Open problems in feedback design for MIMO nonlinear systems

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strumenti musicali

Feedback design for MIMO nonlinear systems - 1/3

- In the late 1960s and early 1970s, the theory of MIMO linear systems reached a high degree of sophistication.
- In the 1980s, a big a collective effort aimed at extending this theory to nonlinear systems took place. Sophisticated tools had been developed, yielding a rather satisfactory understanding of decoupling, inversion, zero dynamics, infinite zero structure for MIMO nonlinear systems.
- The issue of feedback stabilization was thoroughly addressed in the 1990s, but mostly for SISO systems. For MIMO systems this issue was only marginally touched.
- By the early 1990s, a rather sudden blackout occurred in the study of MIMO systems.
- One basic question has always puzzles me since then: why interest in MIMO systems had faded ?
- MIMO systems seem to be important, though.

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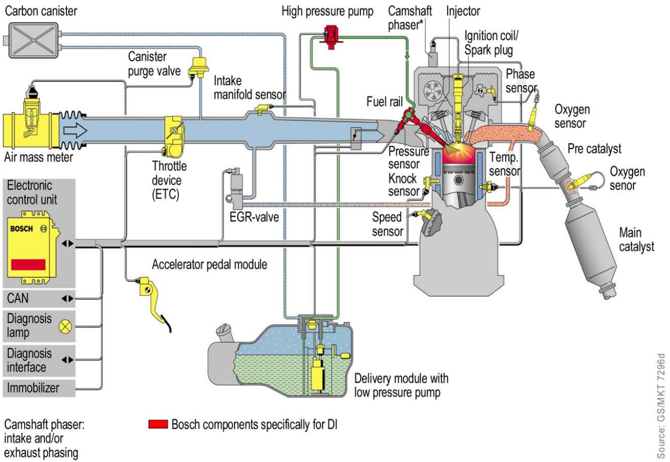

Engine control system: 15 – 25 sensors, 8 -12 control inputs,

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- Typically, this argument was used to "blame" methods based on feedback linearization.
- Until ... it was understood how such methods can be robustified (exemplar, in this respect, are the recent works in which the concept of and extended observer is exploited: see e.g. Han (1995) , Praly-Jiang (1998), Khalil (2008)).
- Thus, the argument in question is false.
- Another, more subtle, argument is that non-trivial MIMO nonlinear systems ("non-trivial" = systems that cannot be handled by trivial extensions of methods developed for SISO systems, such as systems that do not have vector relative degree) are pretty delicate to handle.
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- Consider a system with $n = 3$, two inputs and two outputs and assume

$$L_g h_2(x) = \delta(x) L_g h_1(x)$$

for some $\delta(x)$.

- Define $\phi(x) = L_f h_2(x) - \delta(x) L_f h_1(x)$ to obtain

$$\begin{aligned} \dot{y}_1 &= L_f h_1(x) + L_g h_1(x) u \\ \dot{y}_2 &= L_f h_2(x) + L_g h_2(x) u = \phi(x) + \delta(x)\dot{y}_1 \\ \dot{\phi} &= L_f \phi(x) + L_g \phi(x) u \end{aligned}$$

- Setting

$$\xi_{11} = h_1(x), \quad \xi_{21} = h_2(x), \qquad \xi_{22} = \phi(x)$$

these equations can be rewritten as

$$\begin{aligned} \dot{\xi}_{11} &= a_1(x) + b_1(x) u \\ \dot{\xi}_{21} &= \xi_{22} + \delta(x)[a_1(x) + b_1(x) u] \\ \dot{\xi}_{22} &= a_2(x) + b_2(x) u \end{aligned}$$

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A toy benchmark - 1/2

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- Assume

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is nonsingular for all x.

- This is a system that does not posses a vector relative degree. However, if $\delta(x)$ is bounded, the system can be trivially stabilized by state feedback.
- How can we achieve global stability via output feedback ?
- A related question: how can we characterize observability ?

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MIMO input-affine nonlinear systems (having the same number of input and output components), can be classified as follows:

- The class $\mathcal{S}_0$ of systems in which the zero dynamics algorithm is everywhere regular.
- The sub-class $\mathcal{S}_{\text{INV}} \subset \mathcal{S}_0$ consisting of those systems in which the inversion algorithm is everywhere regular i.e. systems that are uniformly invertible (the inversion algorithm is an extension of the celebrated structure algorithm).
- The sub-sub-class $\mathcal{S}_{\text{IOL}} \subset \mathcal{S}_{\text{INV}}$ consisting of those systems in which it is possible to force, by means of state-feedback, a linear input-output behavior.
- The sub-sub-sub-class $\mathcal{S}_{\text{VRD}} \subset \mathcal{S}_{\text{INV}}$ consisting of those systems for which a vector relative degree can be defined.

A system in the class $\mathcal{S}_0$, if certain vector fields are complete, is globally diffeomorphic to a system described by equations that can be split in a subset of the form

$$\dot{z} = f_0(z,\xi) + g_0(z,\xi)u$$

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\begin{array}{rcl}
\dot{\xi}_{i,1} & = & \xi_{i,1} \\
 & & \cdots \\
\dot{\xi}_{i,r_1-1} & = & \xi_{i,r_1} \\
\dot{\xi}_{i,r_1} & = & \xi_{i,r_1+1} + \delta^1_{i,r_1+1}(x)[a_1(x) + b_1(x)u] \\
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\dot{\xi}_{i,r_2-1} & = & \xi_{i,r_2} + \delta^1_{i,r_2}(x)[a_1(x) + b_1(x)u] \\
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\dot{\xi}_{i,r_{i-1}} & = & \xi_{i,r_{i-1}+1} + \displaystyle\sum_{j=1}^{i-1} \delta^j_{i,r_{i-1}+1}(x)[a_j(x) + b_j(x)u] \\
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It's a horrible form, but we have to live with that !

- In the sub-class $\mathcal{S}_{INV}$ of those systems that are invertible, the equation

$$\begin{pmatrix} a_1(x) \\ \vdots \\ a_m(x) \end{pmatrix} + \begin{pmatrix} b_1(x) \\ \vdots \\ b_m(x) \end{pmatrix} u = v$$

can be solved for u, and the multipliers $\delta^j_{i,k}(x)$ depend on the components of x in a special way.

- In the sub-sub-class $\mathcal{S}_{IOL}$ of those systems in which it is possible to force, by means of state-feedback, a linear input-output behavior, the multipliers $\delta^j_{i,k}(x)$ are independent of x.
- In the sub-sub-sub-class $\mathcal{S}_{VRD}$ of those systems for which a vector relative degree can be defined, the multipliers $\delta^j_{i,k}(x)$ are zero.

In the case of a SISO system, all such sub-classes collapse to a single one.

The classes in question can be identified also in coordinate-free terms

The r_i's characterize what in a linear system is known as infinite zero structure.

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- In the sub-sub-class $\mathcal{S}_{IOL}$ of those systems in which it is possible to force, by means of state-feedback, a linear input-output behavior, the multipliers $\delta^j_{i,k}(x)$ are independent of x.
- In the sub-sub-sub-class $\mathcal{S}_{VRD}$ of those systems for which a vector relative degree can be defined, the multipliers $\delta^j_{i,k}(x)$ are zero.

In the case of a SISO system, all such sub-classes collapse to a single one.

The classes in question can be identified also in coordinate-free terms

The r_i's characterize what in a linear system is known as infinite zero structure.

It's a horrible form, but we have to live with that !

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- A MIMO system having a well-defined vector relative degree and an input-to-state stable inverse (a strongly minimum-phase system) can be asymptotically stabilized, with a guaranteed region of attraction, by dynamic output feedback.
- An extended observer can be designed and hence the stabilizer in question is robust with respect to model uncertainties (but not to measurement noise).
- The extension of such stabilization paradigm to more general classes of MIMO systems is still an open domain of research.
- Interest in stabilizing more general classes of MIMO systems has been triggered by the works of Liberzon, Morse, Sontag (TAC, 2002), in which the authors analyze, in a coordinate-free framework, systems in which state and input are bounded so long as the output is bounded, and decay to zero so long as the output decays to zero. This can be seen as "equivalent" of the property that the inverse of the system is input-to-state stable.

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$$|x(t)| \leq \max\{\beta(|x(0)|, t), \gamma(\|\mathbf{y}^{N-1}\|_{[0,t]})\}$$

  as long as it exists. This is the version, for MIMO systems, of the property of being strongly minimum phase.
- Then, Liberzon proves that if:
  1. the system is uniformly left invertible and strongly minimum phase, and
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  then a static state feedback law $u = \alpha(x)$ exists that globally stabilizes the system.
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- Since this result, no relevant contributions appeared for some time. Recently, interest in improving such stabilization result has resumed.
- If a system belongs to the sub-sub-class in which the multipliers $\delta^j_{i,k}(x)$ are constant, semiglobal stabilization via dynamic output feedback is possible (Wang, A.I. et al. (TAC, 2015)).
- Define a set of dummy output functions

$$\tilde{y}_i = c_{i1}\xi_{i,1} + c_{i2}\xi_{i,2} + \cdots + c_{i,r_i-1}\xi_{i,r_i-1} + \xi_{i,r_i}$$

- Then, the system can be described by equations of the form

$$\begin{aligned} \dot{z} &= f_0(z,\xi) + g_0(z,\xi)u \\ \dot{\tilde{z}} &= F\tilde{z} + G\tilde{y} \\ \dot{\tilde{y}} &= q(z,\xi) + b(z,\xi)u \end{aligned}$$

in which $\xi = \xi(\tilde{z}, \tilde{y})$ and $b(z,\xi)$ is a nonsingular matrix. The property that the multipliers $\delta^j_{i,k}(x)$ are constant is instrumental to this end.

- The system with output $\tilde{y}$ has now vector relative degree $\{1, 1, \ldots, 1\}$.
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- Thus, the minimum phase properties of the original system (if any) are preserved.
- As a consequence, if the system with output y is strongly minimum phase, so is the system with output $\tilde{y}$.
- Hence, global stabilization can be obtained by means of a feedback law $u = \kappa(\tilde{y})$.
- An estimate of $\tilde{y}$ can be obtained by means of a high-gain observer driven by the actual output y. This is not a trivial task, though, because the components of ξ are not just higher-order derivatives of the components of y.
- As a consequence, semiglobal stabilization via dynamic output feedback can be obtained.
- Note that the original system is not required to possess a vector relative degree.
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- Note that the original system is not required to possess a vector relative degree.
- The design of an extended observer for such class of systems is still an open problem.
- The design can be made robust with respect to uncertainties in $q(z, \xi)$ and $b(z, \xi)$, but the actual values of the "multipliers" $\delta^j_{i,k}$ need to be known.

- Thus, the minimum phase properties of the original system (if any) are preserved.
- As a consequence, if the system with output y is strongly minimum phase, so is the system with output $\tilde{y}$.
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- In the benchmark problem

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\begin{aligned}
\dot{\xi}_{11} &= a_1(x) + b_1(x)u \\
\dot{\xi}_{21} &= \xi_{22} + \delta(x)[a_1(x) + b_1(x)u] \\
\dot{\xi}_{22} &= a_2(x) + b_2(x)u \\
y_1 &= \xi_{11} \\
y_2 &= \xi_{21}
\end{aligned}
$$

the multiplier $\delta(x)$ is not constant. Thus, the previous stabilization method is not applicable.

- In general, if $\delta(x)$ is not constant, the system may even fail to be uniformly invertible.
- In fact

$$
\begin{aligned}
y_1^{(1)} &= a_1 + b_1 u \\
y_2^{(2)} &= a_2 + b_2 u + \delta y_1^{(2)} \\
&\quad + \left[\frac{\partial \delta}{\partial \xi_{11}} y_1^{(1)} + \frac{\partial \delta}{\partial \xi_{21}} y_2^{(1)} + \frac{\partial \delta}{\partial \xi_{22}} [a_2 + b_2 u] \right] y_1^{(1)}
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As seen, not any function if x is admissible if the system is assumed to be invertible. Thus, we consider a special dependence on x.

Specifically, we assume that:

- (i) The z variables are missing (the system has a trivial zero dynamics), in which case $x = \xi$
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If $p = 2$, the multipliers $\delta^1_{2,k}(x)$ only appear in the second string, and we assume that they depend on x as follows

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y_2 &= x_{2,1}\\
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The “triangular” dependence on x - 1/2

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The "triangular" dependence on x - 2/2

To simplify a little bit the notation, we set

$$[x_2]_k = \mathrm{col}(x_{2,1}, \ldots, x_{2,k})\,, \qquad x_i = \mathrm{col}(x_{i,1}, \ldots, x_{i,r_i})\,.$$

in which case the equations are rewritten as

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We refer to this as a "triangular dependence on the components of x". Note the absence of z.

- Proposition If a system has 2 inputs and 2 outputs and a trivial zero dynamics, the system is invertible if and only if the dependence of $\delta_{2,k}(x)$ on x is as described above.
- The extension of this result to systems having $p > 2$ and/or non-trivial zero dynamics is yet to be found.

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Such systems can be easily stabilized via full state feedback.

Pick a function $v(x_1)$ so that

$$\begin{aligned}\dot{x}_{1,1} &= x_{1,2}\\ &\cdots\\ \dot{x}_{1,r_1-1} &= x_{1,r_1}\\ \dot{x}_{1,r_1} &= v_1(x_1)\end{aligned}$$

is stabilized and let u be such that

$$a_1(x) + b_1(x)u = v_1(x_1)\,.$$

If this is the case, the second string becomes

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$$\begin{aligned} \dot{x}_{1,1} &= x_{1,2} \\ & \cdots \\ \dot{x}_{1,r_1-1} &= x_{1,r_1} \\ \dot{x}_{1,r_1} &= v_1(x_1) \end{aligned}$$

is stabilized and let u be such that

$$a_1(x) + b_1(x)u = v_1(x_1)\,.$$

If this is the case, the second string becomes

$$\begin{aligned} \dot{x}_{2,1} &= x_{2,2} \\ \dot{x}_{2,2} &= x_{2,3} \\ & \cdots \\ \dot{x}_{2,r_1-1} &= x_{2,r_1} \\ \dot{x}_{2,r_1} &= x_{2,r_1+1} + \delta_{2,r_1+1}(x_1, [x_2]_{r_1})v_1(x_1) \\ & \cdots \\ \dot{x}_{2,r_2-1} &= x_{2,r_2} + \delta_{2,r_2}(x_1, [x_2]_{r_2-1})v_1(x_1) \\ \dot{x}_{2,r_2} &= a_2(x) + b_2(x)u\,. \end{aligned}$$

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This sub-system can be stabilized recursively, via backstepping.

At the end of the backstepping process, a control law $v_2(x_1, x_2)$ can be found such that, if

$$a_2(x) + b_2(x)u = v_2(x_1, x_2)$$

the entire sub-set is stabilized.

Thus, we conclude that if the control u is such that

$$A(x) + B(x)u = \begin{pmatrix} a_1(x) + b_1(x)u \\ a_2(x) + b_2(x)u \end{pmatrix} = \begin{pmatrix} v_1(x_1) \\ v_2(x_1, x_2) \end{pmatrix}$$

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This method though, requires an accurate model of the plant and availability of the full state x.

The second of these two problems can be fixed, because the state x of the system in question turns out to be easily observable.

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- Lemma Set, for $i = 1, 2$,

$$\mathbf{y}_i^j = \mathrm{col}(y_i, y_i^{(1)}, \ldots, y_i^{(j-1)})\,.$$

There exists a map $\Psi : \mathbb{R}^{2r_2} \to \mathbb{R}^{r_1+r_2}$ such that

$$x = \Psi(\mathbf{y}_1^{r_2}, \mathbf{y}_2^{r_2})\,.$$

By definition

$$\begin{aligned} x_{1i} &= y_1^{(i-1)} \\ x_{2i} &= y_2^{(i-1)} \end{aligned} \qquad \text{for } i = 1, \ldots, r_1.$$

So long as x_{2,r_1+1} is concerned, observe that

$$\begin{aligned} x_{2,r_1+1} &= \dot{x}_{2,r_1} - \delta_{2,r_1+1}(x_1, [x_2]_{r_1})[a_1(x) + b_1(x)u] \\ &= y_2^{(r_1)} - \delta_{2,r_1+1}(x_1, [x_2]_{r_1})y_1^{(r_1)}\,, \end{aligned}$$

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The first term $\dot{\psi}_{2,r_1+1}(\cdot)$ is a function $y_1, \ldots, y_1^{(r_1+1)}, y_2, \ldots, y_2^{(r_1+1)}$, while the arguments x_1 and $[x_2]_{r_1+1}$ of $\delta_{2,r_1+2}(\cdot)$ are functions of $y_1, \ldots, y_1^{(r_1)}, y_2, \ldots, y_2^{(r_1)}$.

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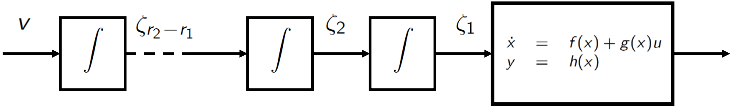
  In particular, this requires the estimation of the derivatives of y_1 from order r_1 to order $r_2 - 1$ and such derivatives, in turn, depend on the input u and a few of its higher order derivatives, up to order $r_2 - r_1 - 1$.

- To circumvent this problem, it is convenient to dynamically extend the system, by adding a chain of $r_2 - r_1$ integrators on both input channels.



Stabilization by output feedback - 1/3

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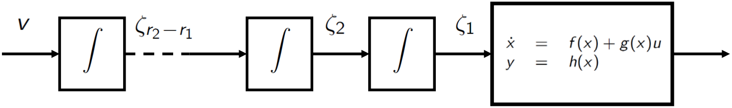

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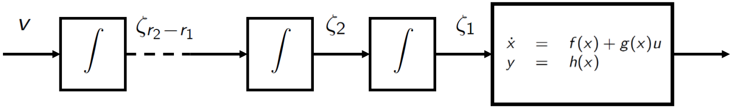

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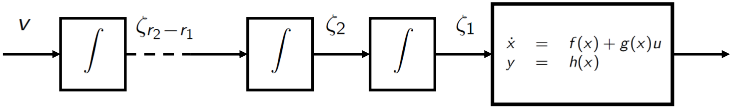

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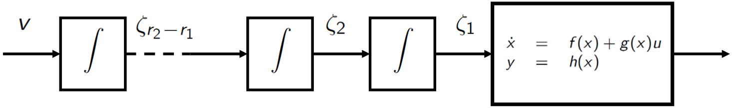

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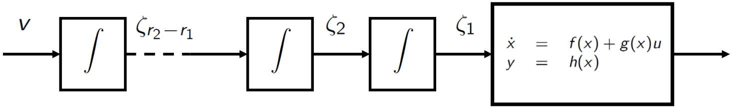
  The components of $\mathbf{y}_1^{r_1}$ and $\mathbf{y}_2^{r_2}$ could be estimated by means of a high-gain observer.
- In this respect, though, it must be stressed that the arguments of $\Psi(\cdot)$ consist of y_1, y_2 and all their higher order derivatives up order $r_2 - 1$.

  In particular, this requires the estimation of the derivatives of y_1 from order r_1 to order $r_2 - 1$ and such derivatives, in turn, depend on the input u and a few of its higher order derivatives, up to order $r_2 - r_1 - 1$.
- To circumvent this problem, it is convenient to dynamically extend the system, by adding a chain of $r_2 - r_1$ integrators on both input channels.



- The system is extended by setting

$$\begin{aligned} u &= \zeta_1 \\ \dot\zeta_1 &= \zeta_2 \\ &\cdots \\ \dot\zeta_{r_2-r_1} &= v\,, \end{aligned}$$

in which $\zeta_i \in \mathbb{R}^2$ and where $v \in \mathbb{R}^2$ plays a role of a new input.
- The system thus extended has a structure similar to that of the system seen before: hence that there exists a feedback law $v = \alpha(x, \zeta)$ that globally asymptotically stabilizes the equilibrium $(x, \eta) = 0$.
- The components of the vector ζ are states of the dynamic extension, hence available for feedback.
- Thus, to implement this feedback law, only the vector x has to be estimated.
- But we know, from the previous analysis, that $x = \Psi(\mathbf{y}_1^{r_2}, \mathbf{y}_2^{r_2})$. Hence, to implement this feedback law, estimates of $\mathbf{y}_1^{r_2}, \mathbf{y}_2^{r_2}$ suffice.
- Such estimates can be generated by means of a standard high-gain observer, because now

$$\begin{aligned} y_1^{(r_2)} &= q_1(x, \zeta) + p_1(x)v \\ y_2^{(r_2)} &= q_2(x, \zeta) + p_2(x)v\,. \end{aligned}$$

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- The zero dynamics are trivial
- The multipliers $\delta^j_{i,k}(x)$ are special "triangular" functions of x.
- The functions $\delta^j_{i,k}(x)$ are assumed to be accurately known, as they are required in the construction of the maps

$$\begin{aligned} u &= \bar{v}(x,\zeta) \\ x &= \Psi(\mathbf{y}_1^{r_2}, \mathbf{y}_2^{r_2}) \end{aligned}$$

The design of a more robust controller is still an open issue.

Note also that, as a byproduct, we have proven that the map $T(x)$ defined in Liberzon (SCL, 2004) is onto.

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- In the case $p > 2$, a special functional dependence of the $\delta^j i, k(x)$'s on the individual components of x that guarantees invertibility is easily found. However, a sharp necessary condition for invertibility is not know yet.
- The previous stabilization method presupposes a trivial zero dynamics. The more general case of systems having a nontrivial zero dynamics has not been handled yet.
- The method of Liberzon, for stabilization via full state feedback, requires a map $T(x)$ to be onto. Conditions ensuring that this map is onto are likely to be related to invertibility and observability (as shown in the case discussed before), but precise conditions have not been determined yet.
- The method of Liberzon provides a full state feedback stabilizing law. However, since the system is strongly minimum phase, it is likely to expect that only outputs (and their higher order derivatives) suffice for stabilization. This issue has not been explored yet.
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Conclusions

- The topic of robust feedback design of MIMO nonlinear systems, that had remained silent for a while, is now experiencing a revival.
- If the system is robustly minimum-phase and the multipliers $\delta^j_{i,k}(x)$ in the normal form are constant, it can be robustly (semiglobally) stabilized via dynamic output feedback.
- In any case "practical" disturbance decoupling and feedback linearization can obtained on a finite interval, in spite of model uncertainties, if an extended observer is used.
- More challenging extensions are have been pursued, notably those addressing special cases in which the multipliers $\delta^j_{i,k}(x)$ in the normal form are not constant.
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Happy Birthday Anders !

Cento di questi giorni !