Optimal Symbol-by-Symbol Costa Precoding for a Relay-Aided Downlink Channel

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***Abstract*—In this article, we consider practical approaches to Costa precoding (also known as dirty paper coding). Specifically, we propose a symbol-by-symbol scheme for cancellation of interference known at the transmitter in a relay-aided downlink channel. For finite-alphabet signaling and interference, we derive the optimal (in terms of maximum mutual information) modulator under a given power constraint. A sub-optimal modulator is also proposed by formulating an optimization problem that maximizes the minimum distance of the signal constellation, and this non-convex optimization problem is approximately solved by semi-definite relaxation. For the case of binary signaling with binary interference, we obtain a closed-form solution for the sub-optimal modulator, which only suffers little performance degradation compared to the optimal modulator in the region of interest. For more general signal constellations and more general interference distributions, we propose an optimized Tomlinson-Harashima precoder (THP), which uniformly outperforms conventional THP with heuristic parameters. Bit-level simulation shows that the optimal and sub-optimal modulators can achieve significant gains over the THP benchmark as well as over non-Costa reference schemes, especially when the power of the interference is larger than the power of the noise.**

***Index Terms*—Costa precoding, dirty paper coding, interference, modulation, relay channel.**

I. Introduction

FROM information theory, it is known since [1] that the achievable rate of a communication channel remains unchanged if the receiver observes the transmitted signal in the presence of additive interference and white Gaussian noise, *provided that the transmitter knows the interference non-causally*. The resulting precoding method is known as "Costa precoding" or "dirty paper coding" (DPC) after the title of [1]. The problem of designing a DPC transmitter is important because the scenario with known interference arises in many contexts, notably, in precoding for inter-symbol interference channels and for the downlink multiuser MIMO wireless channel [2]–[5]. In [4] DPC has been shown to be capacity achieving in non-degraded MIMO broadcast channels. DPC can also be applied in a cooperative two-transmitter two-receiver wireless network [6], in relay-aided broadcast channels [7], and in relay interference channels with a cognitive source [8]. Essentially, an information theoretic strategy for achieving capacity is known; it is precisely the achievability proof in [1] and works as follows: First quantize the interference into a number of bins and then, depending on what bin the interference falls into, choose an appropriate code to encode the message at the transmitter. This approach has been used with success in [9], [10], for example, where sophisticated coding schemes were proposed based on superposition coding [9], lattices and trellis shaping [10]. Trellis and convolutional precoding was used in [11] where the trellis shaping was developed taking into account the knowledge of a noncausal interference sequence.

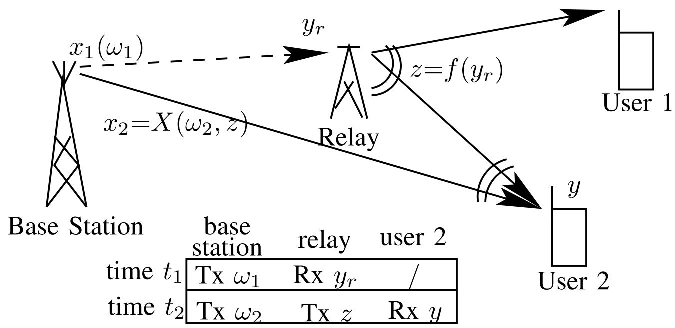

Fig. 1. The base station transmits ω_1 to user 1 during time slot t_1 and ω_2 to user 2 during time slot t_2. The relaying signal $z{=}f(y_r)$ dedicated for user 1 appears as "interference" for user 2. With non-causal knowledge of z, the base station can design a DPC modulator $x_2 = X(\omega_2, z)$ given the information symbol ω_2 and the interference z.

In this work we study practical DPC schemes in the context of a relay-aided downlink channel. Consider a communication network where the base station transmits information symbols ω_1 and ω_2 to user 1 and user 2, respectively, with the aid of a half-duplex relay (a relay that cannot transmit and receive simultaneously). As illustrated in Figure 1, the relay is dedicated to assist user 1 (the weaker/more distant user) whose direct link with the source fails. The base station transmits x_1 (signal for ω_1) during time slot t_1 and x_2 (signal for ω_2) during t_2. The relay listens to the base station during t_1 and transmits $z = f(y_r)$ during t_2, where y_r is the received signal at the relay during t_1 and $f(\cdot)$ is a relay mapping function. The relaying signal z, which is useful for user 1, appears as interference for user 2. Assuming that the relaying function

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$f(\cdot)$ is known at the base station and that the source-relay link is good, the "interference" z will be known non-causally at the base station with high probability, effectively resulting in the Costa problem.

The goal of our work is to obtain an understanding for what one can achieve in small (or a single) dimensions of signals and at low complexity, rather than to achieve the channel capacity. Indeed achieving capacity requires coding over an infinite number of dimensions, as in [1]. More precisely, we consider the design of *one-dimensional*[1] schemes $X(\omega_2, z)$ that map the information symbol $\omega_2 \in \mathbb{Z}$ and an interference symbol $z \in \mathbb{R}$ (known to the base station but *not* to user 2) onto an output symbol $x_2 \in \mathbb{R}$. Thereby, our focus is on symbol-by-symbol modulation rather than on coding. To get a better understanding of how our proposed scheme performs compared to the theoretical limit, we will use the mutual information between the transmitted ω_2 and the received signal at user 2 as the criterion for design.

The well known Tomlinson-Harashima precoder (THP) [12], [13], originally proposed for channels with inter-symbol interference, is a symbol-by-symbol DPC approach and therefore it serves as a good benchmark. The achievable rate for THP has been investigated in [14] and a scaled THP has been invented in [15]. THP with partial channel knowledge has been studied in [16]. Essentially THP (and its variations) subtracts the interference z from the information-bearing symbol and then performs a modulo operation to avoid a power boost. Another reason for introducing THP is that it already has wide applications. For instance, THP has been proposed as a building block for transmitter precoding for the downlink multiuser MIMO channel [17], [18]. Another symbol-by-symbol DPC scheme proposed in [19] minimizes the uncoded symbol error probability by joint design of the modulator and the demodulator. It is omitted in this paper due to the difficulty to evaluate its performance in terms of mutual information.

In our conference paper [20], we have presented the optimal[2] modulator for binary signaling with binary interference based on an exhaustive search over 12 possible mappings, which typically outperforms THP even when the parameters of THP are optimally chosen. Based on these preliminary findings, we propose here a mapping set size reduction method which makes our modulation design strategy applicable to higher order modulations. We also propose a sub-optimal modulator by formulating an optimization problem targeted at maximizing the minimum constellation distance, which is approximately solved by convex optimization after relaxation. A closed-form solution of the sub-optimal modulator is obtained for the case of binary signaling with binary interference, which suffers a minor performance loss compared to the optimal modulator in most of the interesting scenarios. For arbitrary signal and interference distributions, we propose an optimized THP scheme which demonstrates significant gains over heuristic THP in strong and medium interference scenarios. Our proposed DPC schemes are evaluated in terms of mutual information, coded bit-error-rate (BER), as well as energy efficiency, and compared to two non-DPC approaches, namely orthogonal transmission and receiver centric interference cancellation.

The rest of this paper is organized as follows. The system model and design criteria are introduced in Sec. II, where a brief overview of THP is also presented. The optimal modulator and the sub-optimal modulator are discussed in Sec. III, and THP with optimized parameters for Gaussian interference is presented in Sec. IV. Two non-DPC schemes are discussed in Sec. V as a reference. Simulation results are presented in Sec. VI and conclusions are drawn in Sec. VII.

Notations: $\boldsymbol{X}$ denotes a matrix and $\boldsymbol{x}$ denotes a vector. $(\cdot)^T$ indicates matrix/vector transpose and $\mathrm{Tr}(\cdot)$ means the trace of a matrix. $N!$ denotes the factorial of the integer N. $E[\cdot]$ stands for the expected value of a random variable and $P(\cdot)$ denotes the probability of a discrete-valued random variable. $p_y(t)$ indicates the value of the probability density function (pdf) of a continuous-valued variable y at the position where $y = t$. The random variable and its realization will not be explicitly distinguished unless necessary.

II. System Model and Tomlinson-Harashima Precoder

From now on, we consider a discrete, one-dimensional Gaussian channel, and all quantities are real-valued and scalar. As shown in Figure 1, the base station transmits $x_1(\omega_1)$ during time slot t_1. The relay receives $y_r = x_1(\omega_1) + n_r$ during t_1, where n_r is noise, and generates the relaying signal $z = f(y_r)$ dedicated for user 1. During time slot t_2, the base station transmits x_2 to user 2 and the relay transmits z to user 1 through the same channel. Therefore the received signal at user 2 in t_2 can be written as

$$y = x_2 + z + n, \tag{1}$$

where n is noise. The design of the optimal relay mapping function $f(\cdot)$ is interesting and challenging, as discussed in [21], [22]. For example, we can choose the memoryless relaying function proposed in [21] to maximize the generalized signal-to-noise power ratio (GSNR) at user 1, or utilize the constellation rearrangement proposed in [22] to maximize the rate for user 1 if its direct link with the source exists. The joint optimization of the relay function and the modulator in the base station is rather complicated. To simplify the analysis and highlight the insights gained in this paper, hereafter we assume a perfect source-relay link[3] in Figure 1 with a deterministic relay mapping $z = \sqrt{\frac{P_r}{P_x}} x_1$. The DPC modulator in the base station that we envision maps an information symbol ω_2 from an M-ary alphabet ($\omega_2 \in \{0, \ldots, M-1\}$), and the interfering relay symbol $z \in \mathbb{R}$, onto a modulated symbol $x_2 \in \mathbb{R}$, through the (nonlinear) modulator mapping as follows

$$x_2 = X(\omega_2, z).$$

User 2 does not know z, but we shall assume that it knows the probability distribution of z, say $p_z(u)$. This assumption

[1]Extension to inphase/quadrature (narrowband) modulation, or to other orthogonal multiplexing formats is immediate by treating each dimension independently.

[2]Throughout we use "optimal" in the sense of *maximum mutual information* or *minimum error probability*. When not explicitly stated, we refer jointly to both these criteria.

[3]For $P_x \gg \sigma_r^2$, x_1 can be almost perfectly known/estimated at the relay.

is weak if z is drawn from a stationary and ergodic process, because then the base station can provide information about $p_z(u)$ to user 2. We assume that the noise is Gaussian: $n \sim \mathcal{N}(0, \sigma^2)$ where σ^2 is known. Furthermore, we assume that the available average transmit power is fixed to a constant P_x. With the optimization criterion of the mutual information $I(y; \omega_2)$, the problem is then to find the best possible mapping $x_2 = X(\omega_2, z)$ that maximizes $I(y; \omega_2)$, i.e.,

$$X(\omega_2, z) = \arg \max_{X:\ E[x_2^2] \le P_x} I(y; \omega_2), \tag{2}$$

where

$$\begin{aligned} I(y;\omega_2) &= H(\omega_2) - H(\omega_2|y) \\ &= \sum_{\omega_2=0}^{M-1} \int_{-\infty}^{\infty} p_y(y,\omega_2) \log P(\omega_2|y) dy - \sum_{\omega_2=0}^{M-1} P(\omega_2) \log P(\omega_2) \\ &= \sum_{\omega_2=0}^{M-1} \left[\int_{-\infty}^{\infty} p_y(y,\omega_2) \log \frac{p_y(y,\omega_2)}{p_y(y)} dy - P(\omega_2) \log P(\omega_2) \right] \\ &= \sum_{\omega_2=0}^{M-1} P(\omega_2) \int_{-\infty}^{\infty} p_y(y|\omega_2) \log \frac{p_y(y|\omega_2)}{\sum_{\omega_2'=0}^{M-1} p_y(y|\omega_2') P(\omega_2')} dy. \end{aligned} \tag{3}$$

The last equality comes from the fact that $\int_{-\infty}^{\infty} p_y(y|\omega_2) dy = 1, \forall \omega_2$. In practice, $I(y; \omega_2)$ can be easily computed by Monte-Carlo integration. Naturally $p_y(y|\omega_2)$ (and $I(y; \omega_2)$) depends on both the specific modulator mapping $X(\omega_2, z)$ and the distribution $p_z(u)$.

A. Tomlinson-Harashima precoding (THP)

THP is the best known available baseline for comparison and therefore we outline its principle here. THP first maps ω_2 onto a constellation point by modulating it via $x(\omega_2)$, and then subtracts the interference z from it. A modulo operation $\mathrm{mod}(\cdot, \Lambda)$ is then carried out so that the resulting transmitted signal falls into the region $[-\Lambda/2, \Lambda/2]$. Therefore we have

$$\begin{aligned} x_2 &= X(\omega_2, z) = \mathrm{mod}(x(\omega_2) - z, \Lambda), \\ y &= \mathrm{mod}(x(\omega_2) - z, \Lambda) + z + n = x(\omega_2) + k\Lambda + n = x(\omega_2) + e, \end{aligned}$$

where k is an integer which depends both on ω_2 and z. Note that the equivalent noise term $e = k\Lambda + n$ also depends on ω_2. In papers dealing with THP, the following heuristic (and suboptimal) detector is usually used:

$$\hat{\omega_2}_{\text{subopt}} = \operatorname*{argmin}_{\omega_2} |\mathrm{mod}(y, \Lambda) - x(\omega_2)|.$$

To find the minimum error-probability receiver for THP, first note that

$$p_y(y|\omega_2) = \sum_{k=-\infty}^{\infty} P(k|\omega_2) p_n(y - x(\omega_2) - k\Lambda),$$

where the integer k is random with the following conditional distribution:

$$\begin{aligned} P(k|\omega_2) &= P(x(\omega_2) - z \in [-(k+1/2)\Lambda, -(k-1/2)\Lambda]\ |\omega_2) \\ &= P(x(\omega_2) + (k-1/2)\Lambda \le z \le x(\omega_2) + (k+1/2)\Lambda\ |\omega_2) \\ &= F_z(x(\omega_2) + (k+1/2)\Lambda) - F_z(x(\omega_2) + (k-1/2)\Lambda). \end{aligned} \tag{4}$$

In (4), $F_z(t) = \int_{-\infty}^{t} p_z(u) du$ is the cumulative distribution function of z. The maximum a posteriori (MAP) receiver finds the most likely ω_2 when y is received:

$$\begin{aligned} \hat{\omega_2}_{\text{MAP}} &= \operatorname*{argmax}_{\omega_2} P(\omega_2|y) = \operatorname*{argmax}_{\omega_2} p_y(y|\omega_2) \\ &= \operatorname*{argmax}_{\omega_2} \sum_{k=-\infty}^{\infty} P(k|\omega_2) \exp\left(-\frac{(y - x(\omega_2) - k\Lambda)^2}{2\sigma^2}\right), \end{aligned} \tag{5}$$

where the second equality comes from the assumption of equally probable ω_2. In practice the sum in (5) can be truncated to a few terms since $P(k|\omega_2)$ decreases rapidly (exponentially if z is Gaussian) as $|k|$ increases. The difference in performance between the two receivers, however, is usually small except for "unlucky" choices of the mapping $x(\omega_2)$ and Λ, i.e., when $P(k \ne k_0|\omega_2)$ is significant where k_0 satisfies $\mathrm{mod}(y, \Lambda) = y - k_0\Lambda$.

III. Design of the Optimum Modulator

In this section we first find the optimal mapping modulator for binary signaling with binary interference and then generalize it to higher order modulations. A suboptimal modulator by maximizing the minimum distance among constellation points is also proposed by formulating an optimization problem.

A. Optimal mapping for binary signaling with binary interference

For discrete, binary random variables ω_2 and z (over $\mathbb{Z}$ and $\mathbb{R}$, respectively), we assume that

$$P(\omega_2{=}0) = P(\omega_2{=}1) = 1/2,\ \ P(z{=}-\beta) = P(z{=}\beta) = 1/2. \tag{6}$$

That is, the input alphabet is binary ($\omega_2 = 0, 1$) and the interference comes from a scaled BPSK constellation $z = \pm\beta$. Also, ω_2 and z are independent and all combinations of (ω_2, z) are equally likely. Therefore the mapping $X(\omega_2, z)$ can be explicitly written as

$$\begin{aligned} X(\omega_2 = 0, z = -\beta) \triangleq s_0, \quad X(\omega_2 = 0, z = \beta) \triangleq s_1, \\ X(\omega_2 = 1, z = -\beta) \triangleq s_2, \quad X(\omega_2 = 1, z = \beta) \triangleq s_3. \end{aligned} \tag{7}$$

By symmetry (ω_2 and z have symmetric probability densities), we must have $x \in \{-a, -b, b, a\}$ for some positive constants a, b. The problem is then to find suitable (a, b) and to map $s_0, ..., s_3$ onto the set $\{-a, -b, b, a\}$ such that $I(y; \omega_2)$ stated in (3) is maximized. Note that

$$p_y(y|\omega_2) = \sum_{z=\pm\beta} p_{y,z}(y, z|\omega_2) = \sum_{z=\pm\beta} p_y(y|\omega_2, z) P(z), \tag{8}$$

where

$$p_y(y|\omega_2, z) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y - z - X(\omega_2, z))^2}{2\sigma^2}\right).$$

There are $4! = 24$ permutations of the elements in $\{-a, -b, b, a\}$, of which 12 are redundant (a and b are not ordered). The set of all possible mappings (s_0, s_1, s_2, s_3) to be considered are:

$$\begin{aligned} &\text{(I) } (a,-a,b,-b); \quad \text{(II) } (a,-b,b,-a); \quad \text{(III) } (-a,-b,b,a); \\ &\text{(IV) } (-a,-b,a,b); \quad \text{(V) } (-a,b,a,-b); \quad \text{(VI) } (-a,a,b,-b); \\ &\text{(VII) } (-a,a,-b,b); \ \text{(VIII) } (-a,b,-b,a); \ \text{(IX) } (a,b,-b,-a); \\ &\text{(X) } (a,b,-a,-b); \quad \text{(XI) } (a,-b,-a,b); \quad \text{(XII) } (a,-a,-b,b). \end{aligned} \tag{9}$$

The mapping $X(\omega_2, z)$ is a deterministic function that assigns one of the values $\{-a, -b, b, a\}$ to x_2 for each possible pair (ω_2, z). Since the variables ω_2 and z are independent and equiprobable (see (6)), it follows that all four possibilities for x_2, viz. $x_2 \in \{-a, -b, b, a\}$ are equally likely. Thus the power constraint translates into $E[x_2^2] = (a^2 + b^2)/2 \leq P_x$. A straightforward approach, as stated in our preliminary work [20], is to perform an exhaustive search over a fine grid which contains all (a, b) that satisfy this constraint. And for each (a, b) we examine all the 12 mappings to identify the optimal modulation which generates the highest mutual information. This optimization process can be carried out off-line and the result can be stored in a look-up table (indexed by P_x/σ^2 and β^2/σ^2) with resolution as required.

The minimum error-probability receiver for the optimal (maximum mutual information) modulator has a rather simple form. To write it out explicitly, note from (8) that

$$\hat{\omega_2}_{\text{MAP}} = \underset{\omega_2}{\text{argmax}} \sum_{z=\pm\beta} \exp\left(-\frac{(y - z - X(\omega_2, z))^2}{2\sigma^2}\right).$$

When the assumption of a perfect source-relay link does not hold, i.e., when z is not perfectly known at the relay, the conditional probability $p_y(y|\omega_2)$ must be adjusted to reflect the reliability of z. Given the transmit power P_x and source-relay link noise power σ_r^2, the conditional probability (8) should be rewritten as

$$\begin{aligned} p_y(y|\omega_2) = (1 - P_e) &\sum_{z=\pm\beta} p_y(y|\omega_2, z)P(z) \\ +P_e &\sum_{z=\pm\beta} p_y(y|\omega_2, -z)P(z), \end{aligned}$$

where $P_e = Q(\sqrt{P_x/\sigma_r^2})$ is the error probability of detecting the BPSK modulated ω_1 (hence z).

B. Extension to higher order modulation

Despite the fact that the optimization can be done off-line, it is not directly feasible to extend the exhaustive search method proposed in Section III-A to higher-order modulation since the number of possible mappings increases explosively with the order of the modulation. For M-PAM signal with N-PAM interference, in total we have MN combinations for (ω_2, z) and therefore the same number of possible $X(\omega_2, z)$ values. Their amplitudes are symmetric in the real field $\mathbb{R}$ around the origin and hence at most half of them, i.e. $MN/2$, are free to choose under the power constraint. Besides, there are in total $(MN)!$ permutations of the set of MN parameters. Since $MN/2$ of these parameters have no ordering constraint, the number of all possible mappings is $\frac{(MN)!}{(MN/2)!}$. And then for each of these mappings, we still have to do an exhaustive grid search along $MN/2$ dimensions to find the optimal modulator for a particular combination of P_z/σ^2 and P_x/σ^2. For example, in the case of 4-PAM signaling with BPSK interference, there are in total $8!/4! = 1680$ different mappings and we have to do an exhaustive grid search over 4 dimensions. Therefore for higher-order modulation, the number of candidate mappings can become prohibitively large and makes the off-line exhaustive search computationally impractical. In what follows we will present a method which can greatly reduce the number of mappings.

We start with the special case with binary signaling and binary interference, as stated in (6). By comparing all the mappings in (9), we come up with the following observations:

1) Two mappings are said to be equivalent if one can be obtained from the other by exchanging $X(0, z)$ and $X(1, z)$ for all z;
2) Mappings satisfying $X(0, z)X(1, z) > 0$ will result in smaller distance between $\omega = 0$ and $\omega = 1$ in the received signal constellations, and therefore should not be considered;
3) Mappings should satisfy $|X(0, z) - X(1, z)| = |X(0, -z) - X(1, -z)|$.

All the equivalent mappings defined by *Observation* 1) are identical in the sense that a and b are commutable, and therefore we will group them together in a pair of parenthesis. For example, we group the following pairs of equivalent mappings together: (III, IX), (IV, X), (V, XI), and (VI, XII). By applying *Observation* 2), mappings I, II, VII, VIII are excluded. By applying *Observation* 3), mappings (IV, X) and (V, XI) are also excluded. Now we only have ***two*** groups left: (III, IX) and (VI, XII). We then search over a fine grid which contains all (a, b) that satisfy the power constraint, and for each (a, b) we only examine the above mentioned ***two*** mappings (one element from each group, say IX and XII) instead of ***twelve*** as in Section III-A.

For the general cases with uniformly distributed information symbols $\omega \in \{0, \ldots, M - 1\}$ and uniformly distributed interference $z \in \{z_0, \ldots, z_{N-1}\}$ with N-PAM modulation, by defining the modulation vector associated with ω as $\mathcal{X}(\omega) \triangleq [X(\omega, z)|\forall z] = [X(\omega, z_0), \ldots, X(\omega, z_{N-1})]$, the following principles can be applied to reduce the number of mapping candidates:

1) Mappings $X_1(\omega, z)$ and $X_2(\omega, z)$ are equivalent if they have the same vector set[4], i.e., $\{\mathcal{X}_1(\omega)|\forall\omega\} = \{\mathcal{X}_2(\omega)|\forall\omega\}$, where $\mathcal{X}_i(\omega) \triangleq [X_i(\omega, z)|\forall z]$, $i = 1, 2$;
2) For $\omega_i \neq \omega_j$, the elements in received signal constellation subset $\{X(\omega_i, z) + z|\forall z\}$ should be separated as far as possible away from any elements in $\{X(\omega_j, z) + z|\forall z\}$;
3) For each interference pair $(z, -z)$, the subsets $\{X(\omega, z) + z|\forall\omega\}$ and $\{X(\omega, -z) - z|\forall\omega\}$ should be equivalent in the sense that they are symmetric with respect to the origin.

The equivalent mappings defined by the first principle will be grouped together and all the mappings that do not follow the second and the third principles will be deemed "unfavorable" and therefore be dropped. For example, by applying the above principles, the number of mappings for 4-PAM signaling with BPSK interference can be reduced from 1680 down to 133.

C. Maximized minimum distance based sub-optimal modulator

As stated in Section III-B, the off-line optimization can be greatly simplified by reducing the number of mappings.

[4]The assignment of each modulation vector in $\{\mathcal{X}(0), \ldots, \mathcal{X}(M-1)\}$ to an information symbol $w \in \{0, \ldots, M-1\}$ should not affect the achievable rate or symbol error probability.

However, since the optimization in (2) is non-convex, the complexity of a grid search will increase exponentially with the number of searching dimensions. Therefore we propose here a low-complexity sub-optimal modulator based on the criterion of maximized minimum distances among the constellation points.

For uniformly distributed information symbols $\omega \in \{0, \dots, M-1\}$ and uniformly distributed interference z with N-PAM modulation, the distance between received signal constellation points for $\omega_i \neq \omega_j$ (omitting the noise term for simplicity) can be classified into two types

$$\begin{aligned} d_{\text{I}} &= |X(\omega_i, z_m) + z_m - (X(\omega_j, z_m) + z_m)| \\ &= |X(\omega_i, z_m) - X(\omega_j, z_m)|, \\ d_{\text{II}} &= |X(\omega_i, z_m) + z_m - (X(\omega_j, z_n) + z_n)| \\ &= |X(\omega_i, z_m) - X(\omega_j, z_n) + (z_m - z_n)|, \end{aligned} \tag{10}$$

where z_m, z_n are interference symbols.

There are in total $N_{\text{I}} = \frac{M(M-1)N}{2}$ type I distances d_{I} and $N_{\text{II}} = \frac{M(M-1)N(N-1)}{2}$ type II distances d_{II}. By reformulating the mapping $X(\omega, z)$ into a vector

$$\boldsymbol{x} = [X(0, z_0), ..., X(0, z_{N-1}), X(1, z_0), ..., X(M-1, z_{N-1})],$$

and denoting $\boldsymbol{x}(n)$ as the nth element of $\boldsymbol{x}$, we can rewrite (10) as follows

$$\begin{aligned} & d^2_{\text{I},k} = (\boldsymbol{x}(i_k) - \boldsymbol{x}(j_k))^2 = \boldsymbol{x}\boldsymbol{A}_k\boldsymbol{x}^T, \qquad (11) \\ & d^2_{\text{II},l} = (\boldsymbol{x}(i_l) - \boldsymbol{x}(j_l) + \eta_l)^2 = \boldsymbol{x}\boldsymbol{B}_l\boldsymbol{x}^T + 2\boldsymbol{x}\boldsymbol{b}_l^T + \eta_l^2, \\ & i_k, j_k, i_l, j_l \in \{1, ..., MN\},\ k = 1, ..., N_{\text{I}},\ l = 1, ..., N_{\text{II}}, \end{aligned}$$

where $\boldsymbol{b}_l$ are $1 \times MN$ sparse vectors each with only two non-zeros elements $\boldsymbol{b}_l(i_l) = \eta_l$ and $\boldsymbol{b}_l(j_l) = -\eta_l$, and $\boldsymbol{A}_k$ ($\boldsymbol{B}_l$) are $MN \times MN$ sparse symmetric matrices each with only four non-zero elements placed in their diagonal and anti-diagonal positions defined by i_k, j_k (i_l, j_l), i.e.,

$$\boldsymbol{A}_k \text{ or } \boldsymbol{B}_l = \begin{bmatrix} . & . & . & . & . \\ . & 1 & . & -1 & . \\ . & . & \ddots & . & . \\ . & -1 & . & 1 & . \\ . & . & . & . & . \end{bmatrix}, \ \boldsymbol{b}_l^T = \begin{bmatrix} . \\ \eta_l \\ \vdots \\ -\eta_l \\ . \end{bmatrix}. \tag{12}$$

The sub-optimal modulator can therefore be formulated based on (11) as an inhomogeneous quadratically-constrained quadratic program (QCQP) [23] problem,

$$\begin{aligned} \max_{\boldsymbol{x} \in \mathbb{R}^{MN}} \min_{\substack{k=1,\dots,N_{\text{I}} \\ l=1,\dots,N_{\text{II}}}} & \quad \{\boldsymbol{x}\boldsymbol{A}_k\boldsymbol{x}^T, \boldsymbol{x}\boldsymbol{B}_l\boldsymbol{x}^T + 2\boldsymbol{x}\boldsymbol{b}_l^T + \eta_l^2\} \\ \text{subject to} & \quad \boldsymbol{x}\boldsymbol{x}^T \leq MN \cdot P_x. \end{aligned} \tag{13}$$

The solution of (13) will yield constellations with large mutual information, since a constellation that offers a large constellation-constraint mutual information also has a large minimum distance. However, the exact solution of (13) is hard to find since the problem is non-convex. But after reformulation [23] and semi-definite relaxation (SDR) [24] approximation, we can introduce some new matrices

$$\tilde{\boldsymbol{A}}_k = \begin{bmatrix} \boldsymbol{A}_k & 0 \\ 0 & 0 \end{bmatrix},\ \tilde{\boldsymbol{B}}_l = \begin{bmatrix} \boldsymbol{B}_l & \boldsymbol{b}_l^T \\ \boldsymbol{b}_l & 0 \end{bmatrix},\ \boldsymbol{C} = \begin{bmatrix} \boldsymbol{I} & 0 \\ 0 & 0 \end{bmatrix},$$

and therefore obtain the following relaxed version of (13):

$$\begin{aligned} \max_{\boldsymbol{X} \in \mathbb{S}^{MN+1}} & \quad t \\ \text{subject to} & \quad \text{Tr}(\tilde{\boldsymbol{A}}_k\boldsymbol{X}) \geq t,\ k = 1, ..., N_{\text{I}}, \\ & \quad \text{Tr}(\tilde{\boldsymbol{B}}_l\boldsymbol{X}) + \eta_l^2 \geq t,\ l = 1, ..., N_{\text{II}}, \\ & \quad \text{Tr}(\boldsymbol{C}\boldsymbol{X}) \leq MN \cdot P_x,\ \boldsymbol{X} \succeq \boldsymbol{0}, \\ & \quad \boldsymbol{X}(MN+1, MN+1) = 1, \end{aligned} \tag{14}$$

where $\mathbb{S}^n$ denotes the set of $n \times n$ symmetric matrices. Since (14) is an instance of semi-definite programming [23], it can be solved in a numerically reliable and efficient fashion by convex optimization software, e.g. CVX [25]. However, the globally optimal solution $\boldsymbol{X}^*$ to (14) in general has rank greater than 1, and therefore is not a feasible solution to the original problem (13). We can extract from $\boldsymbol{X}^*$ a feasible (normally sub-optimal) solution $\boldsymbol{x}$ to (13) through randomization with provable approximation accuracy, see [24] and references therein for more details.

Note that (13) and (14) are actually a realization of the *Principle* 2) stated in Section III-B. Besides, *Principle* 3) can also be utilized to add extra MN linear constraints to (13). Then following the same procedure of reformulation and relaxation, we can formulate a new optimization problem similar to (14). Detailed discussions on reformulation, relaxation, and approximation are omitted here due to space limitations.

For the special case of $M = N = 2$, by confining ourselves to the selected mappings IX and XII in (9), we can solve (13) analytically (see Appendix A for a detailed derivation), resulting in a closed-form solution for the modulation mapping $X(\omega_2, z)$ as follows:

$$\begin{cases} \text{XII}, a = \sqrt{P_x},\ b = \sqrt{P_x}, & \text{if } P_x \leq \beta^2; \\ \text{XII}, a = \beta,\ b = \sqrt{2P_x - \beta^2}, & \text{if } \beta^2 < P_x < 5\beta^2; \\ \text{IX}, a = \sqrt{P_x - \beta^2} + \beta,\ b = \sqrt{P_x - \beta^2} - \beta, & \text{if } P_x \geq 5\beta^2. \end{cases} \tag{15}$$

This sub-optimal modulation can be carried out on-line given the instantaneous channel conditions.

IV. Optimized THP for Arbitrary Signal and Interference

In Section III we have discussed the modulator design $x = X(\omega, z)$ given information symbols ω from an M-ary alphabet and an interference signal z modulated with N-PAM. We provided the optimal nonlinear mapping based on an exhaustive grid search, and a sub-optimal mapping based on convex optimization and relaxation. For an interference signal with a more general distribution (say Gaussian), however, it appears impractical (at least without approximations) to design the Costa modulator based the methods proposed in Section III. The THP modulation, however, fits for arbitrary signal and interference constellations and therefore can be regarded as a good candidate for such scenarios. The advantage of staying within the framework of THP is twofold. First, there are only two parameters to optimize over, as shown later in this section. Second, THP with heuristic parameter choices (which is commonly used in the literature) is known to provide significant gains over no-interference-cancellation.

Let α be half of the minimum distance between the uniformly distributed constellation points, i.e., $x \in \{-\alpha, \alpha\}$ for

BPSK and $x \in \{-3\alpha, -\alpha, \alpha, 3\alpha\}$ for 4-PAM modulation, and let Λ be the parameter for modulo operations. Then THP modulation with transmit power constraint P_x has two parameters α, Λ to optimize over. THP with the heuristic parameter choice $\Lambda = 2M\alpha$ for M-PAM modulation appears to be customary and is the choice described in Chapter 10 of [2]. This method, referred as heuristic THP hereafter, is rather simple and can be used for general signal constellations and interference distributions. The actual value of Λ (therefore the value of α) is determined by the interference power P_z and the transmit power constraint P_x. For higher-order modulation with equiprobable information symbols, the resulting transmitted signal after modulo operation turns to be approximately uniformly distributed in the region of $[-\Lambda/2, \Lambda/2]$, resulting in a transmit power of $\frac{\Lambda^2}{12}$. Hence we have

$$\Lambda = \sqrt{12P_x}, \qquad \alpha = \Lambda/(2M). \tag{16}$$

However, when the modulation order is small, such as for a BPSK modulated signal, this approximation turns out to be biased. For the case of binary signaling with binary interference, the exact value of α (hence also Λ) can be determined as follows (see Appendix B for a detailed derivation):

$$\begin{cases} \Lambda = 4\alpha; \\ \alpha = \sqrt{P_x - \beta^2}, & \text{if } P_x \geq 2\beta^2; \\ \alpha = \frac{2\beta + \sqrt{5P_x - \beta^2}}{5}, & \text{if } \beta^2/5 \leq P_x < 2\beta^2; \\ \alpha = \frac{2n\beta + \sqrt{(4n^2+1)P_x - \beta^2}}{4n^2+1}, & \text{if } \frac{\beta^2}{4n^2+1} \leq P_x < \frac{\beta^2}{4(n-1)^2+1}, \\ & \text{with integer } n \geq 2. \end{cases} \tag{17}$$

This heuristic parameter choice $\Lambda = 2M\alpha$, however, appears "unlucky" in some specific situations. Therefore we propose to use optimized parameters α, Λ for THP. This optimization can be accomplished via a similar procedure as described in Section III, i.e., performing a grid search over all Λ, α which satisfy the power constraint P_x. Unlike for the optimal modulator where the search dimension increases with the modulation order, the optimization problem here is always two-dimensional. The search over all possible mappings is not necessary either.

V. Non-DPC Benchmarks

We present here two non-DPC approaches as a reference to evaluate our DPC schemes.

A. Relay uses an orthogonal channel

The relay can use an orthogonal channel to help user 1 so that the relaying signal z will not interfere with the reception of x_2, under the same available resource (time, bandwidth, energy) constraints. We use time sharing between the relay and the base station over the same bandwidth to realize the orthogonal transmission. Let $\rho \in [0, 1]$ be the time sharing coefficient for the transmission from the base station to user 2, and assume that the transmitted signal $x_2 = X(\omega_2)$ is uniform M-PAM modulated and subject to the power constraint $E[x_2^2] = P_x/\rho$. Hence the total used energy $\rho \cdot E[x_2^2]$ remains the same as for the other schemes. The mutual information conveyed through this channel is

$$I_\rho(y; \omega_2) = \rho \cdot I(y; \omega_2), \tag{18}$$

where $I(y; \omega_2)$ is calculated according to (3), with

$$p_y(y|\omega_2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-(y - X(\omega_2))^2/(2\sigma^2)).$$

Note that ρ affects the throughput of both user 1 and user 2. Therefore, to choose ρ requires total throughput and fairness considerations. We will choose $\rho = 1/2$ in our simulation for simplicity.

B. Interference cancellation at the receiver

One can also use no precoding at the base station but perform interference cancellation at user 2. When the interference is much stronger than the signal, user 2 can perform successive interference cancellation (SIC) [2]: It first decodes ω_1 treating $x_2 = X(\omega_2)$ as noise, and then subtracts the relaying signal $z(\omega_1)$ from y and uses the remaining signal to decode ω_2. But for moderate and weak interference, SIC will not work. We therefore propose here a new interference cancellation scheme which works for all cases by keeping user 2 receiving signals in both time slot t_1 and t_2. The received signals at user 2 during t_1 and t_2 can be written as

$$y_1 = x_1(\omega_1) + n_1, \; y = X(\omega_2) + z(\omega_1) + n, \tag{19}$$

where n_1 is additive white Gaussian noise and $x_1(\omega_1)$ is the signal for user 1 under average power constraint P_x. The mutual information between the transmitted information symbol ω_2 and the received signals (y_1, y) can therefore be written as

$$I(y_1, y; \omega_2) = \sum_{\omega_2} \int_{y_1} \int_{y} p_{y_1,y}(y_1, y|\omega_2) P(\omega_2) \cdot \log \frac{p_{y_1,y}(y_1, y|\omega_2)}{\sum_{\omega_2'} p_{y_1,y}(y_1, y|\omega_2') P(\omega_2')} dy dy_1, \tag{20}$$

where $p_{y_1,y}(y_1, y|\omega_2)$ can be obtained from the Bayes rule

$$p_{y_1,y}(y_1, y|\omega_2) = \sum_{\omega_1} p_{y_1,y}(y_1, y|\omega_1, \omega_2) P(\omega_1) = \sum_{\omega_1} p_{y_1}(y_1|\omega_1) p_y(y|\omega_1, \omega_2) P(\omega_1). \tag{21}$$

The second equality in (21) comes from the fact that y_1 and y are independent if ω_1 (and therefore x_1 and z) is known. We further get from (19) that

$$p_{y_1}(y_1|\omega_1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y_1 - x_1(\omega_1))^2}{2\sigma^2}\right),$$
$$p_y(y|\omega_1, \omega_2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y - X(\omega_2) - z(\omega_1))^2}{2\sigma^2}\right).$$

The maximum a posteriori receiver given (y_1, y) is therefore

$$\hat{\omega}_{2_{\text{MAP}}} = \underset{\omega_2}{\operatorname{argmax}}\, P(\omega_2|y_1, y) = \underset{\omega_2}{\operatorname{argmax}}\, p_{y_1,y}(y_1, y|\omega_2), \tag{22}$$

where the second equality comes from the assumption of equally probable ω_2.

VI. Numerical Results

In this section we will evaluate our proposed DPC schemes in terms of mutual information, coded BER and energy efficiency. Bit-level Monte-Carlo simulation is used to obtain the results.

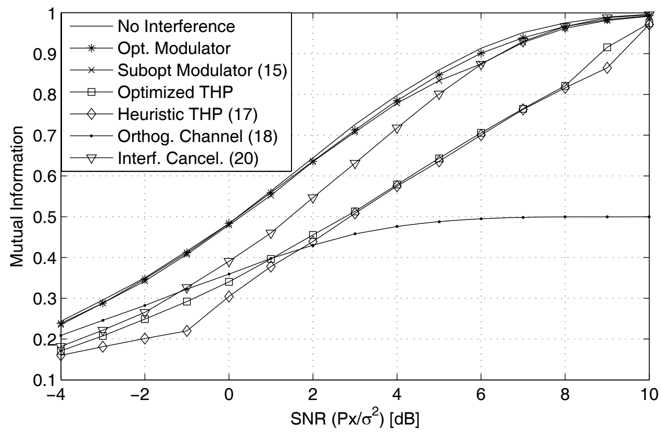

Fig. 2. Mutual information for binary signaling in binary interference, with $\mathrm{INR}(P_z/\sigma^2) = 6$ dB.

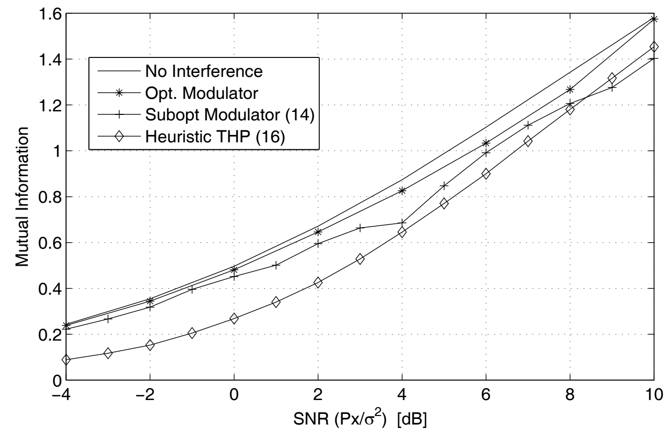

Fig. 3. Mutual information for quaternary signaling in binary interference, with $\mathrm{INR}(P_z/\sigma^2) = 6$ dB.

A. Mutual information

We first evaluate the performance in terms of mutual information. Figure 2 shows $I(y;\omega_2)$ for binary signaling with binary interference, as a function of the signal-to-noise ratio (SNR, P_x/σ^2)[5] at a fixed interference-to-noise ratio (INR, P_z/σ^2) of 6 dB. The optimal modulator and the sub-optimal modulator are slightly worse compared to the no-interference case. THP with optimized parameters (α, Λ), which uniformly outperforms THP with the heuristic parameter choice[6] $\Lambda = 4\alpha$, experiences notable degradation in low to medium SNR regions but converges to the optimal modulator at high SNR. Interference-cancellation performs well in high SNR regions but suffers from the corrupted observation of the interference in low to medium SNR regions. By using orthogonal channels with time sharing $\rho = 1/2$, the performance is relatively good in low SNR regions where the benefits of excluding interference dominate (this is the power-limited regime). In medium and high SNR regions (i.e., in the bandwidth-limited regime), however, the penalty of shortening the transmission time (and therefore less channel use) becomes the bottleneck.

Note that although the mapping parameters for the optimal modulator and for the optimized THP might change with the resolution of the searching grid, the actual performance will only differ slightly. The performance degradation of the optimal modulator compared to the no-interference case varies with the INR, and the maximum loss (less than 1.5 dB in SNR) appears when the INR is about 0 dB, i.e., when $P_z \simeq \sigma^2$, as demonstrated in Figure 4 of [20].

In Figure 3 we present the case for quaternary signaling (4-PAM) with binary interference, focusing on the performance of the optimal modulator and the sub-optimal modulator (14). The optimal modulator, in all SNR regions, suffers only a minor performance degradation compared to the no-interference case and achieves a significant gain (up to 3 dB at low SNR) over heuristic THP. The curve of the sub-optimal modulator is not smooth due to the fact that it is an approximate solution of (13) based on relaxation and randomization, as discussed in Section III-C. The performance of optimized THP, not shown here to improve the readability of this figure, lies in between the curves of the optimal modulator and the heuristic THP.

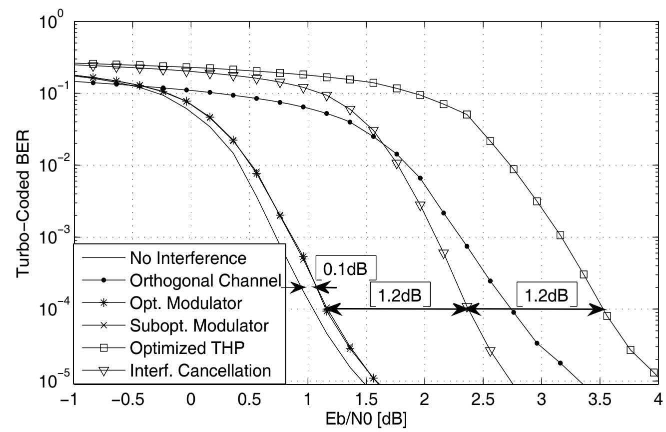

Fig. 4. BER of binary signaling in binary interference with a rate-$1/3$ turbo code with INR $(P_z/\sigma^2) = 6$ dB.

B. Coded bit error rate

Next, we demonstrate that the gains predicted by calculating the mutual information actually do indicate what one can achieve in practice. Towards this end we think of the system in Figure 1 as an inner channel, and concatenate it with a rate-1/3 turbo code [26], [27] (memory length 2, generators 7_8 and 5_8, block length 1000, interleaver from the WCDMA standard, Max-LOG-MAP decoding, 8 iterations). When time sharing (with $\rho = 1/2$) is used, the encoded bit streams have to be punctured to rate $2/3$, with the following puncturing patterns used at the two component encoders

$$\boldsymbol{P}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{P}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

The decoding metrics were computed by evaluating $\log P(\omega_2|y)$ (for the interference cancellation scheme, $\log P(\omega_2|y_1, y)$ is used instead). More precisely, the log-likelihood ratios are used as soft input to the turbo decoder as follows:

$$L(y) = \log \frac{p_y(y|\omega_2 = 1)}{p_y(y|\omega_2 = 0)} = \log \frac{P(\omega_2 = 1|y)}{P(\omega_2 = 0|y)},$$

[5]Strictly speaking, the actual SNR may be less than P_x/σ^2 because the optimal modulator does not necessarily use all available power. Yet we refer to P_x/σ^2 as SNR because this facilitates a well-defined comparison with the no-interference case.

[6]The comparisons in [20] used $\Lambda = 3\alpha$, which results in slightly worse performance for the heuristic THP.

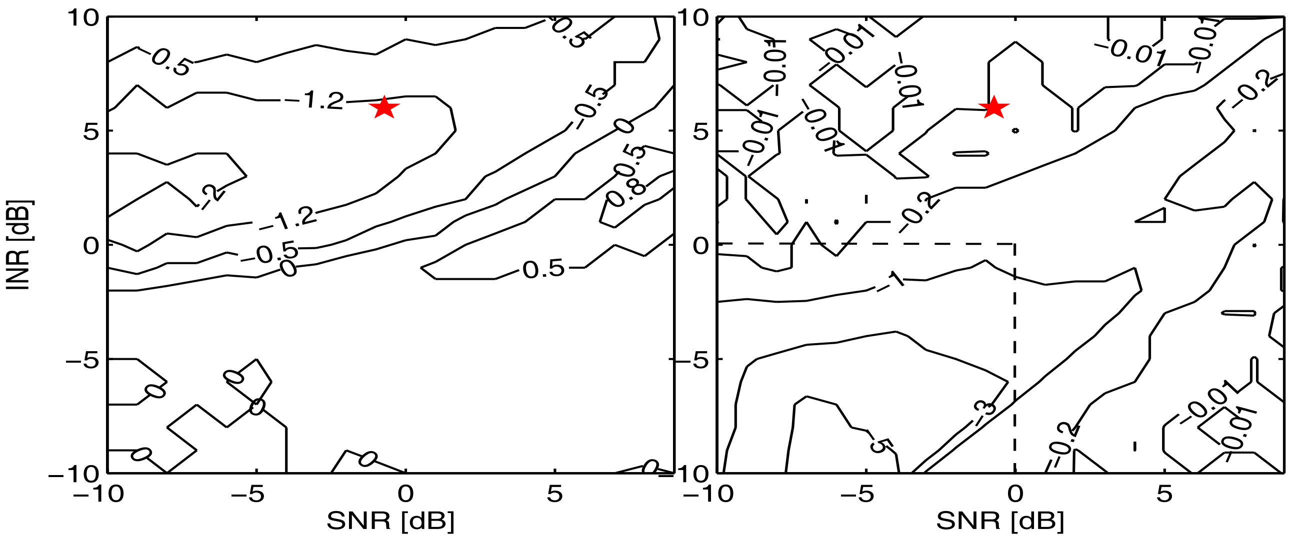

Fig. 5. Energy efficiency (difference in terms of equivalent SNR, in dB) of interference-cancellation (left) and the sub-optimal modulator (right) compared to the optimal modulator. A positive number indicates better energy efficiency.

$$L(y_1, y) = \log \frac{p_{y_1,y}(y_1, y|\omega_2 = 1)}{p_{y_1,y}(y_1, y|\omega_2 = 0)} = \log \frac{P(\omega_2 = 1|y_1, y)}{P(\omega_2 = 0|y_1, y)},$$

where $P(\omega_2{=}0){=}P(\omega_2{=}1){=}1/2$ is assumed. As shown in Figure 4, for a strong interference scenario with INR=6 dB and for a required BER of 10^{-4}, the optimal modulator (and the sub-optimal modulator) suffers only a 0.1 dB loss compared to the no-interference case, and shows a gain of 1.2 dB to the interference-cancellation scheme and 2.4 dB to the optimized THP. These gains approximately equal the difference in required SNR to achieve a mutual information of ~ 0.33 bits/channel use (or slightly larger, since the code is not capacity achieving) as shown in Figure 2. The same conclusion holds for heuristic THP (not shown here to simplify the figure).

C. Energy efficiency

In order to measure the energy efficiency of the different schemes, we defined "equivalent SNR" γ_X as the required SNR for scheme X to achieve the same mutual information as in the no-interference case despite the presence of interference. That is, given an SNR P_x/σ^2 and an INR P_z/σ^2, there exists a constant $\gamma_X(P_x/\sigma^2, P_z/\sigma^2)$ such that

$$I_X(\gamma_X) = I_{\text{no interf.}}(P_x/\sigma^2), \tag{23}$$

where $I_{\text{no interf.}}(P_x/\sigma^2)$ is the end-to-end mutual information with SNR P_x/σ^2 for the no-interference case, and $I_X(\gamma_X)$ is the mutual information for scheme X at an SNR of γ_X.

In Figure 5, we compare the energy efficiencies of the interference-cancellation (left) and of the sub-optimal modulator (right) to the optimal modulator in different channel conditions. Their performance difference in terms of equivalent SNR ($\gamma_{\text{opt. mod.}} - \gamma_X$, in dB) is shown as contour plots. A positive number indicates that the scheme has a better energy efficiency than the optimal modulator. The interference-cancellation scheme achieves slightly better performance than the optimal modulator when the INR is around 0 dB and the SNR is medium or high, with the highest gain being 0.8 dB. When the INR is larger than 0 dB and the SNR is low, the interference-cancellation scheme suffers a performance loss of up to 2 dB. The loss in energy efficiency of the sub-optimal modulator is rather small except in the region indicated by the dashed line where both the SNR and the INR are relatively small.[7] The SNR loss for optimized THP, not shown here due to space limitation, is much larger. The performance difference demonstrated in Figure 4 has been confirmed here, as shown by the $\star$ in Figure 5.

D. Optimized THP with Gaussian interference

We next demonstrate partial interference cancellation at the transmitter via the optimized THP proposed in Section IV. For this purpose we consider transmission of binary (BPSK) and quaternary (4-PAM) symbols on a channel with Gaussian interference. Figure 6 shows the result for both heuristic and optimized THP (computation of the optimal modulator is not directly feasible for this case; cf. Section IV). It is clear that we can gain from optimizing the parameters of THP. For quaternary signaling, the gain is significant especially in low signal-to-interference ratio (SIR, P_x/P_z) regions where interference dominates. This indicates that THP is a fairly effective (yet strictly suboptimal) means for combating Gaussian interference known at the transmitter. The gain achieved by optimizing the parameters of THP is much smaller in the binary case, however.

VII. Summary of Results and Conclusions

In this paper we have studied DPC solutions for a relay-aided downlink channel that partially solve the Costa precoding problem using symbol-by-symbol processing. We started from the simplest scenario of binary signaling with binary interference, and derived the optimal modulator which maximizes the mutual information between the transmitter and

[7] We are more interested in interference dominated channels where DPC is most useful.

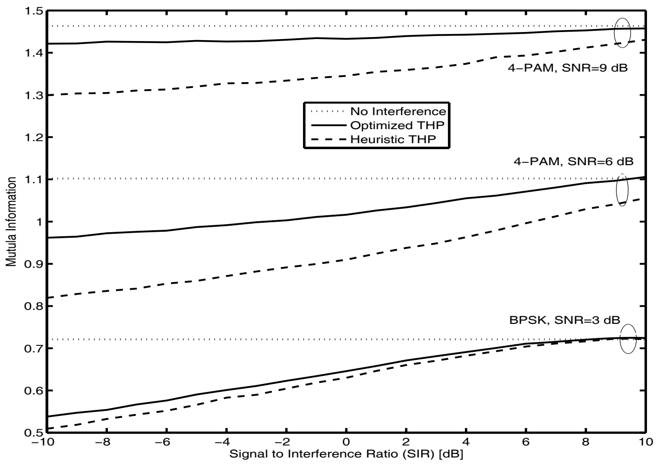

Fig. 6. Mutual information of the optimized THP versus heuristic THP for binary ($M = 2$) and quaternary ($M = 4$) signaling with Gaussian interference and Gaussian noise.

the receiver. By proposing a mapping set size reduction method, we extended this approach to finite-alphabet signaling and interference. We also proposed a sub-optimal modulator based on the maximization of the minimum constellation distance, which was formulated as a QCQP optimization problem and approximately solved by convex optimization after relaxation. A closed-form solution of the sub-optimal modulator was obtained for the case of binary signaling with binary interference, and the performance degradation is very limited compared to the optimal modulator in most interesting scenarios. For arbitrary signal and interference, we proposed an optimized version of THP that outperforms the THP with heuristic parameters. Our proposed DPC solutions were evaluated by simulation of mutual information, coded BER, as well as energy efficiency, and compared to three benchmark schemes, namely THP, orthogonal transmission, and receiver centric interference cancellation. Simulation results showed that both the optimal and sub-optimal modulators typically outperform THP, even when the parameters of the latter are optimally chosen. For example, in high INR scenarios the optimal/sub-optimal modulator can outperform the receiver centric interference cancellation scheme by about 1 dB and outperform the optimized THP by 2$\sim$3dB. Furthermore, we have demonstrated that the gains predicted by our analysis translate directly into energy savings in a turbo coded communication link. Mutual information is therefore a relevant performance measure.

Our study indicates that rather impressive transmitter interference cancellation performance can be achieved in a single dimension. This result serves as motivation to further study low-complexity approaches to the Costa problem. Also, an implementation of DPC schemes in practice will likely rely on operations in a space of small dimensions, so the problem studied here appears to be highly relevant.

Appendix A
Derivation of the Sub-Optimal Modulator

For M=2 and N=2, there are in total four different symbol distances among received signal constellation points, namely,

$$d_1 = |X(0,-\beta) - X(1,-\beta)|,\ d_4 = |X(0,\beta) - X(1,\beta)|,$$
$$d_2 = |X(0,-\beta) - X(1,\beta) - 2\beta|,\ d_3 = |X(0,\beta) - X(1,-\beta) + 2\beta|.$$

As stated in Section III-B, only two mappings IX and XII as stated in (9) need to be considered. We can find from mappings IX and XII that $d_1 = d_4$, and therefore only $\{d_1, d_2, d_3\}$ are used to identify the modulator $X(\omega_2, z)$.

When $P_x \leq \beta^2$, using mapping XII with a=b=$\sqrt{P_x}$ will result in d_2=$d_3 > d_1$=$2\sqrt{P_x}$. The optimal detector which compares $|y|$ with the threshold β will give almost the same performance as the no-interference case.

When P_x>β^2, we can calculate the maximized minimum distance for each mapping and then identify the larger one. Without loss of generality, we assume $a, b \geq 0$ in the following. For mapping IX we have $d_{\min} = \min\{a+b, 2|\beta-a|, 2(\beta+b)\}$. If $\beta \geq a$, we have $2(\beta+b) \geq 2(a+b) > a+b$ which means $d_{\min} = \min\{a+b, 2(\beta-a)\}$. The maximum of $d_{\min}$ is achieved when $a+b = 2(\beta-a)$. Combine this condition with the power constraint $a^2 + b^2 = 2P_x$, we can formulate a new equation of a as $5a^2 - 6\beta a + 2\beta^2 - P_x = 0$, which has two roots

$$a = \frac{3\beta - \sqrt{5P_x - \beta^2}}{5},\ a = \frac{3\beta + \sqrt{5P_x - \beta^2}}{5}.$$

The former root is valid (greater than 0) only if $P_x \leq 2\beta^2$ and the latter root conflicts with the precondition $a \leq \beta$. Hence for $\beta^2 < P_x \leq 2\beta^2$ we have

$$d^*_{\text{IX}} = \frac{4\beta + 2\sqrt{5P_x - \beta^2}}{5}, \tag{24}$$
$$\text{with } a = \frac{3\beta - \sqrt{5P_x - \beta^2}}{5}, b = \frac{\beta + 3\sqrt{5P_x - \beta^2}}{5}.$$

If $\beta < a$, we have $d_{\min} = \min\{a+b,\ 2(a-\beta),\ 2(\beta+b)\}$, whose maximum is achieved when $a+b = 2(a-\beta) = 2(\beta+b)$, i.e. $b = a - 2\beta$. Combine this with the power constraint, we get

$$a^2 - 2\beta a + 2\beta^2 - P_x = 0,$$

which has a valid solution (only if $P_x \geq 2\beta^2$) as follow

$$a = \beta + \sqrt{P_x - \beta^2},\ b = \sqrt{P_x - \beta^2} - \beta.$$

Hence for $P_x \geq 2\beta^2$ we have

$$d^*_{\text{IX}} = 2\sqrt{P_x - \beta^2}, \tag{25}$$
$$\text{with } a = \beta + \sqrt{P_x - \beta^2},\ b = \sqrt{P_x - \beta^2} - \beta.$$

When mapping XII is used, $d_{\min} = \min\{a+b, |2\beta+b-a|\}$. If $2\beta+b-a < 0$, the maximum of $d_{\min}$ is achieved when $a+b = a-b-2\beta$, i.e., $b+\beta$=0 which is impossible. If $2\beta+b-a > 0$, the maximum of $d_{\min}$ is achieved when $a+b = 2\beta+b-a$, i.e. $a = \beta$. Hence $b = \sqrt{2P_x - \beta^2}$ with $d^*_{\text{XII}} = \beta + \sqrt{2P_x - \beta^2}$.

Compared with d^*_{IX} in (24) and (25), d^*_{XII} has greater value for $\beta^2 < P_x < 5\beta^2$ and therefore mapping XII will be selected in this region and mapping IX will be selected when $P_x \geq 5\beta^2$. Together with the finding for $P_x \leq \beta^2$, one can easily conclude the results shown in (15).

Appendix B
Parameters for Heuristic THP

For binary signaling $w \in \{-\alpha, \alpha\}$ with binary interference $z \in \{-\beta, \beta\}$, there are four different combinations/values $(\beta+\alpha, \beta-\alpha, -\beta+\alpha, -\beta-\alpha)$ subject to modulo operation with Λ=4α to ensure $|x| \leq \Lambda/2$. By the assumption of equiprobable signals and interference as stated in (6), only

2 out of these 4 values are of interest due to their amplitude symmetry. Without loss of generality, we just select $\beta+\alpha$ and $\beta-\alpha$.

When $\beta \leq \alpha$, all the values satisfy the requirement $|x| \leq \Lambda/2$ and therefore we have

$$P_x = E[x^2] = \alpha^2 + \beta^2 \geq 2\beta^2.$$

Hence for $P_x \geq 2\beta^2$ we have $\alpha = \sqrt{P_x - \beta^2}$. When $\alpha < \beta \leq 3\alpha$, we have

$$2\alpha < \beta + \alpha \leq 4\alpha; \quad 0 < \beta - \alpha \leq 2\alpha.$$

After modulo operation the resulting x has average power

$$P_x = E[x^2] = 5\alpha^2 - 4\beta\alpha + \beta^2 < 2\alpha^2,$$

which has only one feasible solution $\alpha = \frac{2\beta+\sqrt{5P_x-\beta^2}}{5}$ for $\beta^2/5 \leq P_x < 2\beta^2$. When $(2n-1)\alpha < \beta \leq (2n+1)\alpha$, for $n = 2, 3, ...$, we have

$$2n\alpha < \beta + \alpha \leq (2n+2)\alpha, \quad (2n-2)\alpha < \beta - \alpha \leq 2n\alpha.$$

For n even, the modulo operation will subtract $2n\alpha$ from the above two values and result in $\beta - (2n-1)\alpha$ and $\beta - (2n+1)\alpha$ respectively; for n odd, the modulo operation will subtract $(2n+2)\alpha$ and therefore result in $\beta - (2n+1)\alpha$ and $\beta - (2n-1)\alpha$. Hence we conclude that

$$P_x = E[x^2] = (4n^2+1)\alpha^2 - 4n\beta\alpha + \beta^2 < 2\alpha^2.$$

Similarly, by solving the above equation for α we can get $\alpha = \frac{2n\beta+\sqrt{(4n^2+1)P_x-\beta^2}}{4n^2+1}$ when $\frac{\beta^2}{4n^2+1} \leq P_x < \frac{\beta^2}{4(n-1)^2+1}$. Another solution is infeasible and thus dropped.

Summarize all the above derivation we have proved (17).

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