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Foreword

Svenska $\longrightarrow$

This thesis would never have been started without Harold Seymour Shapiro. It would never have been finished without Ari Laptev. I thank these good men with all my heart. Their help has been truly invaluable. This thesis is my tribute to their unfailing support and friendship. Only a minute fraction of all their intriguing ideas and suggestions have found its reflection in these pages.

The rate of convergence of this work has been at least quadratic.

The cardinality of the set of people I would like to thank is too large. My mathematical friends are too numerous to be enumerated.

Thank you all!

Jockum Aniansson
Stockholm in September
AD MIM

Abstract

This thesis consists of two parts. The first part deals with some integral representations in n-dimensional real Euclidean space, see I. The second part deals with some integral representations in n-dimensional complex Euclidean space, see II and III.

I Multi–dimensional Peano–Sard kernels and a divergence representation.
We prove a divergence representation with increased smoothness and control over the support, valid for compactly supported distributions. We then construct multi–dimensional Peano–Sard kernels with optimal regularity. These kernels are used to represent remainder functionals (error functionals) in e g numerical quadrature or cubature rules. We also obtain a representation theorem for entire analytic functions of exponential order vanishing at a point.

II Fischer kernels, and projectors in some spaces of analytic functions.
This paper is concerned with direct sum decompositions of some classical spaces of analytic functions in n complex variables. In some specific constellations we derive concrete forms for reproducing or representing kernels for certain subspaces of functions. After a Fourier transformation this will explicitly give us the Green's function for some characteristic and non-characteristic Cauchy problems.

III Harmonic projections in Fischer–Fock space.
In this article we show how to decompose any entire analytic function $f(z_1, \cdots z_n)$ uniquely as a sum of two entire functions $f = h + g$, where

$$\triangle h = \triangle_z h = \left(\frac{\partial}{\partial z_1^2} + \cdots + \frac{\partial}{\partial z_n^2} \right) h(z_1 , \cdots , z_n) = 0 ,$$

and the quotient $g(z_1 , \cdots , z_n) / (z_1^2 + \cdots + z_n^2)$ is entire. This is most conveniently done using orthogonal projections within the framework of Fischer–Fock space (Bargmann space) by means of the *Fischer kernels* we determine. Fourier transformation then gives us an explicit solution to the characteristic Cauchy problem for the wave equation.

Key words and phrases. Integral representations. Peano kernel. Sard kernel. Divergence representation. Bargmann–Segal space. Fock space. Reproducing kernel Hilbert space. Clebsch projection. Fischer pair. Characteristic Cauchy problem.

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