Master thesis project: Image reconstruction and compression using rational models

In this work we will investigate the use of rational models for image processing. In particular we will consider reconstruction of images based on given measurements. For example, to recover the function f from the measurements

$$c_k = \int \phi(t_k - s) f(s) ds, \quad k = 1, \ldots, n$$

for a selected convolution kernel ϕ and a set of sample points t_ℓ. A rich theory for rational models which allows for a complete parameterization of low complexity solutions has been developed by Chris Byrnes, Anders Lindquist, etal. (see, e.g., [1]). The theory is well understood for the one dimensional problem and here we will also approach the two dimensional problem using the same methodology.

The main purpose in this theses work is to investigate design methods based on rational functions for image reconstructing, deblurring, and compression [2]. In particular, we will study how the parametrization from [1] can be used for selecting solutions based on design criteria, e.g., when high resolution is desired in a specific area of the image. We will compare this methodology with current methods, such as those building on exponential models. A particular emphasis will be on problems relevant for medical imaging.

References

[1] C. I. Byrnes and A. Lindquist, "The generalized moment problem with complexity constraint," *Integral Equations and Operator Theory*, vol. 56, no. 2, pp. 163–180, 2006.

[2] P. C. Hansen, J. G. Nagy, and D. P. O'leary, *Deblurring images: matrices, spectra, and filtering*, vol. 3. Siam, 2006.