

Freight transport platoon coordination and departure time scheduling under travel time uncertainty

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Abstract

The paper formulates and analyzes a freight transport platoon coordination and departure time scheduling problem under travel time uncertainty. The expected cost minimization framework accounts for travel time cost, schedule miss penalties and fuel cost. It is shown that platooning is beneficial only when scheduled arrival times differ less than a certain threshold. Travel time uncertainty typically reduces the threshold schedule difference for platooning to be beneficial. Platooning in networks is less beneficial on converging routes than diverging routes, due to delay at the merging point. The model provides valuable insights regarding platooning benefits for freight transport planning.

Keywords: heavy-duty vehicle platooning, freight transport, scheduling, travel time uncertainty

1. Introduction

A platoon is an array of vehicles that drive together with short inter-vehicle distances in the same lane. The concept of platooning was originally proposed by California Partners for Advanced Transportation Technology (PATH) (Varaiya, 1993). Since then, the topic has attracted interest from researchers in automatic control, wireless communication and other areas, mostly devoted to maneuvering vehicles efficiently and safely when driving closely to each other. Ideally (i.e., when communication latency is neglected and the entire system functions properly), the vehicles in the platoon act almost simultaneously if equipped with adaptive cruise control (ACC) or cooperative adaptive cruise control (CACC) systems.

The benefits of platooning have been asserted in a number of studies. Hucho (2013) reported a reduction in air drag coefficient by about 22% for the second vehicle in a platoon with 50 m inter-vehicle distance, and about 32% reduction with 20 m distance. This slipstreaming effect occurs because of a reduction in the dynamic pressure at the tail. Even the leading vehicle obtains an air drag reduction of about 4% with 10 m distance. Regarding fuel consumption, experiments show that the following truck may save about 21% fuel with 10 m distance and constant speed 80 km/h, and the leading truck experiences about 7% reduced fuel consumption (Bonnet and Fritz, 2000). Given that vehicle CO₂ emissions are directly proportional to fuel consumption, simulations have showed that a 40% penetration rate of platooning among heavy-duty vehicles on highways can achieve a 2.1% reduction in total vehicle CO₂ emissions with 10 m distance (Tsugawa et al., 2011). The reduction is even greater with shorter distances.

Although there is no uniform legislation world-wide regarding safe driving distances between trucks, with the help of autonomous driving technology the safe distance can be very short. For example, Alam et al. (2014) have reported that when driving at the speed 90 km/h, a 1.2 m inter-vehicle distance can be maintained for two identical heavy-duty vehicles without any risk of collision. Under a worst-case scenario (considering system uncertainties), a 2 m distance will suffice.

To make use of the advantages of platooning, a coordination strategy is needed. By coordination, vehicles may have to adjust their scheduled departure times to form a platoon so that the fuel consumption can be reduced. If the time constraints are soft (with penalties), changing departure times may give rise to other kinds of costs. The scheduling of vehicle departure times so that the vehicles involved can form platoons and reduce fuel consumption without inducing too much extra cost is therefore an important problem, in particular when travel times are uncertain.

The objective of this study is to formulate and analyze the platoon coordination and departure time scheduling problem under travel time uncertainty. No assumptions are made regarding the travel time distribution other than that it is continuous with support on the positive real axis. The paper considers the setting where platooning is

planned in advance by a transport carrier in control of all involved vehicles. The transporter is assumed to minimize an expected transport cost function involving fuel cost, cost related to travel time (drivers' wages, opportunity cost etc.), and schedule miss penalties in relation to a fixed scheduled arrival time. The decision variables considered are the departure times of the vehicles and the binary choice whether they should platoon or drive independently; no constraints in terms of pick-up time windows are assumed. It is assumed here that all vehicles involved are driven manually but are equipped with technology such as cooperative adaptive cruise control (CACC) systems, which facilitates driving with the short inter-vehicle distances required for fuel savings.

Platooning reduces fuel consumption, but may increase schedule miss penalties in cases where scheduled arrival times differ among vehicles. The trade-off between the two cost components determines whether it is more profitable to form a platoon and thus save fuel at the risk of incurring lateness or earliness penalty. The components of the transport cost are not independent of each other, which adds a level of complexity to the scheduling problem. In particular, travel time uncertainty influences both fuel consumption and schedule miss penalties. The paper demonstrates that travel time uncertainty has significant impacts on the benefits of platooning compared to independent driving, which have not been previously acknowledged in the literature.

The schedule miss cost component in the platoon scheduling problem builds on the model for a single trip originally formulated by [Vickrey \(1969\)](#). The model assumes that late or early arrival in relation to a scheduled arrival time are associated with costs that are proportional to the schedule miss, with typically a higher unit cost for late arrivals. [Fosgerau and Karlström \(2010\)](#) derive the optimal departure time in order to minimize expected schedule miss cost. The model is extended to chains of trips in [Jenelius \(2012\)](#), and to commuting for meetings in [Fosgerau et al. \(2014\)](#). The latter study assumes that each traveler seeks to minimize her own cost function in a non-cooperative game setting.

The paper first formulates the platoon scheduling problem for two vehicles with different scheduled arrival times on a common route and shows that the problem of minimizing total cost (schedule penalties, travel time cost and fuel cost) is convex. The impacts of the schedule difference and the travel time uncertainty on the optimal departure time and minimum expected transport cost are analyzed. It is shown that there exists a threshold schedule difference below which platooning is beneficial. The impact of travel time uncertainty on the threshold schedule difference is studied in a numerical example. The problem is then extended to network configurations of diverging and converging routes, respectively. It is shown that platooning on converging routes, where vehicles must coordinate at the merging point, leads to higher schedule miss and driving costs than on diverging routes, where vehicles split after platooning. This distinction is an effect of travel time uncertainty.

The remainder of the paper is organized as follows. Section 2 discusses relevant studies in the literature. Section 3 introduces the transport cost function and describes the modeling of driving cost, schedule miss cost and fuel cost. In Section 4, the scheduling problem for independent and platooning vehicles on a common route is formulated, and the impacts of different parameters are analyzed. A numerical example highlighting the trade-off between fuel savings and schedule miss penalties is given in Section 5. In Section 6, the problem is extended to networks of diverging and converging routes. Section 7 discusses extensions of the model to more vehicles. Section 8 concludes the paper.

2. Literature review

Research on vehicle platooning has so far been mainly driven by the technical requirements for safe and efficient autonomous driving. Some studies have considered vehicle sorting, merging and splitting at network nodes or at highway on-ramps and off-ramps. [Hall and Chin \(2005\)](#) focus on the formation and characteristics of platoons on ramps. They define a set of platoon formation strategies and develop analytical models for calculating performance measures. In the study, vehicles are allowed to form groups by lanes at highways on-ramps. [Contet et al. \(2006\)](#) present an approach for decentralized platoon control. The introduction of a multi-agent system provides both longitudinal and lateral control as well as merge and split capabilities. By simulation with the MaDKit (the Multi-agent Development Kit) platform, the control strategy shows flexibility with respect to obstacle avoidance and reliability. Similarly, [Zhou et al. \(2012\)](#) propose a higher-level longitudinal decentralized control algorithm targeting safety, highway capacity and efficiency improvement.

The introduction of heavy-duty vehicle platooning adds new possibilities for transport carriers to reduce fuel costs and emissions, but also increases the complexity of the transport scheduling and routing problem. Still, there is relatively little research on freight transport planning with platooning as a decision variable. A few studies have considered platoon routing and coordination in networks. In general, there are two types of platoon coordination modes based on the number of vehicles that participate in the system, referred to as off-road coordination and on-road

coordination. In off-road mode, vehicles are selected for platooning in advance, and departure times and routes are planned in order to converge at merging points (the start points of shared paths) and diverge at splitting points. In on-road mode, meanwhile, vehicles search for candidates for platooning in real-time by means of vehicle-to-vehicle communication, especially when approaching intersections. Off-road coordination is particularly suitable for scenarios with low penetration rates of platooning candidates, whereas on-road coordination becomes feasible when there are many platooning candidates.

Larson et al. (2013) develop a “local route and platooning coordinator” (LRPC) for heavy-duty vehicles approaching a common node. The controller compares the fuel consumption following the shortest path and a local control strategy: if the additional fuel required to form the platoon can be compensated by the fuel savings in the platoon, the controller will suggest speed adjustments to facilitate platoon formation. Simulation on the simplified German Autobahn road network shows that the total fuel consumption is decreased by 2-2.5% for a fleet of 300 trucks and over 6% for 2000 trucks. If transporters are willing to accept slightly longer travel times, even greater savings can be achieved. Liang et al. (2014) develop a method to analyze spontaneous platooning by using vehicle probe data, map matching and path inference, where vehicles meet and form groups by coincidence without coordination. Several coordination schemes are proposed to increase the platooning rate and fuel saving by altering vehicle speed profiles and adjusting current travel schedules. Results show that departure coordination gives the largest number of platoons compared with spontaneous platooning and catch-up coordination throughout the day. Integer linear programming formulations for a large-scale, real-world platooning problem without considering deadlines are presented in Larsson et al. (2015). The performance of several heuristics are also compared. The work reveals that savings of 9% are obtained for 200 trucks if all trucks share the same origin.

Previous studies on platoon scheduling have all assumed that vehicle speeds are deterministic and adjustable by the transport planner or controller. Thus, given a certain strategy, the arrival times of a vehicle to merging points and the final destination are perfectly known. In reality, however, non-recurrent factors like congestion, incidents, construction work, severe weather conditions, drivers’ behavior etc. mean that travel times are more or less uncertain ahead of time (e.g., van Lint et al., 2008). Thus, vehicle speeds are only partially controllable, and transporters must add safety margins, or “head starts”, to departure times to ensure that deliveries do not arrive too late.

Furthermore, previous studies have treated scheduled arrival times at destinations as hard constraints, or only indirectly as a constraint on total travel time. In practice, late arrivals are associated with costs that vary depending on the specific transport assignment. For certain transports, such as just-in-time deliveries, lateness deliveries may be associated with significant penalties, including negative impacts on customer relations; for other, less time critical assignments, penalties may be less severe. In addition, there may be economic losses associated with unexpected early arrivals, considering the lost opportunity of performing other assignments. Several studies have evaluated the costs associated with travel time variability, delays and late deliveries as valued by logistics managers and the freight transport industry, and have shown the the costs vary depending on the type of goods transported (de Jong et al., 2014; Danielis et al., 2005; Fowkes et al., 2004). Variations in the on-time criticality of different assignments has an impact on transport planning and should be considered in the platoon scheduling problem together with other cost components.

With respect to planning under uncertainty, there are related applications in other fields of transportation. In public transport planning, slack time is often added between the scheduled arrival for a feeder line and the scheduled departure for a transfer line. Ideally, the arrival of the feeder line and the departure of the transfer line would be synchronized, so that passengers could alight from the feeder line and immediately board the transfer line. If the feeder line is delayed, however, problems occur and an optimization of the slack time is needed. For example, Hall (1985) assumes vehicles are delayed according to an exponential distribution and optimizes the schedules analytically by taking headway into consideration. In flight scheduling, buffer times are often added to the required flight time under optimal conditions so that on-time arrival statistics can be increased. The trade-off is that such schedule padding will lead to higher salary costs (and thus operational costs) and increase the possibility of early arrival (Skaltsas, 2011).

3. Problem formulation

In freight transport planning, vehicles are assigned to transport goods from origins, e.g., freight terminals, to destinations, e.g., city logistic centers or customers, based on specified scheduled delivery times. Given the origin, destination and scheduled arrival time of each transport, departure times are scheduled so that the transport cost is

minimized. The cost typically contains several components, including fuel cost, travel time related cost, and penalties for schedule miss. Analytically, the total cost can be represented as

$$C(\theta) = C_f(\theta) + C_t(\theta) + C_s(\theta), \quad (1)$$

where C_f is the fuel cost, C_t is the travel time cost, C_s is the penalty on schedule miss, and θ represents a set of decision variables for optimal transport cost such as departure time and route choice. Each cost component is modeled in greater detail below.

3.1. Driving cost and schedule miss penalty

Travel time is sensitive to many factors in the transport system, including travel demand, incidents, construction work, and drivers' characteristics. Thus, different vehicles traversing the same route during the same time interval may experience different travel times. Considering that travel time is not fully predictable at the time when departure times are scheduled, it is modeled in this paper as a deterministic part T_c plus a stochastic part τ ,

$$T = T_c + \tau. \quad (2)$$

The constant T_c , which can be acquired from historical data, represents the lower bound of the travel time, while the stochastic part τ represents recurrent and non-recurrent delay (due to congestion, incidents, weather conditions, etc.). In this study, τ is assumed to be a non-negative random variable with mean μ , standard deviation σ , probability density function ϕ and cumulative distribution function Φ . It is assumed that Φ is continuous and invertible on $[0, \infty)$. The distribution of τ is assumed to be known and may be estimated from historical probe data (e.g., [Rahmani et al., 2015](#)). Note that except for the numerical example in Section 5 and the specific distribution (exponential distribution) used for illustrations, all the derivations in the paper are applicable to any travel time distribution which conforms to the aforementioned assumptions.

The travel time cost item represents costs proportional to travel time, including the driver's wage and the opportunity cost of not being able to carry out other transport tasks while driving. The travel time cost is evaluated by the total travel time and the unit cost of travel time w_t , assumed to be constant,

$$C_t = w_t(T_c + \tau). \quad (3)$$

Since τ is unknown at the time when the departure time is scheduled, the realization of τ will in general lead to an early or late arrival with respect to the scheduled arrival time. Some authors, including [Noland and Small \(1995\)](#), use the terms "schedule delay early" and "schedule delay late" to name these two situations. Depending on the type of goods that the vehicle carries as well as customers' requirements, a schedule miss may lead to a penalty if the vehicle arrives early or late. If the actual arrival time is earlier than the scheduled arrival time t_a , an earliness penalty is incurred; otherwise a lateness penalty is incurred.¹ Here, earliness penalty and lateness penalty are collectively referred to as the schedule miss penalty. In line with [Fosgerau and Karlström \(2010\)](#), the unit costs of early and late arrivals are assumed to be constant and equal to w_e and w_l , respectively. With this model, the schedule miss penalty increases proportionally to the deviation from the scheduled arrival time. Given scheduled arrival time t_a and departure time t_d , the schedule miss cost is²

$$C_s = w_e(t_a - t_d - T_c - \tau)_+ + w_l(t_d + T_c + \tau - t_a)_+. \quad (4)$$

Although the model assumes no hard constraints on arrival time, such constraints can be approximated by setting either the marginal lateness penalty w_l very high so that t_a represents a latest acceptable arrival time, or the marginal earliness penalty w_e very high so that t_a represents an earliest acceptable arrival time. Early arrival may be associated with no cost (e.g., if the vehicles may be parked at no cost until the scheduled delivery time) by setting w_e . However, in practice there are likely opportunity costs associated with early arrival since the vehicles could have been used more productively for other tasks.

¹Since the stochastic travel time component τ is a continuous random variable, the probability of arriving exactly at the scheduled arrival time is 0.

²For a number $a \in \mathbb{R}$, $(a)_+$ denotes the maximum $\max\{a, 0\}$.

3.2. Fuel cost

The main difference in fuel consumption between independent driving and platooning lies in the external air drag force reduction for the platoon follower. To highlight this difference, the fuel consumption is modeled as a function of travel time and the air-drag reduction factor φ , i.e., $F := F(T, \varphi)$. If the vehicle is a follower in the platoon, $\varphi = \varphi_p > 0$, otherwise $\varphi = 0$.

The fuel consumption is based on Franceschetti et al. (2013), in which the instantaneous fuel rate F_r is modelled as

$$F_r = \frac{\xi}{\kappa\eta} \left(kN_e V + \frac{0.5c_d\rho A v^3(1-\varphi) + Mv(g \sin \alpha + g c_r \cos \alpha)}{1000\varepsilon\varpi} \right), \quad (5)$$

where ξ is fuel-to-air mass ratio, κ is the heating value of the fuel, η is a conversion factor from grams to litres, k is the engine friction factor, N_e is the engine speed, V is the engine displacement, ρ is the air density, A is the vehicle front area, v is speed, M is the total vehicle weight, g is the gravitational constant, α is the road gradient, C_d and C_r are the coefficient of aerodynamic drag and rolling resistance, ε is vehicle drive train efficiency, and ϖ is an efficiency parameter for the engine.

In real traffic, vehicle speed v is not constant during a long haul, so the speed profile will change over time. For the purpose of simplification, in this study instantaneous speed is replaced with average speed, and the road gradient α is assumed to be 0. When traversing a distance of L , multiplying the fuel rate F_r by the travel time $T_c + \tau = L/v$, the total amount of fuel consumed is

$$F(T_c + \tau, \varphi) = \frac{\xi}{\kappa\eta} \left(kN_e V(T_c + \tau) + 0.5 \frac{c_d\rho A(1-\varphi)}{1000\varepsilon\varpi} \frac{L^3}{(T_c + \tau)^2} + \frac{Mg c_r}{1000\varepsilon\varpi} L \right). \quad (6)$$

Assuming that the road gradient is known, (6) may be generalized by multiplying the resistance functions with appropriate coefficients.

Fuel consumption thus consists of three components: the basic consumption to maintain the operation of the cylinders, and the energy needed to overcome the frictional resistance and air drag, respectively. Because of the air-drag effect, fuel consumption is a nonlinear function of travel time, reflecting that driving at a rather high average speed (short travel time) or encountering heavy traffic congestion (long travel time) will both lead to increased fuel consumption compared to the optimal case. The fuel cost is calculated as the fuel consumption multiplied by the unit fuel cost w_f ,

$$C_f = w_f F(T_c + \tau, \varphi). \quad (7)$$

Assuming two vehicles with identical parameters driving on a common path of length L , the fuel saving by platooning is

$$F_s(T_c + \tau) = F(T_c + \tau, 0) - F(T_c + \tau, \varphi_p) = B \frac{L^3}{(T_c + \tau)^2}, \quad (8)$$

where $B = 0.5\varphi_p\xi c_d\rho A/(1000\kappa\eta\varepsilon\varpi)$. It can be observed from (8) that the main approach to increase the fuel saving by platooning is to extend the common path of the vehicles as much as possible.

4. Platooning on a common route

This section considers the scheduling of multiple vehicles from the same transport carrier with different scheduled arrival times along a common route (see Fig. 1). Section 4.1 derives the optimal scheduling for vehicles driving independently, and Section 4.2 derives the optimal scheduling for two platooning vehicles. The optimal choice between independent driving and platooning is analyzed in Section 4.3.

4.1. Independent transports

For a single vehicle driving in isolation, the total cost consists of four parts: fuel cost, travel time cost, penalty for early arrival, and penalty for late arrival. Let $x = t_a - t_d - T_c$, where x represents the safety margin reserved for unanticipated delay, or ‘‘head start’’. $x = 0$ implies that the vehicle will arrive according to schedule if there is no delay, i.e., if the stochastic component of travel time is 0 and $T = T_c$. If the actual stochastic travel time turns out to be



Figure 1: Common route: Both transports are from A to B.

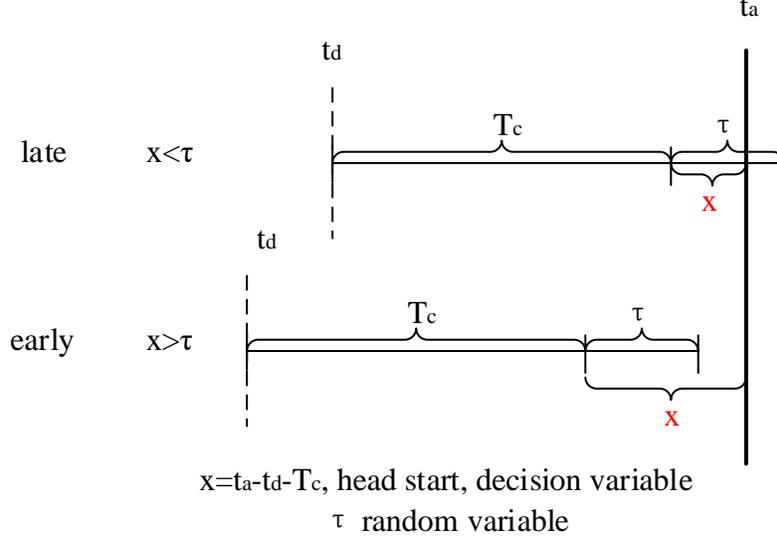


Figure 2: Illustration of model variables.

greater than x , a lateness penalty is incurred; if it is less than x , an earliness penalty is incurred (see Fig. 2). The total cost function of a single vehicle is

$$C(x, \tau) = w_f F(T_c + \tau) + w_t(T_c + \tau) + w_e(x - \tau)_+ + w_l(\tau - x)_+, \quad x \in \mathbb{R}, \tau \in [0, \infty). \quad (9)$$

Since τ is unknown at the time of departure, the head start is chosen to minimize the expected total cost

$$H(x) \equiv \mathbb{E}[C(x, \tau)] = \mathbb{E}[w_f F(T_c + \tau) + w_t(T_c + \tau) + w_e(x - \tau)_+ + w_l(\tau - x)_+], \quad x \in \mathbb{R}. \quad (10)$$

across the distribution of τ . Furthermore, since x only appears in the schedule miss penalty terms, the optimal head start is found by minimizing the expected schedule miss penalty (see the Appendix),

$$\begin{aligned} \min_{x \in \mathbb{R}} h(x) &\equiv \mathbb{E}[w_e(x - \tau)_+ + w_l(\tau - x)_+] \\ &= w_l(\mu - x) + (w_e + w_l) \int_0^x \Phi(z) dz. \end{aligned} \quad (11)$$

An illustration of $h(x)$ as a function of x is given in Fig. 3 assuming that τ is exponentially distributed and $w_l > w_e$. As can be seen, short or negative head starts imply a high expected cost due to late arrival penalty, while a long head start imply high cost due to early arrival penalty.

Since $\Phi(x)$ is an increasing function, $h(x)$ is convex. Hence, there is a unique optimal head start x^* . From the first-order conditions of (11), the optimal head start is chosen so that the probability of early arrival equals the ratio

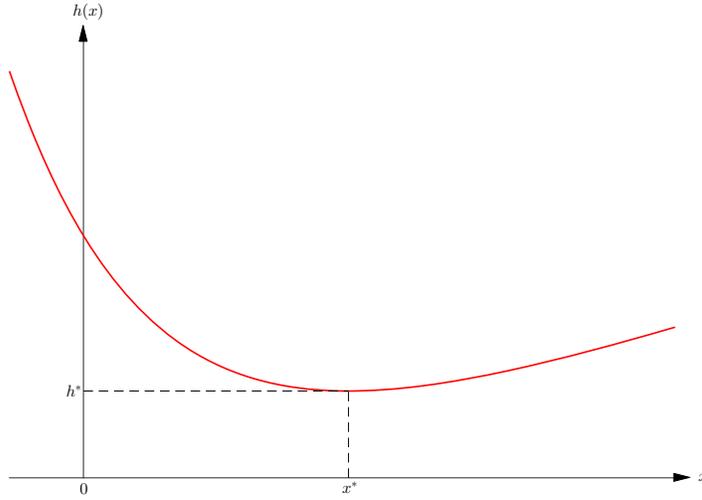


Figure 3: Sketch of expected schedule miss penalty $h(x)$ as a function of head start x for a single transport. Exponential delay distribution, $w_1 > w_e$.

between the marginal lateness penalty and the sum of the marginal earliness and lateness penalties,

$$\Phi(x^*) = \frac{w_1}{w_e + w_1}. \quad (12)$$

The optimal head start is thus always positive given that $w_1 > 0$ and $w_e > 0$. Eq. (12) shows that the larger the lateness penalty cost in relation to the earliness cost, the larger is the optimal head start. As discussed in Section 3.1, unacceptable late arrival may be approximated by setting w_1 very large. In this case, the right hand side of (12) tends to 1, which means that the optimal head start is chosen very large. The same strategy applies if the earliness penalty w_e is set very low. Conversely, unacceptable early arrival may be approximated by setting w_e very high, in which case the right hand side tends to 0 and the optimal head start is chosen close to 0.

Inserting (12) into (11), the minimum expected schedule miss cost is found,

$$h^* \equiv h(x^*) = w_1\mu - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Phi^{-1}(z) dz. \quad (13)$$

The first term represents the expected schedule miss cost with no head start, and the second term is the reduction in cost because of the optimal head start.

A common measure of travel time uncertainty is the standard deviation of the travel time distribution σ . It is intuitive to expect that the optimal head start increases with the standard deviation as a buffer for the increased probability of late arrival. The Appendix shows that this is indeed true,

$$\frac{dx^*}{d\sigma} = \frac{1}{\sigma} \Phi^{-1} \left(\frac{w_1}{w_e + w_1} \right) \geq 0. \quad (14)$$

Even though the head start is increased, the minimum expected schedule miss penalty also increases as the travel time uncertainty increases,

$$\frac{dh^*}{d\sigma} = \frac{1}{\sigma} \left(w_1\mu - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Phi^{-1}(z) dz \right) \geq 0. \quad (15)$$

The Appendix further shows that both x^* and h^* increase linearly with σ .

4.2. Platooning

This section considers two vehicles with scheduled arrival times t_1^a and t_2^a respectively. For simplicity, the cost parameters w_e and w_l and the fuel consumption parameters are assumed to be equal for both vehicles.³ If $t_1^a = t_2^a$, it is clear that forming a platoon is beneficial compared to driving individually because of fuel cost savings. However, if $t_1^a \neq t_2^a$, forming a platoon by departing at the same time will cause one or both vehicles to experience more schedule miss penalty than when driving individually. There is thus a trade-off in whether the schedule miss penalty can be compensated by the fuel savings.

Assume without loss of generality that $t_2^a \geq t_1^a$. Now introduce x_p as the head start relative to the earlier scheduled arrival time, and δ as the difference between the two scheduled arrival times, i.e., $x_p = t_1^a - t_d - T_c$ and $\delta = t_2^a - t_1^a$. The head start relative to the later scheduled arrival time is thus $x_p + \delta$. Across the distribution of the non-recurrent delay τ , the expected total cost for the two vehicles is

$$H_p(x_p) = \mathbb{E} [2w_f F(T_c + \tau) - w_f F_s(T_c + \tau) + 2w_t(T_c + \tau) + w_e(x_p - \tau)_+ + w_l(\tau - x_p)_+ + w_e(x_p + \delta - \tau)_+ + w_l(\tau - (x_p + \delta))_+], \quad x_p \in \mathbb{R}. \quad (16)$$

Note the second term which is the fuel savings from platooning. The optimal head start is found by minimizing the expected schedule miss cost, which is the sum of the costs for the two vehicles given their head starts,

$$\begin{aligned} \min_{x_p \in \mathbb{R}} h_p(x_p) &\equiv \mathbb{E}[w_e(x_p - \tau)_+ + w_l(\tau - x_p)_+ + w_e(x_p + \delta - \tau)_+ + w_l(\tau - (x_p + \delta))_+] \\ &= h(x_p) + h(x_p + \delta). \end{aligned} \quad (17)$$

The cost function is a sum of two convex functions and hence itself convex. Thus, there exists a unique optimal head start x_p^* . For further analysis, it is convenient to introduce the function

$$\Pi(x) = \frac{\Phi(x - \delta) + \Phi(x)}{2}, \quad x \in \mathbb{R}. \quad (18)$$

Function $\Pi(x)$ is a mixture of two invertible cumulative distribution functions and is hence itself an invertible cumulative distribution function, with support on $[0, \infty)$. From the properties of mixture distributions, the distribution associated with $\Pi(x)$ has mean $\mu + \delta/2$ and variance $\sigma^2 + \delta^2/4$. Combining (11), (17) and (18), the expected schedule miss cost given head start x_p is thus

$$h_p(x_p) = 2 \left[w_l \left(\mu - \frac{\delta}{2} - x_p \right) + (w_e + w_l) \int_0^{x_p + \delta} \Pi(z) dz \right], \quad x_p \in \mathbb{R}. \quad (19)$$

Note the similar forms of (11) and (19). As the schedule difference δ tends to zero, the expected cost tends to the cost for two independent transports. An illustration of $h_p(x_p)$ as a function of x_p is given in Fig. 4 for the case where τ is exponentially distributed and $w_l > w_e$.

In the Appendix it is shown that there are two possibilities for the optimal head start depending on the shape of the travel time distribution Φ and the relative marginal costs of late and early arrivals. Under the reasonable assumption that the penalty for late arrivals is higher, i.e., $w_l > w_e$, the optimal head start x_p^* is positive and chosen so that the average probability of early arrival for both vehicles equals the ratio between the marginal lateness penalty and the sum of the marginal earliness and the lateness penalties,

$$\Pi(x_p^* + \delta) = \frac{\Phi(x_p^*) + \Phi(x_p^* + \delta)}{2} = \frac{w_l}{w_e + w_l}. \quad (20)$$

Comparison between (12) and (20) shows that $\Phi(x^*)$ for an independent transport equals the average of $\Phi(x_p^*)$ and $\Phi(x_p^* + \delta)$, as illustrated in Fig. 5. Since Φ is an increasing function, it follows that $x_p^* \leq x^* \leq x_p^* + \delta$, i.e., the optimal

³The analysis is straightforward also in the case when unit schedule miss costs are vehicle-specific. Essentially, arithmetic averages are replaced with weighted averages based on the relative magnitude of the penalties for the two vehicles.

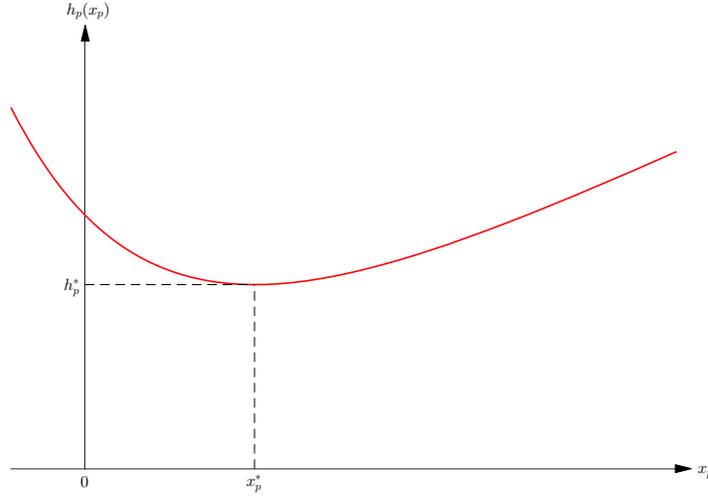


Figure 4: Sketch of expected schedule miss penalty $h_p(x_p)$ as a function of head start x_p for two platooning transports. Common route, exponential delay distribution, $w_1 > w_e$.

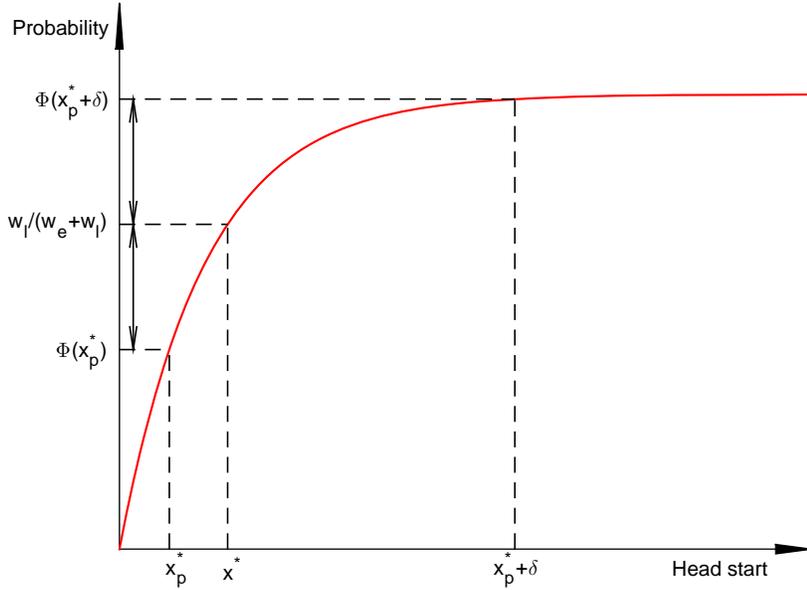


Figure 5: The relationship between optimal head starts x_p^* , $x_p^* + \delta$ for two platooning vehicles and optimal head start x^* for vehicles driving independently. The intervals indicated by the arrows are equal. Common route, exponential delay distribution, $w_1 > w_e$.

head start when platooning is shorter for the first vehicle and longer for the second vehicle compared to independent transports.

The minimum expected schedule miss cost is obtained by inserting (20) in (19):

$$h_p^* \equiv h_p(x_p^*) = 2 \left[w_1 \left(\mu + \frac{\delta}{2} \right) - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Pi^{-1}(z) dz \right]. \quad (21)$$

The similarity with the corresponding minimum cost for the independent driving case (13) is notable. The mean μ and

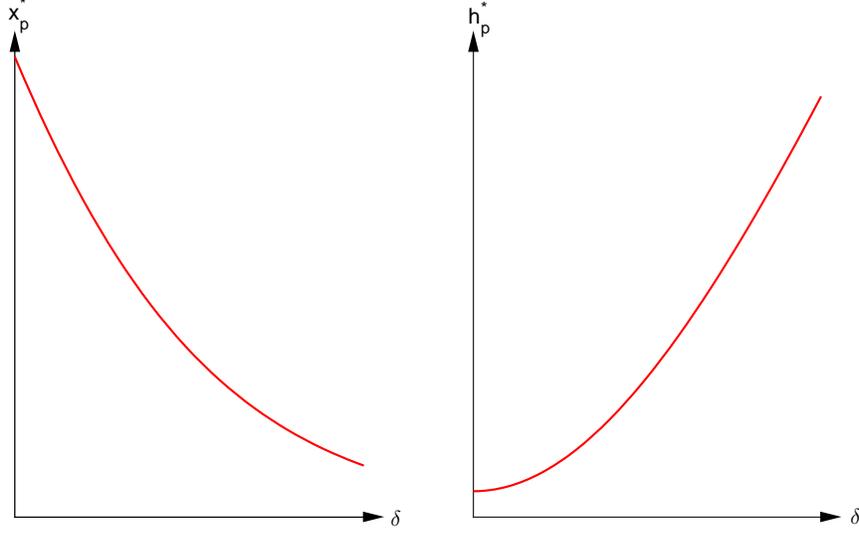


Figure 6: Two platooning vehicles on a common route. a) Optimal head start x_p^* as function of schedule difference δ ; b) Minimum expected schedule miss cost h_p^* as function of δ . Exponential delay distribution, $w_1 > w_e$.

the cumulative distribution function Φ in (13) are replaced in (21) with the corresponding quantities $\mu + \delta/2$ and Π for the mixture distribution representing the platooning case. The first term inside the brackets represents the average expected schedule miss cost with no head start, and the second term is the reduction in cost because of the optimal head start. The factor 2 represents that the cost is for two vehicles.

The time difference δ between the two scheduled arrival times has an impact on the optimal head start. It is shown in the Appendix that the optimal head start decreases as the schedule difference δ increases; specifically,

$$\frac{dx_p^*}{d\delta} = -\frac{\phi(x_p^* + \delta)}{\phi(x_p^*) + \phi(x_p^* + \delta)} < 0. \quad (22)$$

In line with expectations, the expected schedule miss cost increases as the difference between the scheduled arrival times increases,

$$\frac{dh_p^*}{d\delta} = (w_e + w_1) \frac{\Phi(x_p^* + \delta) - \Phi(x_p^*)}{2} \geq 0. \quad (23)$$

The impact of δ on x_p^* and h_p^* is illustrated in Fig. 6.

It is also of interest to consider the impact on the platoon scheduling problem of travel time uncertainty as represented by the standard deviation σ . Intuitively, an increase in travel time uncertainty should increase the optimal head start as well as the minimum schedule miss cost. The Appendix shows that this is correct; the impact on the optimal head start of an increase in travel time uncertainty is

$$\frac{dx_p^*}{d\sigma} = \frac{1}{\sigma} \left[\Pi^{-1} \left(\frac{w_1}{w_e + w_1} \right) - \delta \frac{\phi(x_p^*)}{\phi(x_p^*) + \phi(x_p^* + \delta)} \right] \geq 0. \quad (24)$$

The minimum expected schedule miss cost also increases as the travel time uncertainty increases,

$$\frac{dh_p^*}{d\sigma} = \frac{1}{\sigma} \left(2w_1\mu - (w_e + w_1) \left[\int_0^{\Phi(x_p^*)} \Phi^{-1}(z) dz + \int_0^{\Phi(x_p^* + \delta)} \Phi^{-1}(z) dz \right] \right) \geq 0. \quad (25)$$

As illustrated in Fig. 7, both x^* and h^* for independent transports increase linearly with σ (see (47) and (48) in the Appendix). For platooning transports, the rate of increase is lower for low levels of uncertainty but approaches the same rate as for independent transports for high levels of uncertainty. Note that platooning leads to schedule miss cost

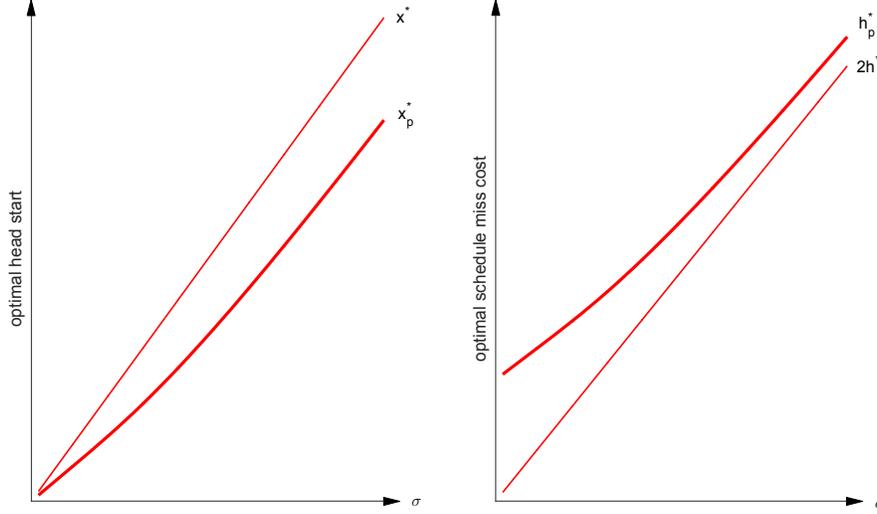


Figure 7: Comparison between platooning and independent driving. a) Optimal head starts x_p^* and x^* as functions of travel time standard deviation σ ; b) Minimum expected schedule miss costs h_p^* and $2h^*$ as functions of σ . Common route, exponential delay distribution.

even when travel times are deterministic (i.e., as σ tends to 0) given that the vehicles have different scheduled arrival times.

In summary, the analysis shows that the scheduling problem of two platooning vehicles can be formulated on the same convex form as for independent vehicles. Thus, an increase in travel time uncertainty increases the optimal head start and the expected schedule miss cost of the transports in the same way. Compared to the independent driving case, the optimal head start is shorter for the first transport and longer for the second transport.

4.3. Trade-off between platooning and independent transports

Compared with the situation that two vehicles depart independently, the gain of platooning is the expected fuel saving

$$\mathbb{E}[F_s] = BL^3 \cdot \mathbb{E} \left[\frac{1}{(T_c + \tau)^2} \right] \geq 0. \quad (26)$$

The scheduling loss from platooning is the increase in expected schedule miss penalty compared to optimal independent transports, obtained from (13) and (21),

$$Q = h_p^* - 2h^* = w_1\delta + 2(w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} [\Phi^{-1}(z) - \Pi^{-1}(z)] dz \geq 0. \quad (27)$$

The net difference in expected total cost is

$$H_p(x_p^*) - 2H(x^*) = Q - w_f \mathbb{E}[F_s]. \quad (28)$$

If the loss is smaller than the gain, i.e., $Q < w_f \mathbb{E}[F_s]$, it is beneficial for the transporter to let the two vehicles platoon. Otherwise, the two vehicles should drive independently. If $\delta = 0$, i.e., both vehicles have the same scheduled arrival time, x_p^* is equal to x^* and $Q = 0$. It follows from (23) that the expected schedule miss penalty when platooning increases as the difference between scheduled arrival times δ increases. In other words, there exists some threshold time difference δ_{\max} such that platooning is beneficial if $\delta \leq \delta_{\max}$ but inefficient if $\delta > \delta_{\max}$.

A relevant question in this context is what impact travel time uncertainty has on the benefit of platooning. Fig. 8 shows the expected fuel saving $\mathbb{E}[F_s]$ and the scheduling loss Q as functions of the travel time standard deviation σ . It follows from (26) that higher travel time uncertainty will lead to lower expected fuel savings. Furthermore, the Appendix shows that the scheduling loss from platooning, which is obtained from (15) and (25), decreases as the travel

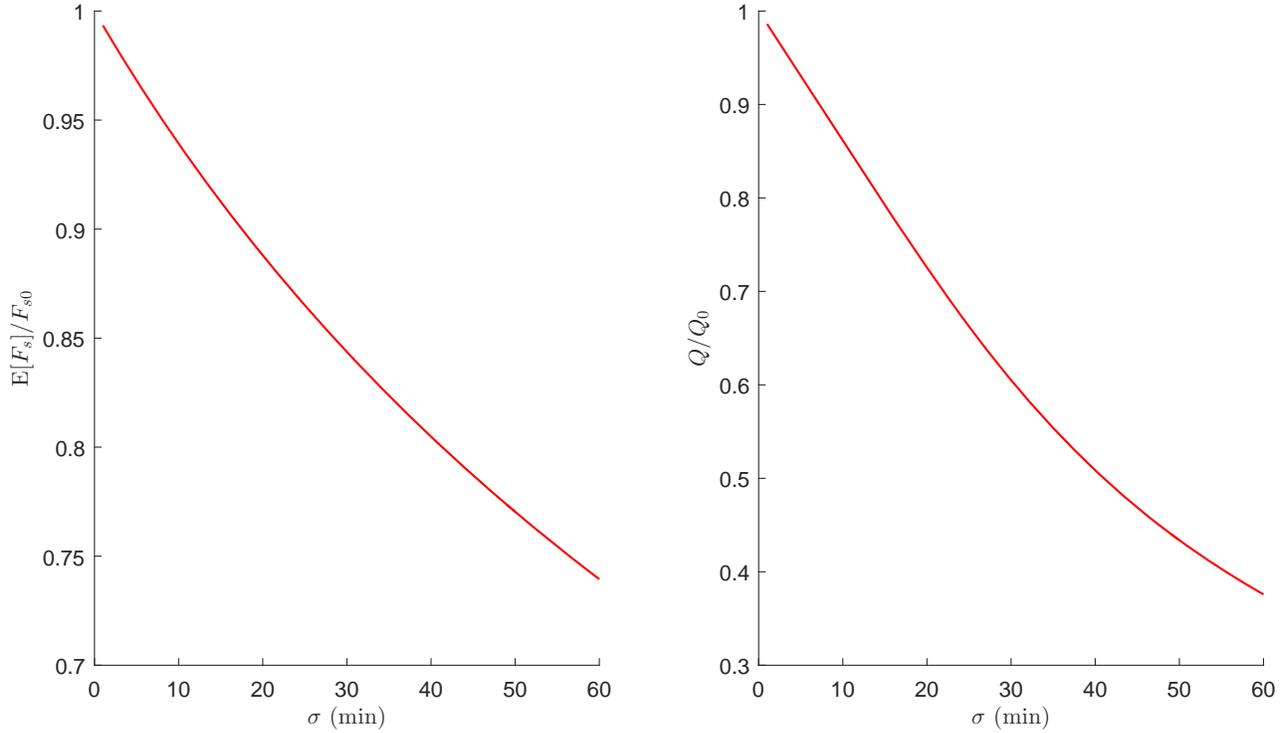


Figure 8: a) The ratio of expected fuel savings from platooning $\mathbb{E}[F_s]$ relative to F_{s0} as a function of delay standard deviation σ ; b) The ratio of scheduling loss from platooning Q (€) relative to Q_0 as a function of σ . Common route, exponential delay distribution, schedule difference $\delta = 100$ min, other parameters given in Section 5. $F_{s0} = 0.02232$ liters/km, which is the fuel saving when $\sigma = 0$. $Q_0 = 1$ €, which is the scheduling loss from platooning when $\sigma = 0$.

time uncertainty increases,

$$\begin{aligned} \frac{dQ}{d\sigma} &= \frac{dh_p^*}{d\sigma} - 2 \frac{dh^*}{d\sigma} \\ &= \frac{w_e + w_1}{\sigma} \left[\int_{\Phi(x_p^*)}^{\frac{w_1}{w_e + w_1}} \Phi^{-1}(z) dz - \int_{\frac{w_1}{w_e + w_1}}^{\Phi(x_p^* + \delta)} \Phi^{-1}(z) dz \right] \leq 0. \end{aligned} \quad (29)$$

Thus, it is in general ambiguous whether higher travel time variability reduces or increases the net benefit of platooning. More detailed analysis is needed to determine the benefit and the threshold schedule difference δ in specific cases. A numerical example is presented in the next section.

5. Numerical example

The impact of the difference in scheduled arrival times δ on the benefits of platooning is studied numerically based on a set of realistic parameter values. The route length is $L = 400$ km and the deterministic part of travel time is $T_c = 300$ min. The stochastic travel time component τ is assumed to follow an exponential distribution.

Following [Franceschetti et al. \(2013\)](#), the driver wage is estimated to $w_t = 0.1877$ €/min (converted from 0.0022 £/s). As for w_e and w_1 , they are much dependent on the sort of goods the vehicle delivers, varying from machine components, chemicals to food products ([Fowkes et al., 2004](#); [Danielis et al., 2005](#)). In this study, we assume $w_e = 0.01$ €/min, $w_1 = 0.05$ €/min, which means being late is much more severe than being early. Moreover, $w_f = 1.5$ €/liter ([Fuel-prices-europe.info, 2015](#)).

Regarding fuel consumption, the air drag reduction coefficient for the platoon follower φ_p is set to 0.32 ([Tsugawa et al., 2011](#); [Liang et al., 2016](#)). Other parameters involved in the fuel consumption model (6) are: $\xi = 1$, $\kappa = 44$ kJ/g,

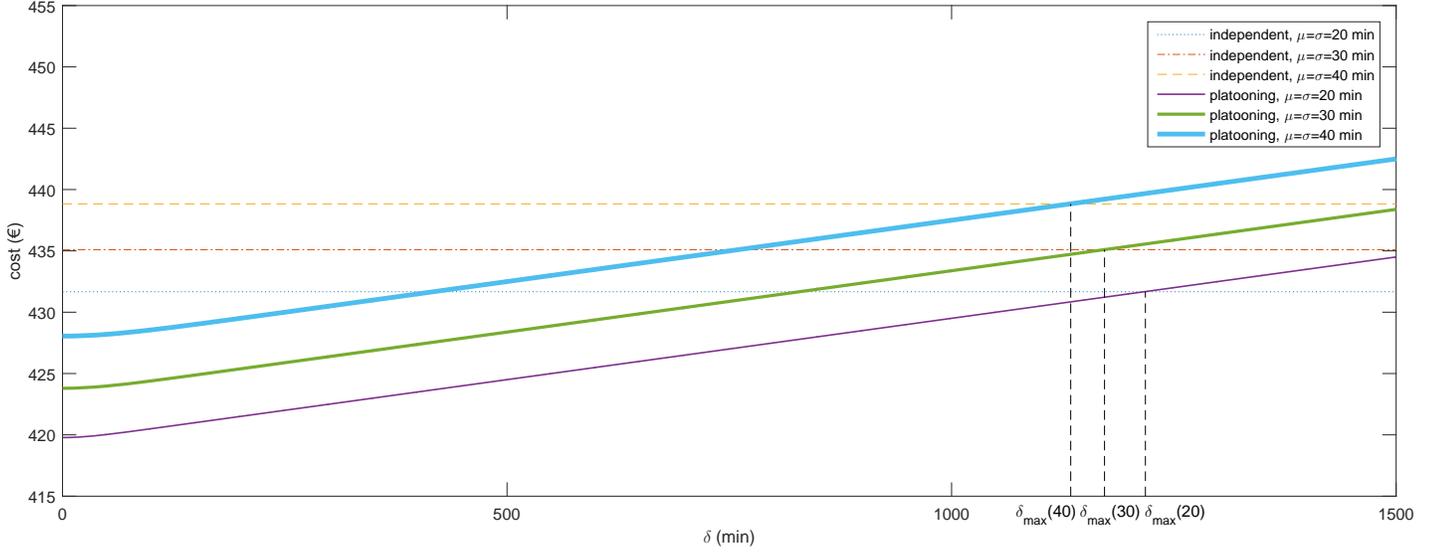


Figure 9: Relationship between schedule difference δ and expected transport cost under platooning and independent driving for different levels of travel time uncertainty. Solid lines show cost H_p^* for two platooning vehicles, dashed lines show cost $2H^*$ for two vehicles driving independently. Common route, exponential distribution, parameters given in Section 5. Intersections between curves represent threshold schedule differences.

$\eta = 737$ g/l, $k = 0.2$ kJ/rev/l, $N_e = 33$ rev/s, $V = 5$ l, $\rho = 1.2041$ kg/m³, $A = 3.912$ m², $g = 9.81$ m/s², $C_d = 0.7$, $C_r = 0.01$, $\varepsilon = 0.4$, $\varpi = 0.9$ (Franceschetti et al., 2013) and $M = 20000$ kg. $B = 4.5192 \times 10^{-8}$ kg·l/(kJ·m).

Based on these parameter values, Fig. 9 shows the impact of δ on the minimum expected transport cost (i.e., including driving cost, schedule miss cost and fuel cost), where the cost for platooning vehicles H_p^* is represented by solid lines and the cost for two vehicles driving independently $2H^*$ by dashed lines. The mean and standard deviation of the delay distribution are assumed to be equal and are varied from 20 min to 40 min. The threshold difference in scheduled arrival times δ_{\max} is determined by the point where the curve for platooning transports crosses the line for independent transports. For $\mu = \sigma = 40$ min, the threshold is around 1134 min; for $\mu = \sigma = 30$ min it is around 1172 min, and even larger for $\mu = \sigma = 20$ min. When μ are 20 min, 30 min and 40 min, the fuel savings are 0.01982 liters/km, 0.01883 liters/km and 0.01796 liters/km respectively, which are 88.79%, 84.37% and 80.48% relative to the case when $\mu = \sigma = 0$. This is also reflected in Fig. 8 a).

The threshold schedule difference δ_{\max} as a function of σ is shown in Fig. 10. In this example, the threshold δ_{\max} decreases to about $\sigma = 250$ min, then slowly increases. The non-linearity is attributed to the form of the fuel consumption model and also the difference between the fuel saving and the scheduling loss from platooning. The results show that travel time uncertainty may reduce the potential for platooning compared to the situation where travel times are deterministic.

6. Platooning in networks

This section considers an extension of the analysis to a network with multiple routes as shown in Fig. 11. The network contains a common corridor (route segment 1) and two branches (route segments 2 and 3). Let L_i denote the length of segment i and let $T_i = T_i^c + \tau_i$ be the travel time on segment i , $i = 1, 2, 3$, where T_i^c is a constant and τ_i is a non-negative random variable with cumulative distribution function Φ_i , probability density function ϕ_i and mean μ_i . The segment travel times may not in general be statistically independent. Without loss of generality, each route segment may consist of multiple links in the underlying road network.

Two different cases are considered: In the first scenario, two transports both start at A but have different destinations, at C and D, respectively. In the second, reversed, scenario, two transports start at different origins C and D, but have a common destination at A.

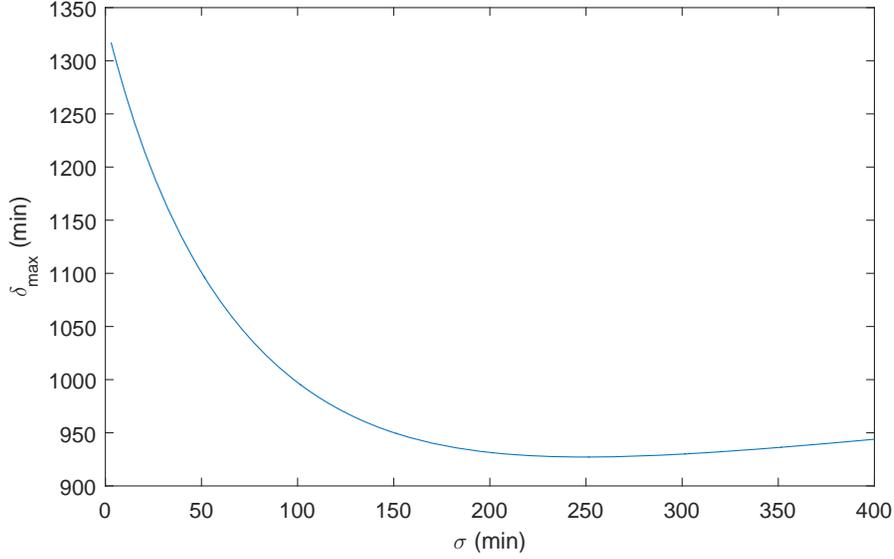


Figure 10: Threshold schedule difference δ_{\max} for positive net benefit of platooning as a function of travel time standard deviation σ . Common route, exponential delay distribution, parameters given in Section 5.

6.1. Diverging routes

Let route a be the route from A to C on segments 1 and 2, and let route b be the route from A to D on segments 1 and 3. Define $\tau_a = \tau_1 + \tau_2$ and $\tau_b = \tau_1 + \tau_3$ as the total stochastic travel time component on route a and b , with cumulative distribution functions Φ_a and Φ_b , respectively.

For a single independent vehicle traversing route a with scheduled arrival time t_a^a , departure time t_a^d and corresponding head start $x_a = t_a^a - t_a^d - T_1^c - T_2^c$, the expected schedule miss cost is equivalent to the common route case and is obtained from (11),

$$\begin{aligned} h_a(x_a) &= \mathbb{E}[w_e(x_a - \tau_a)_+ + w_l(\tau_a - x_a)_+] \\ &= w_l(\mu_1 + \mu_2 - x_a) + (w_e + w_l) \int_0^{x_a} \Phi_a(z) dz, \quad x_a \in \mathbb{R}. \end{aligned} \quad (30)$$

The optimum head start $x_a^* = \Phi_a^{-1}(w_l/(w_e + w_l))$ is obtained from (12) and the minimum expected schedule miss cost from (13),

$$h_a^* \equiv h_a(x_a^*) = w_l(\mu_1 + \mu_2) - (w_e + w_l) \int_0^{\frac{w_l}{w_e + w_l}} \Phi_a^{-1}(z) dz. \quad (31)$$

Corresponding relations hold for a transport on route b .

For platooning vehicles, define $\delta = t_b^a - t_a^a + T_2^c - T_3^c$ as the time difference between the two scheduled arrival times, adjusted by the difference in minimum travel times from node B to each destination. Thus, δ represents the difference in scheduled arrival times at B for the two vehicles. Without loss of generality, it is assumed that routes a and b are labelled such that $\delta \geq 0$. Further, define $x_p = t_a^a - t_a^d - T_1^c - T_2^c$ as the head start for the transport on route a . The head start for the transport on route b is thus $x_p + \delta$. The expected schedule miss cost of two platooning vehicles is

$$\begin{aligned} h_{\text{div}}(x_p) &= \mathbb{E}[w_e(x_p - \tau_a)_+ + w_l(\tau_a - x_p)_+ + w_e(x_p + \delta - \tau_b)_+ + w_l(\tau_b - (x_p + \delta))_+] \\ &= h_a(x_p) + h_b(x_p + \delta), \quad x_p \in \mathbb{R}. \end{aligned} \quad (32)$$

Note the similarity to the common route case in (17). The cost function is convex, and there exists a unique optimal head start x_p^* . Unlike the common route case, the expected schedule miss cost is a sum of two cost functions that are not only shifted in relation to each other but may also have different shapes depending on the travel time distributions

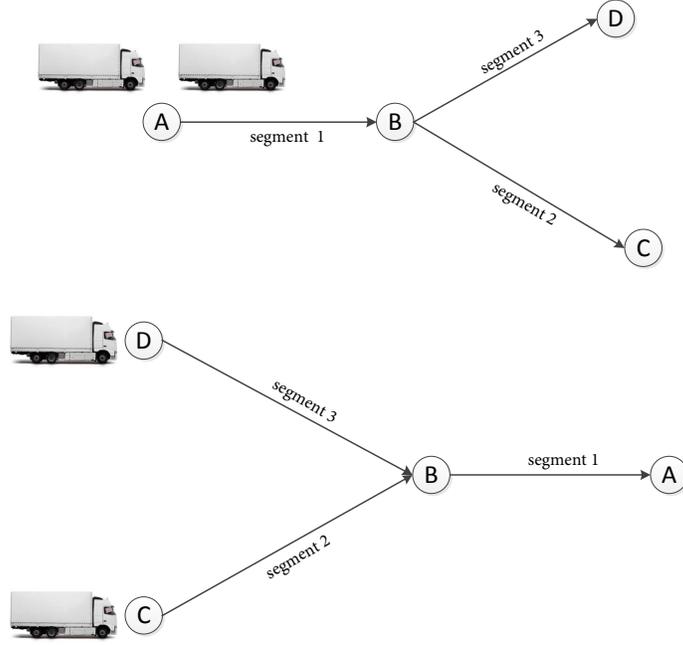


Figure 11: Scheduling and platooning in networks. a) network with diverging routes (from A to C and D); b) network with converging routes (from C and D to A).

of segments 2 and 3. As in Section 4.2, we introduce the mixture cumulative distribution function

$$\Pi_{\text{div}}(x) = \frac{\Phi_a(x - \delta) + \Phi_b(x)}{2}, \quad x \in \mathbb{R}, \quad (33)$$

with mean $\mu_1 + (\mu_2 + \mu_3 + \delta)/2$. By applying the corresponding computations as in Section 4.2, it is found that the optimal head start is chosen so that $x_p^* = \Pi_{\text{div}}^{-1}(w_1/(w_1 + w_e)) - \delta$. Further, the minimum expected schedule miss cost is obtained as

$$h_{\text{div}}^* \equiv h_{\text{div}}(x_p^*) = 2 \left[w_1 \left(\mu_1 + \frac{\mu_2 + \mu_3 + \delta}{2} \right) - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Pi_{\text{div}}^{-1}(z) dz \right]. \quad (34)$$

The scheduling cost is on the same form as for a single vehicle and for two vehicles platooning on a common route. The Appendix shows that the marginal effects of travel time variability have the same signs as in these cases. In other words, the expected schedule miss cost and optimal head start increase with the travel time variability. The impact of the schedule difference δ , however, may differ depending on the delay distributions on the branches. While the optimal head start increases with δ as in the previous cases, the impact on the expected schedule miss cost is

$$\frac{dh_{\text{div}}^*}{d\delta} = (w_e + w_1) \frac{\Phi_b(x_p^* + \delta) - \Phi_a(x_p^*)}{2}. \quad (35)$$

The direction of the marginal effect depends on the relative magnitudes of $\Phi_b(x_p^* + \delta)$ and $\Phi_a(x_p^*)$. A larger schedule difference reduces the expected minimum schedule miss cost in situations where the vehicle on route a is more likely to arrive early than the vehicle on route b , despite the earlier scheduled arrival time. Such an unintuitive situation may occur if the travel time uncertainty is significantly higher on route b than on route a . If the travel time distributions on the branches are similar, in particular if the branches are symmetrical, i.e., $\Phi_a = \Phi_b$, the marginal cost of larger schedule difference is positive as in the common route case.

The fuel saving from platooning depends on the length and travel time of the common segment 1,

$$\mathbb{E}[F_s] = BL_1^3 \cdot \mathbb{E} \left[\frac{1}{(T_1^c + \tau_1)^2} \right]. \quad (36)$$

Meanwhile, the scheduling loss of platooning is

$$Q = h_{\text{div}}^* - h_a^* - h_b^* = w_1 \delta + (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} [\Phi_a^{-1}(z) + \Phi_b^{-1}(z) - 2\Pi_{\text{div}}^{-1}(z)] dz \geq 0. \quad (37)$$

If the scheduling loss is smaller than the fuel saving, i.e., $Q < w_f \mathbb{E}[F_s]$, it is beneficial for the transport carrier to let the two vehicles form a platoon on the common part of the routes. If the travel time distributions on the branches are sufficiently similar, in particular if they are identical, there exists a certain maximum schedule difference δ_{max} below which platooning is beneficial. It can be seen that the longer the common route L_1 , the larger is the threshold δ_{max} . In general, δ_{max} is lower in a diverging network compared to a common route of the same length, since the common segment on which fuel is saved is shorter, while scheduling losses are of the same magnitude. For highly asymmetric branches in terms of travel time uncertainty, there may not exist a unique threshold δ_{max} .

6.2. Converging routes

Another scenario of practical interest is when two transports with a common destination depart from different origins at different departure times. This can be visualized by reversing the directions in Fig. 11. Thus, define route a' as the route from C to A on segments 2 and 1, and b' as the route from D to A on segments 3 and 1. The analysis assumes that the two vehicles form a platoon by meeting at node B, where the vehicle to arrive first waits until the second vehicle arrives. Under this assumption the travel time and fuel consumption on the branches 2 and 3 are not affected by the platoon coordination at B. An alternative assumption would be that the leading vehicle reduces its speed along the branch in order to save fuel and coordinate the arrival at node B. While the fuel consumption may be reduced, such a strategy would lead to the same schedule miss cost as in the analysis here.

For independent transports, it is irrelevant whether the routes are converging or diverging, and the expected schedule miss cost and optimal head starts are identical to the case with diverging routes in Section 6.1. For coordinated transports, the departure times of the two vehicles constitute two decision variables. Compared to the previous cases, there is thus an additional dimension in the scheduling problem.

Define the schedule difference $\delta = t_{b'}^a - t_{a'}^a$ as the difference in scheduled arrival times between the two transports. Without loss of generality, assume that the routes are labeled such that $\delta \geq 0$. It is convenient to define $x_{a'} = t_{a'}^a - T_1^c - T_2^c - t_{a'}^d$ and $x_{b'} = t_{b'}^a - T_1^c - T_3^c - t_{b'}^d$ as the head starts of the first and the second vehicle, respectively, with respect to the first scheduled arrival time.

The schedule miss, i.e., the difference between actual and scheduled arrival time, for each vehicle on a particular day depends on whether it arrives first or last to the merging point. The vehicle traversing route a' arrives last if the actual stochastic travel time difference $\tau_3 - \tau_2$ between the two branches is less than the difference in head starts, which is $x_{b'} - x_{a'}$; otherwise vehicle a' arrives first.⁴ If vehicle a' arrives last, the schedule miss is $\tau_1 + \tau_2 - x_{a'}$; if vehicle a' arrives first, the additional delay from waiting for vehicle b' is $\tau_3 - \tau_2 + x_{a'} - x_{b'}$. The schedule miss for vehicle a' is thus the random variable

$$\tau_1 + \tau_2 + (\tau_3 - \tau_2 + x_{a'} - x_{b'})_+ - x_{a'} = \begin{cases} \tau_1 + \tau_2 - x_{a'} & \text{if } \tau_3 - \tau_2 \leq x_{b'} - x_{a'}, \\ \tau_1 + \tau_3 - x_{b'} & \text{if } \tau_3 - \tau_2 > x_{b'} - x_{a'}. \end{cases} \quad (38)$$

The corresponding analysis applies to vehicle b' .

Unlike the previously considered cases, platooning on converging routes implies that the total travel times from the origins to the destination depend on the head starts $x_{a'}$ and $x_{b'}$, since they influence the waiting time at the merging point. The travel time related cost should therefore be included in the departure time scheduling problem. The travel

⁴Since stochastic travel time components τ_2 and τ_3 are continuous random variables, the probability that the trucks arrive simultaneously is 0 unless the variables are perfectly correlated.

times of the two vehicles are

$$\begin{aligned} T_{a'} &= T_1^c + T_2^c + \tau_1 + \tau_2 + (\tau_3 - \tau_2 + x_{a'} - x_{b'})_+ \\ T_{b'} &= T_1^c + T_3^c + \tau_1 + \tau_3 + (\tau_2 - \tau_3 - x_{a'} + x_{b'})_+, \end{aligned} \quad (39)$$

and the total increase in travel time for both vehicles due to waiting at the merging point is thus $|\tau_3 - \tau_2 + x_{a'} - x_{b'}|$. The optimal head starts are found by minimizing the expected travel time and schedule miss cost across the distributions of stochastic route segment travel times τ_1 , τ_2 and τ_3 ,

$$\begin{aligned} h_{\text{con}}(x_{a'}, x_{b'}) &\equiv \mathbb{E} [w_e(x_{a'} - (\tau_3 - \tau_2 + x_{a'} - x_{b'})_+ - \tau_1 - \tau_2)_+ \\ &\quad + w_1(\tau_1 + \tau_2 + (\tau_3 - \tau_2 + x_{a'} - x_{b'})_+ - x_{a'})_+ \\ &\quad + w_e(x_{b'} + \delta - (\tau_3 - \tau_2 + x_{a'} - x_{b'})_+ - \tau_1 - \tau_2)_+ \\ &\quad + w_1(\tau_1 + \tau_2 + (\tau_3 - \tau_2 + x_{a'} - x_{b'})_+ - x_{a'} - \delta)_+ \\ &\quad + w_t|\tau_3 - \tau_2 + x_{a'} - x_{b'}|], \quad x_{a'}, x_{b'} \in \mathbb{R}. \end{aligned} \quad (40)$$

In general, the scheduling problem depends on the joint distributions of the stochastic route segment travel times. In the analysis here, it is assumed that the route segment travel times are mutually independent. If the travel times are not independent the analysis becomes more complex, but some general observations are made below.

Assuming independent segment travel times, the Appendix shows that the expected travel time and schedule miss cost is a mixture of the costs of platooning on segment 1 with head starts depending on which vehicle arrives last to node B, plus an additional travel time cost term. Specifically,

$$\begin{aligned} h_{\text{con}}(x_{a'}, x_{b'}) &\equiv \mathbb{E} [\Phi_3(x_{b'} - x_{a'} + \tau_2) (h_1(x_{a'} - \tau_2) + h_1(x_{a'} - \tau_2 + \delta)) \\ &\quad + \Phi_2(\tau_3 - x_{b'} + x_{a'}) (h_1(x_{b'} - \tau_3) + h_1(x_{b'} - \tau_3 + \delta)) \\ &\quad + w_t \left(\int_0^{x_{b'} - x_{a'} + \tau_2} \Phi_3(z_3) dz_3 + \int_0^{\tau_3 - x_{b'} + x_{a'}} \Phi_2(z_2) dz_2 \right)], \quad x_{a'}, x_{b'} \in \mathbb{R}. \end{aligned} \quad (41)$$

An illustration of the expected cost function for the case where the stochastic travel time distributions on the branches are identical is given in Fig. 12.

The schedule miss cost terms in (41) penalize large positive or negative head starts in absolute terms, while the driving cost term penalizes large differences in head starts. The Appendix shows that optimal head starts are bounded in magnitude and difference. Unlike the other considered cases, the expected cost function h_{con} is not convex in general, and we are not able to derive closed-form expressions for optimal head starts $x_{a'}^*$, $x_{b'}^*$ and the minimum schedule miss penalty $h_{\text{con}}^* = h_{\text{con}}(x_{a'}^*, x_{b'}^*)$ in the general converging routes case. However, a special case can be considered, in which the delays τ_2 and τ_3 on the two branches are independent and identically distributed with cumulative distribution function Φ_{sym} . The Appendix establishes necessary and sufficient conditions for a symmetrical minimum where the optimal head starts for each vehicle are equal and unique, i.e., $x_{a'}^* = x_{b'}^* = x_{\text{con}}^*$. The optimal head starts, marked in Fig. 12, are then given implicitly from

$$\mathbb{E} [\Phi_{\text{sym}}(\tau) (\Phi_1(x_{\text{con}}^* - \tau) + \Phi_1(x_{\text{con}}^* - \tau + \delta))] = \frac{w_1}{w_e + w_1}. \quad (42)$$

In this case, there is a 50% chance of each vehicle arriving first to the merging point. Furthermore, the optimal head starts are greater than in the diverging routes case. The expected driving and schedule miss cost is

$$h_{\text{con}}^* = 2\mathbb{E} \left[\Phi_{\text{sym}}(\tau) [h_1(x_{\text{con}}^* - \tau) + h_1(x_{\text{con}}^* - \tau + \delta)] + w_t \int_0^\tau \Phi_{\text{sym}}(z) dz \right]. \quad (43)$$

Compared with the diverging routes case, there is in general an additional non-negative schedule miss penalty for each vehicle which arises from the fact that delay may be incurred by waiting for the other vehicle. Furthermore, there is an increase in travel time cost due to the delay at the merging point. Meanwhile, the fuel saving from platooning is the same as for the diverging routes case in (36). Thus, all else equal, platooning is generally less beneficial on converging routes than on diverging routes with the same lengths and travel times.

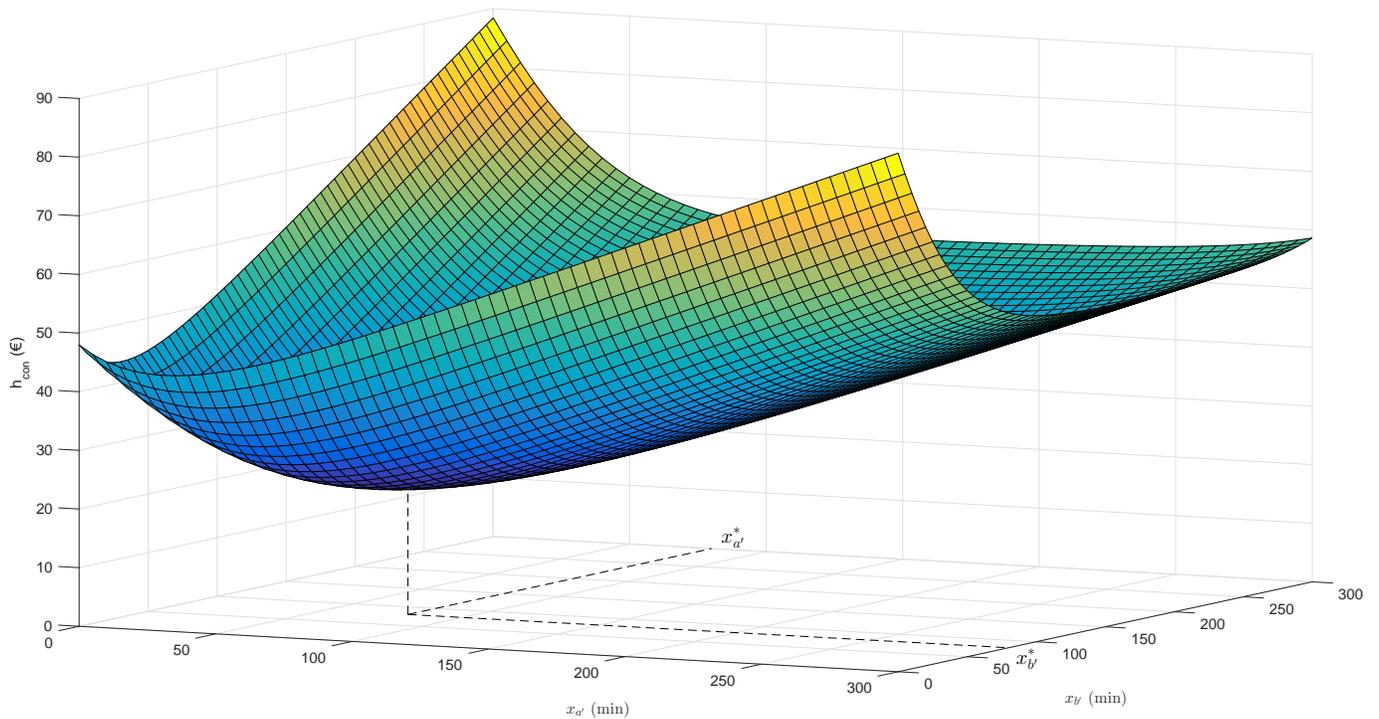


Figure 12: Platooning on converging routes. Expected travel time and schedule miss cost h_{con} as function of head starts $x_{a'}$ and $x_{b'}$. Symmetric branches, exponential distribution, $\sigma_1 = 20$ min, $\sigma_2 = \sigma_3 = 30$ min, $\delta = 60$ min, $w_1 > w_t > w_e$.

Fig. 13 shows the minimum total travel time schedule miss cost on converging and diverging routes with symmetric branches as functions of travel time standard deviation σ_1 on route segment 1. The standard deviation is increased proportionally on all segments assuming that the uncertainty on the branches is 50% higher than on segment 1. The result illustrates that platooning on diverging and converging routes is equivalent under deterministic travel times, i.e., when σ_1 tends to 0. The costs in both cases increase as σ_1 increases, but the rate of increase is higher in the converging routes case due to the additional waiting time.

The analysis above assumes that segment travel times are mutually independent. If travel times on the branches are positively correlated, the difference between arrival times at node B will be smaller, which leads to lower schedule miss costs and travel time costs.⁵ In the extreme case where the travel times on both branches are perfectly correlated, the probability of delay at B is 0, and the scheduling problem is reduced to the common route case (although fuel savings only apply on common segment 1).

7. Discussion: Extensions to more vehicles

The analysis of platooning on a common route and on diverging routes shows that the departure time scheduling problem has the same convex form for two platooning vehicles as for each vehicle driving independently. It can be shown through inductive reasoning that this property can be extended to an arbitrary number of vehicles. Thus, for any number of platooning vehicles there exists a unique optimal departure time with associated minimum expected scheduling loss which is easily computed. Furthermore, the expected fuel costs are straightforward to compute based on the lengths of the common route segments and the number of platooning vehicles.

It can be seen that the analysis of whether two vehicles should platoon can be easily generalized to whether two platoons (each consisting of one or multiple vehicles) should be merged to a single platoon. For a large fleet of vehicles, many combinations of platooning configurations are possible. To find the optimal platoon strategy, however, some

⁵Similar observations were made by Fosgerau et al. (2014) for the case of commuting to meetings.

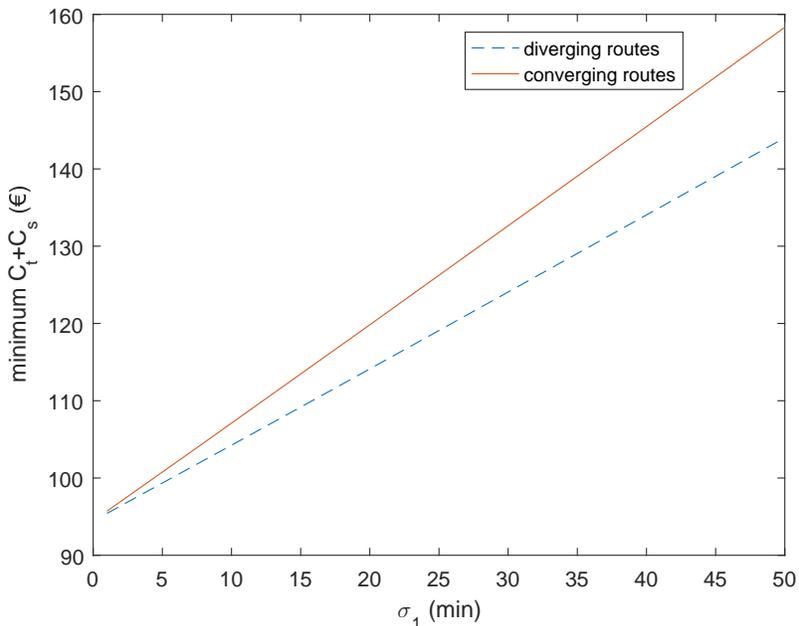


Figure 13: The minimum total expected travel time cost and schedule miss penalty as a function of travel time standard deviation σ_1 . Solid line: Converging routes. Dashed line: Diverging routes. Symmetric branches, exponential distribution, $w_1 > w_e$, $\sigma_1 : \sigma_{\text{sym}} = 2 : 3$, $\delta = 60$ min.

configurations may be ruled out; for example, if it is not beneficial to platoon with a vehicle with a certain scheduled arrival time, it will not be beneficial to platoon with any vehicle for which the difference in scheduled arrival times is even greater. Such principles may reduce the dimensionality of the optimization problem.

Platooning on converging routes represents a more challenging problem to extend to more vehicles. In general, the arrival time at the common destination (and the associated schedule miss penalty) is determined by the arrival time of the last arriving vehicle to the merging point. This observation applies also to the merging of multiple platoons, where the last platoon arriving at the merging point determines the overall schedule miss penalty. In other words, scheduling losses tend to increase more rapidly with the number of vehicles on converging routes compared to common and diverging routes. In any case, heuristic methods may be developed to derive efficient platooning coordination strategies.

8. Conclusions

The paper has formulated and analyzed a platoon coordination and departure time scheduling problem under travel time uncertainty within an expected cost minimization framework. The proposed model explicitly accounts for travel time cost, schedule miss penalties and fuel cost. The platoon scheduling problems for a common route and a network with diverging routes were shown to be on the same functional form as for independent vehicles. Furthermore, they are convex and hence each have a unique optimal departure time and corresponding minimum expected transport cost. The scheduling problem for converging routes is more complex since departure times are interdependent and affect both schedule miss cost and travel time cost through waiting time at the merging point.

It was demonstrated that the objectives of arriving on time and save fuel by platooning are conflicting goals when scheduled arrival times differ between vehicles. For a common route there is a threshold for the difference in scheduled arrival times below which the vehicles should platoon; otherwise, they should drive independently. In a network with diverging routes such a threshold also exists if the travel time distributions on the branches are sufficiently similar, in particular if the branches are symmetrical. Since diverging networks allow platooning only on the common corridor, the threshold schedule difference is smaller than for a common route of the same length. Platooning on converging routes has additional schedule miss and travel time costs compared to the diverging routes case, since each vehicle may

have to wait for the other to arrive at the merging point. Thus, all else equal, travel time uncertainty implies that platooning on converging networks is less beneficial than on diverging networks.

The analysis further showed that minimum expected transport costs and optimal head starts increase with the travel time uncertainty. However, the scheduling loss from platooning compared to driving independently diminishes as the travel time uncertainty increases. The numerical example demonstrated that uncertainty generally reduces the threshold schedule difference for platooning to be beneficial.

The model gives useful insights for transport planners and logistics managers regarding platooning decisions under travel time uncertainty. In particular, it indicates whether platooning is beneficial for pairs of vehicles given their origins, destinations and scheduled arrival times.

Further work should be devoted to extend the model framework to larger number of vehicles, heterogenous cost and fuel consumption parameters, more flexible scheduling loss functions, and more complex network topologies. The model may then serve as a foundation for a planning tool for freight transport platooning scheduling. Large-scale applications, however, require the development of heuristic methods to handle the combinatorial challenges of possible platoon configurations. Another area of development is to incorporate vehicle routing to increase the chances of forming platoons. This additional degree of freedom would introduce an interesting trade-off between routes with large platooning potential but high travel time uncertainty, and routes with small platooning possibilities but low uncertainty.

Currently, the technology for autonomous driving is undergoing rapid development. In the not so distant future, it is foreseeable that at least the following vehicles in a platoon may be allowed to be operated without drivers. The proposed model framework would not need to change significantly to handle such scenarios, since there would still be a trade-off between platooning for saving fuel and driving independently to meet scheduled arrival times. What may change is rather the magnitude of certain model parameters such as the marginal cost of travel time.

This paper considered the setting where a single transport carrier seeks the system optimal strategy regarding platooning decision and head start. In order for large-scale and spontaneous platooning to become reality, there must be a mutual interest among carriers to platoon. A direction for further research is to analyze the setting where multiple carriers seek to minimize their own transport costs, and to assess the potential for platooning between vehicles from different carriers.

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Appendix: Mathematical derivations

A common route: Independent transports

The domain of the probability density function of τ is $[0, \infty)$, whereas x can be any value in the set of real numbers \mathbb{R} . Depending on the sign of $x - \tau$, the penalty function can be written as a piecewise function, which is continuous in the entire domain. Given the mean μ , probability density function ϕ and cumulative distribution function Φ , the expected schedule miss penalty is

$$\begin{aligned}
 h(x) &= \mathbb{E}[w_e(x - \tau)_+ + w_1(\tau - x)_+] \\
 &= \begin{cases} \int_0^\infty w_1(z - x)\phi(z)dz & x \leq 0, \\ \int_0^x w_e(x - z)\phi(z)dz + \int_x^\infty w_1(z - x)\phi(z)dz & x > 0 \end{cases} \\
 &= \begin{cases} w_1(\mu - x) & x \leq 0, \\ w_1(\mu - x) + (w_e + w_1) \int_0^x \Phi(z)dz, & x > 0. \end{cases}
 \end{aligned} \tag{44}$$

Since the integral function $I(x) = \int_0^x \Phi(z)dz$ is increasing and convex, the cost function $h(x)$ is convex. Moreover, $\lim_{x \rightarrow 0} w_1(\mu - x) = \lim_{x \rightarrow 0} w_1(\mu - x) + (w_e + w_1) \int_0^x \Phi(z)dz = w_1\mu$ ensures the continuity of $h(x)$ at $x = 0$. For $x < 0$, the transport will arrive late with probability 1; the smaller x is, the more late penalty will be incurred. For $x \geq 0$,

taking the derivative of $h(x)$ with respect to x and setting it equal to 0, the optimal x is obtained as follows provided that Φ is continuous at x^* ,

$$x^* = \Phi^{-1} \left(\frac{w_1}{w_e + w_1} \right) \geq 0. \quad (45)$$

Given the optimum x^* , the minimum expected penalty can be calculated by substituting (45) into (44):

$$\begin{aligned} h^* = h(x^*) &= w_1 \left[\mu - \Phi^{-1} \left(\frac{w_1}{w_e + w_1} \right) \right] + (w_e + w_1) \int_0^{\Phi^{-1} \left(\frac{w_1}{w_e + w_1} \right)} \Phi(z) dz \\ &= w_1 \mu - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Phi^{-1}(z) dz. \end{aligned} \quad (46)$$

To study the impact of travel time uncertainty on the scheduling problem, the stochastic travel time component τ is written in the form $\tau = \sigma\tau_0$, where τ_0 is a standardized stochastic variable with mean μ_0 , standard deviation 1, and cumulative distribution function Ψ . Then $\mu = \mu_0\sigma$, $\Phi(\tau) = \Psi(\tau/\sigma)$, and $\Phi^{-1}(z) = \sigma\Psi^{-1}(z)$. It follows directly that the optimal head start is increased,

$$\frac{dx^*}{d\sigma} = \Psi^{-1} \left(\frac{w_1}{w_e + w_1} \right) = \frac{1}{\sigma} \Phi^{-1} \left(\frac{w_1}{w_e + w_1} \right) \geq 0. \quad (47)$$

Further, the minimum expected schedule miss cost also increases,

$$\begin{aligned} \frac{dh^*}{d\sigma} &= w_1 \mu_0 - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Psi^{-1}(z) dz \\ &= \frac{1}{\sigma} \left(w_1 \mu - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Phi^{-1}(z) dz \right) = \frac{h^*}{\sigma} \geq 0. \end{aligned} \quad (48)$$

A common route: Platooning

For two platooning vehicles with scheduled arrival time difference δ , the expected schedule miss cost is

$$h_p(x_p) = h(x_p) + h(x_p + \delta), \quad x_p \in \mathbb{R}, \quad (49)$$

which can be split up in three different domains of the head start based on (44),

$$h_p(x_p) = \begin{cases} w_1(2\mu - 2x_p - \delta) & x_p \leq -\delta, \\ w_1(2\mu - 2x_p - \delta) + (w_e + w_1) \int_0^{x_p + \delta} \Phi(z) dz & -\delta < x_p \leq 0, \\ w_1(2\mu - 2x_p - \delta) + (w_e + w_1) \left[\int_0^{x_p} \Phi(z) dz + \int_0^{x_p + \delta} \Phi(z) dz \right] & x_p > 0. \end{cases} \quad (50)$$

The cost function $h_p(x_p)$ is a sum of convex functions and hence itself convex. The piecewise function shows that when $x_p < -\delta$, both transports will arrive late with probability 1 even if there is no delay. In this sense, the expected minimum cost will not be obtained because departing earlier will give a higher probability to arrive on time. It can be easily shown that $h_p(x_p)$ is continuous at $-\delta$ and 0. The function $h_p(x_p)$ decreases in the domain $(-\infty, -\delta]$, which means if the head start is no greater than $-\delta$, the smaller the head start is, the more penalty will be incurred. The first order condition in the domain $(-\delta, 0]$ gives

$$\Phi(x_p + \delta) = \frac{2w_1}{w_e + w_1}, \quad (51)$$

but the right hand side of (51) is always greater than 1 for $w_1 > w_e > 0$. Thus, when $w_1 > w_e$ there is no local minimum in this domain. Therefore, departing with certain late arrival of one vehicle will not yield the minimum

expected penalty. For $x_p > 0$, taking the derivative of (50) with respect to x_p and equating it to 0 gives:

$$\frac{\Phi(x_p^*) + \Phi(x_p^* + \delta)}{2} = \frac{w_1}{w_e + w_1}, \quad (52)$$

or, introducing the mixture cumulative distribution function $\Pi(x) = (\Phi(x - \delta) + \Phi(x))/2$,

$$x_p^* = \Pi^{-1}\left(\frac{w_1}{w_e + w_1}\right) - \delta > 0. \quad (53)$$

$x_p^* > 0$ is applied in the following deduction. The minimum expected schedule miss cost is found by inserting (52) into (50):

$$\begin{aligned} h_p^* &= h_p(x_p^*) = w_1(2\mu - 2x_p^* - \delta) + (w_e + w_1) \left[\int_0^{x_p^*} \Phi(z) dz + \int_0^{x_p^* + \delta} \Phi(z) dz \right] \\ &= w_1(2\mu + \delta) - (w_e + w_1) \left[\delta\Phi(x_p^*) + \int_0^{\Phi(x_p^*)} \Phi^{-1}(z) dz + \int_0^{\Phi(x_p^* + \delta)} \Phi^{-1}(z) dz \right] \end{aligned} \quad (54)$$

or expressed in terms of the mixture distribution Π :

$$\begin{aligned} h_p^* &= w_1(2\mu - 2x_p^* - \delta) + 2(w_e + w_1) \int_0^{x_p^* + \delta} \Pi(z) dz \\ &= 2 \left[w_1 \left(\mu - \Pi^{-1}\left(\frac{w_1}{w_e + w_1}\right) + \frac{\delta}{2} \right) + (w_e + w_1) \int_0^{\Pi^{-1}\left(\frac{w_1}{w_e + w_1}\right)} \Pi(z) dz \right] \\ &= 2 \left[w_1 \left(\mu + \frac{\delta}{2} \right) - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Pi^{-1}(z) dz \right]. \end{aligned} \quad (55)$$

The impact of the difference in scheduled arrival times δ on the optimal head start can be derived from the implicit function

$$P(x_p^*, \delta) = \Phi(x_p^*) + \Phi(x_p^* + \delta) - \frac{2w_1}{w_e + w_1} = 0, \quad x_p^* > 0, \delta \geq 0. \quad (56)$$

The marginal effect on x_p^* of a change in δ is

$$\frac{dx_p^*}{d\delta} = -\frac{\partial P/\partial \delta}{\partial P/\partial x_p^*} = -\frac{\phi(x_p^* + \delta)}{\phi(x_p^*) + \phi(x_p^* + \delta)}, \quad (57)$$

which is negative since the probability density function ϕ is always positive. Since $dx_p^*/d\delta < 0$, the result shows the optimal head start decreases as the scheduled arrival time difference increases. The marginal effect of the difference in scheduled arrival times δ on the minimum expected schedule miss cost is

$$\begin{aligned} \frac{dh_p^*}{d\delta} &= -w_1 + (w_e + w_1)\Phi(x_p^* + \delta) \\ &= (w_e + w_1) \frac{\Phi(x_p^* + \delta) - \Phi(x_p^*)}{2}. \end{aligned} \quad (58)$$

Since Φ is an increasing function, (58) is always positive.

The marginal effect on x_p^* of a change in the travel time standard deviation σ is obtained by rewriting (56) as

$$P(x_p^*, \sigma) = \Psi\left(\frac{x_p^*}{\sigma}\right) + \Psi\left(\frac{x_p^* + \delta}{\sigma}\right) - \frac{2w_1}{w_e + w_1} = 0, \quad x_p^* > 0, \sigma \geq 0. \quad (59)$$

Thus,

$$\frac{dx_p^*}{d\sigma} = -\frac{\partial P/\partial\sigma}{\partial P/\partial x_p^*} = \frac{1}{\sigma} \left[\Pi^{-1} \left(\frac{w_1}{w_e + w_1} \right) - \delta \frac{\phi(x_p^*)}{\phi(x_p^*) + \phi(x_p^* + \delta)} \right]. \quad (60)$$

It follows from the optimality conditions that $\Pi^{-1}(w_1/(w_e + w_1)) > \delta$ given that $w_1 > w_e$. Thus, (60) is positive, which means that the optimal head start larger increases with travel time uncertainty. The marginal effect on the minimum expected schedule miss cost of a change in the travel time standard deviation σ is

$$\begin{aligned} \frac{dh_p^*}{d\sigma} &= 2w_1\mu_0 - (w_e + w_1) \left[\int_0^{\Phi(x_p^*)} \Psi^{-1}(z) dz + \int_0^{\Phi(x_p^* + \delta)} \Psi^{-1}(z) dz \right] \\ &= \frac{1}{\sigma} \left(2w_1\mu - (w_e + w_1) \left[\int_0^{\Phi(x_p^*)} \Phi^{-1}(z) dz + \int_0^{\Phi(x_p^* + \delta)} \Phi^{-1}(z) dz \right] \right). \end{aligned} \quad (61)$$

The derivative is positive since Φ , and hence its inverse Φ^{-1} , is an increasing function. Therefore, the integral function $I(y) = \int_0^y \Phi^{-1}(z) dz$ is increasing and convex. Given that $\mu = I(1)$, this implies $I(y)/\mu \leq y$ for all $y \in [0, 1]$. Rewriting Eq. (61) as

$$\frac{dh_p^*}{d\sigma} = \frac{(w_e + w_1)\mu}{\sigma} \left(\Phi(x_p^*) - \frac{I(\Phi(x_p^*))}{\mu} + \Phi(x_p^* + \delta) - \frac{I(\Phi(x_p^* + \delta))}{\mu} \right), \quad (62)$$

shows that the marginal effect is positive.

A common route: Trade-off between platooning and independent driving

The scheduling loss from platooning is the increase in expected schedule miss penalty compared to optimal independent transports, obtained from (46) and (55),

$$Q = h_p^* - 2h^* = w_1\delta - 2(w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} [\Pi^{-1}(z) - \Phi^{-1}(z)] dz. \quad (63)$$

The scheduling loss is positive, since the platoon scheduling problem involves minimizing a linear combination of two cost functions, whereas the independent transport scheduling problem involves minimizing each cost function separately. From the convexity of the problem it follows that the schedule miss cost under platooning must be at least as high as for two independent transports. The marginal effect on Q from an increase in the travel time standard deviation σ is obtained from (48) and (61),

$$\frac{dQ}{d\sigma} = \frac{w_e + w_1}{\sigma} \left[\int_{\frac{w_1}{w_e + w_1}}^{\frac{w_1}{w_e + w_1}} \Phi^{-1}(z) dz - \int_{\frac{w_1}{w_e + w_1}}^{\Phi(x_p^* + \delta)} \Phi^{-1}(z) dz \right]. \quad (64)$$

Since Φ^{-1} is an increasing function and the two integration intervals are equally long, the term inside the parentheses is negative. In other words, the loss from platooning is decreasing as the travel time uncertainty increases.

Diverging routes

An extension of the common route case is when two vehicles platoon from a common origin to a splitting point, from which they continue to separate destinations. The derivations are analogous to the common route case but are carried out here for completeness. For two platooning vehicles with scheduled arrival time difference δ , the expected schedule miss cost is

$$\begin{aligned} h_{\text{div}}(x_p) &= h_a(x_p) + h_b(x_p + \delta) \\ &= w_1(2\mu_1 + \mu_2 + \mu_3 - 2x_p - \delta) + (w_e + w_1) \left[\int_0^{x_p} \Phi_a(z) dz + \int_0^{x_p + \delta} \Phi_b(z) dz \right], \quad x_p \in \mathbb{R}. \end{aligned} \quad (65)$$

The cost function $h_{\text{div}}(x_p)$ is a sum of convex functions and hence itself convex. As in the common route case, it can be shown that the optimal head start must be positive for $w_1 > w_e$. For $x_p > 0$, taking the derivative of (65) with

respect to x_p and equating it to 0 gives

$$\frac{\Phi_a(x_p^*) + \Phi_b(x_p^* + \delta)}{2} = \frac{w_1}{w_e + w_1}, \quad (66)$$

or, introducing the mixture cumulative distribution function $\Pi_{\text{div}}(x) = (\Phi_a(x - \delta) + \Phi_b(x))/2$,

$$x_p^* = \Pi_{\text{div}}^{-1}\left(\frac{w_1}{w_e + w_1}\right) - \delta > 0. \quad (67)$$

The minimum expected schedule miss cost is found by inserting (66) into (65):

$$\begin{aligned} h_{\text{div}}^* &= h_{\text{div}}(x_p^*) = w_1(2\mu_1 + \mu_2 + \mu_3 - 2x_p^* - \delta) + (w_e + w_1) \left[\int_0^{x_p^*} \Phi_a(z) dz + \int_0^{x_p^* + \delta} \Phi_b(z) dz \right] \\ &= w_1(2\mu_1 + \mu_2 + \mu_3 + \delta) - (w_e + w_1) \left[\delta \Phi_b(x_p^*) + \int_0^{\Phi_a(x_p^*)} \Phi_a^{-1}(z) dz + \int_0^{\Phi_b(x_p^* + \delta)} \Phi_b^{-1}(z) dz \right] \\ &= 2 \left[w_1 \left(\mu_1 + \frac{\mu_2 + \mu_3 + \delta}{2} \right) - (w_e + w_1) \int_0^{\frac{w_1}{w_e + w_1}} \Pi_{\text{div}}^{-1}(z) dz \right] \end{aligned} \quad (68)$$

The impact of the difference in scheduled arrival times δ on the optimal head start can be derived from the implicit function

$$P_{\text{div}}(x_p^*, \delta) = \Phi_a(x_p^*) + \Phi_b(x_p^* + \delta) - \frac{2w_1}{w_e + w_1} = 0, \quad x_p^* > 0, \delta \geq 0. \quad (69)$$

The marginal effect on x_p^* of a change in δ is

$$\frac{dx_p^*}{d\delta} = -\frac{\partial P_{\text{div}}/\partial \delta}{\partial P_{\text{div}}/\partial x_p^*} = -\frac{\phi_b(x_p^* + \delta)}{\phi_a(x_p^*) + \phi_b(x_p^* + \delta)}, \quad (70)$$

which is negative since the probability density functions ϕ_a and ϕ_b are always positive. Thus, the optimal head start decreases as the scheduled arrival time difference increases. The marginal effect of the difference in scheduled arrival times δ on the minimum expected schedule miss cost is

$$\frac{dh_{\text{div}}^*}{d\delta} = (w_e + w_1) \frac{\Phi_b(x_p^* + \delta) - \Phi_a(x_p^*)}{2}. \quad (71)$$

The sign of $dh_{\text{div}}^*/d\delta$ depends on the relationship between $\Phi_b(x_p^* + \delta)$ and $\Phi_a(x_p^*)$. If the distributions on the two branches are such that early arrival is less likely for the vehicle with later scheduled arrival time than for the other vehicle, a larger schedule difference reduces the expected minimum schedule miss cost. For symmetric branches, i.e., $\Phi_a = \Phi_b$, (71) is always positive.

To study the impact of travel time uncertainty on the scheduling problem, the stochastic travel time component on each route segment τ_i , $i = 1, 2, 3$, is written in the form $\tau_i = \sigma \tau_i^0$, where τ_i^0 is a standardized stochastic variable with mean μ_i^0 , standard deviation σ_i^0 , and cumulative distribution function Ψ_i . Then $\mu_i = \mu_i^0 \sigma$, $\Phi_i(\tau_i) = \Psi_i(\tau_i/\sigma)$, and $\Phi_i^{-1}(z) = \sigma \Psi_i^{-1}(z)$. Further, $\Phi_a(\tau) = \Psi_a(\tau/\sigma)$, $\Phi_a^{-1}(z) = \sigma \Psi_a^{-1}(z)$, $\Phi_b(\tau) = \Psi_b(\tau/\sigma)$, and $\Phi_b^{-1}(z) = \sigma \Psi_b^{-1}(z)$.

The marginal effect on x_p^* of a change in the travel time standard deviation σ is obtained by rewriting (69) as

$$P_{\text{div}}(x_p^*, \sigma) = \Psi_a\left(\frac{x_p^*}{\sigma}\right) + \Psi_b\left(\frac{x_p^* + \delta}{\sigma}\right) - \frac{2w_1}{w_e + w_1} = 0, \quad x_p^* > 0, \sigma \geq 0. \quad (72)$$

Thus,

$$\frac{dx_p^*}{d\sigma} = -\frac{\partial P_{\text{div}}/\partial \sigma}{\partial P_{\text{div}}/\partial x_p^*} = \frac{1}{\sigma} \left[\Pi_{\text{div}}^{-1}\left(\frac{w_1}{w_e + w_1}\right) - \delta \frac{\phi_a(x_p^*)}{\phi_a(x_p^*) + \phi_b(x_p^* + \delta)} \right]. \quad (73)$$

It follows from the optimality conditions that $\Pi_{\text{div}}^{-1}(w_1/(w_e + w_1)) > \delta$ given that $w_1 > w_e$. Thus, (73) is positive, which

means that the optimal head start larger increases with travel time uncertainty.

The marginal effect on the minimum expected schedule miss cost of a change in the travel time standard deviation σ is

$$\begin{aligned} \frac{dh_{\text{div}}^*}{d\sigma} &= w_1(2\mu_1^0 + \mu_2^0 + \mu_3^0) - (w_e + w_1) \left[\int_0^{\Phi_a(x_p^*)} \Psi_a^{-1}(z) dz + \int_0^{\Phi_b(x_p^* + \delta)} \Psi_b^{-1}(z) dz \right] \\ &= \frac{1}{\sigma} \left(w_1(2\mu_1 + \mu_2 + \mu_3) - (w_e + w_1) \left[\int_0^{\Phi_a(x_p^*)} \Phi_a^{-1}(z) dz + \int_0^{\Phi_b(x_p^* + \delta)} \Phi_b^{-1}(z) dz \right] \right). \end{aligned} \quad (74)$$

The derivative is positive since Φ_a and Φ_b , and hence their inverses Φ_a^{-1} and Φ_b^{-1} , are increasing functions. Therefore, the integral functions $I_i(y) = \int_0^y \Phi_i^{-1}(z) dz$, $i = a, b$, are increasing and convex. Given that $\mu_1 + \mu_2 = I_a(1)$ and $\mu_1 + \mu_3 = I_b(1)$, this implies $I_a(y)/(\mu_1 + \mu_2) \leq y$ and $I_b(y)/(\mu_1 + \mu_3) \leq y$ for all $y \in [0, 1]$. Rewriting Eq. (74) as

$$\frac{dh_{\text{div}}^*}{d\sigma} = \frac{(w_e + w_1)}{\sigma} \left((\mu_1 + \mu_2) \left[\Phi_a(x_p^*) - \frac{I_a(\Phi_a(x_p^*))}{\mu_1 + \mu_2} \right] + (\mu_1 + \mu_3) \left[\Phi_b(x_p^* + \delta) - \frac{I_b(\Phi_b(x_p^* + \delta))}{\mu_1 + \mu_3} \right] \right), \quad (75)$$

shows that the marginal effect is positive.

Converging routes

Consider two vehicles driving independently from separate origins on branches 2 and 3, respectively, forming a platoon at the merging point, and platooning on the corridor segment 1 to the destination. The expected schedule miss cost is

$$\begin{aligned} h_{\text{con}}(x_{a'}, x_{b'}) &\equiv \mathbb{E} [w_e(x_{a'} - (\tau_3 - \tau_2 - x_{b'} + x_{a'})_+ - \tau_1 - \tau_2)_+ \\ &\quad + w_1(\tau_1 + \tau_2 + (\tau_3 - \tau_2 - x_{b'} + x_{a'})_+ - x_{a'})_+ \\ &\quad + w_e(x_{a'} + \delta - (\tau_3 - \tau_2 - x_{b'} + x_{a'})_+ - \tau_1 - \tau_2)_+ \\ &\quad + w_1(\tau_1 + \tau_2 + (\tau_3 - \tau_2 - x_{b'} + x_{a'})_+ - x_{a'} - \delta)_+ \\ &\quad + w_t |\tau_3 - \tau_2 - x_{b'} + x_{a'}|] \quad x_{a'}, x_{b'} \in \mathbb{R}. \end{aligned} \quad (76)$$

Integration over all distributions and separating the cases $\tau_3 \leq x_{b'} - x_{a'} + \tau_2$ and $\tau_3 > x_{b'} - x_{a'} + \tau_2$ gives

$$\begin{aligned} h_{\text{con}}(x_{a'}, x_{b'}) &= \int_0^\infty \int_0^\infty \phi_1(z_1) \phi_2(z_2) \int_0^{x_{b'} - x_{a'} + z_2} \phi_3(z_3) [w_e(x_{a'} - z_1 - z_2)_+ + w_1(z_1 + z_2 - x_{a'})_+ \\ &\quad + w_e(x_{a'} + \delta - z_1 - z_2)_+ + w_1(z_1 + z_2 - x_{a'} - \delta)_+] dz_1 dz_2 dz_3 \\ &\quad + \int_0^\infty \int_0^\infty \phi_1(z_1) \phi_2(z_2) \int_{x_{b'} - x_{a'} + z_2}^\infty \phi_3(z_3) [w_e(x_{b'} - z_1 - z_3)_+ + w_1(z_1 + z_3 - x_{b'})_+ \\ &\quad + w_e(x_{b'} + \delta - z_1 - z_3)_+ + w_1(z_1 + z_3 - x_{b'} - \delta)_+] dz_1 dz_2 dz_3 \\ &\quad + w_t \int_0^\infty \phi_2(z_2) \int_0^{x_{b'} - x_{a'} + z_2} \phi_3(z_3) (x_2 - z_3 + x_{b'} - x_{a'}) dz_2 dz_3 \\ &\quad + w_t \int_0^\infty \phi_3(z_3) \int_0^{z_3 - x_{b'} + x_{a'}} \phi_2(z_2) (z_3 - z_2 + x_{a'} - x_{b'}) dz_2 dz_3. \end{aligned} \quad (77)$$

Changing the order of integration gives

$$\begin{aligned}
h_{\text{con}}(x_{a'}, x_{b'}) &= \int_0^\infty \phi_2(z_2) \Phi_3(x_{b'} - x_{a'} + z_2) \int_0^\infty [w_e(x_{a'} - z_1 - z_2)_+ + w_1(z_1 + z_2 - x_{a'})_+ \\
&\quad + w_e(x_{a'} + \delta - z_1 - z_2)_+ + w_1(z_1 + z_2 - x_{a'} - \delta)_+] \phi_1(z_1) dz_1 dz_2 \\
&\quad + \int_0^\infty \phi_3(z_3) \Phi_2(z_3 - x_{b'} + x_{a'}) \int_0^\infty [w_e(x_{b'} - z_1 - z_3)_+ + w_1(z_1 + z_3 - x_{b'})_+ \\
&\quad + w_e(x_{b'} + \delta - z_1 - z_3)_+ + w_1(z_1 + z_3 - x_{b'} - \delta)_+] \phi_1(z_1) dz_1 dz_3 \\
&\quad + w_t \left[\int_0^\infty \phi_2(z_2) \int_0^{x_{b'} - x_{a'} + z_2} \Phi_3(z_3) dz_3 dz_2 + \int_0^\infty \phi_3(z_3) \int_0^{z_3 - x_{b'} + x_{a'}} \Phi_2(z_2) dz_2 dz_3 \right].
\end{aligned} \tag{78}$$

By comparing with (17), it can be seen that the two integrands involve the expected schedule miss cost function for platooning on segment 1 given head start $x_{a'} - \tau_2$ and $x_{b'} - \tau_3$, respectively. Thus,

$$\begin{aligned}
h_{\text{con}}(x_{a'}, x_{b'}) &\equiv \mathbb{E} [\Phi_3(x_{b'} - x_{a'} + \tau_2) [h_1(x_{a'} - \tau_2) + h_1(x_{a'} - \tau_2 + \delta)] \\
&\quad + \Phi_2(\tau_3 - x_{b'} + x_{a'}) [h_1(x_{b'} - \tau_3) + h_1(x_{b'} - \tau_3 + \delta)] \\
&\quad + w_t \left(\int_0^{x_{b'} - x_{a'} + \tau_2} \Phi_3(z_3) dz_3 + \int_0^{\tau_3 - x_{b'} + x_{a'}} \Phi_2(z_2) dz_2 \right)] \quad x_{a'}, x_{b'} \in \mathbb{R}.
\end{aligned} \tag{79}$$

It can be demonstrated that any minima of (79) must be interior. For a given head start $x_{a'}$, the first-order optimality conditions for $x_{b'}$ are

$$\begin{aligned}
0 &= \frac{\partial h_{\text{con}}}{\partial x_{b'}} = \mathbb{E} [\phi_3(x_{b'} - x_{a'} + \tau_2) [h_1(x_{a'} - \tau_2) + h_1(x_{a'} - \tau_2 + \delta)] \\
&\quad - \phi_2(\tau_3 - x_{b'} + x_{a'}) [h_1(x_{b'} - \tau_3) + h_1(x_{b'} - \tau_3 + \delta)] \\
&\quad + \Phi_2(\tau_3 - x_{b'} + x_{a'}) [h'_1(x_{b'} - \tau_3) + h'_1(x_{b'} - \tau_3 + \delta)] \\
&\quad + w_t [\Phi_3(x_{b'} - x_{a'} + \tau_2) - \Phi_2(\tau_3 - x_{b'} + x_{a'})]]
\end{aligned} \tag{80}$$

For large positive $x_{b'}$ the right-hand side tends to w_t , which violates the first-order condition given that $w_t > 0$. For large negative $x_{b'}$ the right-hand side tends to $\mathbb{E}[h'_1(x_{b'} - \tau_3) + h'_1(x_{b'} - \tau_3 + \delta)] - w_t$, which is negative and also violates the optimality condition. Thus, there is no optimum involving a large difference in head starts. Now consider $x_{b'} = x_{a'} + \Delta$. The first-order conditions imply

$$0 = \mathbb{E} [\Phi_3(\Delta + \tau_2) [h'_1(x_{b'} - \Delta - \tau_2) + h'_1(x_{b'} - \Delta - \tau_2 + \delta)] + \Phi_2(\tau_3 - \Delta) [h'_1(x_{b'} - \tau_3) + h'_1(x_{b'} - \tau_3 + \delta)]] \tag{81}$$

For large positive $x_{b'}$ the right-hand side is positive; for large negative $x_{b'}$ it is negative, in violation of the optimality condition. There is thus no optimum involving a large positive or negative head start. The corresponding arguments apply to $x_{a'}$. In conclusion, optimal head starts are bounded in magnitude and difference.

Symmetric branches

Consider the special case where the travel times on the branches τ_2 and τ_3 are identically distributed, i.e., $\Phi_2(x) = \Phi_3(x) \equiv \Phi_{\text{sym}}(x)$ and $\phi_2(x) = \phi_3(x) \equiv \phi_{\text{sym}}(x)$ for all $x \in \mathbb{R}$. The expected schedule miss cost simplifies to

$$\begin{aligned}
h_{\text{con}}(x_{a'}, x_{b'}) &= \mathbb{E} [\Phi_{\text{sym}}(x_{b'} - x_{a'} + \tau) [h_1(x_{a'} - \tau) + h_1(x_{a'} - \tau + \delta)] \\
&\quad + \Phi_{\text{sym}}(\tau + x_{a'} - x_{b'}) [h_1(x_{b'} - \tau) + h_1(x_{b'} - \tau + \delta)] \\
&\quad + w_t \left(\int_0^{x_{b'} - x_{a'} + \tau} \Phi_{\text{sym}}(z) dz + \int_0^{\tau - x_{b'} + x_{a'}} \Phi_{\text{sym}}(z) dz \right)].
\end{aligned} \tag{82}$$

As can be seen, the cost function is symmetric in the head starts $x_{a'}$ and $x_{b'}$. Given the convexity of the cost function h_1 , it is reasonable to focus on solutions where head starts $x_{a'}^*$ and $x_{b'}^*$ are equal, i.e., minimize the function

$$h_{\text{con}}^{\text{sym}}(x_{\text{con}}) \equiv h_{\text{con}}(x_{\text{con}}, x_{\text{con}}) = 2\mathbb{E} \left[\Phi_{\text{sym}}(\tau) [h_1(x_{\text{con}} - \tau) + h_1(x_{\text{con}} - \tau + \delta)] + w_t \int_0^\tau \Phi_{\text{sym}}(z) dz \right]. \quad (83)$$

The first-order optimality condition for the head start is

$$\mathbb{E} [\Phi_{\text{sym}}(\tau)(\Phi_1(x_{\text{con}}^* - \tau) + \Phi_1(x_{\text{con}}^* - \tau + \delta))] = \frac{w_1}{w_e + w_1}. \quad (84)$$

independent of the unit travel time cost w_t . The second-order sufficient conditions for a minimum require that the Hessian of h_{con} be positive semi-definite at $(x_{\text{con}}^*, x_{\text{con}}^*)$. It follows from straightforward calculation that this implies

$$0 \leq \mathbb{E} [4w_t\phi_{\text{sym}}(\tau) + \Phi_{\text{sym}}(\tau)[h_1''(x_{\text{con}}^* - \tau) + h_1''(x_{\text{con}}^* - \tau + \delta)] - 4\phi_{\text{sym}}(\tau)[h_1'(x_{\text{con}}^* - \tau) + h_1'(x_{\text{con}}^* - \tau + \delta)] + 4\phi'_{\text{sym}}(\tau)[h_1(x_{\text{con}}^* - \tau) + h_1(x_{\text{con}}^* - \tau + \delta)]] \quad (85)$$

The right-hand side is positive for sufficiently large unit travel time cost w_t , which ensures that $(x_{\text{con}}^*, x_{\text{con}}^*)$ is a minimum.

The optimal symmetrical head start may be compared with the diverging routes case, assuming that both vehicles platoon along the corridor and split to the symmetric branches. According to (52) the optimality condition in this case is

$$\frac{\Phi_{\text{div}}(x_{\text{div}}^*) + \Phi_{\text{div}}(x_{\text{div}}^* + \delta)}{2} = \frac{\mathbb{E} [(\Phi_1(x_{\text{div}}^* - \tau) + \Phi_1(x_{\text{div}}^* - \tau + \delta))]}{2} = \frac{w_1}{w_e + w_1}. \quad (86)$$

where $\Phi_{\text{div}} = \Phi_{\text{sym}} * \Phi_1$ and the expectation is with respect to the distribution of the stochastic branch travel time Φ_{sym} . Given that $\Phi_{\text{sym}}(\tau)$ is increasing and $\mathbb{E}[\Phi_{\text{sym}}(\tau)] = 1/2$ by the properties of cumulative distribution functions, it follows that the optimal head start in the symmetric converging routes case (84) is greater than the optimal head start in the symmetric diverging route case (86).

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