Exchange, Fisheries, and Gradients

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Setting

- **Economic agents** $i \in I$

- Agent i has endowment/ resource holding/ quota/ **user right** $x_i^0 \in \mathbb{X}$ (Euclidean).

- He has **payoff function** $\pi_i : \mathbb{X} \to \mathbb{R} \cup \{-\infty\}$.

- Going alone he takes home $\pi_i(x_i^0)$

Convolution of payoffs

For argument, consider the **problem**

$$\pi_I(x_I) := \sup\left\{\sum_{i\in I}\pi_i(x_i) : \sum_{i\in I} x_i = x_I\right\}$$

with fixed **aggregate endowment** $x_I := \sum_{i\in I} x_i^0$.

Payoff $\pi_i : \mathbb{X} \to \mathbb{R} \cup \{-\infty\}$ is concave usc.

? Who states the problem? Nobody!

Who solves the convolution?

Here, the agents themselves! No coordination!
Recall x^* a **supgradient** of $f : \mathbb{X} \to \mathbb{R} \cup \{+\infty\}$ at x:

$$x^* \in \partial f(x) \iff \quad f(x) + x^*(\hat{x} - x) \geq f(\hat{x}) \text{ for all } \hat{x}.$$

Proposition: *If allocation (x_i) is optimal, then valuation/ pricing is common:*

$$\partial \pi_I(x_I) \subseteq \cap_{i \in I} \partial \pi_i(x_i).$$

Conversely, if (x_i) is an allocation, and $\cap_{i \in I} \partial \pi_i(x_i)$ is non-empty, then (x_i) is optimal and

$$\partial \pi_I(x_I) \supseteq \cap_{i \in I} \partial \pi_i(x_i).$$

?**Who finds a common price? Answer.: the agents themselves**!

How? By bilateral exchanges!

Classic case: The coffee house Lloyds of London

• agent $i \in I$ is a "**name**" = accredited **underwriter** of risk.

• C = finite "complete" list of verifiable contingencies, $\mathbb{X} = \mathbb{R}^C$, and expected payoff

$$E_i \pi_i[x_i] := \sum_{c \in C} \pi_i[c, x_i(c)] \mu_i(c).$$

• **Probability measure** (individual belief) μ_i over C.

• Agents stroll in the coffee house. They meet and discuss pair-wise $\Longrightarrow$ **repeated bilateral trade**.

• Double stochasticity:
1) **random encounters,**
2) **random liabilities**.

Central versus distributed

Practical query: Problem

$$\pi_I(x_I) := \sup\left\{\sum_{i\in I}\pi_i(x_i) : \sum_{i\in I} x_i = x_I\right\}.$$

is hardly stated! Even more hardly solved!

No center! No coordinating agent!

Mitigating mechanism: bilateral direct exchange. (Work with two variables at a time!)

Two complementary views on what comes:
1) **an algorithm** or
2) **a story about agent behavior**.

A technicality, a conflict, a tension

On one hand: Working with few variables a time,

using "**coordinate methods", one needs smoothness.**

On the other hand:

non-smooth functions should be allowed:

Example: $\pi_i(x_i) :=$ optimal value of LP with rhs x_i.

A compromise is needed! (for this randomness helps.)

Bilateral exchange

Agent $i \in I$ meets agent j, have holdings $x_i \in \mathbb{X}$ and $x_j \in \mathbb{X}$ respectively. After direct trade, with no LOG:

$$x_i^{+1} := x_i + \Delta \quad \text{and} \quad x_j^{+1} := x_j - \Delta$$

where

$$\Delta := sd \quad \text{features } \textbf{step-size} \quad s > 0 \quad \text{and } \textbf{direction} \quad d \in \mathbb{X}.$$

First issue:

which direction?

Secondary issue:

which step-size?

Third, issue:

who meets whom?

First issue: which direction?

Proposal: choose supgradients $x_i^* \in \partial\pi_i(x_i)$, $x_j^* \in \partial\pi_j(x_j)$ and posit

$$d = x_i^* - x_j^*.$$

Economic rationale:

$$d_c > 0 \Longleftrightarrow x_{ic}^* > x_{jc}^*.$$

Mathematical rationale: If $x_i^* = \pi_i'(x_i)$ and $x_j^* = \pi_j'(x_j)$ are differentiable, then

$$d = \pi_j'(x_j) - \pi_i'(x_i) \quad \Longrightarrow \quad \pi_i'(x_i; d) + \pi_j'(x_j; -d) = \left\| x_i^* - x_j^* \right\|^2.$$

In that case, $x_i^* \neq x_j^* \Longrightarrow$ system

$$\pi_i(x_i^{+1}) + m_i > \pi_i(x_i) \quad \text{and} \quad \pi_j(x_j^{+1}) + m_j > \pi_j(x_j)$$

is solvable with **side payments** $m_i + m_j = 0$.

An aside: on trade incentives

The **steepest slope**

$$\mathfrak{S}_{ij}(x_i, x_j) := \sup\left\{\pi_i'(x_i; d) + \pi_j'(x_j; -d) : d \text{ feasible and } \quad \|d\| \leq 1\right\}$$

NB: No trade iff the interlocutors see a common price:

$$\text{No trade} \quad \Longleftrightarrow \quad \mathfrak{S}_{ij}(x_i, x_j) = 0 \Longleftrightarrow \partial\pi_i(x_i) \cap \partial\pi_j(x_j) \neq \varnothing.$$

$$\text{Trade} \quad \Longleftrightarrow \quad \mathfrak{S}_{ij}(x_i, x_j) > 0 \Longleftrightarrow \partial\pi_i(x_i) \cap \partial\pi_j(x_j) = \varnothing.$$

How could a common price emerge? Tentative answer: By repeated bilateral exchanges!

Second issue: Which step-sizes?

Recall

$$x_i^{+1} := x_i + sd \quad \text{and} \quad x_j^{+1} := x_i - sd \quad \text{with} \quad s > 0$$

Step-sizes should dwindle but not too fast!

They dwindle over stages $k = 0, 1, 2, \ldots$**.if**

$$\sum_{k=0}^{\infty} s_k^2 < +\infty,$$

but not too fast if

$$\left\{ \sum_{k=0}^{\infty} s_k : \text{agent } i \text{ meets } j \right\} = +\infty \quad \text{for all pairs} \quad i, j.$$

Third issue, the protocol: who meets next whom?

Assumption: Here, completely random matching. Pick any agent pair independently - and with uniform probability $\Longrightarrow$

Alternative protocols:

* periodic, quasi-cyclic encounters, or

* those with largest slope.

Algorithm: Repeated bilateral deals

Exchange construed as algorithm:
Agent i **starts** with some feasible holding x_i such that $\sum_{i \in I} x_i = x_I$.

Select any agent pair i, j in equiprobable, independent manner.

Update their holdings x_i, x_j :

$$x_i^{+1} := x_i + sd \quad \text{and} \quad x_k^{+1} := x_j - sd$$

with step-size $s \geq 0$ and feasible direction $d \in \partial \pi_i(x_i) - \partial \pi_j(x_j)$.

Continue until convergence.

Exchange driven by differential valuations

Recall

$$\text{steepest slope } \mathfrak{S}_{ij}(x_i, x_j) > 0 \Longleftrightarrow \partial\pi_i(x_i) \cap \partial\pi_j(x_j) = \varnothing.$$

In every cluster point (x_i) each agent pair see a common price; that is:

$$\partial\pi_i(x_i) \cap \partial\pi_j(x_j) \neq \varnothing \quad \forall i, j.$$

Standing assumption: **at least one agent has** $x_i \in intX_i$ **and** π_i **is differentiable there**. $\Rightarrow$

$$p \in \cap_{i \in I} \partial\pi_i(x_i).$$

Common prices emerge finally! They are results, not prerequisites!

Economic summary

- Prices need not come from somewhere; they rather emerge.
- Price-taking or maximization is neither necessary nor quite realistic.
- Agents can do without posted (common) prices.
- Agents merely seek own improvements.
- Everybody can contend with idiosyncratic, local information.
- No coordination, central agency, or global knowledge is ever required.

Political summary (on fisheries management)

Common property can causes severe problems:

Dissipation of surplus

Decimation of stocks.

Private property not quite legitimate

Recommendation: 1) user rights instead of property rights
2) **market for user rights.**

Consequence: **resource rent restored, safeguarded and distributed.**