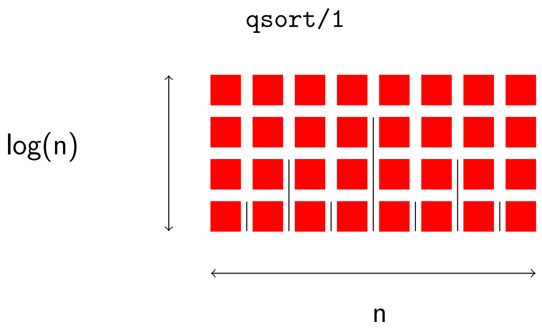
- What is done in each iteration?
- How many iterations do we have?

the recurence relation

$$T_1 = a$$

$$\begin{aligned} T_n &= 2 \times T_{n/2} + nc \\ &= 2 \times (2 \times T_{n/4} + (n/2)c) + nc \\ &= 4 \times T_{n/4} + 2 \times nc \\ &= 8 \times T_{n/8} + 3 \times nc \\ &\vdots \\ &= 2^k \times T_1 + k \times nc \\ &= 2^{lg(n)} \times a + lg(n) \times nc \\ &= n \times a + lg(n)n \times c \end{aligned} \tag{2}$$

complexity of quick-sort



qsort worst case

What if we run qsort on a already ordered list?

complexity of merge-sort

```
def msort([]) do [] end
def msort(l) do
  {a, b} = split(l)
  as = msort(a)
  bs = msort(b)
  merge(as, bs)
end
```

- What is done in each iteration?
- How many iterations do we have?
- What is the run-time complexity?
- Which is best qsort or msort?

complexity of fibonacci

```
def fib(0) do 0 end
def fib(1) do 1 end
def fib(n) do
  fib(n-1) + fib(n-2)
end
```

- What is done in each iteration?
- How many iterations do we have?

the recurence relation

Let's cheat a bit to make it simpler:

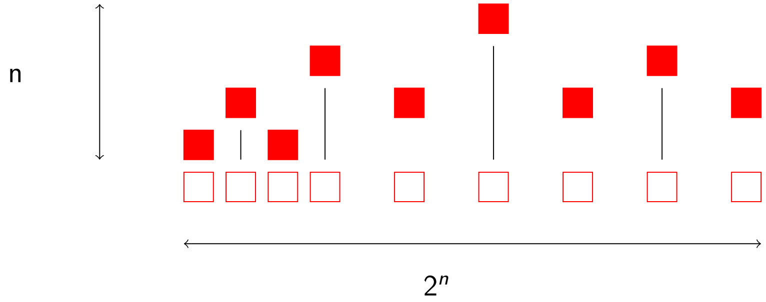
$$T_0 = a$$

$$\begin{aligned} T_n &= 2 \times T_{n-1} + c \\ &= 2 \times (2 \times T_{n-2} + c) + c \\ &= 4 \times T_{n-2} + 3 \times c \\ &= 8 \times T_{n-3} + 7 \times c \\ &\vdots \\ &= 2^n \times T_0 + (2^n - 1) \times c \\ &= 2^n \times a + 2^n \times c - c \end{aligned} \tag{3}$$

The more precise answer is $O(1.6^n)$

complexity of fibonacci

`fibonacci/1`



The smarter implementation is $O(n)$

... an even smart solution is $O(log(n))$

The big question

What is the difference between a smart programmer and a not so smart programmer?

3 billion years?

operations on trees

Let's represent trees as:

```
:nil
{:node, key, value, left, right}
```

- new: create a empty tree
- insert: add an element to the three
- lookup: search for an element
- modify: modify an element

why trees?

Why use trees, why not use lists?

benchmark tree operations

Operations on a tree.

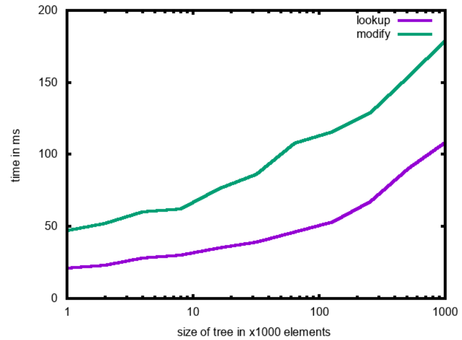


Figure: Execution time in ms of 100.000 calls

why trees?

Why use trees, why not use tuples?

tuples as a key value store

```
def new([a,b,c]) do {a,b,c} end

def lookup({a,_,_}, 1) do a end
def lookup({_, b,_}, 2) do b end
  :

def modify({_,b,c}, 1, v) do {v, b, c} end
def modify({a,_,c}, 2, v) do {a, v, c} end
  :
```

tuples using builtin functions

```
def new(list) do List.to_tuple(list) end

def lookup(tuple, k) do elem(tuple, k) end
def modify(tuple, k, v) do put_elem(tuple, k, v) end
```

The functions put_elem/3 will create a copy of the original tuple!

benchmark tuple operations

Operations on a tuple.

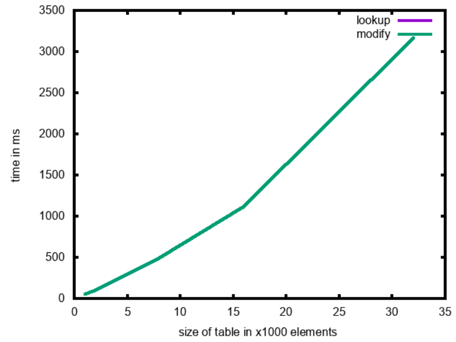


Figure: Execution time in ms of 100.000 calls

compare tuples and trees

Tuple vs tree.

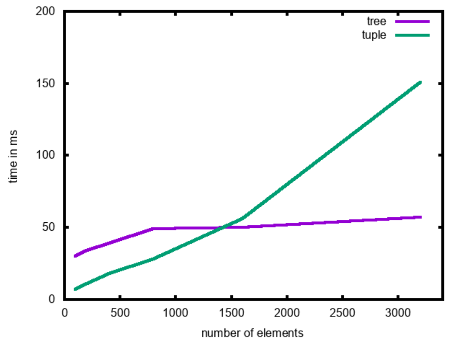


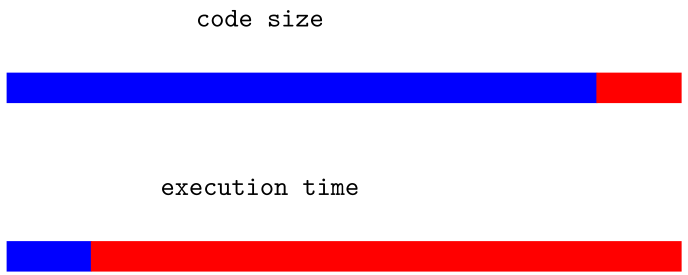
Figure: Modify operations, execution time in ms of 100.000 calls

root of all evil

> *Programmers waste enormous amounts of time thinking about, or worrying about, the speed of noncritical parts of their programs, and these attempts at efficiency actually have a strong negative impact when debugging and maintenance are considered. We should forget about small efficiencies, say about 97 percent of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3 percent.*

Donald Knuth

code vs time



programming rules

- understand the problem before starting coding
- write well structured code that is easy to understand
- use abstractions to separate functionality from implementation
- think about complexity
- benchmark your program
- if needed, optimize