Hypertracking beyond the Nyquist frequency

Dedicated to Anders Lindquist
on the occasion of his 75th birthday

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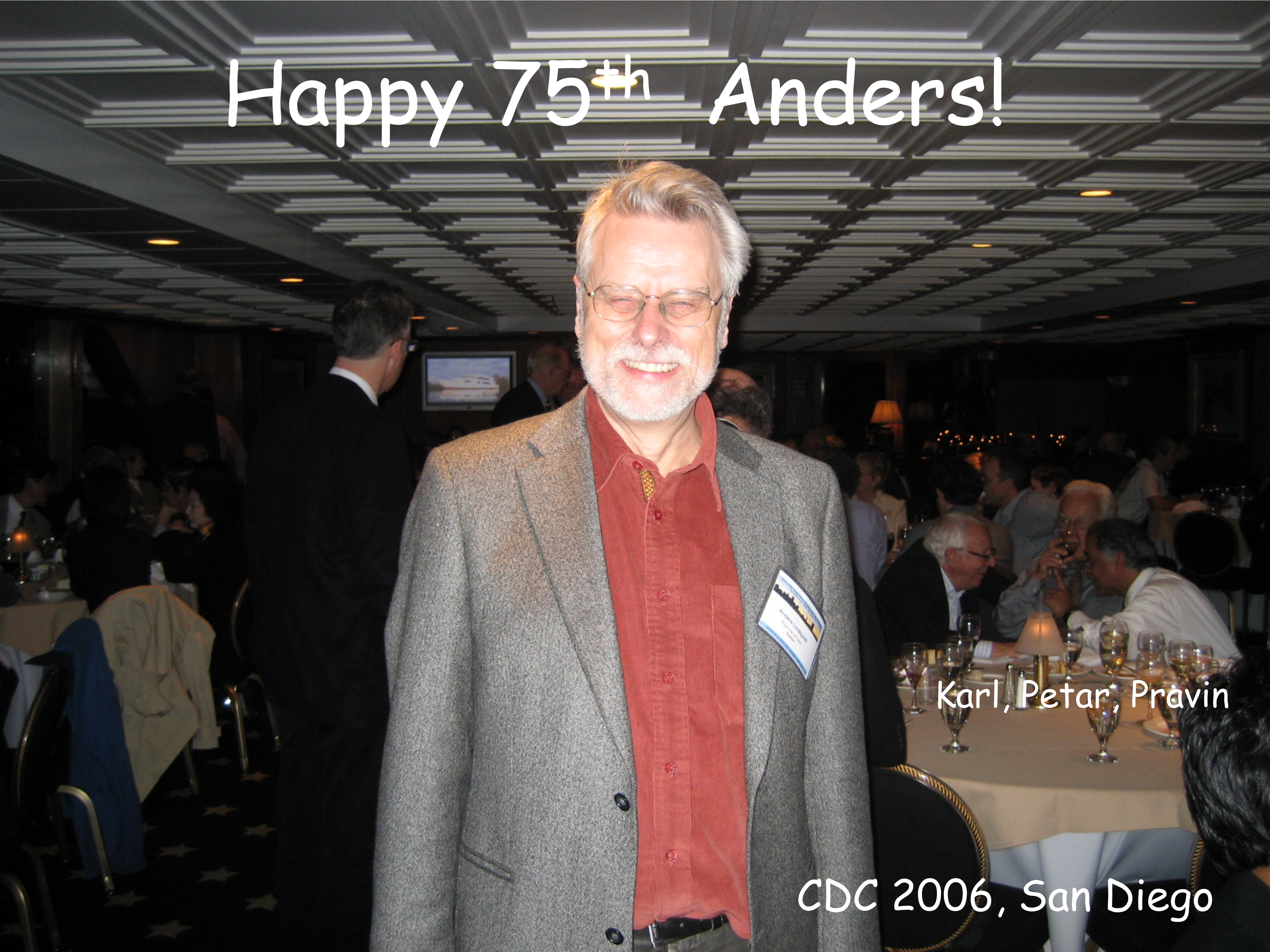
Kyoto University, Emeritus
CentraleSupelec, France


Happy 75th Anders!
Karl, Petar, Pravin
CDC 2006, San Diego

Our encounters

- 1973 in Gainesville, Florida??
- MTNS 85 – again miss (MTNS beer)
- MTNS 89 at Amsterdam
- At many conferences, particularly at MTNS
- Mittag-Leffler Institute “Systems Year” 2003
- Two week visit to Kyoto (2004?)
- YYFest 2010
- My retirement party 2015
- And many other occasions

MTNS 2004, Leuven

More pictures later…

MTNS 2006, Kyoto

Joint work with

Kaoru Yamamoto,
Lund University

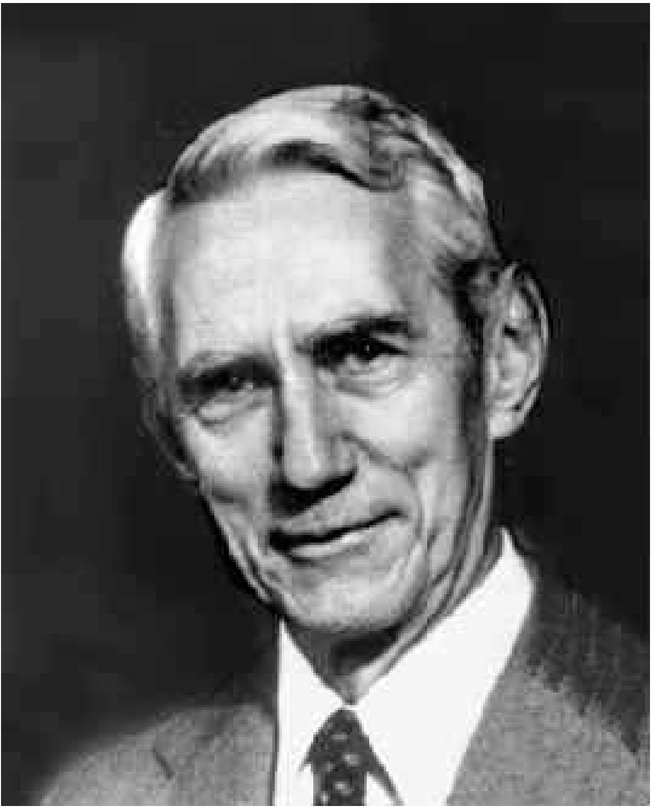
Masaaki Nagahara,
U. Kitakyushu

Topic of this talk

- Processing signals beyond the Nyquist frequency
- ⇒ *Hyper-*tracking

WHAT'S SO SPECIAL ABOUT IT?

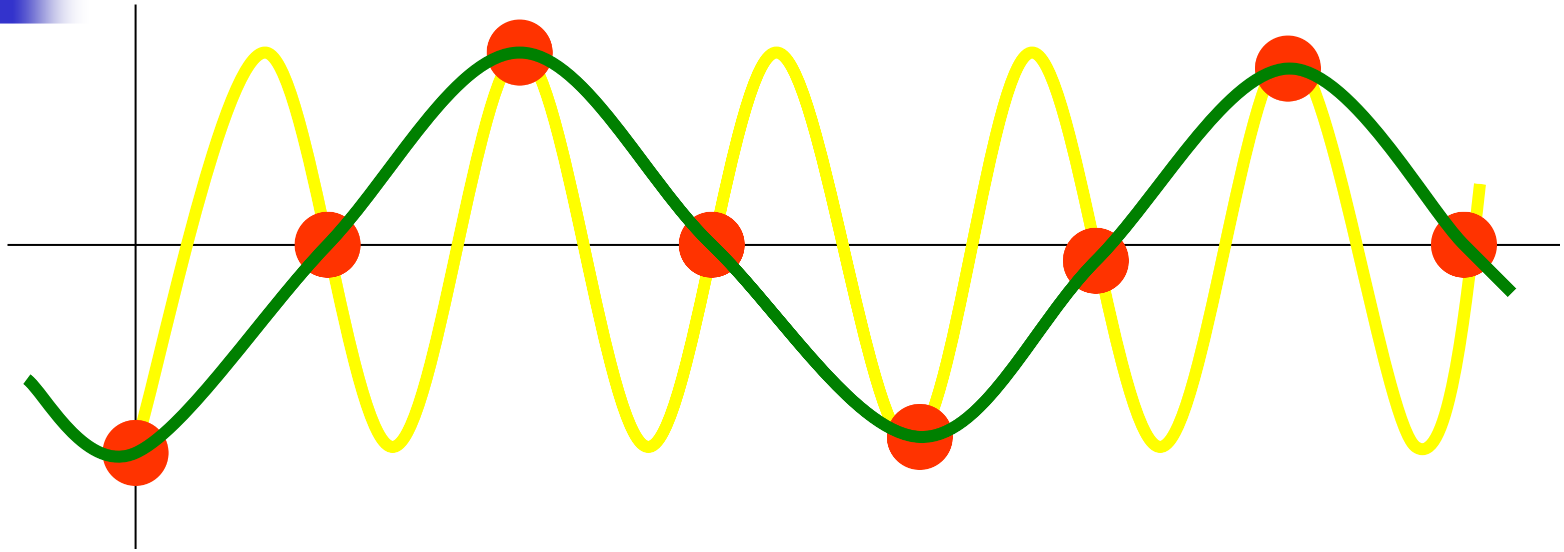
History: Claude Shannon classic



- Communication in the Presence of Noise, C. E. Shannon, Proc. IRE, vol. 37, 1949, pp. 10–21.
- *How fast should we sample in transmitting data through a channel?*
- *⇒ Unique reconstruction below the Nyquist frequency*

Claude Elwood Shannon (1916-2001)

Typical difficulty
Sampling → Aliasing



- High-freq. intersample information can be lost
- If no high-freq. components beyond the Nyquist frequency (=1/2 of sampling freq.) → unique restoration

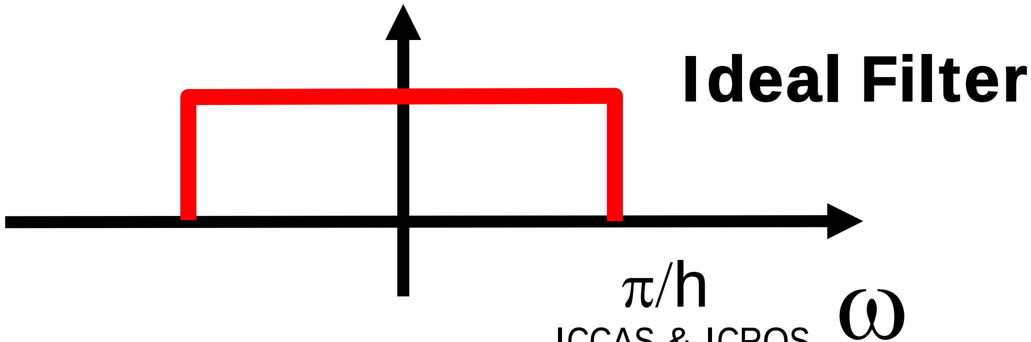
→ Whittaker-Shannon-Someya sampling theorem

Sampling Theorem

- Band limiting hypothesis $\Rightarrow$ unique recovery

$$\hat{f}(j\omega) = 0 \text{ for } |\omega| > \pi/h \Rightarrow$$

$$f(t) = \sum_{n=-\infty}^{\infty} f(nh) \frac{\sin \pi(t/h - n)}{\pi(t/h - n)}$$



To recap,

- Perfectly bandlimited $\Rightarrow$ perfect reconstruction
- This is *below the Nyquist frequency*

Common belief induced

- One should limit signals below the Nyquist frequency
- We can do nothing about the signals beyond the Nyquist frequency
- Hence we should limit our bandwidth below the Nyquist frequency by a low-pass filter

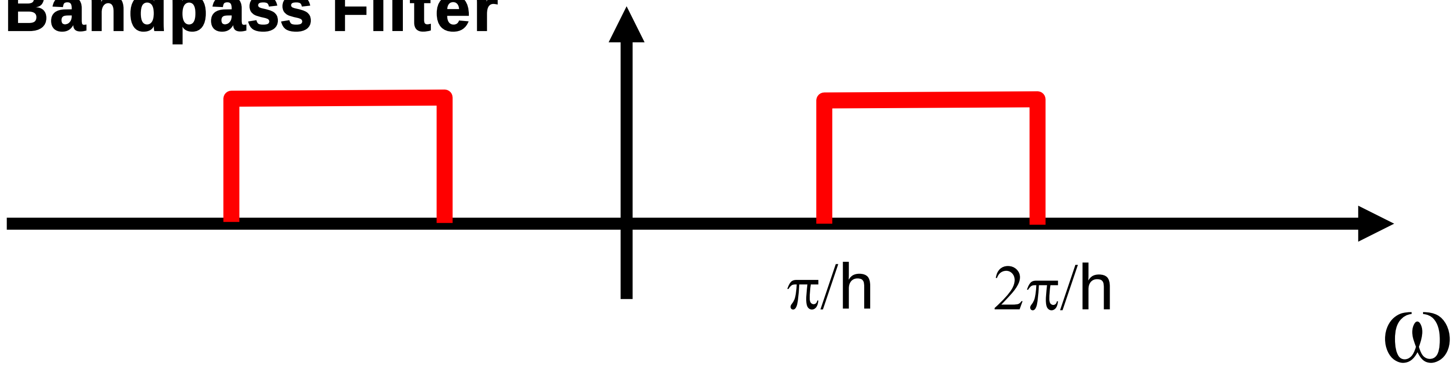
Is this so?

An Alternative Theorem

$\hat{f}(j\omega) = 0$ for $0 \leq |\omega| \leq \pi/h$
and $|\omega| \geq 2\pi/h \Rightarrow$

$$f(t) = \sum_{n=-\infty}^{\infty} f(nh)(2\text{sinc}(2(t-nh)) - \text{sinc}(t-nh)).$$

Bandpass Filter



That is,

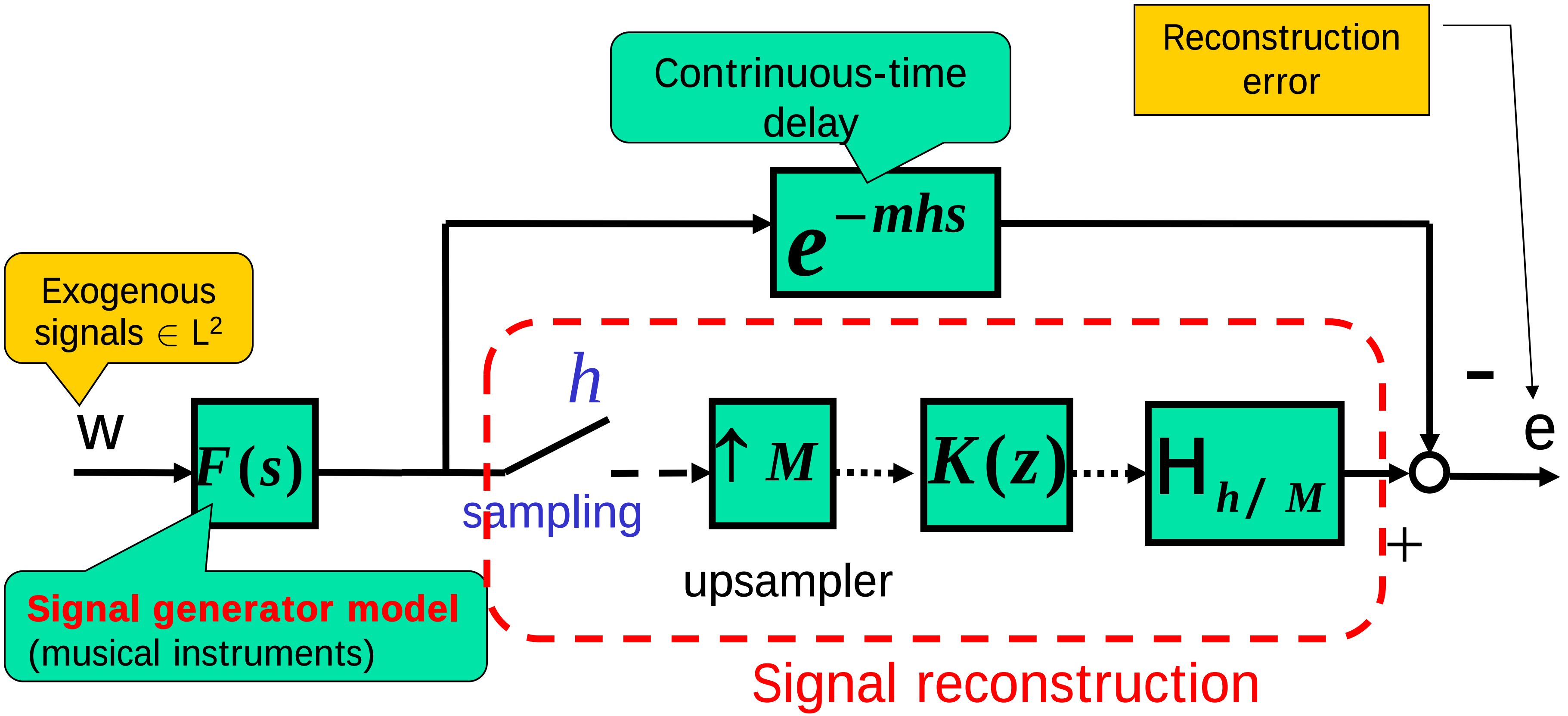
- Low-pass, perfect band-limiting hypothesis is not the only choice
- Everything hinges upon the underlying model

A new approach through sampled-data control

A new recipe (YY, since 1995)

- Introduce a signal generator model $F(s)$
- This need not be fully band-limiting
- The signal class filtered by $F(s)$ is to be reconstructed
- This can be done optimally via sampled-data H^∞-control

Sampled-data Design Model



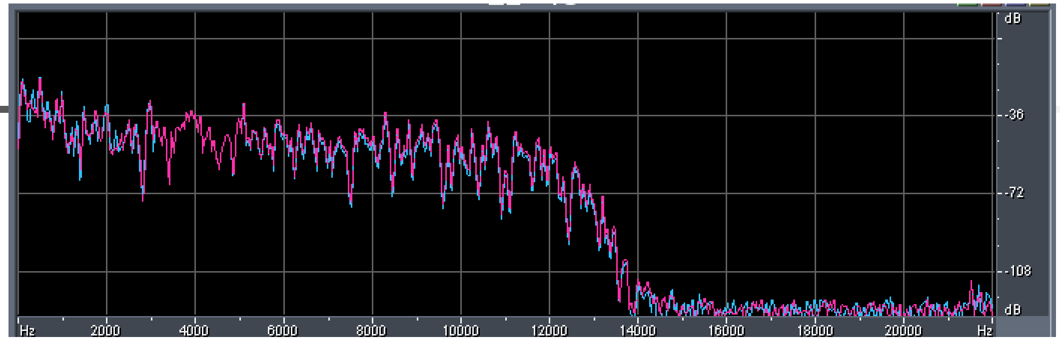
Problem: Find K[z] satisfying

$$\|T_{ew}\| < \gamma$$

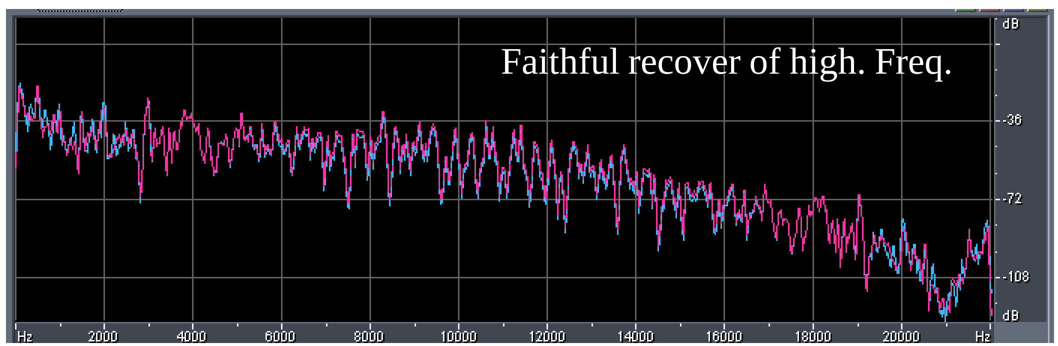
Sampled-data H^∞ control problem

A commercial success in sound processing (since 2006)

MDLP4(66kbps)



After "YY"



More natural high freq. response

By the courtesy of SANYO Corporation

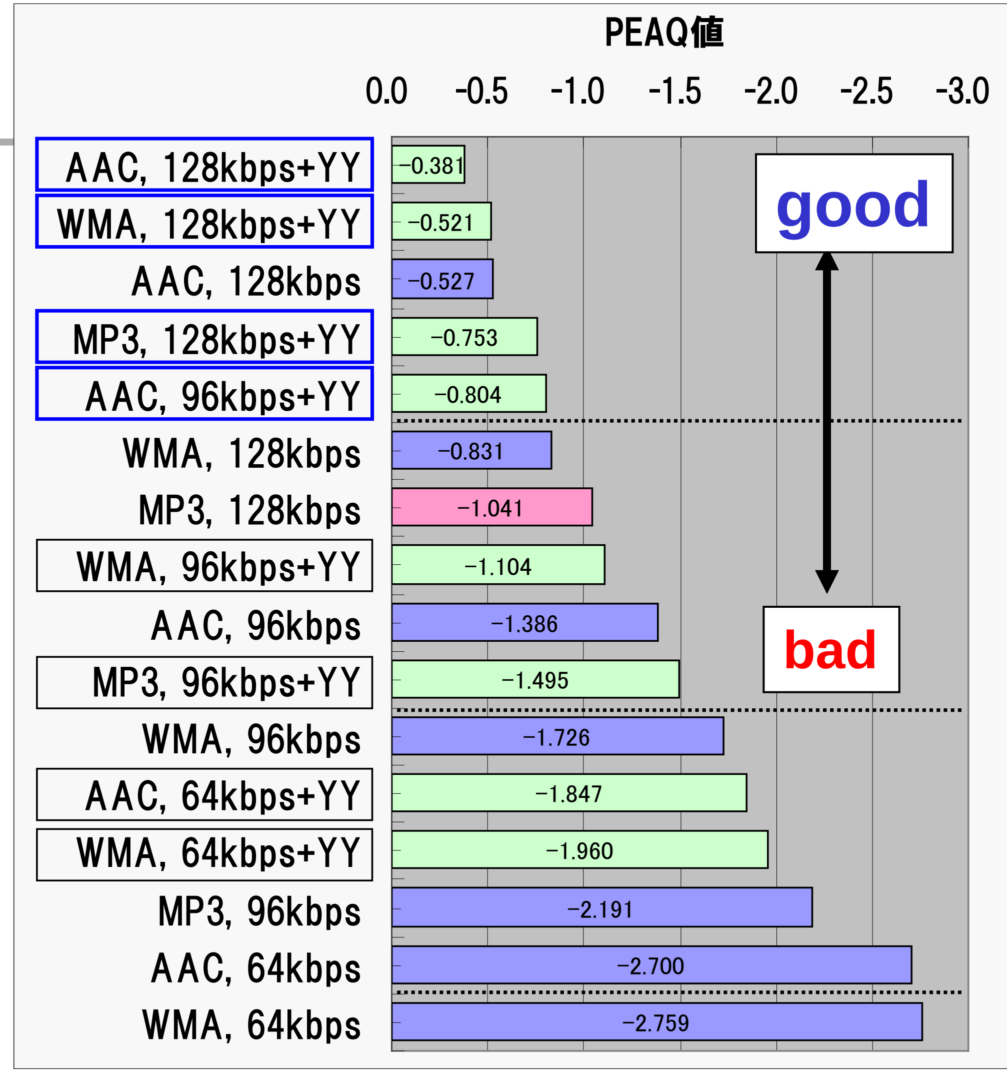
This "YY filter" is implemented in custom LSI sound chips by SANYO Coop., and being used in MP 3 players, mobile phones, voice recorders. The cumulative sales have exceeded 65 million chips.

Effect evaluation on compressed audio via PEAQ program

- Tested on 100 compresed music sources via PEAQ (Perceptual Evaluation of Audio Quality)
- PEAQ values:
  - 0…indistinguishable from CD
  - -1…distinguishable but does not bother the listener
  - -2…not disturbing
  - -3…disturbing
  - -4…very disturbing
- Note how YY improves the sound quality

http://en.wikipedia.org/wiki/PEAQ

By the courtesy of SANYO corporation



Compression formats: MP3, AAC, WMA
Bitrates: 64kbps, 96kbps, 128kbps
Showing average values

Result

Moving image demo
(next page)

Left: Bicubic – no high freq. beyond Nyquist freq
Right: yy (reconstructed high freq.)

Comparison


Left: Bicubic – no high freq. beyond Nyquist freq
Right: yy (reconstructing high freq.)
Proposed

New question

Can we do the same in control?

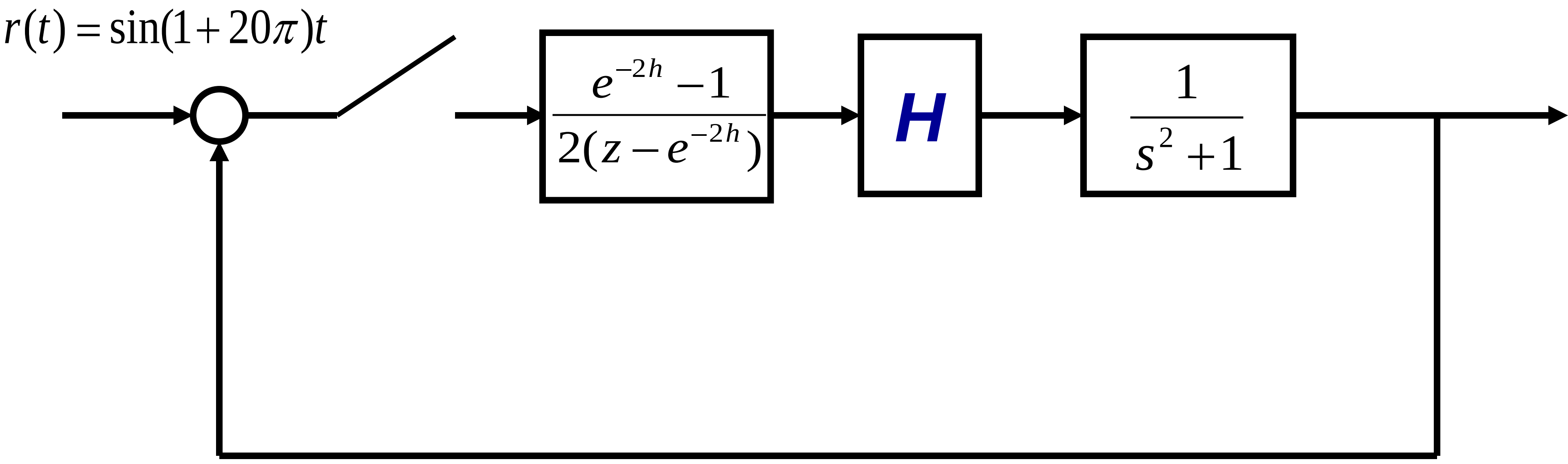
Many practical demands

- Hard disk drives, mechanical systems
- Often high-freq. disturbances (due to winds)
- Sampling frequency is often limited by a physical limitation; not high enough
- Can we reject such high-freq. disturbances? (beyond the Nyquist frequency)

Tracking Problem

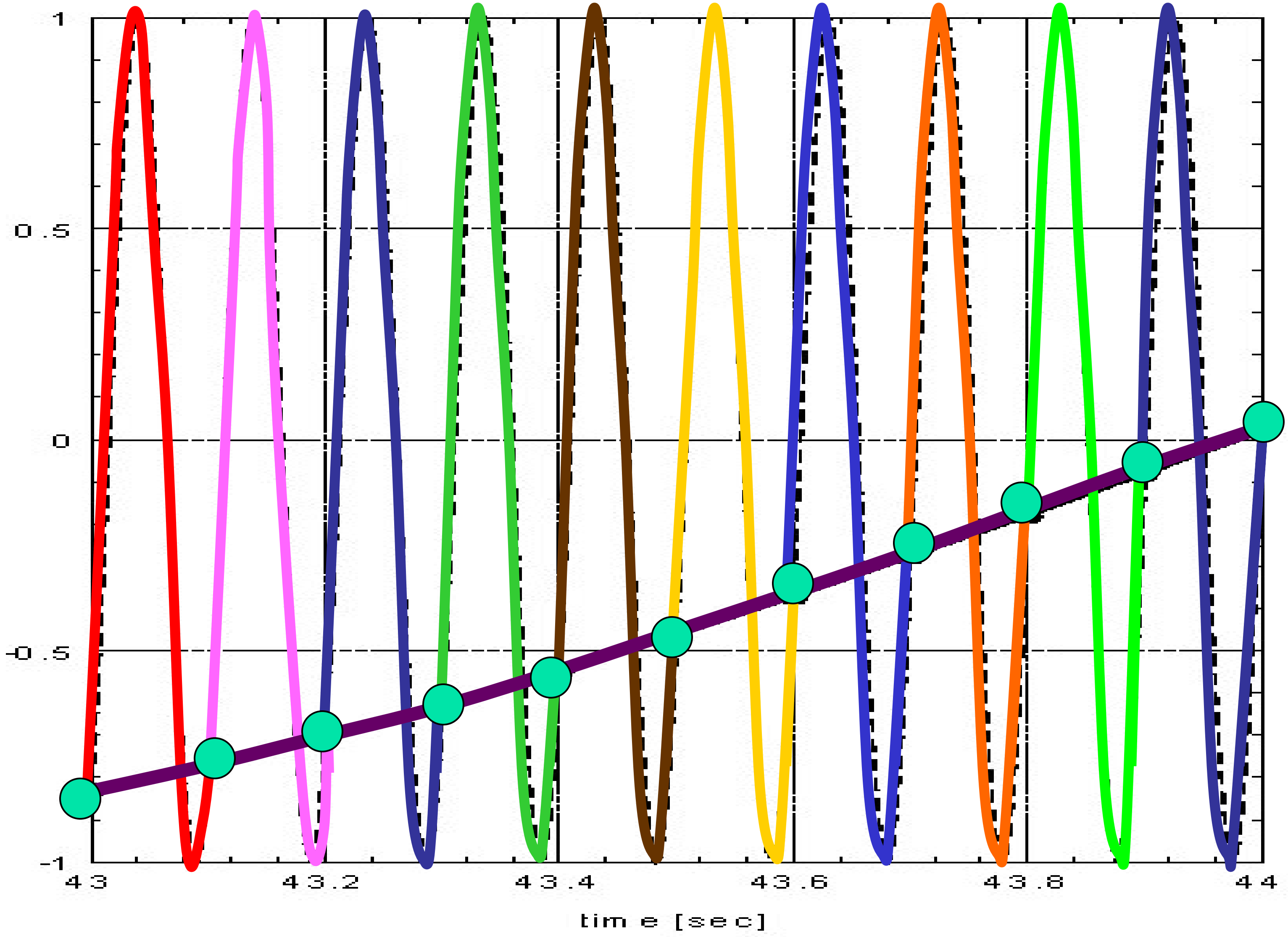
- Can we track a reference with frequency higher than the Nyquist frequency?

A Hint: Counterexample?



Response

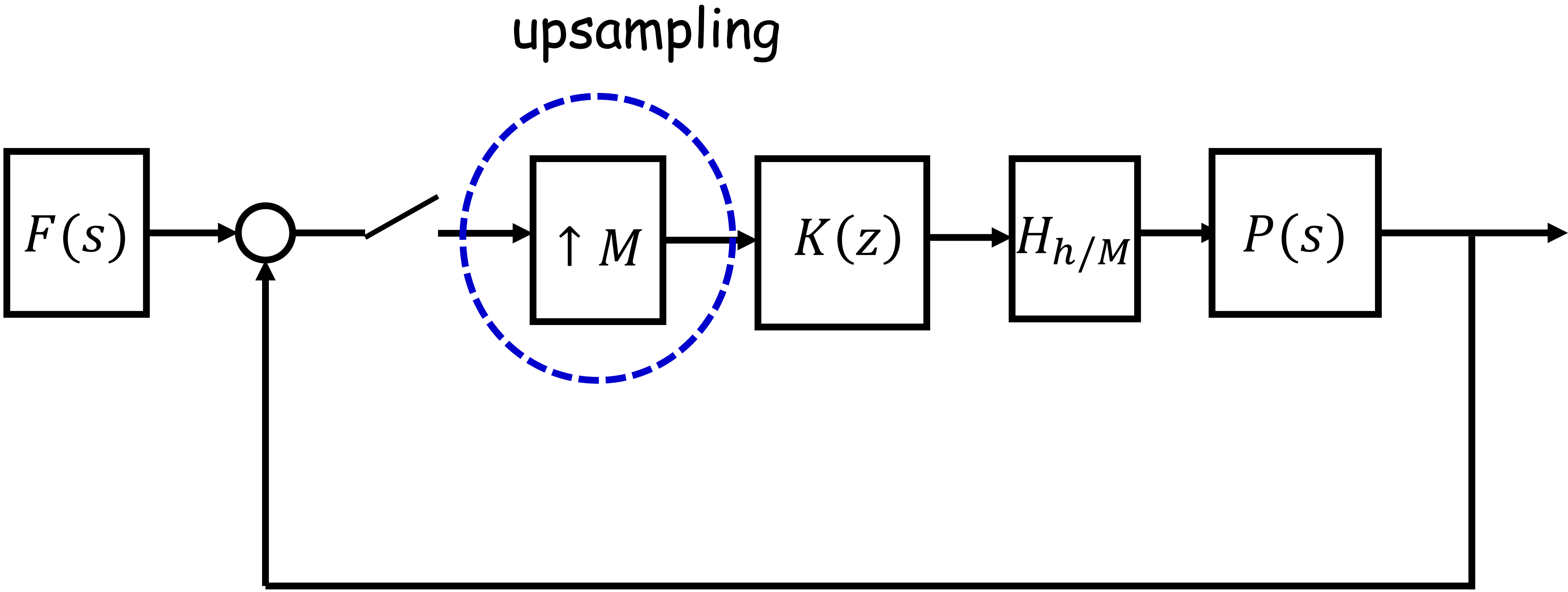
$$v(\theta) = \sin(1 + 20\pi)\theta$$



Recipe

- We need upsampling to take care of the intersampling behavior
- We need a proper weighting in the high frequency range (i.e., right signal model)

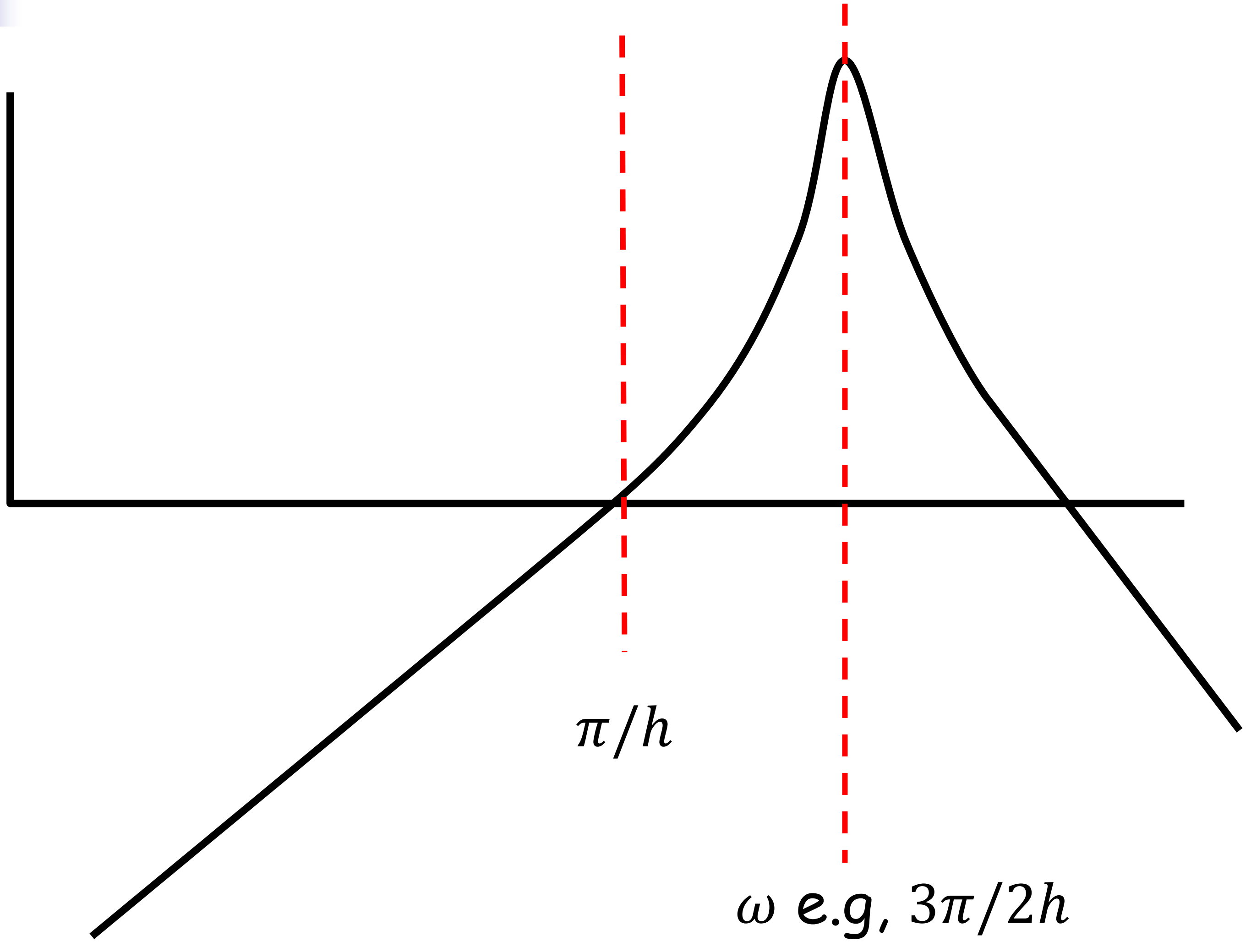
Basic construction

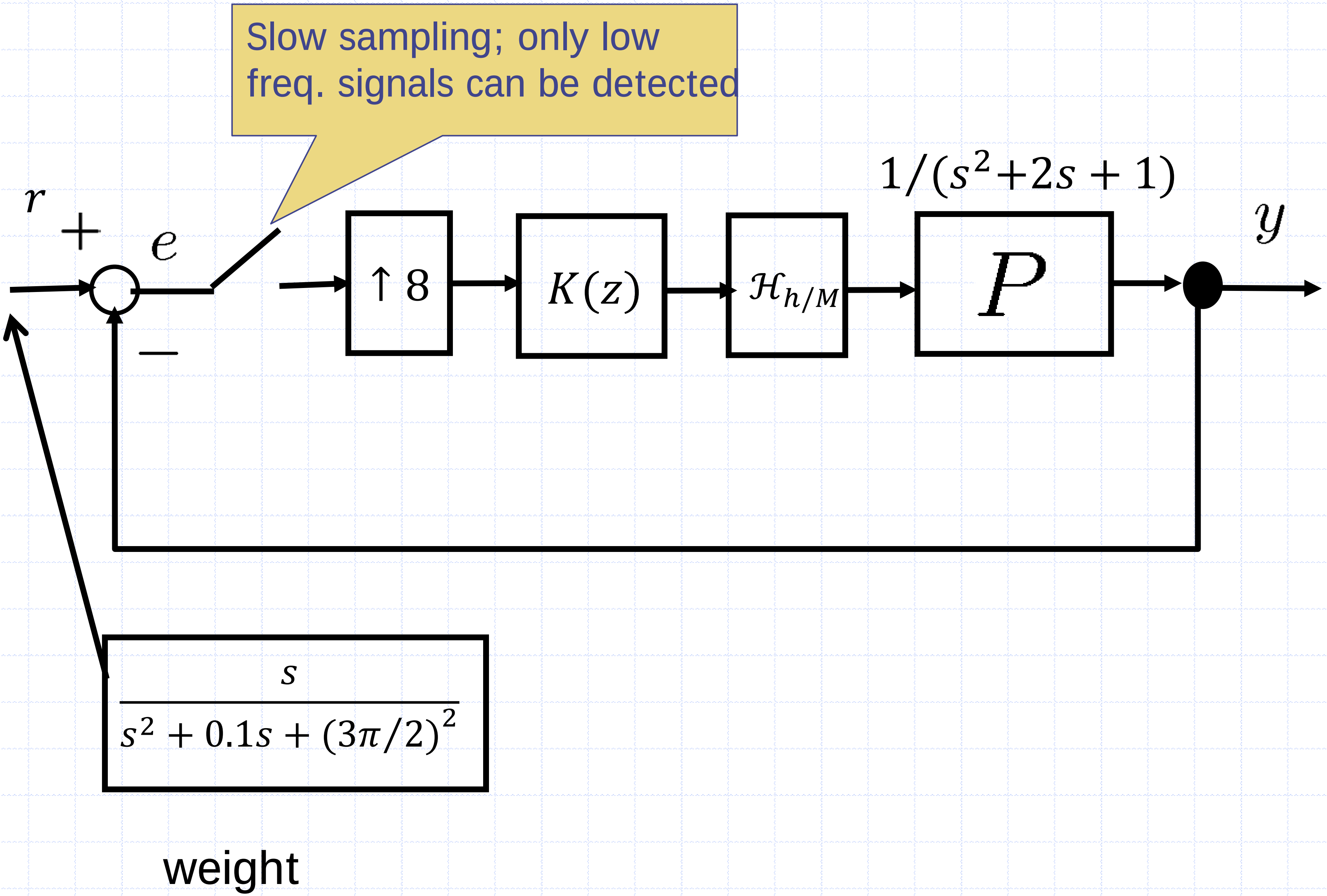


Example

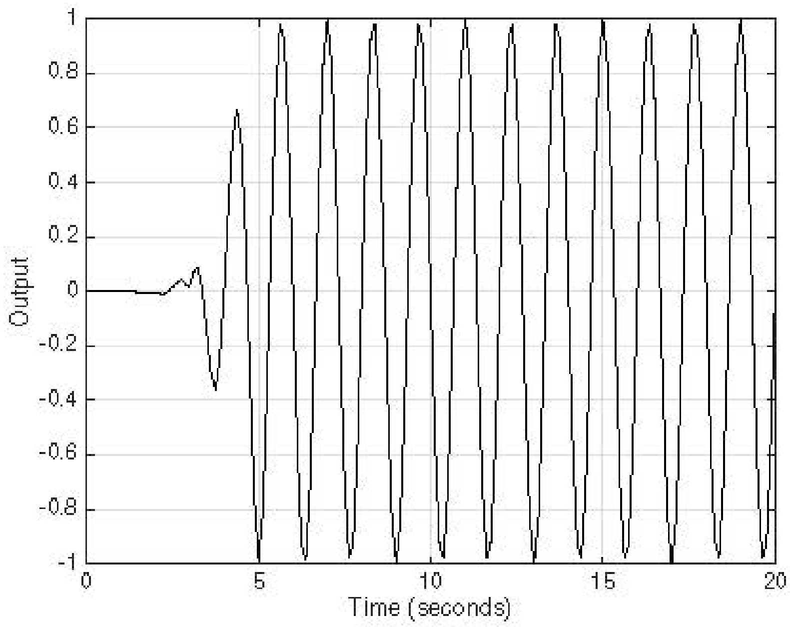
- $P(s) = \frac{1}{s^2+2s+1}$
- $h = 1$, Nyquist freq. $=\pi$
- $r(t) = \sin(3\pi/2)t$
- $F(s) = \frac{s}{s^2+0.1s+(3\pi/2)^2}$, peak at $(3\pi/2)$

signal model $F(s)$ (weighting)

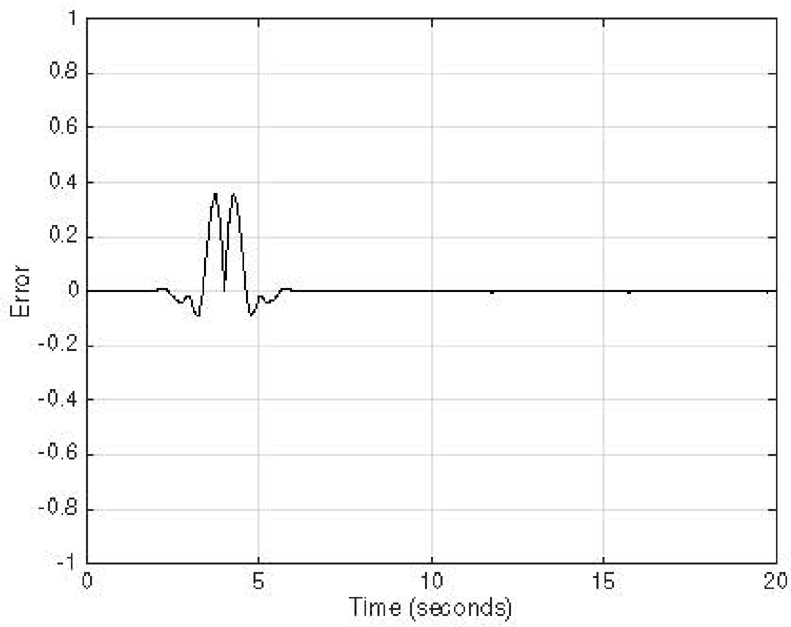



Slow sampling; only low freq. signals can be detected
r
$+$
e
$-$
$\uparrow 8$
$K(z)$
$\mathcal{H}_{h/M}$
$1/(s^2+2s+1)$
P
y
$\frac{s}{s^2+0.1s+(3\pi/2)^2}$
weight

Example - results



$y(t)$

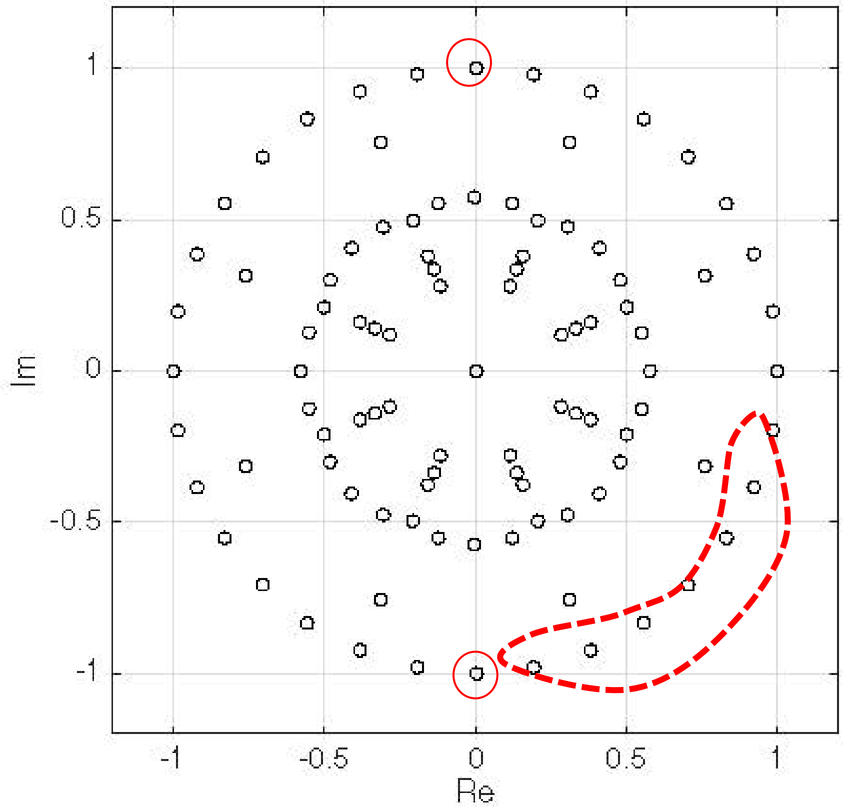


$e(t)$

Earlier results presented at the CDC 2016, Las Vegas

Poles in the controller

This gives rise to an approx. internal model along with the upsampler+fast hold



Poles arising From ↑ 8 upsampling; necessary to track intersample signals

Poles at $e^{\pm j(3/2\pi)} = \pm j$.

Next questions

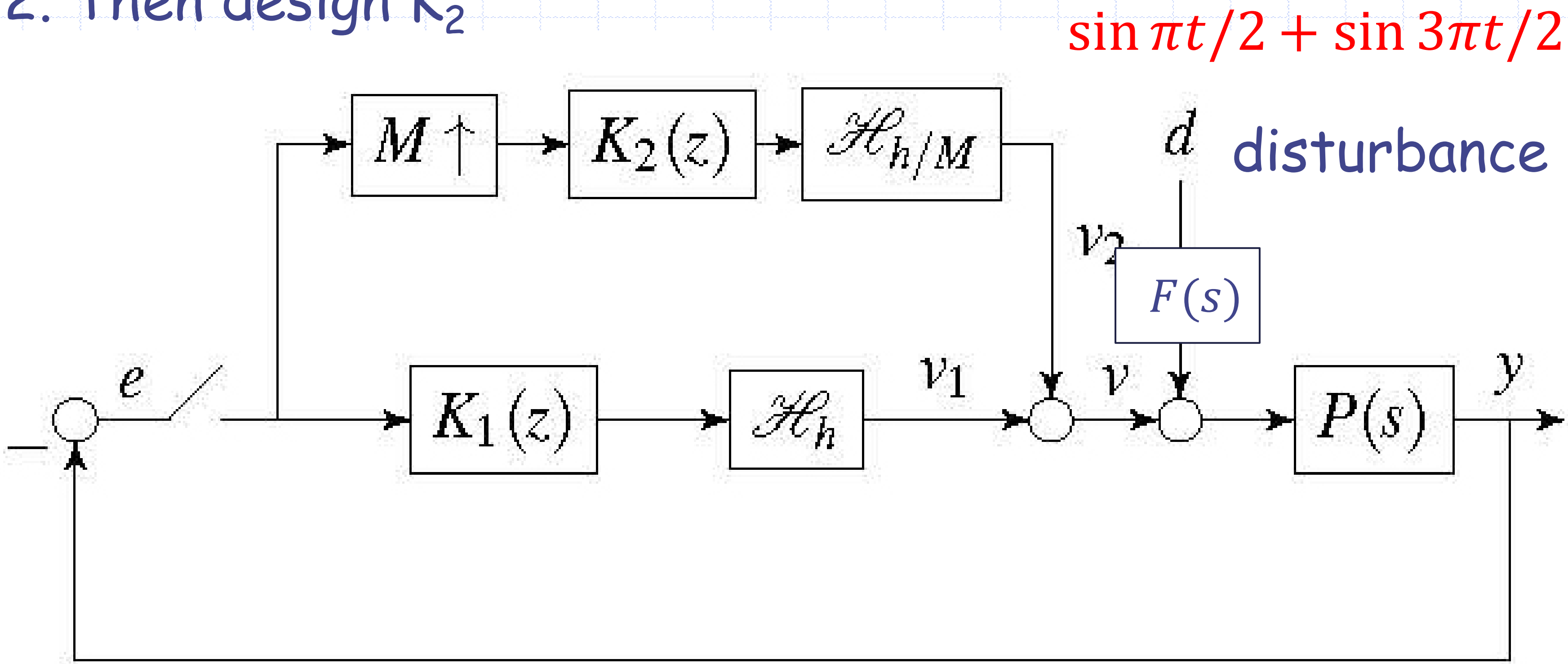
- Disturbance rejection
- More than one reference or disturbance signals?
- robustness

More than one signals

- Standard recipe: weights at two frequencies
  - May work, but
- Fails if they are symmetric against the Nyquist freq. π
- Sampling cannot distinguish two signals

Two step design:

1. Design K_1 for low frequency
2. Then design K_2



Two step design configuration

$$F(s) = {}^{50s}\!/_{(s^2+0.2s+\omega_1{}^2)(s^2+0.1s+\omega_2{}^2)}$$

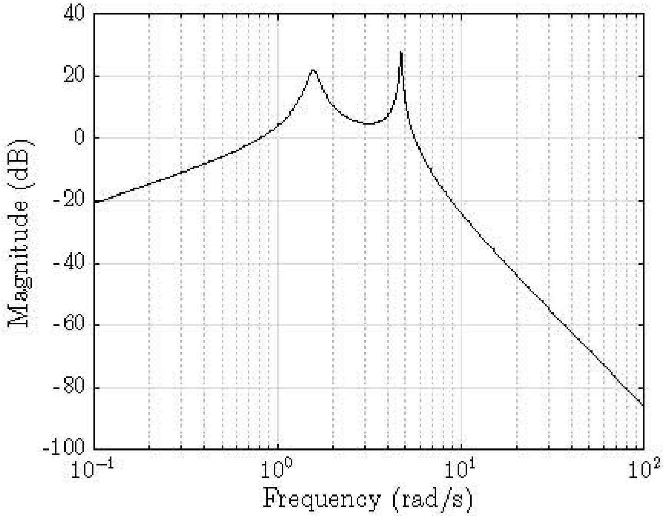


Fig. 4. Weighting function $F(s)$.

Weighting function

Presented at the 1st IEEE CCTA, Hawaii, 2017

Output $y(t)$

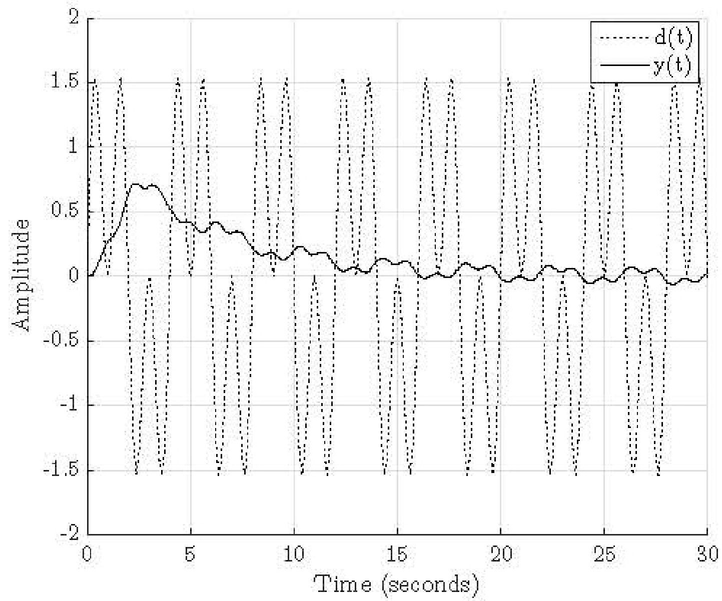


Fig. 8. System output (solid) against disturbance $\sin(\pi/2)t + \sin(3\pi/2)t$ (dotted).

Conclusion

- Tracking/rejection are possible for signals beyond the Nyquist frequency
- Not limited by the Shannon paradigm
- Crucial elements:
  - a. Physical model
  - b. appropriate weighting
  - c. upsampling (multirate processing)
- Possible applications: edge detection from low-freq. data, e.g., optical tomography

Happy 75^{th} Anders!

MTNS 2006

MTNS 2006, Kyoto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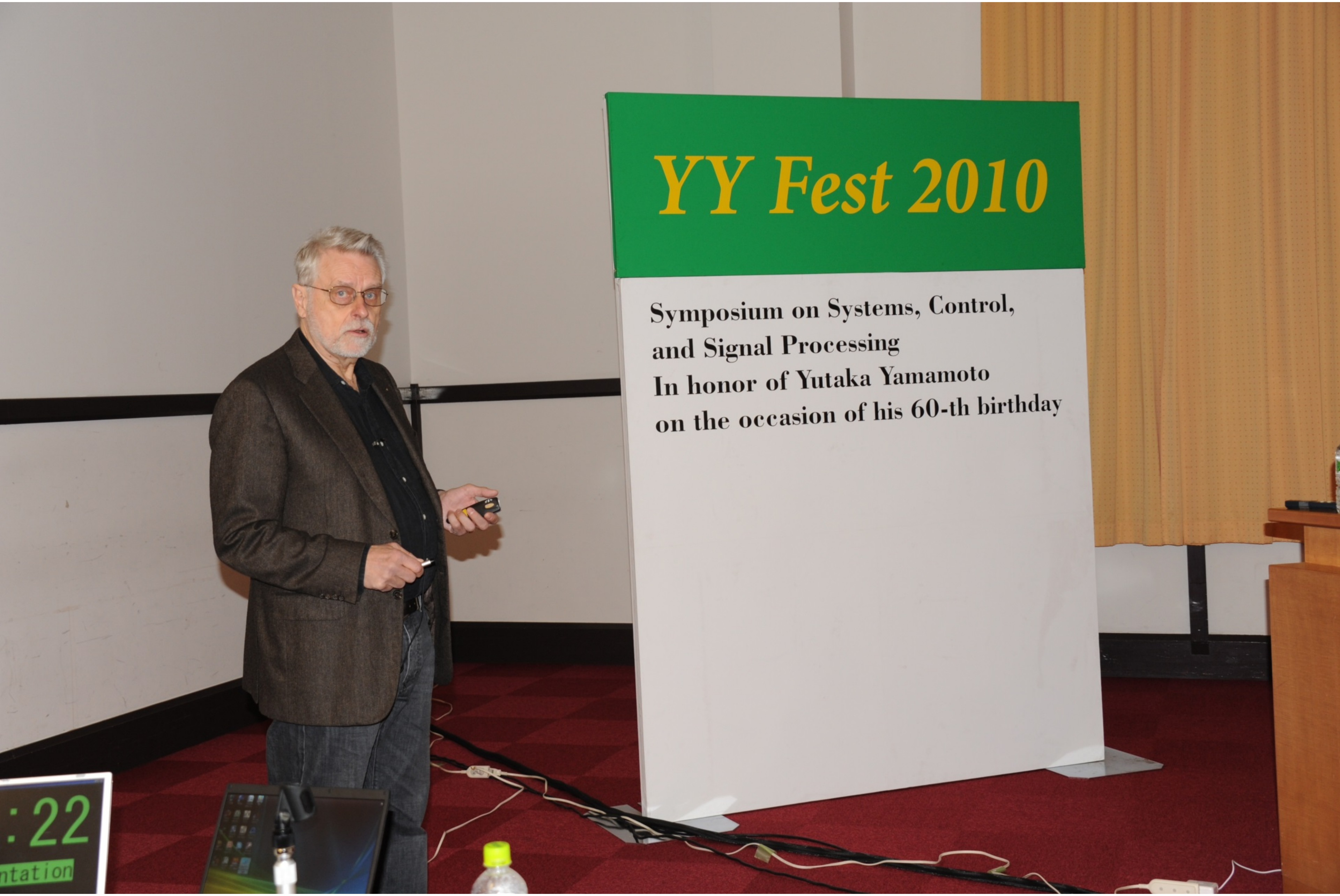
YY Fest 2010
Symposium on Systems, Control,
and Signal Processing
In honor of Yutaka Yamamoto
on the occasion of his 60-th birthday
:22
ntation

YYFest 2010


At my retirement party,
Kyoto, Shimogamo-saryo


Thank you Anders for your
Long-term contributions to our community,
Many happy returns
Of the day!