Network structure and travel patterns: explaining the geographical disparities of road network vulnerability

Erik Jenelius
Div. of Transport and Location Analysis, KTH
jenelius@infra.kth.se

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Abstract

Inevitably, links in the road network are sometimes disrupted because of adverse weather, technical failures or major accidents. Link closures may have different economic and societal consequences depending on in which regions they occur (regional importance), and users may be affected differently depending on where they travel (regional exposure). In this paper we investigate in what way these geographical disparities depend on the road network structure and travel patterns. We propose aggregate supply-side (link redundancy, network scale, road density, population density) and demand-side (user travel time, traffic load) indicators and combine them in statistical regression models. Using the Swedish road network as a case study, we find that regional importance is largely determined by the network structure and the average traffic load in the region, whereas regional exposure is largely determined by the network structure and the average user travel time. Our findings show that the long-term vulnerability disparities stem from fundamental properties of the transport system and the population densities. Quantitatively, they show how vulnerability depends on different variables, which is of interest for robust network design.

KEYWORDS: vulnerability, reliability, regional, geography, network, transport, road

1 Introduction

Inevitably, links in the road network are sometimes closed because of adverse weather conditions (e.g., heavy snowfall, storm and cloudburst), physical breakdowns (due to, e.g., landslides and flash floods), accidents, or even antagonistic attacks. In the worst cases, such disturbances may impair the ability for people to receive emergency medical care. Beside the threat to life and health, disruptions of the transport system may cause substantial economic and social strains. For people, this includes impaired abilities to get to work in time, to drop off and pick up children from daycare and school, to do the shopping, etc. For companies, the impacts may include delayed deliveries and supplies (with possible ripple effects), increased freight costs, delayed or cancelled business meetings, etc.

It is the aim of road network vulnerability analysis to assess the likelihood, as well as the economic and societal consequences, of such events. In many areas, for example in USA (Transportation Research Board, 2008), UK (Department for Transport, 2004) and Sweden (Miljödepartementet, 2007), the strains on the road infrastructure are predicted to increase due to the climate change, which makes vulnerability assessments all the more urgent.

In recent years the fields of road network vulnerability and transport reliability have received increasing attention; see, e.g., the special issues edited by Lam (1999) and Sumalee and Karauchi (2006), and the books edited by Bell and Cassir (2000), Iida and Bell (2003) and Murray and Grubesic (2007). Several authors have noted that there is a need for methods to assess the consequences of severe, albeit seemingly unlikely, disruptions of the transport system (e.g., Berdica, 2002; Taylor and D'Este, 2004; Taylor et al., 2006; Jenelius et al., 2006).

In particular, much work has been focused on developing methods to identify important (the terms critical or significant are also used) road links, i.e. links where disruptions would be particularly severe. Taylor et al. (2006) use three different measures of diminished accessibility to evaluate the consequences: the increase in generalised travel cost, the relative decrease in the Hansen integral accessibility index, and the increase of a "remoteness" index specially developed for the regional and remote parts of Australia. Jenelius et al. (2006) use the increase in generalised travel cost to define various measures of link importance, which are then applied to the road network of northern Sweden. Scott et al. (2006) propose a Network Robustness Index to identify important links in highway networks, which is defined as the change in vehicle travel time that is incurred when the link is closed. Sohn (2006), Chen et al. (2007) and Qiang and Nagurney (2008) also present similar approaches.

While the previous research has been focused on identifying critical locations in the road network, little attention has been given to the structural factors underlying these vulnerabilities. In the general field of network analysis, the relationship between network structure and vulnerability has been studied for a long time. For example, in his work on designing reliable communications networks, Baran (1964) noted that distributed, mesh-like networks are more robust to random failures of nodes or links than centralized, hub-and-spoke networks. The recent interest in complex networks has led to several similar studies of both real and model networks, see e.g. Cohen et al. (2000), Holme et al. (2002) and Holmgren (2006). As Patuelli et al. (2007) note, physical constraints tend to restrict the topology of transportation networks and make them nearly planar, which makes them sensitive to failures. With a user perspective on road network vulnerability, however, not only the structural vulnerability of the network but also the traffic and travel patterns must be considered.

Unlike most previous work on road network vulnerability, the focus of the present paper is at a regional level rather than at link level. We adopt two complementary views on vulnerability: From a supply perspective, we say that a region is particularly important if the consequences, in terms of increases in travel time, of a disruption somewhere in this region are particularly severe for the overall network traffic. From a demand perspective, a region is particularly exposed if the consequences of a disruption somewhere in the road network are particularly severe for the network users in this region. The regional exposure and importance measures used in this paper are based on and extended from measures introduced by Jenelius et al. (2006).

By definition, the importance of a particular road link depends on the traffic flow using the link and on the availability of good alternative routes for that traffic. Correspondingly, the exposure of a particular network user depends on the length of the trip and on the availability of alternative routes. The aim of this paper is to investigate to what extent these relationships can be generalized to the regional level. Using the Swedish road network as a case study, we analyse how well aggregate supply-side and demand-side variables such as road density and average travel time together predict the differences in exposure and importance between different municipalities.

The contributions of the paper are both qualitative and quantitative: Qualitatively, our results give a better understanding of the factors that determine the regional variations of road network vulnerability. The results show that the disparities are manifestations of fundamental properties of the transport system, which suggests that road investments of typical size will have little impact on them. Quantitatively, the results reveal through what variables and to what extent the vulnerability of a network can be regulated, which is of interest for the field of optimal network design. Not least, they also provide simple, yet accurate, estimates of the vulnerability of a region based on aggregate, easily obtained data for the region.

The rest of the paper is organized as follows. In Section 2 we introduce our measures of regional exposure and importance. In Section 3 we propose and theoretically motivate explanatory variables and formulate statistical regression models to estimate the exposure and importance of a region. The case study is presented in Section 4, followed by a concluding discussion in Section 5.

2 Methods and measures

2.1 General assumptions

The basic event that underlies our regional measures of vulnerability is the closure of a randomly chosen road segment of unit length. We use the expected consequences of this event to represent the long-term vulnerability of the road system. The approach is based on two assumptions: First, the closure probability per unit road length is assumed to be the same for all roads. Within our setting, it is equivalent to closing a randomly chosen link with probabilities proportional to the lengths of the links. In reality, factors such as traffic load, road type and various characteristics of the link and its surroundings would influence the closure probabilities. With data on these relationships, our general approach does not prevent refining the probability model accordingly.

Second, we only consider closures of one road link at a time, based on the assumption that single road closures are the most common incidents and that regional variations of such events are fairly representative also for more extensive incidents. In reality, of course, links located close to each other are sometimes closed by the same event, e.g. a flood or an earthquake. However, the number of link combinations that would have to be considered when studying closures of more than one link makes such an analysis computationally intractable for large networks. In any case, simultaneous closures of two links located far from each other can generally be regarded as independent and studied one at a time.

The consequences of the link closure are operationalized by the increase in user travel time. The travel time serves as a crude indicator of accessibility and can easily be generalized to include changes in monetary costs and travel distance as well. All users are assumed to minimize their travel time when choosing what route to take from their origin to their destination. We assume that the users respond to the closure by either changing routes or, if more beneficial, by delaying their trip until the link is reopened. In reality, it is likely that some users would change travel mode or destination or choose not to travel at all. Such a decision would mean that the user perceives the cost (or disutility) of such an adjustment to be lower than the cost of the delay of the originally intended trip. By assuming inelastic demand we therefore get an upper bound on the consequences of the closure.

2.2 Regional exposure

The measures of regional exposure used here are based on the average-case demand-weighted exposure introduced by Jenelius et al. (2006). Their measure involved calculating the expected increase in travel time per trip starting in the region when a randomly chosen link in the entire network is closed. In this paper we weight the closure probabilities by the lengths of the links, which we believe gives a more practically interesting measure. The effect is of course that long links have more influence in the measure, which has some but no dramatic impact on the results. Two measures of regional exposure are used in this paper:

- *Expected total exposure (TE):* the expected total increase in vehicle travel time *for all trips* starting within the region during the closure.
- *Expected user exposure (UE):* the expected average increase in travel time *per trip* starting within the region during the closure.

As the name implies, expected user exposure should be of direct interest for the individual users in a region. Expected total exposure, on the other hand, represents the expected socio-economic costs for the region, since travel time changes are generally the dominating component in economic valuations. Hence, total exposure should be of interest for regional authorities with the welfare and efficiency of their own region in mind. To translate the travel times into monetary units, some appropriate value of time should be used.

At the most basic level we consider an origin i, a destination j and a closure of link k. The time from the closure until the traffic situation has returned to the initial, fully functional state is denoted τ. The total increase in vehicle travel time for all users between OD pair (i, j) during this time is denoted ΔT_{ij}^k, and the average travel demand (vehicles) per unit time is denoted x_{ij}. Let r denote a region, let ℓ_k be the length of link k and let $w_k = \ell_k / \sum_\kappa \ell_\kappa$ be the closure probability of link k. Then the expected total exposure *TE* of region r is

$$TE_r = \sum_k w_k \sum_{i \in r} \sum_{j \neq i} \Delta T_{ij}^k, \tag{1}$$

and the expected user exposure *UE* of region r is

$$UE_r = \frac{\sum_k w_k \sum_{i \in r} \sum_{j \neq i} \Delta T_{ij}^k}{\sum_{i \in r} \sum_{j \neq i} x_{ij} \tau}. \tag{2}$$

2.3 Regional importance

To measure the importance of a region we generalize the demand-weighted link importance measure of Jenelius et al. (2006). We calculate the expected effects for all users of the transport system of a closure somewhere in the region, where the probability of closure per unit road length is constant. Let $v_k = \ell_k / \sum_{\kappa \in r} \ell_\kappa$ be the closure probability of link k located in region r. Then the importance I of region r is

$$I_r = \sum_{k \in r} v_k \sum_i \sum_{j \neq i} \Delta T_{ij}^k. \tag{3}$$

The importance of a region is a measure of how the total efficiency of the road transport system is affected by road closures in the region in the question. The difference between regional exposure and importance is, simply put, that exposure expresses how dependent the region is on the whole road transport system, while importance expresses how dependent the whole road transport system is on the region.

2.4 Travel time model

The model of how travel times are affected by a road closure used here was introduced and explained in greater detail by Jenelius (2008). A benefit of the model is that it incorporates closures regardless of whether there are alternative routes available or not. It is applicable to very large, mainly uncongested road networks, where computation time and memory consumption are important issues. In particular, it is assumed that the closure of a link does not affect the travel time on any other link. This approximation should be valid for most of the Swedish road network used in the case study, which is largely uncongested. In densely populated areas, however, the model most likely underestimates the delays caused by a closure. This limitation should be kept in mind when considering the results of the case study presented below. There is, principally, no problem to apply a more elaborated model to calculate the travel time increases.

During the closure, there may be either no or at least one alternative route from origin i to destination j. If there are no alternative routes, the best a user can do is to wait until link k is reopened. Henceforth, a link of this kind will be called a *cut link*. Assuming that the travel demand is constant over time, a user wishing to depart during the closure will on average be delayed $\tau/2$ time units. The total demand during the closure is $x_{ij}\tau$ and the total increase in vehicle travel time during this period is

$$\Delta T_{ij}^{k} = \frac{x_{ij}\tau^{2}}{2} \quad \text{if } k \text{ is a cut link.} \tag{4}$$

If there are alternative routes, a user can choose to travel along the new shortest route or to wait until link k is reopened if this means reaching the destination faster. Let Δt_{ij}^{k} denote the difference in travel time between the new and the original shortest route, which we assume is known to the users. It can be shown that the total increase in vehicle travel time during the closure in any case is

$$\Delta T_{ij}^{k} = \begin{cases} \frac{x_{ij}\tau^{2}}{2} & \text{if } \Delta t_{ij}^{k} \geq \tau, \\ x_{ij}\Delta t_{ij}^{k}\left(\tau - \frac{\Delta t_{ij}^{k}}{2}\right) & \text{if } \Delta t_{ij}^{k} < \tau. \end{cases} \tag{5}$$

It is clear from (4) and (5) that the benefit from short alternative routes will increase with the closure duration τ, since the consequences will then be linear in τ, while they will be quadratic in τ if there are no short alternative routes. Thus, the duration of the closure will in general affect the exposure and importance of a region, both in absolute numbers and in relation to other regions.

For non-cut links, Jenelius et al. (2006) calculate the total increase in vehicle travel time per time unit as $x_{ij}\Delta t_{ij}^{k}$, independent of the closure duration. This measure assumes that all users use the new shortest route during the entire closure. We will use this simpler model as well to test the generality of our results.

3 Explanatory models for regional exposure and importance

In this section we introduce indicators and models for estimating the exposure and importance of regions in the road network. It is intuitively clear that the exposure and importance of a region depend on both the structure and on the usage of the regional road network, i.e. on both supply and demand factors. It should be noted that while both regional exposure and importance involve the entire transport system, in terms of the location of the incident for the former and the extent of the consequences for the latter, the explanatory models are based solely on properties of the own region.

3.1 Supply-side variables: Network structure

3.1.1 Link redundancy and network scale

It is well known that the topology of a network has a significant impact on its ability to withstand node and link failures (Baran, 1964; Holme et al., 2002). The most fragile network structure is the tree, where every link is a cut link. When more links are added, i.e. when the link redundancy increases, more alternative routes become available. More alternative routes should, in turn, mean shorter best alternative routes, in terms of the number of links in the routes. To measure the link redundancy of the road network we use the beta index (see Haggett and Chorley, 1969), defined for region r as

$$\beta_r = \frac{M_r}{N_r}, \tag{6}$$

where M_r and N_r are the number of undirected links and nodes in the region, respectively[1]. The beta index tends to 1 for large trees, 1.5 for hexagonical (honeycomb) grids, 2 for square grids and 3 for triangular grids. The latter are the planar networks with the highest possible redundancy. To test the robustness of our results, we also use the alpha index (Haggett and Chorley, 1969), defined as

$$\alpha_r = \frac{M_r - N_r + 1}{2N_r - 5}, \tag{7}$$

which ranges from 0 for trees to 1 for triangular grids.

For the road network (and all networks with distances associated with the links), the effects of link failures depend not only on the topology but also on the physical scale of the network, since increasing all link lengths would increase all trip lengths correspondingly. An intuitive measure of the scale of the regional road network is the average link length, $\bar{\ell}_r$. To test the robustness of our results, we also use the average link travel time, $\bar{t}_r$, as an alternative measure.

3.1.2 Road density

A simpler indicator of the network structure is the road density R_r, i.e. the total length of the regional road network divided by the area of the region. If the network topology is similar in different regions, higher road density should in general imply a higher availability of short alternative routes. Road density is easier to calculate and to acquire data for than the beta index and the average link length above since no explicit network representation is needed.

3.1.3 Population density

It is reasonable to assume that regional road density is highly positively correlated with regional population density (inhabitants per unit area), as has been shown at the national level by Glover and Simon (1975). By using the population density P_r as the supply-side variable for the region, we have thus completely eliminated the actual structure of the network from the model. As the model becomes more abstract, however, the causality also becomes less obvious: while it may be reasonable to say that high regional exposure and importance are caused by low link redundancy, the dependency on population density is more indirect and possibly circular.

[1] In the case study a network representation with directed links is used to allow for asymmetric travel times. The beta index is then calculated by dividing the number of directed links by 2.

3.2 Demand-side variables: Travel patterns

3.2.1 Exposure: User travel time

The exposure of a single user depends on the duration of the trip in two ways. First, travel time is strongly correlated with travel distance, which makes long trips more likely to be affected by link failures. Second, a long travel time may indicate that the average speed in the regional road network is low, which suggests that the increase in travel time due to a link closure will be large even if the increase in travel distance is moderate. Therefore, the average travel time of the users in the region before the link closure, denoted $\bar{T}_r$, should be an important factor for regional user exposure.

The total exposure of a region is the sum of the user exposure over all trips starting in the region during the closure. If user exposure depends on the average user travel time, it is reasonable to assume that total exposure depends on the total travel time of all users starting during some period of time, denoted T_r.

3.2.2 Importance: Traffic flow

The importance of a link depends on the link flow, i.e. the amount of traffic using the link. For regional importance, the relevant demand side factor should be the average regional traffic flow, defined as

$$\bar{f}_r = \frac{\sum_{k\in r} \ell_k f_k}{\sum_{k\in r} \ell_k}, \tag{8}$$

where f_k is the traffic flow (vehicles per unit time) on link k. This quantity can also be interpreted as the vehicle kilometers travelled (VKT) per unit time divided by the total length of the road network in the region.

3.3 Regression models

We use regression models to analyze the dependence of the continuous vulnerability measures on the continuous explanatory variables introduced above. The regression models for each vulnerability measure are formed by combining one of the supply-side variables with the appropriate demand-side variable.

It is reasonable to assume that the independent variables influence the vulnerability measures in a multiplicative way: at the link level, for example, the consequences of a link failure depends on the average increase in travel time for the users of the link (the availability of short alternative routes), multiplied with the total number of users of the link (the traffic flow). Furthermore, the independent as well as the dependent variables are all non-negative; when the user travel time or the average regional traffic flow tend to zero, the corresponding vulnerability measure must tend to zero as well. As the dependent variables increase, on the other hand, we should expect larger variation in the vulnerability measures, which empirical analyses confirm. For these reasons we use multiplicative regression models, i.e. models on the form $Y_r = 10^{\gamma_0} \cdot z_{r1}^{\gamma_1} \cdot z_{r2}^{\gamma_2} \cdot \ldots \cdot 10^{\varepsilon_r}$, or, taking the logarithm of both sides,

$$\log Y_r = \gamma_0 + \gamma_1 \log z_{r1} + \gamma_2 \log z_{r2} + \ldots + \varepsilon_r, \tag{9}$$

where $z_{r1}, z_{r2}, \ldots$ are the independent variables, Y_r is the dependent variable, and $\gamma_0, \gamma_1, \ldots$ are parameters to be estimated. Only γ_0 depends on the units used for the variables. The error terms ε_r are assumed to be independent, identically distributed $N(0, \sigma^2)$. The issue of possible spatial correlation between the regions is discussed in Section 4.

For example, a model for regional user exposure using road density and average user travel time as explanatory variables is formulated as

$$\log UE_r = \gamma_0 + \gamma_R \log R_r + \gamma_{\bar{T}} \log \bar{T}_r + \varepsilon_r. \tag{10}$$

All model specifications can be found in the Appendix.

4 Case study: The Swedish road network

We have tested the explanatory power of the proposed models of regional exposure and importance in a case study of the Swedish road network. The measures have been calculated for every municipality in Sweden for a 30 minutes closure duration. We have also tested the generality of our results by using alternative regional partitions, explanatory variables, closure durations and travel time models, as explained in Section 4.4.

4.1 Data

In order to calculate the regional exposure and importance measures, the following data are necessary: a network representation with nodes, links and centroids (i.e. origin/destination nodes), the length and travel time of every link, the travel demand between every centroid pair, and an appropriate regional coding of every link and centroid. We have obtained the first three sets of data from the Swedish national travel demand model system SAMPERS (see Beser and Algers, 2001). The SAMPERS system divides Sweden into zones in which all trips begin and end, each zone comprising about 1,000 inhabitants. The travel demand between different zones is calculated using nested logit choice models which have been estimated on travel surveys. The travel time of each link is calculated with user equilibrium traffic assignments, which means that initial congestion is considered in our study. The OD travel demand matrix used in this study represents the annual daily average travel demand and includes only trips made by car.

For computational reasons, SAMPERS has divided the Swedish transport system into five complementary regional submodels. We have obtained a detailed representation of the entire Swedish road transport system by merging these regional submodels while retaining all interregional trips. The resulting road network consists of 77,733 nodes (including 8,764 centroids or origin/destination nodes) and 173,930 directed links, and represents a very fine level of detail.

In the original network representations in SAMPERS, roads are divided into shorter sections with varying characteristics such as different volume-delay functions. With our model a closure of any of these links would have the same consequences, but how these more or less arbitrary divisions are made affects the network structure indicators introduced in Section 3.1.1. Therefore, we have replaced links connected in series with a single link (with equivalent travel time and flow) and kept only nodes that mark the dead ends of roads or where more than two road sections join. This gives a unique, and in a sense the most fundamental, representation of the network.

To obtain the municipality in which every node and link is located, we have imported the node coordinates into a GIS and projected them onto municipality maps. A link from node i to node j is defined as being located in the municipality in which node i is located.

The calculations were performed with specially developed software written in C++/C#. Using a 2.0 GHz Pentium III 1 GB laptop, calculating the exposure and importance of every municipality takes about 12 hours.

4.2 Distributions of explanatory variables

Sweden has a total area of 449 964 km^2, a population of 9.2 million people and an average population density of 22.2 inhabitants per km^2 (data as of February 2008, from Statistics Sweden). The

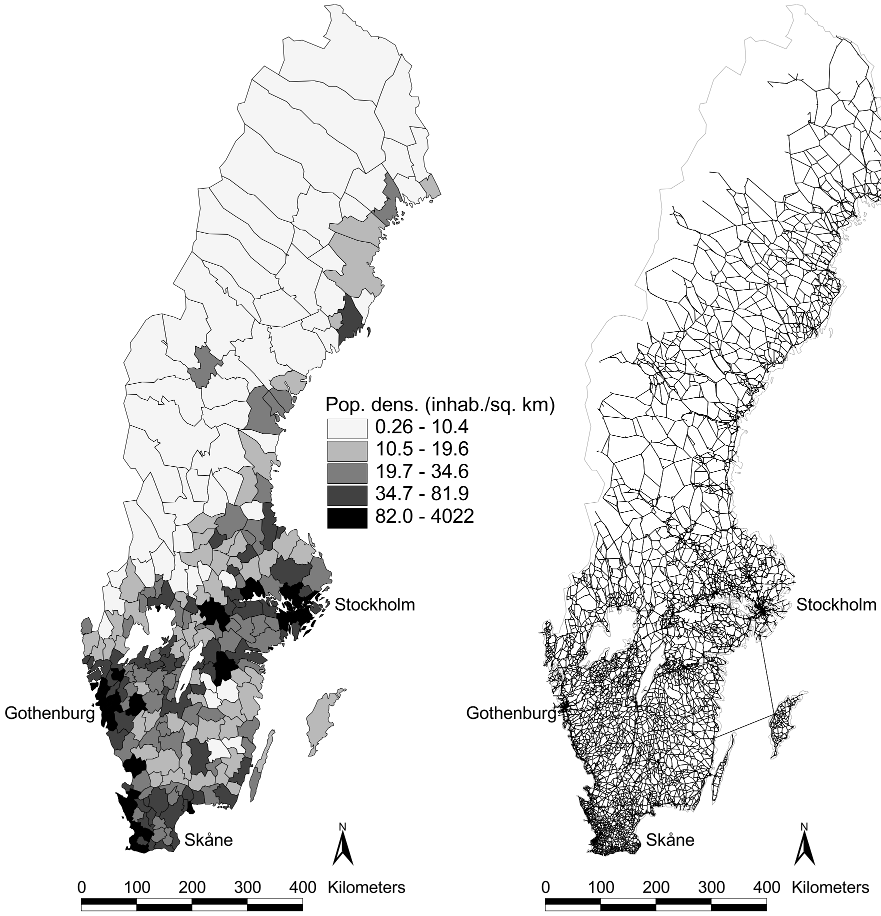

Figure 1: Left: Population density of the Swedish municipalities (inhabitants/km^2), data from Statistics Sweden. Categories represent quintiles. Right: The road network representation used in the case study.

country was in 2001 divided into 289 municipalities. The population density of the municipalities for that year is shown to the left in Figure 1. As can be seen, the population is concentrated to the southern parts of the country, in particular around the urban centra Stockholm, Gothenburg and the Skåne region. The distribution of the population is reflected in the density of the road network, which is shown to the right in Figure 1. Indeed, the regression model $\log R_r = \gamma_0 + \gamma_P \log P_r + \varepsilon_r$ shows high explanatory power (adj. $R^2 = 0.82$, est. $\gamma_P = 0.39$) in line with Glover and Simon (1975).

The beta index β_r for link redundancy varies between 1.04 and 1.55 for different municipalities (mean 1.35, median 1.37). The main geographical trend is that the link redundancy is the highest in the southern parts of the country, in particular the Skåne region, but the networks in and around Stockholm and Gothenburg have remarkably low redundancy considering their heavy traffic loads. The average link length $\bar{\ell}_r$ shows roughly the opposite patttern and is the highest to the north, so that both average link length and link redundancy contribute to the vulnerability in the same direction. Average link length ranges from 0.42 km to 11.0 km (mean 3.02 km, median 2.74 km).

The average user travel time $\bar{T}_r$ for every municipality is shown to the left in Figure 2. Although the travel times vary between municipalities, there are no clear geographical trends. It can be noted, however, that many municipalities in the mountainous northwestern parts and around Stockholm have relatively long travel times. The total travel time for all users starting during a certain interval, T_r, is very nearly proportional to the population size of the region, since the

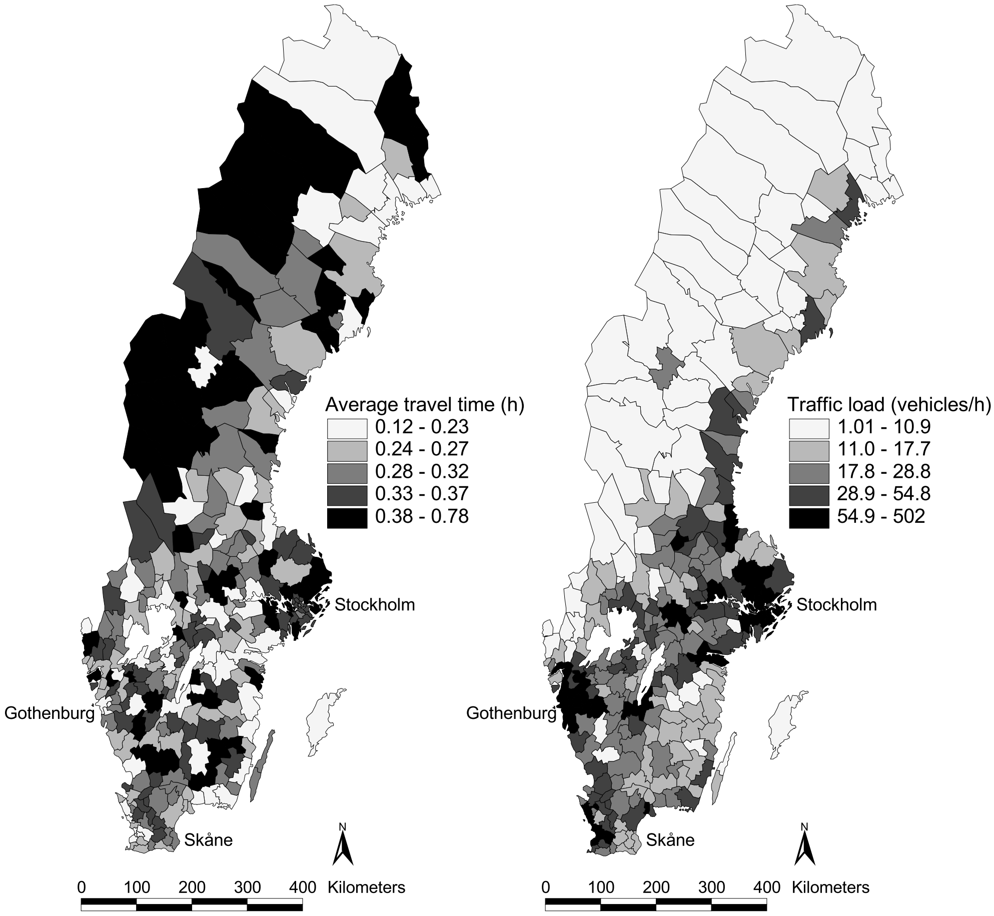

Figure 2: Left: Average user travel time of the Swedish municipalities (h). Right: Average regional traffic flow (vehicles/h). In both maps, categories represent quintiles.

population size strongly determines the total number of trips being made. As would be expected, the average regional traffic flow $\bar{f}_r$, shown to the right in Figure 2, is highly correlated with the population density (the regression model $\log \bar{f}_r = \gamma_0 + \gamma_P \log P_r + \varepsilon_r$ yields adj. $R^2 = 0.84$, est. $\gamma_P = 0.63$). Thus, on one hand the road network is denser in densely populated regions, which should lead to shorter increases in travel time for each individual user. On the other hand, the average regional traffic flow is higher, which makes the number of users affected by a link failure greater.

4.3 Results

The left part of Figure 3 shows the user exposure of each municipality for a 30 minutes closure. There is a clear geographical pattern in the distribution in that the southern parts of Sweden are considerably less exposed than the northern parts. In particular, many of the most exposed municipalities are located in the northwest, where the population and the road network are very sparse and the travel times are often long.

The regression model for user exposure using the beta index β_r and the average link length $\bar{\ell}_r$ as supply-side variables and the average user travel time $\bar{T}_r$ as demand-side variable has an explanatory power of adj. $R^2 = 0.90$. Using the simpler supply-side variable road density gives adj. $R^2 = 0.84$, while using population density gives adj. $R^2 = 0.78$. The supply-side variables appear to have more influence than the average user travel time on the user exposure, in the sense

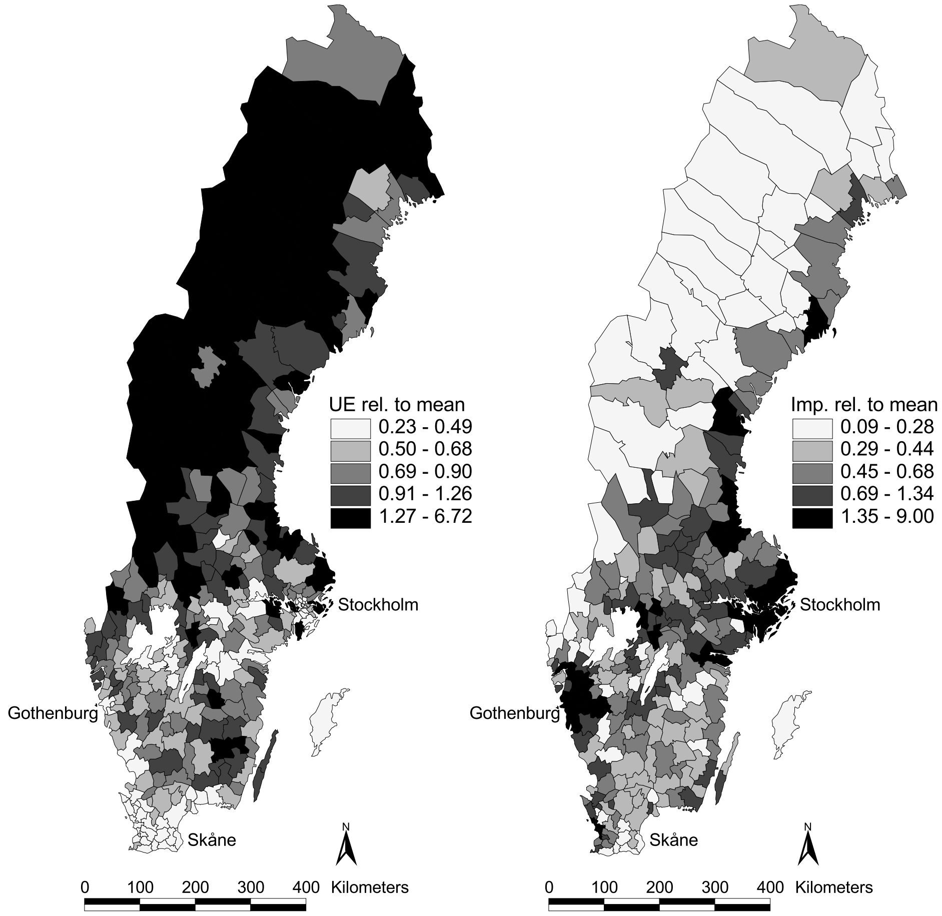

Figure 3: Left: Expected user exposure of the Swedish municipalities relative to the mean $7.8 \cdot 10^{-6}$ h. Right: Importance of the Swedish municipalities relative to the mean 1.1 h. In both maps, the closure duration is 30 minutes; categories represent quintiles.

that the t-statistics for the supply-side parameters are larger in magnitude, and that models based only on supply-side variables have higher explanatory power than the model based only on average user travel time. Figure 4 shows the calculated user exposure of every municipality versus the predicted value with the first two models. In these and all following results, all parameters are highly significant. Estimation results for all models are reported in the Appendix.

The corresponding models for total exposure, based on total user travel time T_r instead of average user travel time, yield adj. $R^2 = 0.95$, 0.92 and 0.88, respectively. In fact, since total user travel time is nearly proportional to population size, total exposure can be explained solely based on population size and density, yielding adj. $R^2 = 0.87$. The total travel time has larger influence than the supply-side variables on the total exposure.

The importance of each municipality for a 30 minutes closure is shown to the right in Figure 3. The geographical distribution displays strong similarities to those of the population density and the average traffic flow. We can see that the denser networks in densely populated regions do not fully compensate for the high traffic flow there, which makes these network regions particularly important. An exception is the Skåne region to the south. The three regression models based on average traffic flow as demand-side variable give adj. $R^2 = 0.95$, 0.87 and 0.81, respectively. As an illustration of the need to consider both network structure and travel patterns when estimating regional vulnerability, estimating importance only from the beta index and average link length gives adj. $R^2 = 0.41$, and estimation based only on average traffic flow gives adj. $R^2 = 0.77$,

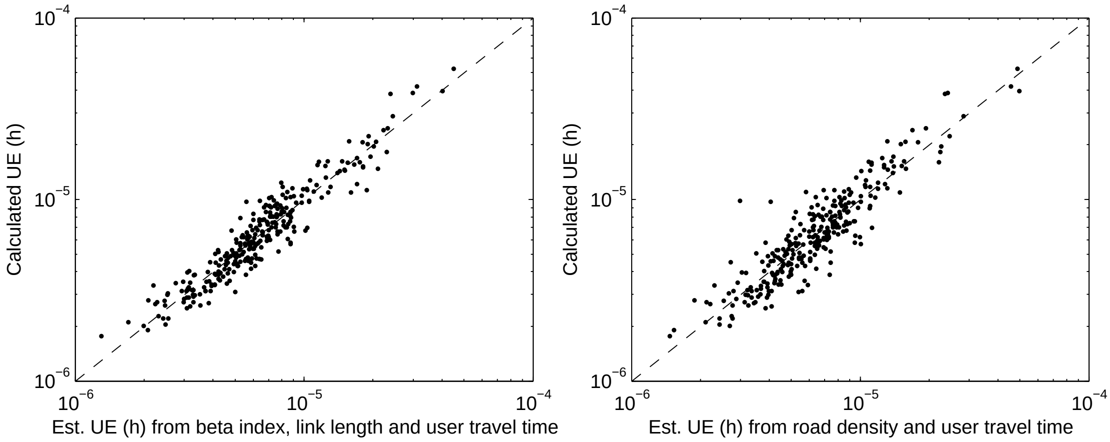

Figure 4: Calculated user exposure (h) against estimated user exposure (h) for every municipality. Left: estimate based on beta index, average link length and average user travel time (adj. $R^2 = 0.90$). Right: estimate based on road density and average user travel time (adj. $R^2 = 0.84$). 30 minutes closure, logarithmic scales.

significantly poorer than using all variables. This also illustrates that the average traffic flow has larger influence than the supply-side variables on the importance. Figure 5 shows the calculated importance versus the predicted value with the first and the third models.

The estimates for the same supply-side variable in the models for different vulnerability measures are usually similar. For example, the parameter for road density is estimated as -0.599 in the user exposure model, -0.555 in the total exposure model, and -0.621 in the importance model. Furthermore, the parameters for the demand-side variables are often close to and sometimes not significantly different from 1, indicating linear relationships with the vulnerability measures.

The residuals from the models show signs of some spatial correlation, and the pattern is quite consistent for different models. This is not to be unexpected, since adjacent regions in the road transport system are typically interconnected, both through the physical road network and through the traffic crossing the regional borders. The finding suggests that a spatial error term could be a suitable refinement of the regression model, see e.g. Anselin (1988). We have also repeated the analysis on a coarser geographic scale, as described in the next Section.

4.4 Effects of alternative assumptions

The effects of using the alpha index (see Section 3.1.1) rather than the beta index as indicator of link redundancy are small; the explanatory power of the models as measured by adj. R^2 decreases by about 0.01. Using the average link travel time rather than the average link length has a slightly larger negative effect.

We have calculated the regional exposure and importance measures for a 48 hour closure as well. We find that the closure duration has a large impact on the distributions of the vulnerability measures, and the regression models have considerably lower explanatory powers. This is because cut links (i.e. links without alternative routes) have much greater impact on the expected consequences of long closures than of short closures. The distribution of cut links in the network, in turn, is quite random and has little connection with properties of the surrounding network such as the road density. It is therefore unlikely that there exists any accurate indicator of the expected consequences of a long closure that does not explicitly involve considering and identifying the cut links.

We have studied the impact of the travel time model by also using the model of Jenelius et al.

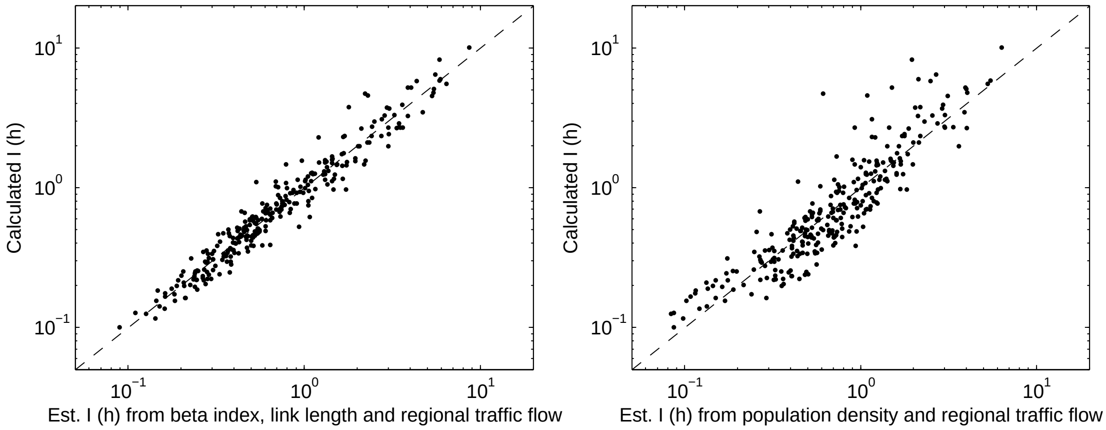

Figure 5: Calculated importance (h) against estimated importance (h) for every municipality. Left: estimate based on beta index, average link length and average regional traffic flow (adj. $R^2 = 0.95$). Right: estimate based on population density and average regional traffic flow (adj. $R^2 = 0.81$). 30 minutes closure, logarithmic scales.

(2006), see Section 2.4. This model only considers non-cut links and is independent of the closure duration. With this model the explanatory variables are still applicable, but the residuals are in general larger and the explanatory powers therefore lower. The reason for this may be that this model is more sensitive to local circumstances of the network, since the consequences of a link closure depend only on the alternative routes and are not bounded by the closure duration in any way.

Finally, we have investigated the impact of the regional partitioning of the network by repeating the calculations for counties instead of municipalities. Sweden is divided into 21 counties, each comprising between 1 and 26 municipalities. We find that the explanatory power of the models are generally better on the more aggregated county level. For example, the models based on road density give adj. $R^2 = 0.84$ for user exposure, 0.97 for total exposure and 0.93 for importance. The spatial correlation between the counties is insignificant. These results are intuitively reasonable; there is of course a lower limit on the size of the regions, in relation to the scale of the road network and the travel distances of the users, below which these indicators based only on the regions themselves become too inaccurate. The findings here suggest that at the municipal level we are approaching this lower limit, at least for some of the municipalities.

5 Conclusion

We have investigated how the network structure and the travel patterns affect the long-term vulnerability of a road network. We have found that the properties that make a particular road link important (the link flow and the availability of alternative routes) and a particular user exposed (the trip length and the availability of alternative routes) can be generalized to larger regions in the road network. The regional availability of alternative routes, i.e. the network structure, is represented by three different indicators of increasing simplicity and abstraction: the link redundancy in combination with the network scale, the road density, and the population density. The regional travel patterns are represented by the average user travel time for user exposure, the total user travel time for total exposure, and the average traffic flow for importance.

The vulnerability measures tend to increase linearly with the travel pattern variables. Hence, doubling the average user travel time in a region will double the exposure, and doubling the average traffic load will double the importance. As expected, the explanatory power of the models decreases as the indicator of network structure becomes more implicit. Still, it remains remarkably high even when an indirect indicator such as population density is used. User exposure depends mainly on the network structure, whereas total exposure and importance depend mainly on the travel patterns.

In principle, the regression models indicate what measures should be taken to reduce the exposure or importance of a region: increasing the link redundancy, reducing the link lengths, reducing the user travel times or reducing the traffic load. In practice, however, road investments of typical size will likely have little influence on these aggregated variables except on a very long time scale. Our results therefore indicate that the long-term geographical disparities of vulnerability cannot be easily regulated but are manifestations of fundamental properties of the transport system, also reflecting geographical disparities in population densities. Hence, the consequences of link closures will likely continue to be the highest if they occur in densely populated areas, and the consequences for individual users will continue to be the highest in sparsely populated areas with long travel times.

Our results are thus mainly of theoretical interest, not least for the field of optimal transport network design, where robustness or non-vulnerability can be an objective. The regression models give valuable information about what variables can be adjusted to increase the robustness of the network, and the effectiveness of different actions. Regarding already existing networks, it may be more worthwhile to approach vulnerability from a worst-case perspective and focus on longer closure durations. Our results then indicate that cut links are responsible for the largest consequences (see also Jenelius, 2008). Providing alternative routes locally around the most heavily used cut links would be an effective way to reduce the worst-case vulnerability of the road network.

In this paper we have only studied one particular national road network, and it is too early to claim that the relationships found here exist generally in road networks. Our hypothesis, however, is that road transport systems are sufficiently similar in different parts of the world that the general relationships indeed should be universal, albeit with varying values for the parameters in the models. It would be most interesting to perform the same kind of analysis on road networks from other locations and compare the results.

As discussed in Section 2, the methods used in the paper has some limitations. Travel time is a very simple indicator for the impacts of a road closure on the network users; some more refined measure of accessibility or economic losses may be more appropriate. Furthermore, we assume that link closures do not cause additional congestion in the surrounding network, which is unrealistic in urban areas. The modelling of link closure probabilities only takes link lengths into account, not traffic flow, road type, etc. The same closure duration is assumed for all links, whereas in reality incidents are likely resolved more rapidly in urban areas. There is nothing in the analyses that prevents such factors to be added in future studies. In general, we expect that the introduced indicators will remain relevant when such factors are taken into account, though parameter values may be different, and slight modifications may give higher explanatory power. This is an area for further research.

Appendix: Model specifications and estimation results

Road density and population density
Variables: road density (km^{-1}), population density (km^{-2})
$\log R_r = \gamma_0 + \gamma_P \log P_r + \varepsilon_r$

Adj. R^2: 0.82, log likelihood: 185.6

Parameter	Estimate	Std. error	t-statistic
γ_0	-0.624	0.0173	-36.10
γ_P	0.394	0.1077	36.57

Regional traffic load and population density
Variables: aver. regional traffic load (h^{-1}), population density (km^{-2})
$\log \bar{f}_r = \gamma_0 + \gamma_P \log P_r + \varepsilon_r$

Adj. R^2: 0.84, log likelihood: 63.47

Parameter	Estimate	Std. error	t-statistic
γ_0	0.445	0.0264	16.84
γ_P	0.633	0.0165	38.45

Regional user exposure model 1
Variables: user exposure (h), beta index, aver. link length (km), aver. user travel time (h)
$\log UE_r = \gamma_0 + \gamma_\beta \log \beta_r + \gamma_{\bar{\ell}} \log \bar{\ell}_r + \gamma_{\bar{T}} \log \bar{T}_r + \varepsilon_r$

Adj. R^2: 0.90, log likelihood: 316.5

Parameter	Estimate	Std. error	t-statistic
γ_0	-4.782	0.0303	-157.7
γ_β	-2.088	0.1483	-14.08
$\gamma_{\bar{\ell}}$	0.695	0.0208	33.46
$\gamma_{\bar{T}}$	0.828	0.0434	19.07

Regional user exposure model 2
Variables: user exposure (h), road density (km^{-1}), aver. user travel time (h)
$\log UE_r = \gamma_0 + \gamma_R \log R_r + \gamma_{\bar{T}} \log \bar{T}_r + \varepsilon_r$

Adj. R^2: 0.84, log likelihood: 243.1

Parameter	Estimate	Std. error	t-statistic
γ_0	-4.717	0.0293	-161.2
γ_R	-0.599	0.0210	-28.58
$\gamma_{\bar{T}}$	0.981	0.0543	18.06

Regional user exposure model 3
Variables: user exposure (h), population density (km^{-2}), aver. user travel time (h)
$\log UE_r = \gamma_0 + \gamma_P \log P_r + \gamma_{\bar{T}} \log \bar{T}_r + \varepsilon_r$

Adj. R^2: 0.78, log likelihood: 199.0

Parameter	Estimate	Std. error	t-statistic
γ_0	-4.367	0.0333	-131.2
γ_P	-0.248	0.0108	-22.95
$\gamma_{\bar{T}}$	0.903	0.0645	14.00

Regional total exposure model 1
Variables: total exposure (h), beta index, aver. link length (km), total user travel time during 1 h (h)
$\log TE_r = \gamma_0 + \gamma_\beta \log \beta_r + \gamma_{\bar{\ell}} \log \bar{\ell}_r + \gamma_T \log T_r + \varepsilon_r$

Adj. R^2: 0.95, log likelihood: 310.1

Parameter	Estimate	Std. error	t-statistic
γ_0	−4.942	0.0450	−109.8
γ_β	−1.958	0.1477	−13.26
$\gamma_{\bar{\ell}}$	0.655	0.0254	25.82
γ_T	0.977	0.0144	67.90

Regional total exposure model 2
Variables: total exposure (h), road density (km^{-1}), total user travel time during 1 h (h)
$\log TE_r = \gamma_0 + \gamma_R \log R_r + \gamma_T \log T_r + \varepsilon_r$

Adj. R^2: 0.92, log likelihood: 249.3

Parameter	Estimate	Std. error	t-statistic
γ_0	−4.872	0.0384	−126.8
γ_R	−0.555	0.0231	−24.07
γ_T	0.940	0.0167	56.19

Regional total exposure model 3
Variables: total exposure (h), population density (km^{-2}), total user travel time during 1 h (h)
$\log TE_r = \gamma_0 + \gamma_P \log P_r + \gamma_T \log T_r + \varepsilon_r$

Adj. R^2: 0.88, log likelihood: 198.2

Parameter	Estimate	Std. error	t-statistic
γ_0	−4.656	0.0418	−111.3
γ_P	−0.251	0.0140	−17.92
γ_T	1.019	0.0234	43.62

Regional importance model 1
Variables: importance (h), beta index, aver. link length (km), aver. regional traffic flow (h^{-1})
$\log I_r = \gamma_0 + \gamma_\beta \log \beta_r + \gamma_{\bar{\ell}} \log \bar{\ell}_r + \gamma_{\bar{f}} \log \bar{f}_r + \varepsilon_r$

Adj. R^2: 0.95, log likelihood: 280.0

Parameter	Estimate	Std. error	t-statistic
γ_0	−1.423	0.0492	−28.93
γ_β	−3.441	0.1646	−20.91
$\gamma_{\bar{\ell}}$	0.706	0.0396	17.82
$\gamma_{\bar{f}}$	1.046	0.0195	53.66

Regional importance model 2
Variables: importance (h), road density (km^{-1}), aver. regional traffic flow (h^{-1})
$\log I_r = \gamma_0 + \gamma_R \log R_r + \gamma_{\bar{f}} \log \bar{f}_r + \varepsilon_r$

Adj. R^2: 0.87, log likelihood: 147.8

Parameter	Estimate	Std. error	t-statistic
γ_0	−1.566	0.0391	−40.09
γ_R	−0.621	0.0426	−14.57
$\gamma_{\bar{f}}$	1.013	0.0267	37.88

Regional importance model 3
Variables: importance (h), population density (km^{-2}), aver. regional traffic flow (h^{-1})
$\log I_r = \gamma_0 + \gamma_P \log P_r + \gamma_{\bar{f}} \log \bar{f}_r + \varepsilon_r$

Adj. R^2: 0.81, log likelihood: 99.95

Parameter	Estimate	Std. error	t-statistic
γ_0	-1.245	0.0329	-37.86
γ_P	-0.305	0.0361	-8.464
$\gamma_{\bar{f}}$	1.127	0.0522	21.59

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