INCORPORATING DYNAMICS AND INFORMATION IN A CONSEQUENCE MODEL FOR ROAD NETWORK VULNERABILITY ANALYSIS

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1. INTRODUCTION

Issues of transport reliability and road network vulnerability are now receiving increasing and much needed attention. An important question is how to model and assess the consequences of a disrupting event in the network, here in terms of increased travel time. As in most transport modeling, there is a trade-off between the size of the area to be studied and the level of detail in the analysis that is feasible for computational reasons. At one end of the scale, there are the micro-simulation models, which fully capture the dynamic aspects of an incident. These models, however, require highly detailed data, careful calibration and long computation times to produce useful results. At the other end, there are the static equilibrium models, which calculate the travel times based on the assumption that the users have perfect information about the network conditions and that traffic quickly adapts to a new user optimal situation (e.g., Nicholson and Du 1997). In their simplest form, no congestion effects are accounted for either (Jenelius et al. 2006, Taylor and D'Este 2004). In return, these models require relatively few data and short computation times, which make them attractive for studies in very large networks.

One issue that has been largely overlooked when static equilibrium models have been applied is the duration of the disruption. In reality, there usually occurs a period of disorder and suboptimal behavior before a state resembling an equilibrium settles. If the duration of the disruption is short in relation to this transient period, the static models likely underestimate the consequences of the disruption. Also, the presence of good alternative routes increases in value with the duration of the disruption.

In this paper we propose a dynamic extension of the user equilibrium model without congestion (i.e., shortest paths or all-or-nothing assignment). The model handles both the case when there are alternative routes and the case when there are no alternative routes during the closure. The purpose of the extended model is to give a better estimate of the consequences of a road closure, while requiring as little extra computations as possible compared to the basic model (henceforth called the "traditional" model). The new model is thus intended to be applicable in studies of every link in very large networks. In contrast to the traditional model, the new model does not assume that the users immediately find new optimal routes after a disruption. Instead, it is based on the fact that it takes some time before the users are informed of the event and can act accordingly. During this period of information spreading, there is a transition from a suboptimal situation to a new equilibrium.

2. THE MODEL

2.1. Basic assumptions

Consider, for simplicity, a single origin o and destination d and assume that the travel demand from o to d is x vehicles per unit time, constant and inelastic. Assume also that there is originally at least one route between o and d, that the travel time is independent of the traffic volume and that all users choose to travel along the shortest route r_{orig}.

Suppose now that link k is located along the route r_{orig}. At $t = 0$, this link is unexpectedly closed for all traffic. The process of informing the users of the closure takes a certain time, independent of the duration of the closure. At time t, the share $a(t)$ of all users who start their trip have been informed of the closure. We assume that $a(t) = 0$ for $t < 0$. At some time t_{clinf} and henceforth, all users have been informed of the closure and $a(t) = 1$ for $t \geq t_{\text{clinf}}$. Between 0 and t_{clinf}, $a(t)$ is nondecreasing.

At t_{open}, the link is reopened and a similar information process begins. At time t, the share $b(t)$ of all users who start their trip have been informed of the reopening. We assume that $b(t) = 0$ for $t < t_{\text{open}}$, $b(t)$ is nondecreasing for $t \in [t_{\text{open}}, t_{\text{opinf}})$ and $b(t) = 1$ for $t \geq t_{\text{opinf}}$. If $t_{\text{open}} \geq t_{\text{clinf}}$, all users will be informed of the closure when the link reopens. If $t_{\text{open}} < t_{\text{clinf}}$, however, some share of the users will still be unaware of the closure. Both these cases are handled by the model. When the users have departed, they cannot receive any new information (thus ignoring the possibility of information via the radio).

During the closure, there may be either no or at least one alternative route from o to d. We will consider these two cases in turn, starting with the latter.

2.2. Case 1: Alternative routes

Let τ_{orig} denote the travel time of the original shortest route r_{orig}. When link k is closed, the traffic must find new routes. The users who have been informed of the closure will choose the route that is now the shortest, denoted r_{short}. At time t, this share of the users is $a(t)$. Let τ_{short} denote the travel time of r_{short}. The uninformed users, meanwhile, will travel along r_{orig} until they reach the near end of link k. From there they will be required to take the shortest route to d. Let r_{long} denote this route from o to d and let τ_{long} denote the travel time of r_{long}. It always holds that $\tau_{\text{long}} \geq \tau_{\text{short}} \geq \tau_{\text{orig}}$. In some cases, r_{short} may be equal to r_{long}.

When $t \geq t_{\text{open}}$ and link k has been reopened, it is again possible to take route r_{orig}. The behavior of the users will depend on what information they have. First, all those who have been informed of the reopening will choose r_{orig}. Second, those who have been informed neither of the closure nor the reopening will also choose r_{orig}. Third, those who have been informed of the closure but not of the reopening will choose r_{short}. At time t, this last group constitutes the share $a(t)(1 - b(t))$.

The total increase in vehicle travel time during the interval $[0, t_{\text{opinf}})$ is thus

$$\begin{aligned}\Delta T_k &= x \int_0^{t_{\text{open}}} (a(t)(\tau_{\text{short}} - \tau_{\text{orig}}) + (1 - a(t))(\tau_{\text{long}} - \tau_{\text{orig}}))dt + \\ &+ x \int_{t_{\text{open}}}^{t_{\text{opinf}}} a(t)(1 - b(t))(\tau_{\text{short}} - \tau_{\text{orig}})dt.\end{aligned} \tag{1}$$

If we introduce $A = \int_0^{t_{\text{open}}} a(t)dt$ and $B = \int_{t_{\text{open}}}^{t_{\text{opinf}}} a(t)(1 - b(t))dt$, we get

$$\Delta T_k = x[(A + B)(\tau_{\text{short}} - \tau_{\text{orig}}) + (t_{\text{open}} - A)(\tau_{\text{long}} - \tau_{\text{orig}})]. \tag{2}$$

The traditional model assuming perfect information about the current network status is obtained in the limit when $0 \leftarrow t_{\text{clinf}}$ and $t_{\text{open}} \leftarrow t_{\text{opinf}}$. In this case, $A = t_{\text{open}}$, $B = 0$, and the total increase in travel time would be

$$\Delta T_k^{\text{trad}} = x t_{\text{open}}(\tau_{\text{short}} - \tau_{\text{orig}}). \tag{3}$$

It is easily shown that $\Delta T_k \geq \Delta T_k^{\text{trad}}$. Thus, the new estimate of the consequences of a road closure is at least as high as the traditional estimate.

2.3. Case 2: No alternative routes

A link that has no alternative routes around it when closed will henceforth be called a *cut link*. When such a link is closed, the users who have been informed of the closure will delay their departure until they receive the information that the link is opened again. Meanwhile, we assume that the users who are uninformed of the closure will travel to the closed link, find that it is closed and return home to wait for the information that the link is open again. Thus, the informed and the uninformed users will receive the same increase in travel time (we do not consider the extra cost of traveling back and forth).

The increase in travel time during the closure consists of two parts: the delay from the desired departure time until the link is reopened, and the delay from the reopening until the user is informed of it. For the first part, we note that since the travel demand per unit time is constant, a user will on average be delayed $t_{\text{open}}/2$ time units. For the second part, we wish to calculate the time until the user is informed of the reopening. Observe that $b(t)$, $t \geq t_{\text{open}}$ represents the cumulative probability distribution of being informed of the reopening at t time units after t_{open}. The expected delay for each user is thus $\int_{t_{\text{open}}}^{t_{\text{opinf}}} (1 - b(t))dt$. Since the total number of users wishing to depart during the closure is $x t_{\text{open}}$, the increase in vehicle travel time during this period is

$$\Delta T_k^{\text{close}} = x \left[\frac{t_{\text{open}}^2}{2} + t_{\text{open}} \cdot \int_{t_{\text{open}}}^{t_{\text{opinf}}} (1 - b(t))dt \right]. \tag{4}$$

When link k has been reopened, it is again possible to take route r_{orig}. The users who have been informed of the reopening, as well as those who are uninformed of both the closure and the reopening, will make their trip as planned. However, those who have been informed of the closure but not yet of the reopening will delay their journey. Consider such a user who was supposed to depart at time $t \geq t_{\text{open}}$. Then $b_t(s) = (b(s) - b(t))/(1 - b(t))$, $s \geq t$ represents the cumulative probability distribution of being informed of the reopening at s time units after t. The expected delay is thus

$$\int_{t}^{t_{\text{opinf}}} (1 - b_t(s))ds = \int_{t}^{t_{\text{opinf}}} \frac{1 - b(s)}{1 - b(t)} ds. \tag{5}$$

Since the share of these users at t is $a(t)(1 - b(t))$, the increase in vehicle travel time during this period is

$$\Delta T_k^{\text{open}} = x \int_{t_{\text{open}}}^{t_{\text{opinf}}} a(t)(1-b(t)) \left(\frac{1}{1-b(t)} \int_t^{t_{\text{opinf}}} (1-b(s))ds \right) dt = x \int_{t_{\text{open}}}^{t_{\text{opinf}}} a(t) \int_t^{t_{\text{opinf}}} (1-b(s))dsdt . \tag{6}$$

Finally, with notations $E(t) = \int_t^{t_{\text{opinf}}} (1-b(s))ds$, $C = E(t_{\text{open}})$ and $D = \int_{t_{\text{open}}}^{t_{\text{opinf}}} a(t)E(t)dt$, the total increase in vehicle travel time is

$$\Delta T_k = \Delta T_k^{\text{close}} + \Delta T_k^{\text{open}} = x\left[\frac{t_{\text{open}}^2}{2} + t_{\text{open}} C + D \right]. \tag{7}$$

If we assume perfect information, the users would only be delayed until the link reopens, and

$$\Delta T_k^{\text{trad}} = \frac{x t_{\text{open}}^2}{2} . \tag{8}$$

3. LINEAR FUNCTIONS

The simplest and perhaps also most reasonable assumption is that the functions $a(t)$ and $b(t)$ are linear during the information spreading processes. Thus, let

$$a(t) = \begin{cases} 0 & t < 0 \\ \dfrac{t}{t_{\text{clinf}}} & 0 \le t < t_{\text{clinf}} \\ 1 & t \ge t_{\text{clinf}} \end{cases} \quad \text{and} \quad b(t) = \begin{cases} 0 & t < t_{\text{open}} \\ \dfrac{t - t_{\text{open}}}{t_{\text{opinf}} - t_{\text{open}}} & t_{\text{open}} \le t < t_{\text{opinf}} \\ 1 & t \ge t_{\text{opinf}} \end{cases} . \tag{9}$$

For brevity, we will here only consider the case when $t_{\text{open}} \ge t_{\text{clinf}}$, i.e., all users have been informed of the closure before the reopening. The case when $t_{\text{open}} < t_{\text{clinf}}$ yields slightly more complicated expressions. When $t_{\text{open}} \ge t_{\text{clinf}}$,

$$A = \int_0^{t_{\text{open}}} a(t)dt = \int_0^{t_{\text{clinf}}} \frac{t}{t_{\text{clinf}}} dt + \int_{t_{\text{clinf}}}^{t_{\text{open}}} 1 \cdot dt = t_{\text{open}} - \frac{t_{\text{clinf}}}{2} , \tag{10}$$

$$B = \int_{t_{\text{open}}}^{t_{\text{opinf}}} a(t)(1-b(t))dt = \int_{t_{\text{open}}}^{t_{\text{opinf}}} 1 \cdot \frac{t_{\text{opinf}} - t}{t_{\text{opinf}} - t_{\text{open}}} dt = \frac{t_{\text{opinf}} - t_{\text{open}}}{2} , \tag{11}$$

$$E(t) = \int_t^{t_{\text{opinf}}} (1-b(s))ds = \int_t^{t_{\text{opinf}}} \frac{t_{\text{opinf}} - s}{t_{\text{opinf}} - t_{\text{open}}} ds = \frac{(t_{\text{opinf}} - t)^2}{2(t_{\text{opinf}} - t_{\text{open}})} ,\ t_{\text{open}} \le t \le t_{\text{opinf}} , \tag{12}$$

$$C = E(t_{\text{open}}) = \frac{t_{\text{opinf}} - t_{\text{open}}}{2} , \tag{13}$$

$$D = \int_{t_{\text{open}}}^{t_{\text{opinf}}} a(t)E(t)dt = \int_{t_{\text{open}}}^{t_{\text{opinf}}} 1 \cdot \frac{(t_{\text{opinf}} - t)^2}{2(t_{\text{opinf}} - t_{\text{open}})} dt = \frac{(t_{\text{opinf}} - t_{\text{open}})^2}{6} . \tag{14}$$

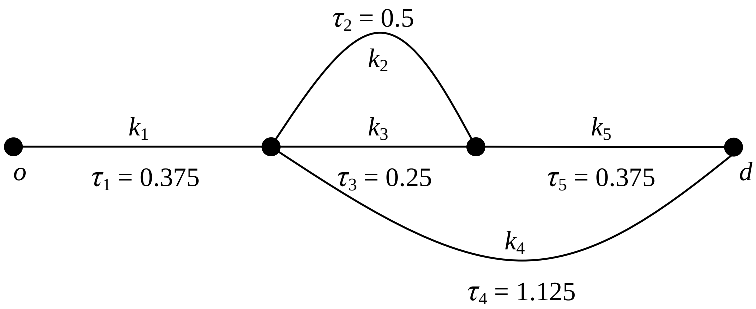

Figure 1 Example network

For the case when there are alternative routes, inserting (10) and (11) in (2) gives

$$\Delta T_k = \frac{x}{2}[(t_{\text{open}} + t_{\text{opinf}} - t_{\text{clinf}})(\tau_{\text{short}} - \tau_{\text{orig}}) + t_{\text{clinf}}(\tau_{\text{long}} - \tau_{\text{orig}})]. \tag{15}$$

For the case when there are no alternative routes, inserting (13) and (14) in (7) gives

$$\Delta T_k = \frac{x}{6}[t_{\text{open}}^2 + t_{\text{open}} t_{\text{opinf}} + t_{\text{opinf}}^2]. \tag{16}$$

4. EXAMPLE

Consider the small example network in Figure 1. The original shortest route r_{orig} from o to d is (k_1, k_3, k_5). Suppose that the duration of the closure is 12 hours, that it takes 6 hours to inform all users of the closure and 2 hours to inform them of the reopening. That is, let $t_{\text{clinf}} = 6$, $t_{\text{open}} = 12$ and $t_{\text{opinf}} = 14$. Let the travel demand x from o to d be 500 vehicles per hour and assume that $a(t)$ and $b(t)$ are linear.

Suppose that link k_5 is closed. Then by using link k_4 it is still possible to reach d. In this case, r_{short} is (k_1, k_4) while r_{long} is (k_1, k_3, k_3, k_4). That is, the uninformed users have to traverse link k_3 in both directions (which we assume takes equally long time) on their route. We get $\tau_{\text{orig}} = \tau_1 + \tau_3 + \tau_5 = 1.0$ hour, $\tau_{\text{short}} = \tau_1 + \tau_4 = 1.5$ hours, $\tau_{\text{long}} = \tau_1 + 2 \cdot \tau_3 + \tau_4 = 2.0$ hours, and insertion together with t_{clinf}, t_{open} and t_{opinf} in (15) gives

$$\Delta T_{k_5} = 500 \cdot 8 = 4{,}000 \text{ vehicle hours.} \tag{17}$$

With the assumption of perfect information, (3) gives that the increase would have been

$$\Delta T_{k_5}^{\text{trad}} = 500 \cdot 6 = 3{,}000 \text{ vehicle hours.} \tag{18}$$

Suppose instead that the cut link k_1 is closed, so that there are no alternative routes to d. Inserting t_{open} and t_{opinf} in (16) gives

$$\Delta T_{k_1} = 500 \cdot (84 + 2/3) = 42{,}333 \text{ vehicle hours.} \tag{19}$$

With the assumption of perfect information, (8) gives that the increase would have been

$$\Delta T_{k_1}^{\text{trad}} = 500 \cdot 72 = 36{,}000 \text{ vehicle hours.} \tag{20}$$

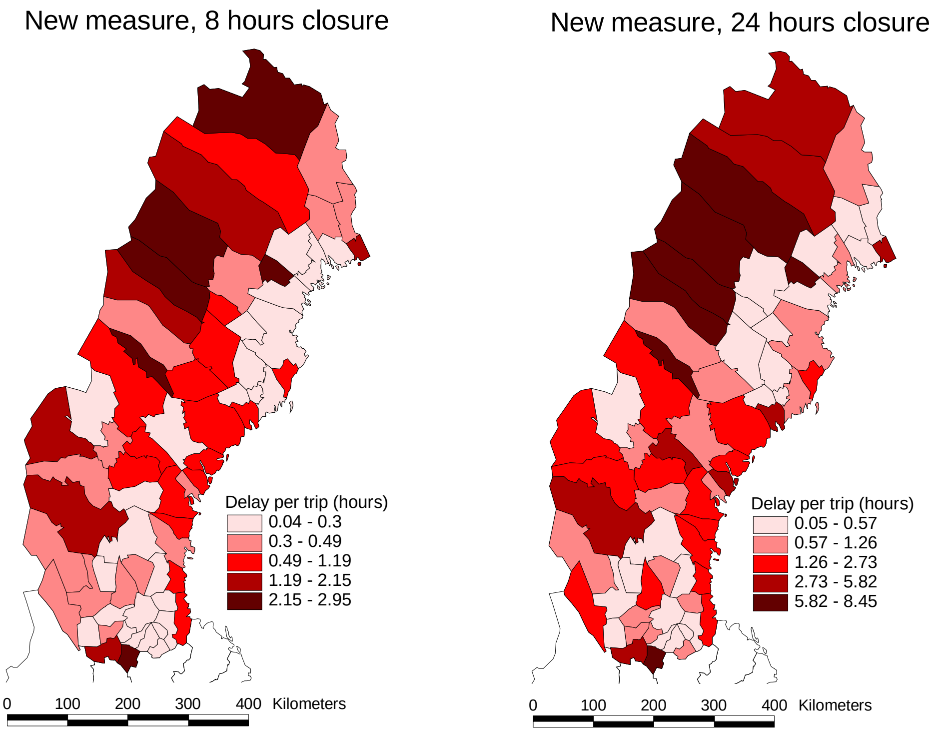

Figure 2 The worst possible consequences for each municipality of closing one link according to the new measure and for two closure durations

5. APPLICATION: NORTHERN SWEDEN

We have compared the new model with the traditional model in a case study on the largely uncongested, rural road network of northern Sweden (Jenelius et al. 2006). The network consists of 14,922 links and 5,553 nodes, including 1,146 centroids. For each of the 69 municipalities in the region we have calculated the worst possible consequences of closing a single link. The consequences are measured as the average increase in travel time per trip starting in the municipality during the affected period $[0, t_{\text{opinf}})$. This increase gives a measure of the exposure of the municipality to the worst-case scenario that this link should be closed. The measure has been calculated for two closure durations, 8 and 24 hours. In both cases we have assumed that the closure information process takes 6 hours and the reopening information process takes 3 hours.

For long closures such as these, the assumption of constant demand (per unit time) is not very realistic. However, if the variation during the day is not too extreme the average demand during the closure should be acceptable to use. In this study we have assumed the same average demand for both closure durations, based on an estimated average weekday demand matrix used by the Swedish Road Administration.

The calculations show that the rankings of the municipalities according this exposure is very similar using the new and the traditional models. In particular, it is generally the same link that comes out as being the most important for a municipality using both measures. This is because the most important link under the studied durations is for most municipalities a cut link, and for such links the new and old measures only differ by a term that is the same for all links, compare (7) and (8).

The duration of the closure has a larger impact on the rankings of the municipalities, as shown in Figure 2. The longer the duration, the more valuable is the possibility to choose an alternative route instead of having to wait for the link to reopen. This can be seen from (16) where the consequences are proportional to the square of the closure duration t_{open}, compared to (15) where the proportionality is linear. For longer closures, the municipalities where there are no alternative routes when the most important link is closed are consequently ranked higher.

That the most important links are often cut links when the closure duration is 8 hours or longer suggests that the focus of vulnerability studies should primarily be on such links. If a new road would be built to provide alternatives for a cut link, the consequences would generally be reduced greatly even if the alternative route would mean a long detour. The study also shows that prioritization among the municipalities is somewhat sensitive to what closure duration is assumed. Preferably, this should be based on observed data from previous incidents. One could then build an empirical probability distribution function for the duration and base the analysis on this, using the expected value or different quantiles. At the present, however, we lack the necessary data for such a procedure.

6. VARIATIONS AND EXTENSIONS

There are several possible variations on the assumptions of the described model. We will outline a few possibilities.

6.1. Long travel times

In the described model it is assumed that the travel time from o to the closed link k is insignificant in relation to the duration of the closure. It is possible to include the travel time τ_{ok} between o and k explicitly if we assume that the users receive no new information during this time.

The users departing in the interval $[-\tau_{ok}, \min(t_{\text{open}} - \tau_{ok}, 0))$ will arrive at k while it is closed. If there are alternative routes, these users will have to take the route r_{long} instead of r_{orig}. Conversely, the uninformed users departing in the interval $[\max(t_{\text{open}} - \tau_{ok}, 0), t_{\text{open}})$ will arrive after k is reopened and will be able to take route r_{orig} instead of r_{long}. For all other departure times the routes will be the same as in the described model.

Similar arguments will give the consequences when there are no alternative routes. In all cases, the total increase in vehicle travel time will be larger than in the described model.

6.2. Beforehand information

The described model assumes that the link is suddenly opened again with no beforehand information. Alternatively, we may assume that the information process $b(t)$ begins at some time $t_{\text{opbgn}} < t_{\text{open}}$. This may change the response of the users starting between t_{opbgn} and t_{open}.

Suppose first that there are alternative routes. A user starting at $t \in [t_{\text{opbgn}}, t_{\text{open}})$ who has been informed of the time of the reopening can choose whether to take route r_{short} or to wait until r_{orig} is available. The delay will be $\min(\tau_{\text{short}} - \tau_{\text{orig}}, t_{\text{open}} - t)$, compared to $(\tau_{\text{short}} - \tau_{\text{orig}})$ in the described model. Similarly, a user who is uninformed of the closure and arrives at the closed link can choose whether to continue on route r_{long} or to wait until r_{orig} is available. The delay will be $\min(\tau_{\text{long}} - \tau_{\text{orig}}, t_{\text{open}} - t)$, compared to $(\tau_{\text{long}} - \tau_{\text{orig}})$ in the described model. Thus, the increase in vehicle travel time will here be smaller.

Suppose then that there are no alternative routes. Because of the earlier information, the time from the reopening until a user is informed of the reopening will be shorter than in the described model. Hence the increase in vehicle travel time will be smaller.

The beforehand information can also be combined with the long travel times, which gives slightly more cases to consider.

7. CONCLUSION

We have presented a model for calculating the increase in travel time caused by a road closure. The model extends the user equilibrium model without congestion (shortest paths) by incorporating slow information spreading and dynamic user response. By including the closure duration in the model it is possible to also handle the case when some users have no possibility to reach their destinations during the closure.

With the traditional model two shortest path calculations per OD pair are necessary, one from o to d in the undamaged network and one from o to d with link k closed. With the new model only one more shortest path needs to be calculated, from the near end of k to d with link k closed. If linear functions are chosen for $a(t)$ and $b(t)$, only two extra parameters need to be estimated, t_{clinf} and t_{opinf}.

Because of the assumption of no congestion, the travel times of each OD pair can be considered independently. Without this assumption, the model would have to be modified (since the route travel times become time-dependent) and the calculations would be much more time consuming.

The case study shows that for relative comparisons and prioritizations at aggregated levels the traditional model may be sufficient, since the new model does not alter the general picture much. As the example shows, however, the magnitude of the consequences of a closure may differ considerably between the two models. We believe that the new model is closer to the truth than the traditional model. Thus, when the analysis calls for more accurate estimates of the consequences, such as in specific scenario analyses and estimations of the socio-economic costs, the new measure should be preferred. That being said, however, these models should ideally be only the first phase of a full vulnerability analysis, to be followed by more refined studies of the areas that was highlighted in the initial phase.

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