Efficiency of Semi-Autonomous Platooning Bus Services in High-Demand Transit Corridors

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Abstract

The paper investigates the efficiency of serving high demand transit corridors with semi-autonomous bus platoons in both bus and BRT services. Platooning could make it possible to provide higher capacity than with conventional buses by forming virtual long buses out of multiple smaller vehicles, which could be particularly relevant in scenarios with large variation in demand between peak and off-peak hours. The problem is formulated as a constrained optimization problem to minimize total system cost, which includes waiting cost, access cost, riding cost, operating cost and capital cost. For single period with fixed demand, both analytical solutions and numerical examples are provided. Sensitivity analysis is carried out with regard to demand levels and capacity upper bound. The problem is generalized to a two-period problem considering peak and off-peak demand. Numerical results are provided with sensitivity analysis regarding average demand level and ratio of peak/off-peak demand. Furthermore, the impact of a lower bound on service headway is investigated. The result shows that semi-autonomous vehicle platooning is competitive in medium and high demand scenarios, with the potential of reduced users' cost and operator's operating cost at the expense of additional rolling stock cost. The minimum headway, restricted vehicle size, and higher demand ratio all make semi-autonomous platooning more advantageous.

Keywords: Semi-autonomous vehicle, platooning, public transport, operational research, BRT

1. Introduction

Autonomous vehicles have the potential to increase traffic safety and road capacity, reduce the driver cost of commercial transport, provide independent mobility for non-drivers, support vehicle sharing, etc. (Litman, 2017). The rapid advances in vehicle automation technology provide opportunities to realize the benefits, but also give rise to a series of challenges (e.g., system design, business models, social implications), especially in the early deployment stages. Within public transport, a main benefit of automated vehicles lies in the reduction of labor cost. Until now, the research focus has mostly been on the potential of using fully autonomous vehicles (Level-5 automation based on the definitions given in Watzenig and Horn (2017)) as a demand-responsive supplement to the current public transport system, e.g., autonomous taxis and autonomous on-demand bus services with flexible routes (Fagnant et al., 2015; Scheltes and de Almeida Correia, 2017).

In high-demand areas, White (2016) points out that it is impractical to serve demand with small vehicle capacities without fixed routes, since the vehicle kilometers traveled (VKT) per fully autonomous car (or bus) may be higher than its conventional counterpart when serving the same demand. For high-demand scenarios, bus rapid transit (BRT) systems can deliver passengers efficiently on segregated roads with corresponding infrastructure. BRT can become a competitive public transport mode unless the demand is low and sparsely distributed. Compared with traditional bus transit in mixed traffic, BRT can avoid congestion to a large extent Tirachini et al. (2010).

A few studies have considered replacing or combining conventional buses with fully autonomous buses on fixed lines. For example, Bergqvist and Åstrand (2017) propose a linear programming model to find

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out the optimal combination of conventional buses (capacity 70) and electrical fully autonomous minibuses (capacity 15) to operate four bus lines in Stockholm during both peak and off-peak hours. The result shows significant reduction in costs and emissions by introducing autonomous minibuses, although the investment costs are not considered due to data unavailability. Bösch et al. (2018) analyze the cost structure of various types of vehicles with and without full automation and conclude that the current line-based public transportation system will remain economically competitive only where travel demand is dense; elsewhere shared and pooled autonomous vehicles could be more efficient alternatives. Zhang et al. (2018) evaluate the efficiency of autonomous bus services in a general trunk-and-branches network. The results show that fully autonomous buses can reduce both users' costs and the operator's cost significantly, even if the rolling stock cost of autonomous vehicles is high.

Full automation is expected to be available in the market not before 2030 due to safety concerns and other reasons (Watzenig and Horn, 2017). Meanwhile, the adoption of semi-autonomous (Level-4 automation) buses is predicted to be realized within 3 to 10 years (Antonio Loro Consulting Inc., 2016). Until now, existing literature on public transport automation has focused on fully autonomous vehicles, with semi-autonomous buses barely being investigated. The objective of this study is to fill this research gap by addressing the technology selection and design of a bus line considering rapid transit and semi-automation.

Unlike fully autonomous buses, semi-autonomous buses need to operate as platoons. In this context, a platoon is a string of vehicles which drive with short inter-vehicle distances, without drivers in the follower vehicles but with a driver in the leading vehicle. Vehicle-to-vehicle communication and adaptive cruise control (ACC) or cooperative adaptive cruise control (CACC) technologies are used to maintain safety. Platooning is more actively studied for heavy-duty vehicles for the purpose of fuel saving, with coordination schemes ranging from scheduling to speed profiles adjustment (Zhang et al., 2017a,b; van de Hoef, 2018). Bus platoon coordination strategies have been developed by Liu et al. (2018) and Lam and Katupitiya (2013). Zhang et al. (2018) evaluate the efficiency of semi-autonomous bus services compared to conventional and fully autonomous buses in trunk-and-branches networks. Results indicate that semi-automation is most feasible in networks with long corridors and many branches to maximize the opportunities for platooning.

By platooning, the labor cost can be reduced, and the platoon capacity can be adjusted to demand without introducing significant operating cost variations. In particular, platooning could make it possible to provide higher capacity in high-demand corridors than with conventional buses by forming virtual long buses out of multiple smaller vehicles. This could be especially relevant in scenarios with large variation in demand between peak and off-peak hours. In such scenarios, a single vehicle size is not well suited to serve both peak and off-peak demand and the service quality is achieved only through the adjustment of service frequency. With platooning, meanwhile, the vehicles could drive independently during off-peak and form platoons during peak hours, thereby not requiring as drastic differences in service frequency.

This paper investigates the potential of serving passengers with semi-autonomous bus platoons in both bus and BRT services in high-demand corridors, by comparing with the services with conventional vehicles. The problem is formulated as a constrained optimization problem to minimize total system cost, which includes waiting cost, access cost, riding cost, operating cost and capital cost. The decision variables are the service headway, vehicle size and number of vehicles in the platoon (i.e., platoon length). The model extends earlier analytical approaches for optimal supply, technology selection and pricing by Jansson (1980), Tirachini et al. (2010) and Börjesson et al. (2017) by introducing automation technology and platoon driving in the operating and capital cost functions. Since bus speeds are typically low, fuel savings are negligible and not considered in this study.

For a single constant demand period, both analytical solutions and numerical examples are provided. The problem is then generalized to a multiple demand period problem considering peak and off-peak demand with numerical results obtained. Analysis is carried out to identify the efficiency of semi-autonomous vehicles under various demand circumstances. A more realistic case study with a lower bound on headways is also presented.

The remainder of the paper is structured as follows. Section 2 introduces the modeling of individual cost components, Section 3 determines the analytical solutions for one-period scenario, and Section 4 presents the formulation of multi-period problem. The numerical case study and sensitivity analysis is carried out in Section 5. Section 6 concludes the paper.

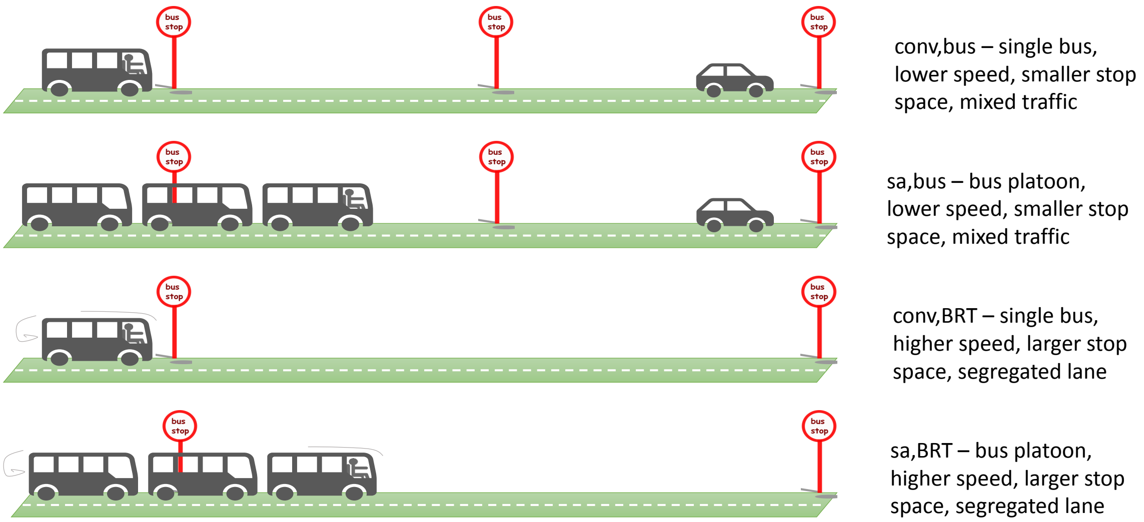

Figure 1: Four types of services, utilizing conventional or semi-autonomous buses within bus or BRT mode.

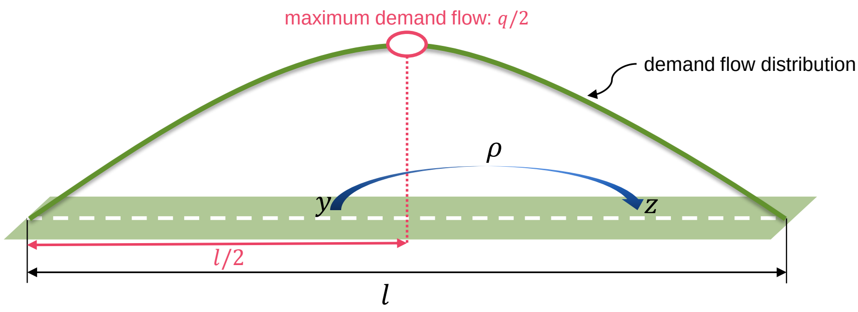

Figure 2: Network configuration and demand distribution.

2. Problem formulation

The objective is to investigate the efficiency of semi-autonomous buses in high-demand public transport systems. The problem is to minimize total system cost, which is the sum of passengers' cost and the bus operator's cost, by optimizing bus size, service frequency and platoon length (the number of vehicles contained in a platoon). The passengers' cost includes access time cost, waiting cost and riding cost, whereas the bus operator's cost is composed of operating cost and capital cost.

As a variant of traditional bus transit service, bus rapid transit (BRT) is also considered in this study. In contrast to bus transit, BRT provides higher driving speed and accordingly requires extra capital cost (mainly infrastructure cost and land cost) and longer distance between stops. There are two types of vehicles, conventional buses and semi-autonomous buses, which can be used in either a traditional bus transit service or in a BRT service. To summarize, semi-autonomous buses differ from conventional buses in lower operating cost and additional rolling stock cost.

Assuming that the bus fleet is homogeneous, there are four combinations of possible modes in total. In the following, we use i and j to distinguish between different vehicle types and service types respectively, where $i \in \{\text{conv}, \text{sa}\}$, $j \in \{\text{bus}, \text{BRT}\}$. The four types of services are illustrated in Fig. 1.

Operations are considered along a corridor of length l (see Fig. 2). The hourly directional demand q is assumed to be uniformly distributed along the route, i.e., the density of demand from any point y to any other point z ($0 \leq y < z \leq l$) is a constant, denoted by ρ. Therefore, the demand flow at any point x ($0 \leq x \leq l$) can be calculated by adding up all passengers who board the bus upstream of x and alight downstream of x. The detailed derivation is presented in Appendix A. The maximum demand flow q_{m} appears at the midpoint of the route (i.e., at $l/2$), where $q_{\text{m}} = q/2$.

2.1. Cost components

2.1.1. Access time cost

The access time cost is modeled as a function of the distance between two consecutive stops d^j. Assume that on average users need to walk $d^j/4$ from the origin to the boarding stop and $d^j/4$ from the alighting stop to the destination. Therefore, the total walking distance is $d^j/2$ on average. Given the walking speed v_{walk}, the hourly total access time cost incurred by all users is

$$C^j_{\text{access}} = c_{\text{access}} \frac{d^j}{v_{\text{walk}}} q, \tag{1}$$

where c_{access} is the value of access time and q is the directional demand. Since the stop spacing d^j is a service-dependent parameter, the access time cost depends only on the transport service (bus or BRT) and not on the vehicle type (conventional or semi-autonomous).

2.1.2. Waiting cost

Generally, low-frequency services are usually provided with a timetable, which helps passengers to adjust their departure time from home (or origin) properly, so passengers will follow the schedule to reduce their waiting time. By contrast, for high-frequency services, passengers arrive to the bus stop randomly at an almost constant rate. Focusing on high-frequency services and assuming that passengers arrive uniformly to the stop, the average waiting time is half of the service headway, i.e., $h/2$. Therefore, the hourly waiting cost is

$$C_{\text{wait}} = c_{\text{wait}} hq, \tag{2}$$

where c_{wait} is the value of waiting cost. It can be seen that the waiting cost does not involve any mode-dependent or vehicle-specific parameters, which means that it only relies on the decision variable h.

2.1.3. Riding cost

Assume that the boarding and alighting time is short enough to be ignored compared to the driving time, or that the average speed considering the stop-and-go process can be used to capture the boarding and alighting time. Ideally, the riding cost is proportional to the in-vehicle time, which is a direct function of the mode-dependent driving speed v^j, $j \in \{\text{bus}, \text{BRT}\}$. However, crowding may cause discomfort and the occupancy rate should be taken into account to measure this externality.

Following Jara-Díaz and Gschwender (2003), we model the value of in-vehicle time c_{iv} as a linear function of the occupancy rate ϕ: $c_{\text{iv}}(\phi) = c_{\text{ride}} + c_{\text{dcf}}\phi$, where c_{ride} is the unit fixed riding cost and c_{dcf} is the discomfort-related cost. The hourly bi-directional riding cost is then (see Appendix A for the derivation)

$$C^{ij}_{\text{ride}} = \frac{2ql}{v^j} \left(\frac{c_{\text{ride}}}{3} + \frac{2}{15} \frac{qhc_{\text{dcf}}}{Ns} \right), \tag{3}$$

where l is the length of the bus line, N is the number of vehicles in a bus platoon, i.e., platoon length ($N^{\text{conv}} = 1$ for conventional vehicle and $N^{\text{sa}} \geq 1$ for semi-autonomous vehicle), s is the size of each vehicle, and Ns is the capacity of the bus platoon. Therefore, the difference in riding cost between conventional and semi-autonomous vehicles lies in the respective headway-to-capacity ratios: $h^{\text{sa}}/(Ns^{\text{sa}})$ and $h^{\text{conv}}/s^{\text{conv}}$.

2.1.4. Operating cost

The hourly operating cost is modeled as a linear function of the vehicle size s. The linear relationship is derived statistically in Jansson (1980) and is applied in many subsequent studies. For each vehicle, the hourly operating cost is represented by

$$g^i_{\text{oper}}(s) = \begin{cases} c_{\text{oper}} + b_{\text{oper}} s & i = \text{conv} \quad \text{or} \quad i = \text{sa, platoon leader}, \\ (1 - \eta^{\text{sa}}) c_{\text{oper}} + b_{\text{oper}} s & i = \text{sa, platoon follower}, \end{cases} \tag{4}$$

where c_{oper} is the fixed operating cost and b_{oper} is the marginal operating cost with respect to vehicle size. The parameter η^{sa} is the fixed operating cost reduction coefficient due to semi-autonomous driving, $0 \leq \eta^{\text{sa}} \leq 1$. Note that only the platoon followers experience reduced operating cost.

To serve the demand with conventional vehicles, the hourly operating cost of the system is directly related to the operating fleet size, which is the total number of vehicles needed. However, semi-autonomous vehicles are operated jointly, which makes the system hourly operating cost dependent on the number of platoons rather than the fleet size. The general form of operating cost can be written as

$$C_{\text{oper}}^{ij} = \frac{2l\{[1 + (N-1)(1-\eta^i)]c_{\text{oper}} + Nb_{\text{oper}}s\}}{v^j h}, \tag{5}$$

where $N^{\text{conv}} = 1$, $\eta^{\text{conv}} = 0$.

2.1.5. Capital cost

The hourly rolling stock cost per vehicle is also modeled as a linear function of the vehicle size s (Jansson, 1980). For each conventional vehicle and semi-autonomous vehicle, the hourly rolling stock cost is represented by

$$g_{\text{cptl}}^i(s) = \begin{cases} c_{\text{cptl}} + b_{\text{cptl}}s & i = \text{conv}, \\ (1+\beta^i)c_{\text{cptl}} + b_{\text{cptl}}s & i = \text{sa}, \end{cases} \tag{6}$$

where c_{cptl} is the unit fixed rolling cost and b_{cptl} is the marginal capital cost with respect to vehicle size. The parameter β^i is the additional capital cost due to semi-autonomous vehicles, $\beta^{\text{conv}} = 0$, $\beta^{\text{sa}} \geq 0$.

The capital cost includes infrastructure cost, land cost and rolling stock cost. Since BRT requires segregated lanes to ensure higher vehicle diving speed than traditional bus transit, we assume a fixed fraction of capital cost to capture the extra infrastructure and land needed by BRT, whereas this part is 0 for traditional bus transit. Therefore, the hourly capital cost in transport mode j using vehicle type i is

$$C_{\text{cptl}}^{ij} = c_0^j + \frac{2lN[(1+\beta^i)c_{\text{cptl}} + b_{\text{cptl}}s]}{v^j h}, \tag{7}$$

where c_0^j is the sum of infrastructure and land cost, and $c_0^{\text{bus}} = 0$.

3. Single demand period cost minimization

For each combination of vehicle type $i \in \{\text{conv}, \text{sa}\}$ and service type $j \in \{\text{bus}, \text{BRT}\}$, the objective is to minimize the total generalized cost,

$$\begin{aligned} C_{\text{tot}}^{ij} &= c_{\text{access}} \frac{d^j q}{v_{\text{walk}}} + c_{\text{wait}} hq + \frac{2ql}{v^j}\left(\frac{c_{\text{ride}}}{3} + \frac{2}{15}\frac{qhc_{\text{dcf}}}{Ns}\right) + c_0^j \\ &+ \frac{2lN[(1-\eta^i)c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s] + 2l\eta^i c_{\text{oper}}}{v^j h}, \end{aligned} \tag{8}$$

subject to

$$2Ns - hq \geq 0, \tag{9}$$

$$s_{\text{ub}} - s \geq 0, \tag{10}$$

$$N - 1 \geq 0, \tag{11}$$

where (9) ensures that the maximum demand flow is served, (10) limits the vehicle size to its upper bound and (11) ensures that each platoon contains at least one vehicle.

By using the Karush-Kuhn-Tucker conditions, we obtain five feasible cases with the general formulation (8)-(11). Since one of the main differences in the formulation between different service modes is the platoon length N, we can divide the analytical results into two categories: services with conventional vehicles and services with semi-autonomous vehicles. Note that constraint (11) is always active for conventional vehicles. Since the main advantage of semi-autonomous buses is the potential to extend the service capacity without incurring proportionally higher additional operating cost, the demand level is a critical exogenous factor. Therefore, the solutions are further divided into several regimes based on the critical values of q.

3.1. Analytical solution for conventional vehicles

For services with conventional vehicles, there are three regimes of demand q levels. The thresholds between each regime, denoted as $q_{1,2}^{\text{conv},j}$ and $q_{2,3}^{\text{conv},j}$, are given by

$$q_{1,2}^{\text{conv},j} = \frac{15 s_{\text{ub}}^2 v^j c_{\text{wait}}(b_{\text{oper}} + b_{\text{cptl}})}{4 l c_{\text{dcf}}[c_{\text{oper}} + c_{\text{cptl}}]}, \tag{12}$$

$$\begin{aligned} q_{2,3}^{\text{conv},j} = \max \Bigg\{ & \frac{15 l [c_{\text{oper}} + c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}}) s_{\text{ub}}] - 15 v^j c_{\text{wait}} s_{\text{ub}}}{4 l c_{\text{dcf}}}, \\ & \frac{30 v^j s_{\text{ub}}^2 c_{\text{wait}}}{15 l [c_{\text{oper}} + c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}}) s_{\text{ub}}] - 8 l c_{\text{dcf}} s_{\text{ub}}}, \\ & \frac{2 v^j s_{\text{ub}}^2 c_{\text{wait}}}{l [c_{\text{oper}} + c_{\text{cptl}}]} \Bigg\}. \end{aligned} \tag{13}$$

Note that $q_{2,3}^{\text{conv},j}$ exists (i.e., regime 3 can be reached) only when

$$s_{\text{ub}} < \frac{15(c_{\text{oper}} + c_{\text{cptl}})}{8 c_{\text{dcf}} - 15(b_{\text{cptl}} + b_{\text{oper}})}. \tag{14}$$

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The three demand regimes can be described as follows.

Demand regime 1: in the low demand regime $q \in (0, q_{1,2}^{\text{conv},j})$, the optimal solution appears when both the service capability constraint (9) and the vehicle capacity constraint (10) are inactive. Thus, the highest occupancy rate (which occurs at the mid-point of the corridor) is below 1 and the bus size is below the upper bound s_{ub}. The optimal bus size and headway are:

$$s = \sqrt{\frac{4 q l c_{\text{dcf}}[c_{\text{oper}} + c_{\text{cptl}}]}{15 v^j c_{\text{wait}}(b_{\text{oper}} + b_{\text{cptl}})}}, \quad h = \sqrt{\frac{2 l [c_{\text{oper}} + c_{\text{cptl}}]}{q v^j c_{\text{wait}}}}, \tag{15}$$

which indicates that s increases and h decreases proportionally to the square root of q.

Demand regime 2: in the medium demand regime $q \in [q_{1,2}^{\text{conv},j}, q_{2,3}^{\text{conv},j})$, the optimal solution is obtained when (9) is inactive and (10) is active. Therefore, $s = s_{\text{ub}}$ and the optimal headway is

$$h = \sqrt{\frac{30 l s_{\text{ub}}[c_{\text{oper}} + c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}}) s_{\text{ub}}]}{15 v^j s_{\text{ub}} c_{\text{wait}} q + 4 q^2 l c_{\text{dcf}}}}. \tag{16}$$

As can be seen, h decreases more rapidly as q increases, and the occupancy rate is still below 1.

Demand stage 3: theoretically, in the high demand regime $q \in [q_{2,3}^{\text{conv},j}, +\infty)$, all the three constraints (9)-(11) become active. Thus $s = s_{\text{ub}}$ and $h = 2 s_{\text{ub}}/q$, which indicates h and q become inversely proportional since the occupancy rate becomes 1. However, it is possible that h may be too short to be practical.

3.2. Analytical solution for semi-autonomous vehicles

Similarly for services with semi-autonomous vehicles, there are three regimes of demand. The transition thresholds $q_{1,2}^{\text{sa}j}$ and $q_{2,3}^{\text{sa}j}$ are given by

$$q_{1,2}^{\text{sa},j} = \frac{15 s_{\text{ub}}^2 v^j c_{\text{wait}}(b_{\text{oper}} + b_{\text{cptl}})}{4 l c_{\text{dcf}}[c_{\text{oper}} + (1 + \beta^{\text{sa}}) c_{\text{cptl}}]}, \tag{17}$$

$$q_{2,3}^{\text{sa},j} = \frac{15 c_{\text{wait}} s_{\text{ub}} v^j [(1 + \beta^{\text{sa}}) c_{\text{cptl}} + (1 - \eta^{\text{sa}}) c_{\text{oper}} + (b_{\text{cptl}} + b_{\text{oper}}) s_{\text{ub}}]}{4 c_{\text{dcf}} c_{\text{oper}} \eta^{\text{sa}} l}. \tag{18}$$

The three regimes can be described as follows:

Demand regime 1: in the low demand regime $q \in (0, q_{1,2}^{\text{sa},j})$, the optimal condition appears when (9) and (10) are inactive and (11) is active, i.e., $N = 1$. The optimal vehicle size and headway are

$$s = \sqrt{\frac{4 q l c_{\text{dcf}}[c_{\text{oper}} + (1 + \beta^{\text{sa}}) c_{\text{cptl}}]}{15 v^j c_{\text{wait}}(b_{\text{oper}} + b_{\text{cptl}})}}, \quad h = \sqrt{\frac{2 l [c_{\text{oper}} + (1 + \beta^{\text{sa}}) c_{\text{cptl}}]}{q v^j c_{\text{wait}}}}. \tag{19}$$

The evolution of the service characteristics are the same as with conventional vehicles above, since there is only a single vehicle in the platoon, except for the additional rolling stock cost coefficient β^{sa}.

Demand regime 2: in the medium demand regime $q \in [q_{1,2}^{\text{sa},j}, q_{2,3}^{\text{sa},j}]$, (10) and (11) are active, i.e., $N = 1$ and $s = s_{\text{ub}}$, while (9) remains inactive. The optimal headway is

$$h = \sqrt{\frac{30 l s_{\text{ub}}[c_{\text{oper}} + (1+\beta^{\text{sa}})c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}{15 v^j s_{\text{ub}} c_{\text{wait}} q + 4q^2 l c_{\text{dcf}}}}, \tag{20}$$

which is analogous to that of conventional vehicles. The occupancy rate is still below 1 in this regime.

Demand regime 3: in the high demand regime $q \in (q_{2,3}^{\text{sa},j}, +\infty)$, (10) remains active, i.e., $s = s_{\text{ub}}$. Whether constraints (9) and (11) are binding or non-binding depend on the relationship between s_{ub} and $s_{\min}^{\text{sa}}$, where

$$s_{\min}^{\text{sa}} = \frac{15[(1-\eta^{\text{sa}})c_{\text{oper}} + (1+\beta^{\text{sa}})c_{\text{cptl}}]}{8c_{\text{dcf}} - 15(b_{\text{cptl}} + b_{\text{oper}})}. \tag{21}$$

If $s_{\text{ub}} > s_{\min}^{\text{sa}}$, (9) and (11) become inactive. The optimal platoon length and headway are

$$N = \sqrt{\frac{4ql\eta^{\text{sa}} c_{\text{oper}} c_{\text{dcf}}}{15 c_{\text{wait}} v^j s_{\text{ub}}[(1-\eta^{\text{sa}})c_{\text{oper}} + (1+\beta^{\text{sa}})c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}}, \quad h = \sqrt{\frac{2l\eta^{\text{sa}} c_{\text{oper}}}{c_{\text{wait}} q v^j}}, \tag{22}$$

which indicates that semi-autonomous vehicles start to form platoons ($N > 1$) and the headway becomes independent of s_{ub} but dependent on the cost saving coefficient η^{sa}. Also, the occupancy rate $\phi(x)$ becomes independent of q. The highest occupancy rate (which occurs at the corridor mid-point) is below 1,

$$\phi\left(\frac{l}{2}\right) = \frac{2Ns}{hq} = \sqrt{\frac{8c_{\text{dcf}} s_{\text{ub}}}{15[(1-\eta^{\text{sa}})c_{\text{oper}} + (1+\beta^{\text{sa}})c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}}. \tag{23}$$

If $s_{\text{ub}} = s_{\min}^{\text{sa}}$, the solution (22) does not change except that the highest occupancy rate reaches 1, which also triggers constraint (9) to become active.

If $s_{\text{ub}} < s_{\min}^{\text{sa}}$, semi-autonomous vehicles always form platoons with fully occupied vehicles in the high demand regime. The optimal platoon length and headway are

$$N = \frac{1}{s_{\text{ub}}}\sqrt{\frac{l\eta^{\text{sa}} c_{\text{oper}} q}{2c_{\text{wait}} v^j}}, \quad h = \sqrt{\frac{2l\eta^{\text{sa}} c_{\text{oper}}}{c_{\text{wait}} v^j q}}. \tag{24}$$

Still, in regime 3, it is possible that h becomes too short or N becomes too large to make the solution practical. The optimal N is continuous between $s_{\text{ub}} < s_{\min}^{\text{sa}}$ and $s_{\text{ub}} \geq s_{\min}^{\text{sa}}$.

4. Multiple demand periods cost minimization

We now consider the more realistic scenario in which, there are $M > 1$ discrete demand levels during a studied period of time, denoted by q_k, $1 \leq k \leq M$. Given the relative length of each demand period r_k with respect to the total period length of t, the cost can be averaged to an hourly basis. Thus, $r_1 + \ldots + r_M = 1$. Without loss of generality, we assume the demand levels are sorted in ascending order, i.e., $q_1 \leq q_2 \leq \ldots \leq q_M$. Therefore, the peak-hour corresponds to period M with demand level q_M. The average hourly demand $\bar{q}$ is easily calculated as $\bar{q} = r_1 q_1 + \ldots + r_M q_M$.

From the operator's perspective, the rolling stock needs to be sufficient to serve the peak demand q_M. The peak hour fleet size (i.e., the number of vehicles in operation on the road simultaneously, different from rolling stock) is expected to be larger than the off-peak fleet size, and the peak headway is expected to be smaller than off-peak headway (see Fig. 3). Assuming that the vehicles are utilized evenly during the studied period, i.e., that every vehicle has the same utilization rate, the differences among demand levels q_k $(1 \leq k \leq M)$ will result in different utilization rates. For instance, if $q_M \gg q_{M-1} \geq \ldots \geq q_1$, the utilization rate of each vehicle will be much lower than in the scenario where $q_M \approx q_{M-1} \approx \ldots \approx q_1$, which may be inefficient.

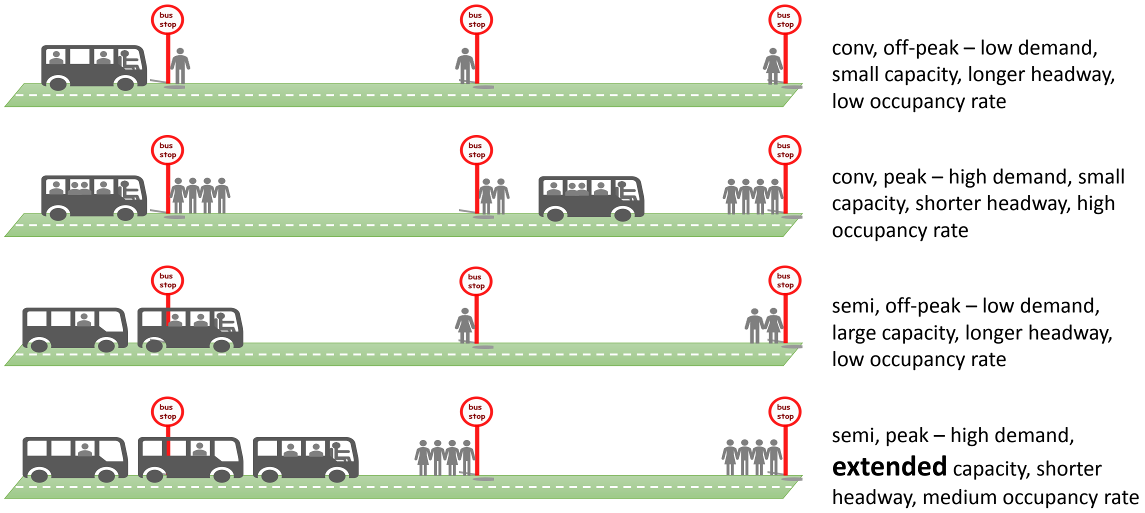


Figure 3: Service operations during off-peak and peak hours with conventional and semi-autonomous vehicles.

When serving multiple demand levels, semi-autonomous vehicles differ from conventional vehicles in the sense that semi-autonomous vehicles are capable of expanding the platoon capacity besides adjusting service headways, which is the only adjustment enabled by conventional vehicles. However, when it comes to rolling stock cost, running with semi-autonomous vehicles cannot avoid the inefficiency of vehicle usage caused by enormous demand leaps in peak hours.

The users' hourly access cost $\overline{C}^{ij}_{\text{access}}$, waiting cost $\overline{C}^{ij}_{\text{wait}}$ and riding cost $\overline{C}^{ij}_{\text{ride}}$ are

$$\overline{C}^{ij}_{\text{access}} = c_{\text{access}} \frac{d^j}{2v_{\text{walk}}} 2\bar{q}, \quad \overline{C}^{ij}_{\text{wait}} = c_{\text{wait}} \sum_{k=1}^{M} r_k h_k q_k, \quad \overline{C}^{ij}_{\text{ride}} = \frac{2l}{v^j} \sum_{k=1}^{M} r_k q_k \left(\frac{c_{\text{ride}}}{3} + \frac{2}{15} \frac{q_k h_k c_{\text{dcf}}}{N_k s} \right), \tag{25}$$

respectively. From the users' perspective, the main difference between peak and off-peak hours lies in the service headway. Furthermore, for service with semi-autonomous vehicles, the platoon length N_k^{sa} may vary between periods as a response to demand variations.

The operator's operating cost $\overline{C}^{ij}_{\text{oper}}$ and capital cost $\overline{C}^{ij}_{\text{cptl}}$ are calculated by

$$\overline{C}^{ij}_{\text{oper}} = \frac{2l}{v^j} \sum_{k=1}^{M} r_k \frac{\{[1 + (N_k - 1)(1 - \eta^i)] c_{\text{oper}} + N_k b_{\text{oper}} s\}}{h_k}, \tag{26}$$

$$\overline{C}^{ij}_{\text{cptl}} = c_0^j + \frac{2lN_M[(1 + \beta^i) c_{\text{cptl}} + b_{\text{cptl}} s]}{v^j h_M}, \tag{27}$$

where h_M is the peak-hour headway. The capital cost depends on the number of vehicles needed to serve the peak hour (i.e., peak vehicle), which is $2lN_M/(v^j h_M)$.

The objective is to minimize the generalized cost considering multiple demand levels for each combination of vehicle type $i \in \{\text{conv}, \text{sa}\}$ and service type $j \in \{\text{bus}, \text{BRT}\}$:

$$\begin{aligned} \overline{C}^{ij}_{\text{tot}} = {} & c_{\text{access}} \frac{d^j \bar{q}}{v_{\text{walk}}} + c_{\text{wait}} \sum_{k=1}^{M} r_k h_k q_k + \frac{2l}{v^j} \sum_{k=1}^{M} r_k q_k \left(\frac{c_{\text{ride}}}{3} + \frac{2}{15} \frac{q_k h_k c_{\text{dcf}}}{N_k s} \right) + c_0^j \\ & + \frac{2lN_M[(1 + \beta^i) c_{\text{cptl}} + b_{\text{cptl}} s]}{v^j h_M} + \frac{2l}{v^j} \sum_{k=1}^{M} r_k \frac{\{[1 + (N_k - 1)(1 - \eta^i)] c_{\text{oper}} + N_k b_{\text{oper}} s\}}{h_k}, \end{aligned} \tag{28}$$

subject to

$$2N_k s - h_k q_k \geq 0, \tag{29}$$

Table 1: Parameters

Parameter	Value	Parameter	Value	Parameter	Value
b_{oper}	0.75 SEK/hour/vehicle	c_{oper}	334.6 SEK/hour/vehicle	v^{bus}	15 km/h
b_{cptl}	1.01 SEK/hour/vehicle	c_{cptl}	14.24 SEK/hour/vehicle	q	4000 passenger/hour
c_0	45310 SEK/hour	d^{BRT}	0.8 km	s_{ub}	64 passenger
c_{access}	66.1 SEK/hour	d^{bus}	0.4 km	β^{sa}	0.2
c_{wait}	79.35 SEK/hour	l	15 km	r_1	9/13
c_{ride}	56.28 SEK/hour	v_{walk}	4 km/h	r_2	4/13
c_{dcf}	28.14 SEK/hour	v^{BRT}	30 km/h	q_2/q_1	3
η^{sa}	0.63			q_1	4000 passenger/hour

$$N_k - 1 \geq 0, \tag{30}$$

and (10), where the constraints ensure the service serves the maximum demand flow at every demand level.

As can be seen from the objective function, the roles of off-peak demand levels $q_1, q_2, \ldots, q_{M-1}$ are equivalent, in contrast to the peak demand q_M. More specifically, all off-peak demand levels affect the decision-making process mainly through the determination of off-peak headways $h_1, h_2, \ldots, h_{M-1}$, whereas the rolling stock cost only relies on the peak demand. Since the specific values of off-peak demand levels and their respective relative durations $r_1, r_2, \ldots, r_{M-1}$ influence the vehicle size s and platoon length N_k together with peak-related parameters, a case study with one off-peak demand level and one peak demand level should be sufficient to capture the impacts of multiple demand levels on the system design without complicating the problem.

5. Numerical analysis

The numerical results for both one-period scenario and two-period scenario are obtained by using the gradient-based nonlinear programming solver provided by MATLAB R2017b with the interior-point optimization algorithm to find the local minimum, combined with a scatter-search global search solver to find the global optimum (MathWorks, 2018).

5.1. Parameters

The parameters are given in Table 1. Most parameter values in the numerical study are based on ATC (2006)[1] and Tirachini et al. (2010). The stop spaces, walking speed, and driving speeds are from Tirachini et al. (2010). The nominal peak/off-peak ratio is from Börjesson et al. (2017), who use 4 peak hours and 9 off-peak hours per day.

The users' cost contains access cost, waiting cost and riding cost, and the operator's cost is comprised of capital cost and operating cost. The capital cost includes three parts, namely infrastructure cost (e.g., management centers, bus lanes, stations), land cost and rolling stock cost (i.e., vehicle capital cost). The user cost parameters $c_{\mathrm{access}}, c_{\mathrm{wait}}, c_{\mathrm{ride}}$ are from ATC (2006). Moreover, the discomfort cost c_{dcf} are based on the finding in Börjesson et al. (2017), which states that observations show that the in-vehicle time cost increases by 50% if the bus is full. Therefore, we use $c_{\mathrm{dcf}} = c_{\mathrm{ride}}/2$.

Operator cost parameters: The vehicle capital cost parameters c_{cptl}, b_{cptl} and operating cost parameters c_{oper}, b_{oper} are derived from linear regression based on relevant data in ATC (2006), which is given in Table. 2. The operating cost is the sum of on-vehicle crew cost (time-related), direct operating cost (distance-related), overhead and profit margin. By dividing the annual capital cost with the equivalent hours per year (see Table 3), the hourly capital cost can be obtained. The distance-related cost can be converted to time-related cost, given the constant speed 15 km/h. Taking the overhead and profit margin into account, the hourly operating cost is calculated. For example, the hourly operating cost for mini bus is $(211.53 + 3.9742 \times 15) \times (1 + 21\%) \times (1 + 6\%) = 347.8$ SEK, whereas the hourly capital cost for mini bus is $101.278 \times 1000/3000 = 33.8$ SEK.

The linear regression model gives $c_{\mathrm{oper}} = 334.60$ SEK/bus/hour, among which the on-vehicle costs are eliminated, which gives the nominal value of $\eta^{\mathrm{sa}} = 211.53/334.60 = 0.63$.

[1] Units are converted from AUD to SEK (1 AUD=6.41 SEK).

Table 2: Original data used in the model. The size consists of seated and standing capacity. The overhead and profit margin are based on the total operating cost, which is the sum of the on-vehicle crew cost and the distance-related direct operating cost. Data source: ATC (2006).

Type	Size (pax.)	Annual capital cost (kSEK)	On-vehicle crew costs (SEK/bus/hour)	Vehicle (direct operating) costs (SEK/bus/km)	Overhead (%)	Profit margin (%)
Mini	19	101.278	211.53	3.9742	21	6
Midi	40	174.352	211.53	4.9357	21	6
Rigid standard	64	237.17	211.53	5.769	21	6
Rigid long	81	249.349	211.53	6.3459	21	6
Articulated	101	374.344	211.53	7.2433	21	6

Table 3: Infrastructure and land cost

Land cost (mSEK/hectare)	Width (m)	Length (km)	Land economic life (years)
57.69	10	15	125
Infrastructure cost (mSEK/km)	Asset life (years)	Maintenance cost (SEK/hour)	Equivalent hours per year
64.1	50	1891	3000

Infrastructure and land cost parameters: Assume infrastructure cost and land cost are independent of the bus size. We refer to Tirachini et al. (2010) for the calculation of c_0[2]. The annual capital cost is calculated by

$$C = (A - B)\frac{r}{1 - \frac{1}{(1+r)^n}}, \tag{31}$$

where *A* is the purchase price, *B* is the residual value, *r* is the discount rate and *n* is the asset life. Therefore, the annual infrastructure cost and the annual land cost are calculated by (see Table 3) $C_{\text{infra}} = (64.1 \times 15 - 0) \times 0.07/(1 - 1/(1 + 0.07)^{50}) = 69.6701$ mSEK, and $C_{\text{land}} = (57.69 \times 15000 \times 10 \times 10^{-4} - 0) \times 0.07/(1 - 1/(1 + 0.07)^{125}) = 60.5874$ mSEK, respectively. Given the equivalent hours of one year, the annual capital cost can be further converted to hourly capital cost. Adding up the infrastructure cost, the land cost and the infrastructure maintenance cost, we obtain $c_0 = (C_{\text{infra}} + C_{\text{land}})/3000 + 1891 = 45310$ SEK.

5.2. *Single demand period scenario*

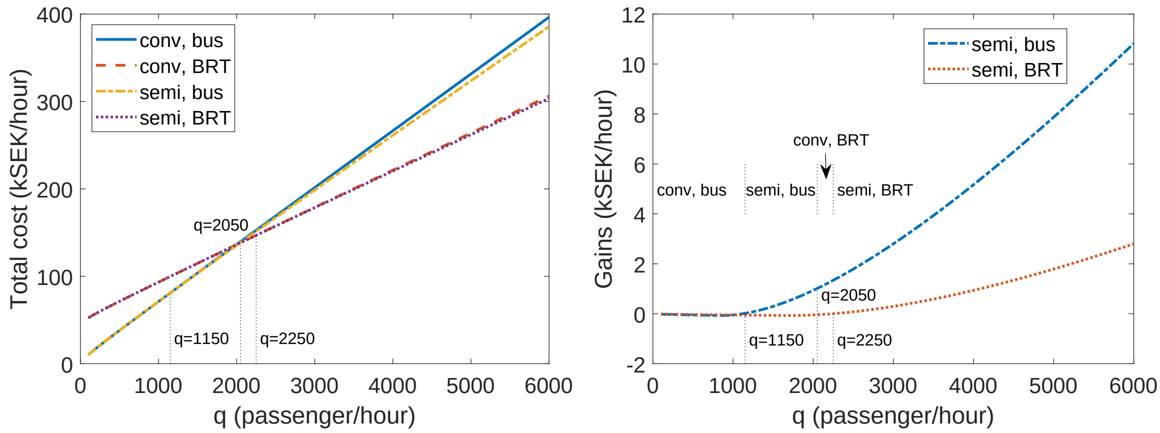
5.2.1. *Impact of demand level*

We study the impact of demand level by varying the directional demand q from 100 passenger/hour to 6000 passenger/hour, while other parameters are fixed. The total cost and gains from adopting semi-autonomous vehicles in bus and BRT systems are given in Fig. 4, where positive gains mean that semi-autonomous vehicles are advantageous compared to conventional vehicles and negative gains mean the opposite. Generally, bus services (with conventional buses and semi-autonomous buses) are preferred in low-demand scenarios, while BRT services with the two bus types exhibit advantages when the demand is high. More specifically, bus service with conventional buses is the most beneficial when the demand is below 1150 passenger/hour; as the demand grows, bus service with semi-autonomous buses becomes advantageous until it reaches 2050 passenger/hour. Above this level of demand, BRT service with conventional buses becomes competitive; eventually, when the demand is above 2250 passenger/hour, BRT service with semi-autonomous buses surpasses the other three modes in cost efficiency (see Fig. 4(b)).

In terms of the service characteristics, it will be helpful to refer to the critical values obtained in Section 3. Given the parameters, the critical demand values are $q_{1,2}^{\text{conv,bus}} = 219$ passenger/hour, $q_{1,2}^{\text{conv,BRT}} = 437$ passenger/hour, $q_{1,2}^{\text{sa,bus}} = 217$ passenger/hour, $q_{1,2}^{\text{sa,BRT}} = 434$ passenger/hour, $q_{2,3}^{\text{sa,bus}} = 814$ passenger/hour and $q_{2,3}^{\text{sa,BRT}} = 1628$ passenger/hour. According to Fig. B.11 in Appendix B, $q_{2,3}^{\text{conv,bus}}$ and $q_{2,3}^{\text{conv,BRT}}$ are far beyond the tested range (or may not exist). The critical values are marked in Fig. 5(b) ($q_{1,2}^{\text{sa,bus}}$ and $q_{1,2}^{\text{sa,BRT}}$ are close to $q_{1,2}^{\text{conv,bus}}$ and $q_{1,2}^{\text{conv,BRT}}$ respectively, and thus are omitted in the marking).

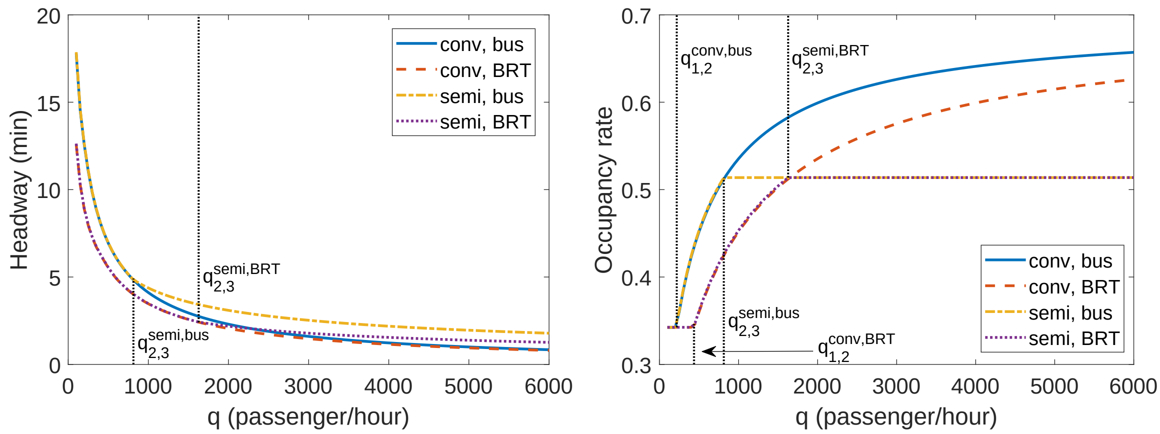
In regime 1, the vehicle size in bus systems are larger than the vehicle size in BRT systems, which leads to a longer headway. As demand increases, semi-autonomous vehicles soon enter regime 3 (the demand range from regime 2 to regime 3 is not large), and then services with semi-autonomous vehicles result in

[2]The discount rate is set to 7%.



(a) (b)

Figure 4: Single-period results of total cost and gains from adopting semi-autonomous vehicles with respect to demand q, parameters according to Table. 1.



(a) (b)

Figure 5: Single-period results of (a) headway and (b) occupancy rate with respect to demand q, parameters according to Table. 1.

longer headways than services with conventional vehicles as demand increases. This is mainly due to the platooning effect, which extends the platoon capacity by combining multiple semi-autonomous vehicles in medium and high demand scenarios. The platoon capacity of semi-autonomous vehicle platoon in bus system can be up to 2.7 times the upper bound of conventional vehicle capacity s_{ub} when $q = 6000$ passenger/hour. Meanwhile, the occupancy rate of semi-autonomous vehicles is fixed to 0.51 when the demand is above $q^{\text{sa,bus}}_{2,3}$ or $q^{\text{sa,BRT}}_{2,3}$, whereas the conventional vehicle occupancy rate gradually increases as the demand grows.

The user and operator cost and gains are shown in Fig. 6. It can be seen from Fig. 6(a) that user cost is the predominant component of the total cost, except for in the low demand region. In high demand regions, the user cost of a certain service can be 3 to 6 times the operator cost. Generally, bus service users incur higher cost than BRT service users, mainly because of the lower riding speed. In medium and high demand regions, semi-autonomous vehicle users gain benefits against conventional vehicle users, due to less crowding effect. Although within the tested demand range the operator cost of BRT services are higher than that of bus services due to the land cost for segregated lanes, the inclinations of the curves imply that the bus operator cost will eventually surpass the BRT operator cost as the demand continues to grow. Fig. 6(b) indicates that the user gains become positive before semi,bus/semi,BRT service becomes the most favorable, which means for semi-autonomous vehicle services to be advantageous (in terms of total cost), the user gains must

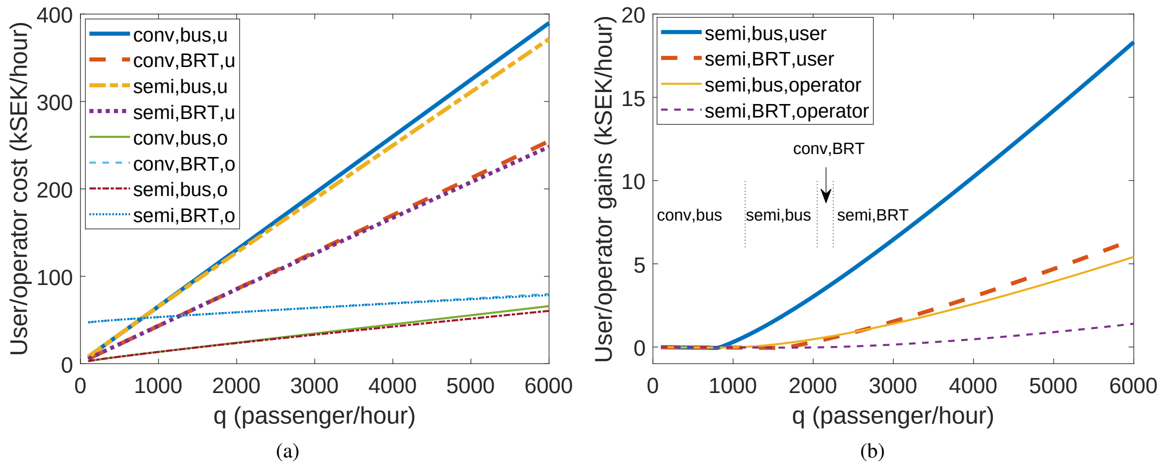


Figure 6: Single-period results of (a) user cost and operator cost, (b) user gains and operator gains with respect to demand q, parameters according to Table. 1.

compensate for the extra operator cost caused by the additional rolling stock expenditure.

Combining Fig. 5 and Fig. 6, when the demand is high, the passengers' waiting cost with semi-autonomous vehicles will be higher than with conventional vehicles, due to longer headways. However, the total users' cost is lower with semi-autonomous vehicles, because the riding cost becomes lower. This implies that the user cost saving can be sensitive to the value of in-vehicle time c_{ride}.

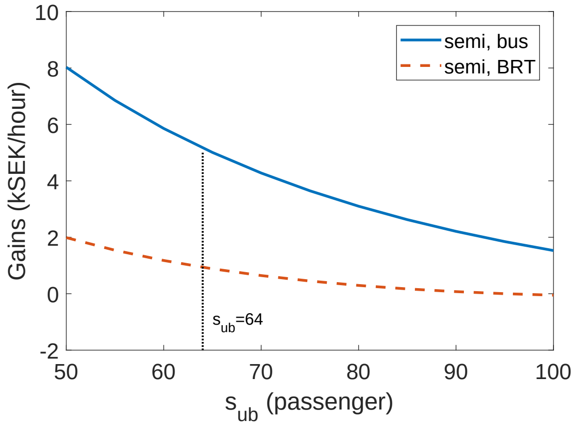
5.2.2. *Impact of capacity upper bound*

The impact of the capacity upper bound s_{ub} is demonstrated in Fig. 7. The tested demand $q = 4000$ passenger/hour is greater than the critical values $q_{1,2}^{\mathrm{conv,bus}}$, $q_{1,2}^{\mathrm{conv,BRT}}$, $q_{2,3}^{\mathrm{sa,bus}}$ and $q_{2,3}^{\mathrm{sa,BRT}}$, given that $s_{\mathrm{ub}} \in [50, 100]$. Therefore, according to (16) and (22) in Section 3, the headways of conventional vehicles in both bus system and BRT system will decrease as s_{ub} decreases, whereas the service headway of semi-autonomous vehicles in both bus system and BRT system will stay unchanged with respect to upper bound variations. The headways of conventional vehicle services are shortened by about 30 sec as s_{ub} varies from 100 to 50. Moreover, the platoon length $N^{\mathrm{sa},j}$ will increase as s_{ub} decreases. If the vehicle size is restricted to be smaller, there will be more necessity for semi-autonomous vehicles to form platoons, or the platoon length tends to be longer, which means services with semi-autonomous vehicles will be more favorable. For example, BRT service with semi-autonomous vehicles costs 51.2 SEK/hour more than the same service mode with conventional vehicles given $s_{\mathrm{ub}} = 100$, but saves 1988.6 SEK/hour if $s_{\mathrm{ub}} = 50$. The occupancy rates of all services increases as stricter restrictions are imposed, but the impacts on conventional vehicle services are more dramatic than on semi-autonomous vehicle services.

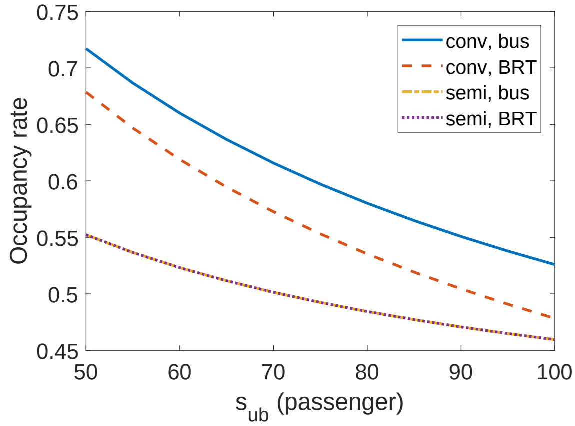
5.3. *Two demand periods scenario*

5.3.1. *Impact of demand level*

The service characteristics and gains of using semi-autonomous vehicles with respect to average demand $\bar{q}$ are presented in Fig. 8. Note that semi-autonomous vehicles can extend the platoon capacity during peak hours, whereas the capacity of conventional vehicles does not change (see Fig. 8(b)). By comparing the results obtained here with the results in the section above, it can be seen that the headway patterns during both off-peak and peak hours are similar with the patterns in the one demand level scenario, only different in the magnitudes. Interestingly, the off-peak headways are not remarkably longer than peak headways, except for in low demand regions. Moreover, the platoon capacities of semi-autonomous vehicles during the two periods are not significantly distinct. The occupancy rates during peak hours are obviously higher than that during off-peak hours. Apparently, the users' cost far outweighs the service provider's operating cost in the decision-making process, which leads to almost indistinguishable headways and platoon capacities during two periods. The differences in occupancy rates are mostly caused by q_2/q_1, given that headways and platoon capacities do not vary greatly.



(a)



(b)

Figure 7: Single-period results of (a) gains by using semi-autonomous vehicles and (b) occupancy rate with respect to capacity upper bound s_{ub}, parameters according to Table. 1.

Fig. 8(d) demonstrates that the increase of average demand results in more loss (negative gains) in capital cost to implement semi-autonomous vehicles from service provider's perspective (note that the capital cost is fixed during peak/off-peak hours). For example, given $\bar{q} = 4000$ passenger/hour, the hourly losses in capital costs for the implementation in bus system and BRT system are 2.3 kSEK/hour and 0.92 kSEK/hour respectively, which are equivalent to 6.9 mSEK/year and 2.76 mSEK/year respectively. The rest of the gains (in total cost, peak/off-peak user cost, peak/off-peak operating cost in both bus system and BRT system) are all positive in medium and high demand regions, which means semi-autonomous vehicles potentially save both users' cost and service provider's operating cost in medium and high demand regions. Given $\bar{q} = 4000$ passenger/hour, the hourly total cost savings in bus system and BRT system are 4.67 kSEK/hour and 0.80 kSEK/hour respectively.

In terms of the winners and losers during the two periods, the gains of users are similar during peak and off-peak hours: although longer waiting times are incurred during off-peak hours, passengers have better riding experiences. However, the gains in bus systems are higher than that in BRT systems. For the service provider, the operating cost gains are higher during peak hours than during off-peak hours. This is because the difference in number of drivers is larger in peak hours than in off-peak hours. For example, from Fig. 8(a) we can find that in bus systems, given $\bar{q} = 9692$ passenger/hour, the headway is 0.3416 min and 1.031 min for conventional and semi-autonomous vehicles, respectively, in bus systems during the peak, and 0.7658 min and 1.785 min, respectively, during the off-peak. Correspondingly, the number of drivers are 176, 58, 78 and 34, respectively (note that the round-trip travel time is 1 hour, and there is only one driver in each platoon). Thus, 118 more drivers are required during peak hours and only 44 more drivers are required during off-peak hours for conventional vehicles.

5.3.2. *Impact of peak/off-peak demand ratio*

The impact of the peak/off-peak demand ratio is studied by fixing the off-peak demand at 4000 passenger/hour while varying q_2/q_1 from 1 to 5. The test result in Fig. 9 shows that the off-peak user gains and operating gains in both bus system and BRT system do not change with the variation of q_2/q_1 (or the variation of q_2)[3]. Larger gap between q_2 and q_1 will result in more additional capital cost, if semi-autonomous vehicles are adopted. As mentioned before, to satisfy the needs of travelers, the service provider should purchase more vehicles to operate during peak hours, which leads to more losses in capital cost, considering semi-autonomous vehicles are 20% more expensive. If q_2/q_1 is extremely high, there will be too many vehicles running during peak hours and very few vehicles running during off-peaks, which leads to the inefficient

[3]However, it is not rigorous to conclude that they are independent of q_2/q_1 given that we have not obtained the analytical solution in this scenario and numerical tests only contain finite number of trials.

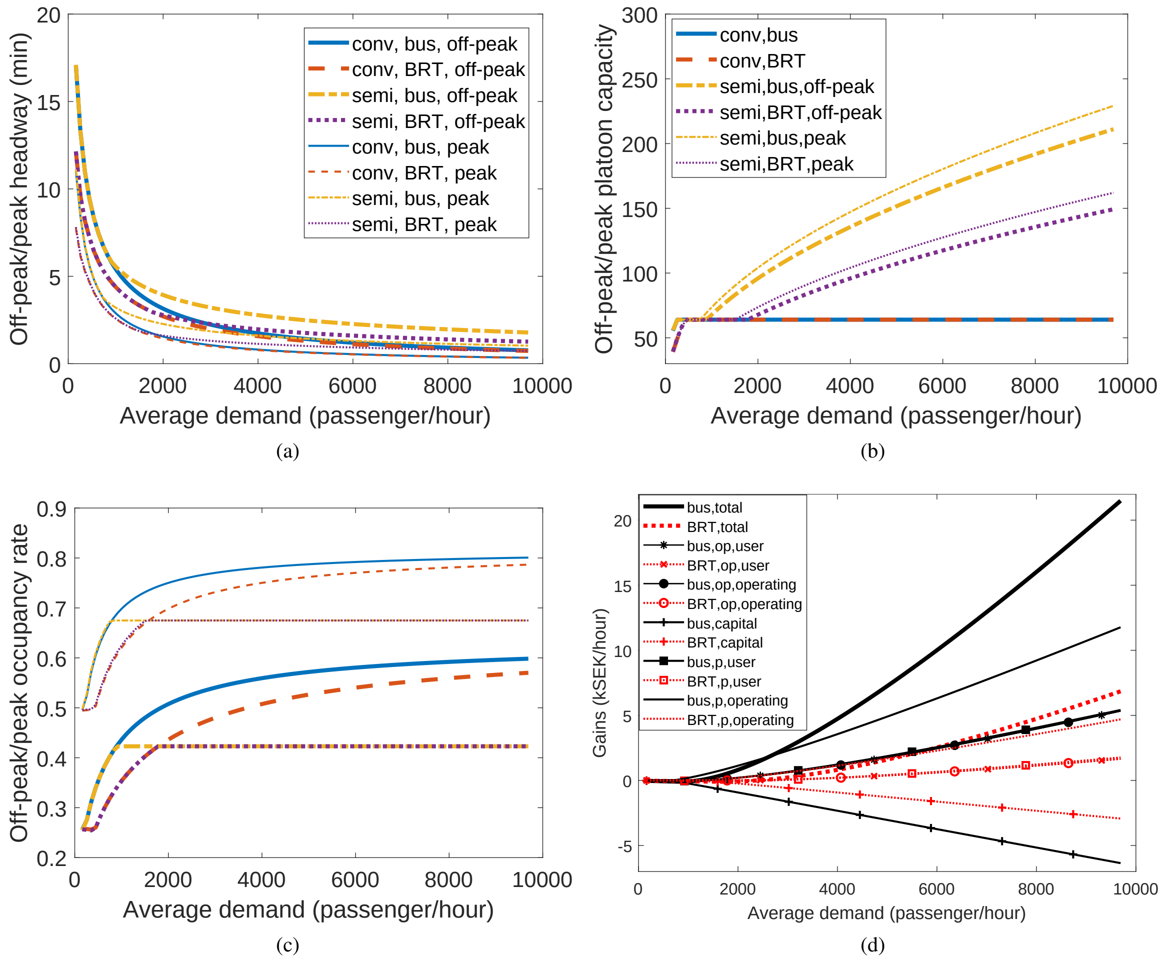


Figure 8: Two-period results of (a) headway, (b) platoon capacity, (c) occupancy rate, and (d) gains of using semi-autonomous vehicles with respect to average demand $\bar{q}$, $q_2/q_1 = 3$, other parameters according to Table. 1. (a) and (c) use the same line styles to distinguish different scenarios. In (d), 'p' is short for peak and 'op' is short for off-peak.

utilization of rolling stocks. Larger q_2/q_1 also yields higher total cost gains, mainly because medium and high demand levels favor semi-autonomous vehicles, as mentioned in the one-period analysis. The required additional capital cost is made up for by the users' benefits and especially the operating gains.

5.4. *Headway lower bound*

For practical reasons (e.g., service provider's budget), the extremely short headways for high demand levels in the previous numerical analysis are not likely to occur in real world. Short headways may also give rise to problems such as bus bunching (different from deliberate platooning) and traffic congestion. Therefore, a more realistic case study with headway restrictions is carried out, in which the lower bounds are set to be 3 min and 2 min for bus services and BRT services, respectively. The results are shown in Fig. 10.

Compared with the unrestricted headway scenarios discussed above, conventional vehicles in both the bus system and the BRT system fail to serve the maximum load when the demand (or average demand) is beyond a certain value. This phenomenon can be clearly observed from the occupancy rate curves in Fig. 10(a)-(b). As the curve hits 1, the vehicles will be fully occupied and if the demand continues to increase, there will be no feasible solutions to the optimization problem with restricted headway. The total cost curves terminate at the same demand levels as the corresponding occupancy rate curves. Therefore, the gains from implementing semi-autonomous vehicles cannot be calculated in the entire tested demand domain. In terms of the specific occupancy rate values of semi-autonomous vehicles, the dramatic change introduced

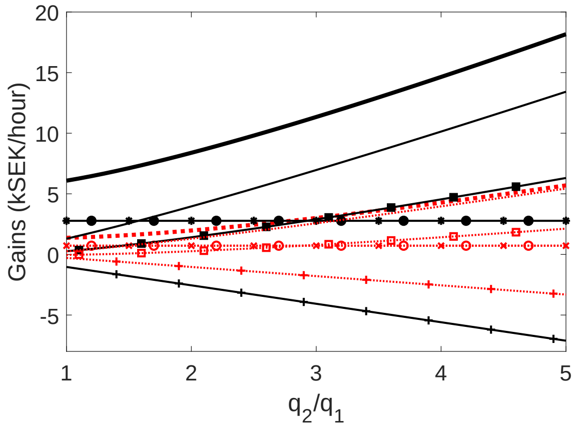


Figure 9: Two-period results of total cost with respect to peak/off-peak ratio q_2/q_1 (off-peak demand $q_1 = 4000$ passenger/hour), other parameters according to Table. 1. The line styles are the same as in Fig. 8(d).

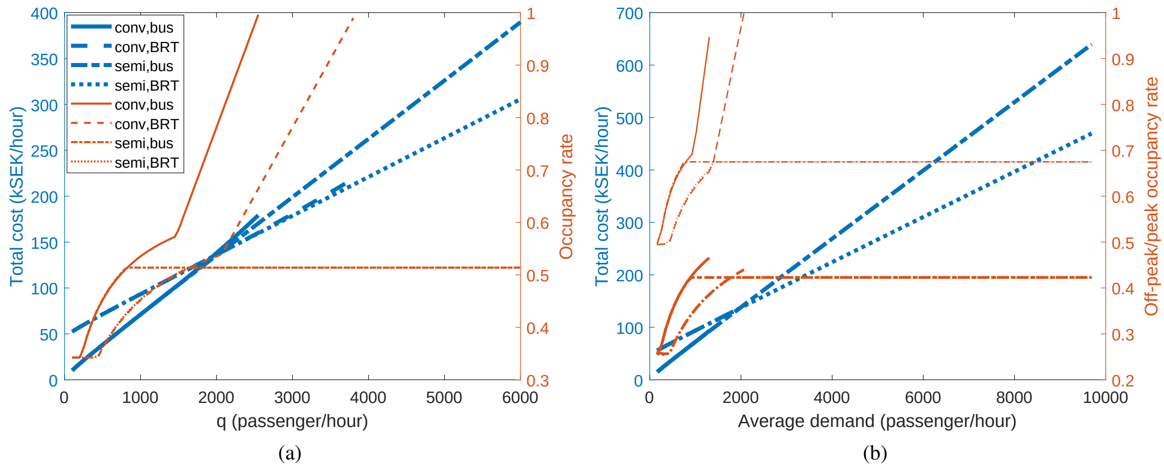


Figure 10: Results of total cost and occupancy rate with respect to (a) demand q (one-period analysis) and (b) average demand $\bar{q}$ (two-period analysis). Bus headway is restricted to no less than 3 minutes and BRT headway is restricted to no less than 2 minutes, $q_2/q_1 = 3$, other parameters according to Table. 1. (a) and (b) share the line styles, except in (b) the wider occupancy rate lines are for off-peak hours and the thinner for peak hours.

by headway restrictions happens in the peak hour (see Fig. 10(b) and Fig. 8(c)). With headway restrictions, the eventual peak occupancy rate is lower, because when the headway restrictions become active, semi-autonomous vehicles will have to extend the platoon capacity to serve the maximum load. As a result, the platoon length in the restricted headway scenario is longer than in the unrestricted headway scenario and semi-autonomous vehicles are thus less occupied.

6. Conclusion

The paper investigates the efficiency and applicability of semi-autonomous vehicles for bus services and BRT services in high demand corridors. A nonlinear optimization model with inequality constraints is formulated in order to minimize total system cost in four scenarios, namely bus service with conventional vehicles, bus service with semi-autonomous vehicles, BRT service with conventional vehicles and BRT service with semi-autonomous vehicles. Assuming that semi-autonomous vehicles are able to save labor cost through forming platoons at the cost of extra rolling stock expenditure, we obtain analytical solutions for the four scenarios given a single demand level. The critical demand values between different operational regimes are analytically identified. Numerical analyses are conducted for both the single demand period case and the peak/off-peak demand period case, concerning the impacts of variations in demand, vehicle capacity upper bound, cost coefficients, etc.

The single demand period results show that the performance of semi-autonomous bus platoons are largely dependent on the demand level. More specifically, bus service with conventional buses is the most beneficial when the demand is below 1150 passenger/hour; as the demand grows, bus service with semi-autonomous buses becomes advantageous until it reaches 2050 passenger/hour. At higher demand levels, BRT service with conventional buses becomes competitive; eventually when the demand is above 2250 passenger/hour, BRT service with semi-autonomous buses surpasses the other three modes in cost efficiency.

As demand increases, the formation of semi-autonomous bus platoons becomes favored, since combining extending capacity and increasing headway becomes more effective than with only headway adjustment. Besides, the upper bound of the vehicle size also affects the performance of semi-autonomous vehicles significantly. For example, BRT service with semi-autonomous vehicles costs 51.2 SEK/hour more than the same service mode with conventional vehicles given $s_{\mathrm{ub}} = 100$, but saves 1988.6 SEK/hour if $s_{\mathrm{ub}} = 50$. In addition, the cost coefficients in extra rolling stock and reduced labor cost have similar influence on semi-autonomous vehicles in bus systems and in BRT systems.

The peak/off-peak period analysis shows that the peak/off-peak demand variation changes the results quantitatively. However, it is mainly the magnitudes of the demand levels that determine whether semi-autonomous vehicles are beneficial. Being aware of the service provider's limited budget, the numerical case study shows that a lower bound on headways makes conventional vehicles fail to serve the demand and favors semi-autonomous bus platooning.

In general, semi-autonomous vehicles bring benefits when the demand is above certain levels, depending on in which system it will be adopted. Compared with conventional vehicles, semi-autonomous bus platoons potentially offer better riding experience for being less crowded, though the waiting time may be slightly longer, which could be ignored by travelers especially in the targeted high-frequency services. In this sense, semi-autonomous bus platoons can be implemented in highly demanded areas where commuting trips occur frequently since crowding creates extraordinary discomfort and sense of insecurity for commuters. Furthermore, since bus platooning is originally designated to undertake the role of trains, this technology can also be promising in areas where railways are not applicable or not affordable. For example, weak soil structure and complicated groundwater system may lead to disproportionate amount of construction investment. Besides, the disruption from railway construction work may hinder the implementation of trains, especially in a city without proper urban planning but already has a large population.

The performance of bus platoons is subject to the cost of automation, which is assumed to be 20% in this study. However, the current cost of automation is much higher than this number. Therefore, mass production along with gradually matured technology is required to overcome the cost barrier.

This study is carried out without considering ticket revenues, taxes or subsidies from the government. It is still unclear how investors are willing to contribute and how passengers are ready to pay or accept this new technology. Future research is needed around these topics.

Acknowledgement

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Appendix A. Demand characteristics and derivation of the riding cost

Considering a corridor of length l where the hourly directional demand q is uniformly distributed along the line, there exists the following relationship between the point-to-point density ρ and the demand:

$$\int_0^l \int_y^l \rho dz dy = q, \tag{A.1}$$

which gives $\rho = 2q/l^2$. The directional hourly demand flow at position x is

$$q(x) = \int_x^l \int_0^x \rho \mathrm{d}y \mathrm{d}z = \frac{2q}{l^2}(lx - x^2), \tag{A.2}$$

The maximum demand flow appears at $x = l/2$, and is $q_{\mathrm{m}} = q/2$. The occupancy rate at x, which is dependent on the headway h, is

$$\phi(x) = \frac{2qh}{l^2 Ns}(lx - x^2), \tag{A.3}$$

where Ns is the capacity of the bus platoon ($N = 1$ for conventional bus and $N \geq 1$ for semi-autonomous bus). The in-vehicle cost as a function of x becomes

$$c_{\mathrm{iv}}(x) = c_{\mathrm{ride}} + \frac{2qhc_{\mathrm{dcf}}}{l^2 Ns}(lx - x^2). \tag{A.4}$$

Since the driving speed is assumed to be constant, the time-related cost can be converted to distance-related cost:

$$c_{\mathrm{iv}}^{km}(x) = \frac{1}{v}\left[c_{\mathrm{ride}} + \frac{2qhc_{\mathrm{dcf}}}{l^2 Ns}(lx - x^2)\right]. \tag{A.5}$$

The total riding cost is obtained:

$$\begin{aligned}
C_{\mathrm{ride}} &= 2\int_0^l \int_0^z \rho \int_y^z c_{\mathrm{iv}}^{km}(x)\mathrm{d}x\mathrm{d}y\mathrm{d}z \\
&= \frac{4q}{vl^2}\int_0^l \int_0^z \left(c_{\mathrm{ride}}(z-y) + \frac{2qhc_{\mathrm{dcf}}}{l^2 Ns}\left[\frac{l(z^2-y^2)}{2} + \frac{y^3 - z^3}{3}\right]\right)\mathrm{d}y\mathrm{d}z \\
&= \frac{4q}{vl^2}\int_0^l \left(\frac{1}{2}c_{\mathrm{ride}}z^2 + \frac{1}{3}\frac{2qhc_{\mathrm{dcf}}}{lNs}z^3 - \frac{1}{4}\frac{2qhc_{\mathrm{dcf}}}{l^2 Ns}z^4\right)\mathrm{d}z \\
&= \frac{2ql}{v}\left(\frac{c_{\mathrm{ride}}}{3} + \frac{2}{15}\frac{qhc_{\mathrm{dcf}}}{Ns}\right).
\end{aligned} \tag{A.6}$$

Appendix B. Analytical solution for single demand period

The Lagrangian of the constrained optimization problem given in (8–11) is

$$\begin{aligned}
\mathcal{L}(h, N, s, \lambda_1, \lambda_2, \lambda_3) = {} & c_{\mathrm{access}}\frac{d^j q}{v_{\mathrm{walk}}} + c_{\mathrm{wait}}hq + \frac{2ql}{v^j}\left(\frac{c_{\mathrm{ride}}}{3} + \frac{2}{15}\frac{qhc_{\mathrm{dcf}}}{Ns}\right) + c_0^j \\
& + \frac{2lN[(1-\eta^i)c_{\mathrm{oper}} + (1+\beta^i)c_{\mathrm{cptl}} + (b_{\mathrm{oper}} + b_{\mathrm{cptl}})s] + 2l\eta^i c_{\mathrm{oper}}}{v^j h} \\
& - \lambda_1(2Ns - hq) - \lambda_2(s_{\mathrm{ub}} - s) - \lambda_3(N-1),
\end{aligned} \tag{B.1}$$

which gives optimality conditions

$$c_{\mathrm{wait}}q + \frac{4q^2 lc_{\mathrm{dcf}}}{15v^j sN} - \frac{2lN[(1-\eta^i)c_{\mathrm{oper}} + (1+\beta^i)c_{\mathrm{cptl}} + (b_{\mathrm{oper}} + b_{\mathrm{cptl}})s] + 2l\eta^i c_{\mathrm{oper}}}{v^j h^2} + \lambda_1 q = 0 \tag{B.2}$$

$$-\frac{4q^2 lhc_{\mathrm{dcf}}}{15N^2 v^j s} + \frac{2l[(1-\eta^i)c_{\mathrm{oper}} + (1+\beta^i)c_{\mathrm{cptl}} + (b_{\mathrm{oper}} + b_{\mathrm{cptl}})s]}{v^j h} - 2\lambda_1 s - \lambda_3 = 0 \tag{B.3}$$

$$-\frac{4q^2 lhc_{\mathrm{dcf}}}{15v^j Ns^2} + \frac{2lN(b_{\mathrm{oper}} + b_{\mathrm{cptl}})}{v^j h} - 2\lambda_1 N + \lambda_2 = 0 \tag{B.4}$$

$$\lambda_1(2Ns - hq) = 0 \tag{B.5}$$

$$\lambda_2(s_{\mathrm{ub}} - s) = 0 \tag{B.6}$$

$$\lambda_3(N-1) = 0 \tag{B.7}$$

$$\lambda_1, \lambda_2, \lambda_3 \geq 0 \tag{B.8}$$

in addition to constraints (9)–(11).

Appendix B.1. Solution cases

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$, $N = 1$ gives

$$s = \sqrt{\frac{4qlc_{\text{dcf}}[c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}}]}{15v^j c_{\text{wait}}(b_{\text{oper}} + b_{\text{cptl}})}}, \quad h = \sqrt{\frac{2l[c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}}]}{qv^j c_{\text{wait}}}} \tag{B.9}$$

Given the specific parameters, the solution is feasible. The minimum objective value is

$$\begin{aligned} C_{\text{tot}}^{ij*} &= c_{\text{access}}\frac{d^j q}{v_{\text{walk}}} + \frac{2qlc_{\text{ride}}}{3v^j} + c_0^j \\ &+ \sqrt{\frac{8c_{\text{wait}}ql[c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}}]}{v^j}} + \frac{4ql}{v^j}\sqrt{\frac{2(b_{\text{oper}} + b_{\text{cptl}})c_{\text{dcf}}}{15}}. \end{aligned} \tag{B.10}$$

Case 2: $\lambda_1 = 0$, $s = s_{\text{ub}}$, $N = 1$ gives

$$h = \sqrt{\frac{30ls_{\text{ub}}[c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}{15v^j s_{\text{ub}} c_{\text{wait}} q + 4q^2 lc_{\text{dcf}}}}, \tag{B.11}$$

The minimum objective value is

$$\begin{aligned} C_{\text{tot}}^{ij*} &= c_{\text{access}}\frac{d^j q}{v_{\text{walk}}} + \frac{2qlc_{\text{ride}}}{3v^j} + c_0^j \\ &+ \frac{2}{v^j}\sqrt{\frac{2ql[c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}](4c_{\text{dcf}}ql + 15c_{\text{wait}}s_{\text{ub}}v^j)}{15s_{\text{ub}}}}. \end{aligned} \tag{B.12}$$

Case 3: $\lambda_1 = 0$, $s = s_{\text{ub}}$, $\lambda_3 = 0$ gives

$$h = \sqrt{\frac{2l\eta^i c_{\text{oper}}}{c_{\text{wait}} q v^j}}, \tag{B.13}$$

$$N = \sqrt{\frac{4ql\eta^i c_{\text{oper}} c_{\text{dcf}}}{15c_{\text{wait}} v^j s_{\text{ub}}[(1-\eta^i)c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}}, \tag{B.14}$$

It can be seen from the results that when the vehicle size reaches the upper bound s_{ub}, the headway is independent of s_{ub}. Moreover, the occupancy rate at the highest loaded point (i.e., $l/2$) in this case is

$$\phi\left(\frac{l}{2}\right) = \frac{2Ns}{hq} = \sqrt{\frac{8c_{\text{dcf}} s_{\text{ub}}}{15[(1-\eta^i)c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}}, \tag{B.15}$$

which is independent of q. The minimum objective value is

$$\begin{aligned} C_{\text{tot}}^{ij*} &= c_{\text{access}}\frac{d^j q}{v_{\text{walk}}} + \frac{2qlc_{\text{ride}}}{3v^j} + c_0^j \\ &+ \frac{4ql}{v^j}\sqrt{\frac{2c_{\text{dcf}}[(1-\eta^i)c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}{15s_{\text{ub}}}} + \sqrt{\frac{8ql\eta^i c_{\text{oper}} c_{\text{wait}}}{v^j}}. \end{aligned} \tag{B.16}$$

Case 4: $2Ns - hq = 0$, $s = s_{\text{ub}}$, $N = 1$ gives

$$h = \frac{2s_{\text{ub}}}{q}, \tag{B.17}$$

$$\lambda_1 = \frac{ql[(1-\eta^i)c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}{2v^j s_{\text{ub}}^2} - \frac{4qlc_{\text{dcf}}}{15v^j s_{\text{ub}}} - c_{\text{wait}}, \tag{B.18}$$

The minimum objective value is

$$C_{\text{tot}}^{ij*} = c_{\text{access}}\frac{d^j q}{v_{\text{walk}}} + \frac{2qlc_{\text{ride}}}{3v^j} + c_0^j + 2c_{\text{wait}}s_{\text{ub}} + \frac{8qlc_{\text{dcf}}}{15v^j} + \frac{ql[c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}{v^j s_{\text{ub}}}. \tag{B.19}$$

Case 5: $2Ns - hq = 0$, $s = s_{\text{ub}}$, $\lambda_3 = 0$ gives

$$h = \sqrt{\frac{2l\eta^i c_{\text{oper}}}{c_{\text{wait}}v^j q}}, \quad N = \frac{1}{s_{\text{ub}}}\sqrt{\frac{l\eta^i c_{\text{oper}}q}{2c_{\text{wait}}v^j}}, \tag{B.20}$$

$$\lambda_1 = \left(\frac{ql[(1-\eta^i)c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}{2v^j s_{\text{ub}}} - \frac{4qlc_{\text{dcf}}}{15v^j}\right)\sqrt{\frac{2c_{\text{wait}}v^j}{l\eta^i c_{\text{oper}}q}}, \tag{B.21}$$

The minimum objective value is

$$C_{\text{tot}}^{ij*} = c_{\text{access}}\frac{d^j q}{v_{\text{walk}}} + \frac{2qlc_{\text{ride}}}{3v^j} + c_0^j + \sqrt{\frac{8qlc_{\text{wait}}c_{\text{oper}}\eta^i}{v^j}} + \frac{8qlc_{\text{dcf}}}{15v^j} + \frac{ql[(1-\eta^i)c_{\text{oper}} + (1+\beta^i)c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}]}{v^j s_{\text{ub}}}. \tag{B.22}$$

Note that Case 4 is only feasible for conventional vehicle and Case 5 is only feasible for semi-autonomous vehicle.

Appendix B.2. Conventional vehicles

The optimal solution can be divided into several regimes with respect to demand q. For conventional vehicles, the feasible cases are Case 1, Case 2 and Case 4. Accordingly, three regimes will be generated. Being aware of the continuity of the evolution, the boundary conditions between Case 1 and Case 2, Case 2 and Case 4 can be obtained. For Case 1 and Case 2, if we set (B.9) to s_{ub}, the transition point of q from stage 1 to stage 2 is obtained:

$$q_{1,2}^{\text{conv},j} = \frac{15s_{\text{ub}}^2 v^j c_{\text{wait}}(b_{\text{oper}} + b_{\text{cptl}})}{4lc_{\text{dcf}}[c_{\text{oper}} + (1+\beta^{\text{conv}})c_{\text{cptl}}]}. \tag{B.23}$$

Similarly, by equating (B.11) with (B.17), we obtain

$$q_{2,3-1}^{\text{conv},j} = \frac{15l[c_{\text{oper}} + (1+\beta^{\text{conv}})c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}] - 15v^j c_{\text{wait}}s_{\text{ub}}}{4lc_{\text{dcf}}}. \tag{B.24}$$

However, regime 3 can be reached only if $s_{\text{ub}}^{\text{conv}} < s_{\text{max}}^{\text{conv}}$, where

$$s_{\text{max}}^{\text{conv}} = \frac{15(c_{\text{oper}} + c_{\text{cptl}})}{8c_{\text{dcf}} - 15(b_{\text{cptl}} + b_{\text{oper}})}. \tag{B.25}$$

Otherwise, λ_1 in (B.18) will be negative, which will make Case 4 infeasible. Note that for $\lambda_1, \lambda_2 \geq 0$ in Case 4, q should fulfill the following requirements:

$$q \geq \frac{30v^j s_{\text{ub}}^2 c_{\text{wait}}}{15l[c_{\text{oper}} + (1+\beta^{\text{conv}})c_{\text{cptl}} + (b_{\text{oper}} + b_{\text{cptl}})s_{\text{ub}}] - 8lc_{\text{dcf}}s_{\text{ub}}} = q_{2,3-2}^{\text{conv}}, \tag{B.26}$$

$$q \geq \frac{2v^j s_{\text{ub}}^2 c_{\text{wait}}}{l[c_{\text{oper}} + (1+\beta^{\text{conv}})c_{\text{cptl}}]} = q_{2,3-3}^{\text{conv}}. \tag{B.27}$$

Therefore, the transition point of q between regime 2 and regime 3 is

$$q_{2,3}^{\text{conv},j} = \max\{q_{2,3-1}^{\text{conv},j}, q_{2,3-2}^{\text{conv},j}, q_{2,3-3}^{\text{conv},j}\}. \tag{B.28}$$

To give an impression about the minimum demand q required for conventional vehicles to reach regime 3 with respect to the capacity upper bound s_{ub}, we plot the result in Fig. B.11. Given the nominal values in Table. 1, $s_{\text{max}}^{\text{conv}} = 26.3$, regime 3 will not be reached.

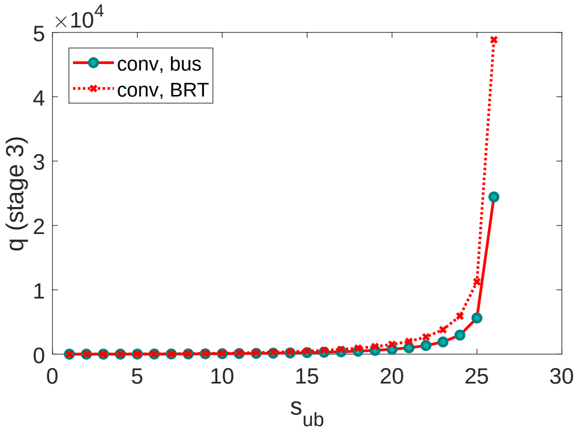

Figure B.11: Minimum q required for conventional vehicles to reach regime 3 with respect to s_{ub}, parameters according to Table. 1. The tolerance is 1e-5 (i.e., $\lambda_1, \lambda_2 \geq -1 \times 10^{-5}$ is regarded to suffice).

Appendix B.3. Semi-autonomous vehicles

For semi-autonomous vehicles, the feasible cases are Case 1, Case 2, Case 3 and Case 5. Correspondingly, there should be four regimes in total, which result in three transition points of q. Similar with $q_{1,2}^{\mathrm{conv},j}$, the transition point $q_{1,2}^{\mathrm{sa},j}$ is also calculated by setting (B.9) to s_{ub}, which is

$$q_{1,2}^{\mathrm{sa},j} = \frac{15 s_{\mathrm{ub}}^2 v^j c_{\mathrm{wait}} (b_{\mathrm{oper}} + b_{\mathrm{cptl}})}{4 l c_{\mathrm{dcf}} [c_{\mathrm{oper}} + (1 + \beta^{\mathrm{sa}}) c_{\mathrm{cptl}}]}. \tag{B.29}$$

By equating the right-hand sides of (B.11) and (B.13) (or equating (B.14) to 1), the transition demand on the boundary between stage 2 and 3 is calculated

$$q_{2,3}^{\mathrm{sa},j} = \frac{15 c_{\mathrm{wait}} s_{\mathrm{ub}} v^j [(1 + \beta^{\mathrm{sa}}) c_{\mathrm{cptl}} + (1 - \eta^{\mathrm{sa}}) c_{\mathrm{oper}} + (b_{\mathrm{cptl}} + b_{\mathrm{oper}}) s_{\mathrm{ub}}]}{4 c_{\mathrm{dcf}} c_{\mathrm{oper}} \eta^{\mathrm{sa}} l}. \tag{B.30}$$

Moreover, (B.14) and (B.20) should be equivalent at the boundary between regime 3 and regime 4, which requires $s_{\mathrm{ub}}^{\mathrm{sa}} = s_{\mathrm{min}}^{\mathrm{sa}}$, where

$$s_{\mathrm{min}}^{\mathrm{sa}} = \frac{15[(1 - \eta^{\mathrm{sa}}) c_{\mathrm{oper}} + (1 + \beta^{\mathrm{sa}}) c_{\mathrm{cptl}}]}{8 c_{\mathrm{dcf}} - 15 (b_{\mathrm{cptl}} + b_{\mathrm{oper}})}. \tag{B.31}$$

To ensure a meaningful $s_{\mathrm{min}}^{\mathrm{sa}}$, a positive denominator should be the premise, i.e., $8c_{\mathrm{dcf}} - 15(b_{\mathrm{cptl}} + b_{\mathrm{oper}}) > 0$. Given the parameter values in Table 1, $s_{\mathrm{min}}^{\mathrm{sa}}$=10.6. As can be seen from (B.15), the occupancy rate of semi-autonomous buses $\phi(l/2)$ will be fixed regardless of the variations in q when it enters regime 3 (i.e., $q \geq q_{2,3}^{\mathrm{sa}}$). However, Case 5 indicates that $\phi(l/2)$ should be 1, which could be different from the value in (B.15). If $s_{\mathrm{ub}}^{\mathrm{sa}} > s_{\mathrm{min}}^{\mathrm{sa}}$, (B.21) is not feasible any more due to the violation of Constraint (B.8) and Case 5 will become infeasible; if $s_{\mathrm{ub}}^{\mathrm{sa}} = s_{\mathrm{min}}^{\mathrm{sa}}$, Case 5 and Case 3 become essentially the same, and the occupancy rate $\phi(l/2)$ will be fixed to 1; if $s_{\mathrm{ub}}^{\mathrm{sa}} < s_{\mathrm{min}}^{\mathrm{sa}}$, $\phi(l/2)$ in (B.15) becomes greater than 1, which is against Constraint (9), so Case 3 becomes infeasible and Case 5 becomes feasible. Therefore, there are actually only three regimes for semi-autonomous vehicles, which are divided by $q_{1,2}^{\mathrm{sa}}$ and $q_{2,3}^{\mathrm{sa}}$.

References

Antonio Loro Consulting Inc., 2016. Planning for automated vehicles in Edmonton. https://www.edmonton.ca/city_government/documents/RoadsTraffic/2016_automated_vehicles_report.pdf, accessed: 2018-10-12.

ATC, 2006. National guidelines for transport system management in Australia. Background material 5.

Bergqvist, O., Åstrand, M., 2017. Bus line optimisation using autonomous minibuses.

Börjesson, M., Fung, C. M., Proost, S., 2017. Optimal prices and frequencies for buses in Stockholm. Economics of Transportation 9, 20–36.

Bösch, P. M., Becker, F., Becker, H., Axhausen, K. W., 2018. Cost-based analysis of autonomous mobility services. Transport Policy 64, 76–91.

Fagnant, D. J., Kockelman, K. M., Bansal, P., 2015. Operations of shared autonomous vehicle fleet for Austin, Texas, market. Transportation Research Record: Journal of the Transportation Research Board (2536), 98–106.

Jansson, J. O., 1980. A simple bus line model for optimisation of service frequency and bus size. Journal of Transport Economics and Policy, 53–80.

Jara-Díaz, S., Gschwender, A., jan 2003. Towards a general microeconomic model for the operation of public transport. Transport Reviews 23 (4), 453–469.

Lam, S., Katupitiya, J., 2013. Modeling and control of a platoon of autonomous buses. In: Intelligent Vehicles Symposium (IV), 2013 IEEE. IEEE, pp. 958–963.

Litman, T., 2017. Autonomous vehicle implementation predictions. Victoria Transport Policy Institute.

Liu, T., Ceder, A., Rau, A., 2018. Scheduled platoons of public transport autonomous modular vehicles. https://www.events.tum.de/frontend/index.php?page_id=3316, accessed: 2018-10-12.

MathWorks, 2018. MATLAB and Simulink. https://se.mathworks.com/.

Scheltes, A., de Almeida Correia, G. H., 2017. Exploring the use of automated vehicles as last mile connection of train trips through an agent-based simulation model: An application to Delft, Netherlands. International Journal of Transportation Science and Technology 6 (1), 28–41.

Tirachini, A., Hensher, D. A., Jara-Díaz, S. R., 2010. Restating modal investment priority with an improved model for public transport analysis. Transportation Research Part E: Logistics and Transportation Review 46 (6), 1148–1168.

van de Hoef, S., 2018. Coordination of heavy-duty vehicle platooning. Ph.D. thesis, KTH Royal Institute of Technology.

Watzenig, D., Horn, M. (Eds.), 2017. Automated Driving: Safer and More Efficient Future Driving. Springer.

White, P., 2016. The roles of "conventional" and demand-responsive bus services. In: Paratransit: Shaping the Flexible Transport Future. Emerald Group Publishing Limited, pp. 307–330.

Zhang, W., Jenelius, E., Badia, H., 2018. Efficiency of semi-autonomous and fully autonomous bus services in trunk-and-branches networks, working paper.

Zhang, W., Jenelius, E., Ma, X., 2017a. Freight transport platoon coordination and departure time scheduling under travel time uncertainty. Transportation Research Part E: Logistics and Transportation Review 98, 1–23.

Zhang, W., Sundberg, M., Karlström, A., 2017b. Platoon coordination with time windows: an operational perspective. Transportation Research Procedia 27, 357–364.