Seminar, Thursday March 26, 1998, 10.15 – 11.15.

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Harmonic projection in Fischer–Fock space.

Abstract. *Fischer space* or *Fock space* $\mathcal{F} = \mathcal{F}(\mathbf{C}^n)$ is a Hilbert space of entire analytic functions in $\mathbf{C}^n$ with a Gaussian weight, and inner product

$$\langle f , g \rangle = \int_{\mathbf{C}^n} f(z)\,\overline{g(z)}\, \exp(-|z|^2)\, dV/\pi^n, \qquad \text{where } dV = dx_1\, dy_1 \cdots dx_n\, dy_n$$

denotes $2n-$dimensional Lebesgue measure. All monomials z^α turn out to be orthogonal in $\mathcal{F}$.

Let $P(z)$ be a polynomial in $z = (z_1, \cdots, z_n)$. The formal adjoint in $\mathcal{F}$ of the unbounded operator "multiplication by $P(z)$" is the partial differential operator $P(D)$, where $D = (D_1, \cdots, D_n)$, $D_j = d/dz_j$.

Harold Shapiro and Donald Newman showed that every element f in $\mathcal{F}$ admits a unique decomposition $f = g + h$, where $P(z)$ divides $g(z)$ and $P(D)\,h(z) = 0$.

We perform this decomposition explicitly when $P(z) = z_1^2 + \cdots + z_n^2$, so that $h(z)$ is (complex) harmonic. Bessel functions inevitably occur, but also Kummer's confluent hypergeometric function.

This could be applied e.g. to obtain the solution to the characteristic Cauchy problem for the wave operator with data on the forward light cone.