Importance and exposure in road network vulnerability analysis

ACCEPTED for publication by TRANSPORTATION RESEARCH part A

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ABSTRACT

The reliability and vulnerability of critical infrastructures have attracted a lot of attention recently. In order to assess these issues quantitatively, operational measures are needed. Such measures can also be used as guidance to road administrations in their prioritisation of maintenance and repair of roads, as well as for avoiding causing unnecessary disturbances in the planning of roadwork. The concepts of link importance and site exposure are introduced. In this paper, several link importance indices and site exposure indices are derived, based on the increase in generalised travel cost when links are closed. These measures are divided into two groups: one reflecting an “equal opportunities perspective”, and the other a “social efficiency perspective”. The measures are calculated for the road network of northern Sweden. Results are collected in a GIS for visualisation, and are presented per link and municipality. In view of the recent great interest in complex networks, some topological measures of the road network are also presented.

KEYWORDS: travel reliability, vulnerability index, contingency planning, road network topology.
1. Introduction

1.1 Background

For at least a decade, there has been a research interest in questions of vulnerability and reliability of transport networks. Events that have spurred this interest are the earthquake in Kobe in 1995 and the terrorist acts of September 11, 2001. Issues of the vulnerability and reliability of large-scale systems have been studied for a long time before that in, e.g., military logistics as well as complex technical systems such as electronic circuits, industrial production processes, etc. In networked systems, however, the issue has arisen quite recently, simultaneously with the increasing number of global connections in the economy (“globalisation”), the rise of the Internet and the threats of computer viruses, worms and other kinds of attacks on computers and servers. In retrospect, it has become more and more evident that interactions and interdependencies between members, parts and subsystems can be represented by graphs, and hence the theories of graphs can be applied for vulnerability and reliability analysis. The development of more powerful and available computers has also made it feasible to employ the computationally often heavy algorithms of discrete and combinatorial mathematics, which are associated with graphs.

Given the importance of the transport networks for journeys to work, for production logistics (notably the just-in-time production philosophy), and for business travel, the reliability of transport networks is a key interest from the point of view of transport system users and hence planners at all levels, both in the public and private sectors. For business, the reliability of transports is actually, together with regularity and safety, one of the three most valued aspects of transport quality, and together with cost and time the most important aspect of transports altogether (SIKA, 2000a; b).

For the purpose of road management, prioritisation for road maintenance and repair, contingency planning, and for the assessment of regional disparities, it would thus be helpful with a general “vulnerability index” attached to links and nodes.

In incident and contingency planning, and the planning of road works, there should be an awareness about the impacts of the reduced capacity on links. Increased efforts should be made on links that are more critical to the system. Similarly, increased awareness and efforts are called for about the degree of vulnerability in different places in the system. In view of the strategic goals of the Swedish Road Administration (SRA) of a “positive regional development” and “an accessible transport system”, relevant questions to ask are: “Which regions are most susceptible to disruptions in the transport system?” and: “Which links are most critical to the operation of the whole system?” In order to make such distinctions, general measures of the criticality of links in the network, and the vulnerability of sites, municipalities etc. need to be developed and tested.

1.2 Aim of the paper

In the following sections, we first attempt to define what it is that we want to measure, and then we develop such indices or measures. We argue that the concept of vulnerability can be divided into two parts, one containing the probability of a hazardous event
and the other, which we call exposure, containing the consequences of the event in a
certain place. Exposure is thus site-dependent. Similarly, following Nicholson and Du
(1994), we call the consequences for a collection of sites of a failing link or group of
links the importance of that link/group of links. As a measure of the consequences of
failure we use the increase in generalised travel cost. In order to reflect the two perspec-
tives of “equal opportunities” and “social efficiency”, for the latter we weight this in-
crease by travel demand.

With the equal opportunities perspective, all roads are equally significant, regardless
how often they are utilised. Each trip is equally important, which reflects the character-
istic of roads as ”public goods”: people should be given equal opportunities everywhere.
In contrast, with the social efficiency perspective, the roads used the most are consid-
ered more significant, because they serve more people and businesses and thus generate
more positive social and economic spin-offs. Which perspective to use in a specific
situation is a matter of political judgement.

The measures are calculated for the road network of northern Sweden (Norrland plus
the county of Dalarna)¹, and are then aggregated over nodes and/or links to yield meas-
ures for, in turn, single nodes and links, municipalities, and the whole network.
Norrland (with Dalarna) is geographically the larger part of the country, but at the same
time the most sparsely populated part. The majority of people, businesses and large cit-
ies are located in the southern part of Sweden. For example, the largest city in our study
area, Umeå, is only number 11 in size in Sweden with 109,000 inhabitants, and out of
the 43 cities with a population of more than 50,000 in all of Sweden, only eight are lo-
cated there. The traffic, in general, is thus moderate and the network not liable to con-
gestion.

The paper is outlined as follows: in the next section, the concepts and definitions of vul-
erability, reliability and criticality are discussed. In section 3, our model and measures
are presented, together with some of the formulas used. The presentation of the case
study starts in section 4 with our assumptions, a short description of the geographical
context, after which the data sources are discussed. The details of the calculations are
presented in section 4.3. In view of the recent big interest in the properties of complex
networks, in section 4.4 we calculate some topological measures of the road network
and compare it with other networks. We then present the most interesting results from
the case study in section 4.5, in which the most exposed municipalities to an average-
case (random) and a worst-case scenario, as well as the most important links, are identi-
fied. We then conclude (section 5) and discuss the results and the caveats associated
with our method (section 6), and last we suggest some options for further research in
section 7. Definitions and all formulas are supplied in the Appendix.

2. Definitions of concepts

2.1 Vulnerability, reliability and risk

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¹ The Palt area, used by the SRA for regional policy analysis.
The concept of vulnerability does not yet have a commonly accepted definition, according to several authors (Einarsson and Rausand, 1998; Berdica, 2002a; Holmgren, Å., 2004; Taylor and D’Este, 2004), and the meaning of the term may depend on the context. Holmgren, Å. (2004) uses “sensitivity to threats and hazards” as a working definition, however stressing that the negative events “substantially reduce [the] ability [of the system] to maintain its intended function” (our emphasis). Other authors stress the infrequent occurrence of the events, such that “vulnerability is a susceptibility for rare, big risks” (Laurentius, 1994, cited in Berdica, 2002a), or the sudden, unpredicted occurrence (Swedish Department of Defence, 1998).

Abrahamsson (1997) notes that in the vulnerability concept there is implicitly the notion of “little strokes fell great oaks”, i.e., a relatively small incident can, if it happens in an unfortunate (critical) place and time, cause major damage or even the failure of the whole system by chain reactions. If the incident is a blow of a hydrogen bomb, we would hardly call the system vulnerable if it fails. In case of moderate strains, networks are therefore, by construction, not vulnerable. If individual links, or contiguous areas of links fail, in most cases there is still a way through the network—this is the fundamental idea of a network. Vulnerabilities appear when the network (or transport system) is put under pressure, when the capacity reaches its maximum, and a small further stress could cause a major damage by magnifying itself and cascade through the system, possibly until it collapses.

Berdica (2002a) defines “vulnerability” as “a susceptibility to incidents that can result in considerable reductions in road network serviceability”. Serviceability of a link/route/road network, in turn, “describes the possibility to use that link/route/road network during a given period”. In the terms of Husdal (2004), vulnerability is “the non-operability of the network under certain circumstances”. Conversely, he states that reliability “describes the operability of the network under varying strenuous conditions (i.e., the ability to continue to function)”. These definitions are quite similar to the one of Holmgren above, given that the term serviceability (non-operability) expresses the ability to maintain the intended function of the transport network (or transport system). The meaning of both serviceability and operability is approximately the same as “capacity” (Chen et al., 2002) or “performance” (Nicholson and Du, 1997), which all introduce the possibility of links and networks to be partially degraded (as in the case of congestion, accidents, road works etc.).

With these definitions of vulnerability, however, we still have the problems of describing the performance in terms that can be quantified, and defining what a “considerable reduction” (“non-operability”) is, not to mention how to assess the “susceptibility” of the road network. To begin with, it has to be made clear what the scope of the vulnerability investigation is: single links, a part of the network, interactions between the network and individual links, or the whole transport system including other modes as well, planning authorities etc.

The use of the word vulnerability in the context of the road transport network (or system) has sometimes been confused with the reliability of the same system. For example, Berdica (2002a) proposes that vulnerability in the road transport network can be seen as the complement of reliability. Some of the confusion with the concepts of vulnerability...
and reliability might be clarified if the perspective of the observer is introduced. For example, Immers et al. (2004) regard reliability rather as a user-oriented quality of the transportation system and not as a characteristic of the system itself. They define reliability as the “degree of certainty with which a traveller is able to estimate his own travel time”, which depends on the probability distribution and stability of travel times, on available information and on alternative travel options. Reliability is deficient when our expectations are not fulfilled on a regular basis. On the other hand, there are purely theoretical interpretations of reliability, like for example connectivity reliability and terminal reliability (the probability of an existing path between two nodes).

Reliability could thus be viewed from at least two angles. In the view of the individual traveller, it could be seen as a binary decision—either the system is operative or not. But in the aggregate view, regarding all travellers, there will be a portion of them finding the system operative who will use it, while others will not, i.e., people have different thresholds for reliability. Lastly, there is also the theoretical probability of connectivity (regardless of travel time).

A number of objective reliability measures, focusing on different aspects of road transportation have been proposed and used in recent years (see, e.g., Bell and Iida, 1997; Berdica, 2002a; Chen et al., 2002; Nicholson, 2003). We already mentioned connectivity and terminal reliability; other measures are travel time reliability and capacity reliability (Chen et al., 2002; Immers et al., 2004). However, as Berdica (2002a) notes, these reliability measures are too much concerned with probabilities, and she argues that the magnitude of the consequences should be more emphasised.

Vulnerability is related to the concept of risk. According to a generally accepted perception, risk includes two components: probability and consequence. The consequences usually concern life, health and environment. If these issues are assigned monetary values, consequences can also be expressed as costs. The risk associated with a harmful event \( X \), \( R(X) \), then is a combination of the probability of the event, \( \Pr(X) \), and the cost, \( C(X) \). A common way of operationalising the risk is to calculate the product of the two components, \( R(X) = \Pr(X) \cdot C(X) \) (Einarsson and Rausand, 1998; Holmgren, Å., 2004).

In view of the above, we argue that vulnerability can be treated in the same way as risk, and that the concept of vulnerability should be dissociated into one component of probability, and one component of consequence. Sarewitz et al. (2003) also point to the disadvantages of including the probability of failure in vulnerability studies. Estimating the probabilities of extreme events such as natural disasters and terrorist attacks is very difficult. The probabilities are predicted from historical data, which implies that the circumstances around the event remain the same at all times, and that all causal connections are known.

2.2 Exposure, criticality and importance

For very rare events where the estimation of the probability is not feasible, the term conditional vulnerability is often used (namely, given that a hazardous event occurs). We suggest the alternative term exposure for conditional vulnerability.
D’Este and Taylor (2003) are on the same track when they argue that “vulnerability” should only be concerned with consequence. They state that a node is vulnerable “if loss (or substantial degradation) of a small number of links significantly diminishes the accessibility of the node, as measured by a standard index of accessibility”. This definition is identical to our use of the term exposure, which we prefer for the sake of clarity.

The criticality of a certain component (link, node, groups of links and/or nodes) in the network involves both the probability of the component failing and the consequences of that failure for the system as a whole. The more critical the component, the more severe is the damage to the system when that component is lost. If the probability of an incident is high, the component (link etc.) is weak, and if the consequences are great, the component is important. If it is both weak and important, the component is critical (cf. Nicholson and Du, 1994).

Following the line of argument about conditional vulnerability and exposure above, it would be natural to use an expression like conditional criticality for the importance of a link or component. We get a division of the concepts as shown in Table 1.

<table>
<thead>
<tr>
<th>Composite concept, A*B</th>
<th>Concept A</th>
<th>Concept B</th>
</tr>
</thead>
<tbody>
<tr>
<td>risk</td>
<td>probability</td>
<td>consequence</td>
</tr>
<tr>
<td>vulnerability</td>
<td>probability</td>
<td>exposure, or conditional vulnerability</td>
</tr>
<tr>
<td>criticality</td>
<td>weakness (probability)</td>
<td>importance, or conditional criticality</td>
</tr>
</tbody>
</table>

In the formulations in the beginning of section 2.1, vulnerability is defined in terms of reduced “serviceability”, “operability” (or even “reliability”), and “accessibility”. These concepts all concern the function of the network, but reliability and accessibility approach the issue from the demand side, whereas serviceability/operability focus on the supply side. Although these are all complex concepts, we argue that a reasonable measure of the reduced serviceability/operability/accessibility is the increase in generalised cost of travel (time, distance, money, etc.) for the users of the network. The cost of travel (weighted by demand or not) is surely a strong indicator of the function and the possibility to use the network, and it is one of the standard indices of accessibility (for an individual) given in D’Este and Taylor (2003).

3. Model and measures

3.1 Measuring importance

Our approach is similar to the one outlined in Taylor and D’Este (2004). We assume that the hazardous event is a link or a group of links, collectively called an element, being completely disrupted or closed, which forces all travellers on those links to take other, less advantageous routes². The travellers are assumed to behave according to the

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² The failure of a node can be seen as being equivalent to the failure of the element consisting of all links connected to the node.
user equilibrium principle, i.e., they choose a route from their origin to their destination that minimises their travel cost. This means that the travel cost between each origin-destination (OD) pair is uniquely defined. At the most basic level we look at an origin node \( i \), a destination node \( j \) and an element \( e \). In the following, the origins/destinations are called demand nodes. We denote the cost of travel from demand node \( i \) to demand node \( j \) when element \( e \) has failed by \( c_{ij}^{(e)} \), and let \( c_{ij}^{(0)} \) represent the cost of the initial, undamaged network. The quantity
\[
\Delta c_{ij}^{(k)} = c_{ij}^{(e)} - c_{ij}^{(0)}
\]
is the basis from which measures of importance and exposure are derived by aggregating over the appropriate sets of links and nodes.

Beside the increase in travel cost, there is another, more severe, consequence that must be taken into account. When an element \( e \) is closed, the network may be divided into several disconnected parts. The travel costs between nodes in different parts then become infinite. Although serious, inability to travel is hardly infinitely worse than any finite increase in travel cost, and we would want some finite measure of the consequences of this event. For this purpose we introduce the concept of unsatisfied demand \( u_{ij}^{(e)} \), defined as
\[
\begin{cases}
  x_{ij} & \text{if } c_{ij}^{(e)} = \infty, \\
  0 & \text{if } c_{ij}^{(e)} < \infty,
\end{cases}
\]
where \( x_{ij} \) is the travel demand from demand node \( i \) to demand node \( j \). The measure represents the number of trips from \( i \) that are unable to reach \( j \) due to the closed element \( e \). At aggregated levels, we generally measure the unsatisfied demand relative to the total demand. How to value unsatisfied demand in relation to increased travel cost is still an open question, however, or a matter of political decisions.

For simplicity, let us focus on the case where a single link is closed. The closure will then result in either finite increases in travel cost (possibly zero) or positive unsatisfied demand. In the following, a link that causes finite increases in travel cost when closed is called a non-cut link, and the set of non-cut links is denoted \( E^{\text{nc}} \) while the set of all links is denoted \( E \), and the set of cut links is denoted \( E^c = E \setminus E^{\text{nc}} \) (the set of \( E \) outside \( E^{\text{nc}} \)). Measures based on increased travel cost must be limited to the non-cut links, whereas measures based on unsatisfied demand are well-defined for all links.

The importance of a link can be calculated with respect to a single demand node, a group of demand nodes (such as those in a municipality) or the whole network. Consider first a non-cut link \( k \). In the case of the whole network, we aggregate \( \Delta c_{ij}^{(k)} \) over all OD pairs. Each OD pair is assigned a weight \( w_{ij} \) that reflects its significance in relation to the other pairs. The importance of link \( k \) with regard to the whole network is then
Importance_{net} (k) = \frac{\sum_{j \in \mathbb{E}_d} w_{ij} (c_{ij}^{(k)} - c_{ij}^{(0)})}{\sum_{j \in \mathbb{E}_d} w_{ij}}, \quad k \in E^{nc}.

For \( k \in E^c \), \( \text{Importance}_{net}(k) = \infty \).

If the weights \( w_{ij} \) are all equal, all origins and destinations are equally important. This is the equal opportunities perspective that represents the role of the road system to provide access to all parts of the region (referred to as “global importance” in the Appendix). If the travel demand \( x_{ij} \) is used as weight, the severity of an increase in travel cost between two nodes depends on the traffic between them. This measures the capability of the road system to provide socially and economically efficient transports where the demand is the highest (“demand-weighted importance” in the Appendix).

Correspondingly, the importance pertaining to unsatisfied demand of any link \( k \) with regard to the whole network (relative to total demand) is

\[
\text{Importance}_{\text{net}}^{\text{uns}} (k) = \frac{\sum_{j \in \mathbb{E}_d} u_{ij}^{(k)}}{\sum_{j \in \mathbb{E}_d} x_{ij}}, \quad k \in E.
\]

For \( k \in E^{nc} \), \( \text{Importance}_{\text{net}}^{\text{uns}}(k) = 0 \).

### 3.2 Measuring exposure

Similarly, exposure to a certain event can be calculated for a single demand node, a group of demand nodes or the whole network. When measuring the increase in travel cost for a municipality \( m \), \( \Delta c_{ij}^{(k)} \) is aggregated over all origins \( i \) in the municipality and all destinations \( j \) in the entire network. One simple event is the failure of a single, randomly chosen link. We then measure the expected increase in travel cost,

\[
\text{Exposure}_{\text{rand}} (m) = \frac{\sum_{k \in E^c} \sum_{i \in V^d_m} \sum_{j \in \mathbb{E}_d} w_{ij} (c_{ij}^{(k)} - c_{ij}^{(0)})}{L^{nc} \sum_{i \in V^d_m} \sum_{j \in \mathbb{E}_d} w_{ij}},
\]

where \( L^{nc} \) is the number of non-cut links and \( V^d_m \) is the set of demand nodes located in municipality \( m \). This “average-case” scenario represents the simplest possible model of failures that have some probability distribution across the region, for example those caused by extreme weather. Another event is the failure of the most important link for the municipality. The exposure of municipality \( m \) is then the maximum value over all non-cut links,

\[
\text{Exposure}_{\text{max}} (m) = \max_{k \in E^{nc}} \frac{\sum_{i \in V^d_m} \sum_{j \in \mathbb{E}_d} w_{ij} (c_{ij}^{(k)} - c_{ij}^{(0)})}{\sum_{i \in V^d_m} \sum_{j \in \mathbb{E}_d} w_{ij}}.
\]
This “worst-case” scenario would correspond to an attack by an informed terrorist who wishes to cause as much damage as possible to municipality $m$ with only one strike. It is similar to the situation studied in Bell (2000) within a game theory setting. As before, the weights $w_{ij}$ can be chosen to reflect either the equal opportunities perspective (“global exposure” in the Appendix) or the social efficiency perspective (“demand-weighted exposure”). The exposure pertaining to unsatisfied demand of each municipality is the importance, expressed as unsatisfied demand from a node in the municipality to anywhere in the network, aggregated over all links using the average or the maximum aggregation rules. For the formulas, see the Appendix.

3.3 Inelastic demand

For computational reasons, the travel demand in our model is independent of the link costs (“supply”), i.e., we assume the demand to be inelastic. Thus, we assume that the duration of the closure is long enough for a new user equilibrium to form, but short enough to not affect the travel demand significantly. In terms of the traditional four-step transport modelling process (trip generation, trip distribution, mode choice and route choice), only the route choice is affected. Inelastic demand should be a reasonable assumption if the trip, as for the major part of our case, is a trip to work, which normally has to be pursued no matter what happens. With elastic demand, travellers would alter their travel decision, as well as their destination and/or mode choice, when facing increases in travel cost or other disturbances in the network.

For an accurate assessment of the economic consequences of link failures, it would be more satisfactory to use elastic demand in the calculations, like in Nicholson (2003). Berdica and Eliasson (2004) also suggest an accessibility-based reliability measure, where changes in accessibility (consumer surplus) is meant to be calculated by the “rule-of-a-half” (see, e.g., Neuburger, 1971, p. 56 ff.). This approach presupposes an elastic, negatively sloping demand function, approaching zero when travel cost increases. Our approach could be justified by the fact that we are looking for general indices for the comparison of links and locations, and for the prioritisation of actions and measures by the road authorities—not primarily for use in cost-benefit calculations at this stage. An inconvenience with the inelastic approach, though, is that we are forced to treat those links separately that completely disconnect the network in two parts.

4. Case study: the road network of northern Sweden

In the following case study, the travel time between the demand nodes is used as the generalised cost of travel, $c_{ij}$. A perhaps more realistic model would be to use a linear combination of travel time and distance. The drawback of this, however, is that a value of travel time (VoT) would have to be assumed, which normally differs from person to person and from time to time. In addition, a distance-dependent cost for the vehicle would have to be assigned. In principle, however, there is nothing in our measures that prevents including distance dependence in the link costs.

We assume that the travel times are independent of the load on the links. This approximation should be reasonable for the network we study, since the region is sparsely populated and most links have fairly low initial traffic loads: the median load is 450 ve-
vehicles per day, summed over both directions, with a mean of 1,804 and a maximum of 35,742 (this latter on a two-lane motorway). One out of six (1,070) links have a calculated load of zero. The travel time of each link is obtained by dividing the length by the free-flow speed from the volume-delay function. The assumption that travel times are independent of traffic load allows us to use a simpler and faster algorithm to find the fastest routes between origins and destinations. The presence of network congestion would necessitate an iterative network assignment procedure, since link costs change for each iteration, and therefore would require considerably more computing time.

4.1 Geographic context

Northern Sweden is generally more sparsely populated than the southern part, which historically has to do with the lack of arable land and the cold climate, and geologically with the Fennoscandian mountain chain along the border to Norway in the north-west. The region also borders on Finland in the north-east and the Botnian Sea and Gulf along the east coast. In the northernmost and western parts, much of the land is marsh and mountains, unusable for cultivation. These are traditionally the grounds of the native, reindeer-breeding Sami people, who also populates the tundras of Norway, Finland and Russia. However, the areas along the coast of the Botnian Sea, as well as the counties of Dalarna and Gävleborg in the south and Jämtland in the mid-west, have become more populated due to both the climate and the access to arable land. The population density of the municipalities in the region is shown in Figure 1.

From an economic point of view, the attraction of the northernmost parts of the region has historically mostly been its natural resources: valuable ores (iron, copper, gold and silver) have been found there, as well as fur and pine forests, and the great rivers have been used for the construction of dams for power generation and for transportation of timber to the sawmills on the coast. The scenic beauty in the mountains attracts tourists and outdoor activities. Nowadays the timber is transported by road and the ore by freight trains, but the importance of the waterways for communication is still visible in that many roads and railways follow the river valleys, and that the inland settlements are often located on the shores of rivers, large lakes and lake systems. There are two main roads going in the north-south direction: the E 4 European highway following the coast and the inland national road 45. Along the rivers, the European highways E 10, E 12 and E 14 and a few national roads are going in the north-west-to-south-east direction, starting in Norway and ending on the Botnian Sea coast.

The combination of heavy road transports and the climate, which in spring causes great

Figure 1: Population density of the municipalities of northern Sweden, as of September 30, 2004. Data from Statistics Sweden and National Atlas of Sweden.
difficulties while the ground below the road is thawing, makes the question of vulnerability relevant also from an economic production perspective. The thawing process starts from above, and converts the ground into something like a quagmire, threatening to break up the road surface. It is neither technically nor economically feasible to construct roads that withstand thaw completely. During a period in spring that can last several months, the bearing capacity of the road is severely reduced, and restrictions have to be put on the volume and time of day for road use. Of course, these restrictions hit the production possibilities in this area.

4.2 Network and travel data

The Swedish national travel demand model system SAMPERS (Beser and Algers, 2001) is used by authorities to forecast the effects of different transportation policies, economic development and infrastructure investments. The SAMPERS model system includes a network representation of the Swedish road system. For local and regional trips, Sweden is divided into approximately 8,500 zones where all trips begin and end, each zone comprising approximately 1,000 inhabitants. The centre of gravity of each zone is represented by a centroid node that is attached to the road network with two connector links, one in each direction.

The SAMPERS model is divided into five regional submodels. In this study we use the road network of the submodel Palt, which focuses on the six northernmost counties of Sweden: Dalarna, Gävleborg, Västernorrland, Jämtland, Västerbotten and Norrbotten. This area also represents the middle and northern planning regions of SRA. In this part the network is represented at its most detailed level, reflecting the situation in 2001, with 19,392 nodes (including 1,208 centroids) and 42,956 directed links (including 2,416 connectors). In the rest of the country the network is represented by the national road network, which is coarser with 7,597 nodes (including 191 centroids) and 17,796 directed links (including 382 connectors).

The network data contains the length, volume-delay function (the relation between traffic load and travel time), type (from European highway to tertiary county road), number of lanes, and county of each link. The data also contains the co-ordinates of each node. By importing the data into a GIS and combining it with other geospatial information, we have obtained the municipality and parish in which each link and node is located.

Figure 2: Traffic load on the links in the simplified road network, from a network assignment by SRA.

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3 Geographic Information System
The travel demand is measured in number of vehicles on an annual daily average. The vehicles include personal cars for private and company use, personal cars in commercial traffic, and heavy trucks with and without trailer. The users only include Swedish travellers within the confines of Sweden, not trips abroad or transit trips. We have also available the results from an assignment of traffic on the road network performed by SRA using SAMPERS. The model includes possible congestion effects, and generalised cost is based on a linear combination of travel time and distance. More precisely, the travel time is valued at 136 SEK (about 19 USD) per hour and the distance at 1.30 SEK (0.18 USD) per kilometre. Figure 2 shows the traffic load on each link in the network according to the assignment. The network here is a simplified version of the original network, which will be introduced in the next section.

4.3 Calculations

4.3.1 Simplifying the network

In order to reduce the time consumption of the travel time calculations, we first simplify the network as much as possible without changing its important characteristics, even during link closures. The first step is to make the network undirected. Analysis of the network data shows that for each link there is another link of the same length and travel time going in the opposite direction. It is therefore possible to substitute the two directed links by one undirected link, which is assigned the length, travel time and the sum of the traffic loads of the old links.

As the second step we remove all the centroids and connectors, since these are not part of the actual road network. Instead, we mark the nodes closest to the centroids (in travel time) within the detailed part of the network as demand nodes. Thus, for each centroid within this region (1,208 centroids) we mark its only neighbour as a demand node. For each centroid outside the region (191 centroids) we perform a shortest path search to find the closest node within the region. During the process, we keep a record of which demand node each centroid is mapped on. Since all demand has been moved inside the study region, we can now remove the coarse parts of the network.

There are two kinds of nodes in the network that can be removed without affecting the travel times and distances between the demand nodes. The first kind is the “dead-end” nodes with only one neighbour. No shortest paths between the demand nodes will pass through these nodes, unless they are demand nodes themselves. The second kind is the “joint” nodes with two links connected to them. Unless demand nodes, they can be replaced with a direct link between the two neighbours. The length and travel time of the new link is set to the sum of the lengths and travel times of the old links, respectively, while the traffic load is preserved. Using an iterative procedure, all nodes of these two kinds are removed.

The last step is to remove a few nodes that are not connected to the rest of the network. After this we arrive at the final network, with $N = 4,470$ nodes, out of which $N^d = 1,136$ are demand nodes, and $L = 6,362$ undirected links, out of which $L^c = 168$ are cut links.
4.3.2 Calculating demand and travel times

From the data we know the demand between every pair of centroids in the original network. For the simplified network, however, we need to know the demand between every pair of demand nodes. To obtain the latter we use the many-to-one map between centroids and demand nodes that was maintained during the simplification process. For every centroid \( h \), let \( d(h) \) be the demand node that \( h \) is mapped on. Then, for every pair of centroids \( h_i \) and \( h_j \), one of the following actions is chosen: If at least one of the centroids is located inside the study area, the demand from \( h_i \) to \( h_j \) is added to the demand from \( d(h_i) \) to \( d(h_j) \). If both centroids are located outside the region, or if \( d(h_i) = d(h_j) \), the demand is not added. The demand is stored in a \( N_d \times N_d \) matrix \( X = (x_{ij}) \). The original demand data is almost symmetrical with respect to start and end nodes, and we make the demand matrix completely symmetrical by adding its transpose and dividing by two, so that \( x_{ij} = x_{ji} \) for all \( i, j \).

The travel times between all demand nodes are calculated using a combination of Dijkstra’s shortest paths algorithm (Dijkstra, 1959) and a semi-dynamic updating algorithm (Buriol et al., 2004). The algorithm of Dijkstra (1959) is implemented with approximate buckets, which is highly efficient for this particular kind of network (Zhan and Noon, 1998). Starting from demand node \( i \), we perform a shortest paths search to all other nodes using the algorithm of Dijkstra (1959) with the travel times of the links as weights. For each link \( k \) in the network, we then take one of two possible actions: If \( k \) is not in the shortest paths tree rooted at \( i \), the travel times are unaffected by the removal of \( k \) and no recalculations are necessary. If \( k \) is in the shortest paths tree, we remove \( k \) by assigning an infinite travel time to it and update the shortest paths tree using the algorithm of Buriol et al. (2004). This reduces the computational burden significantly compared to finding the shortest paths from scratch after each link removal. The link is then replaced and the shortest paths are restored. Since the shortest paths tree contains \( N-1 \) links, the shortest paths will be updated \( N-1 \) times. This procedure is then repeated for each demand node, giving a total of \( N^d \) original shortest paths searches and \( N^d(N-1) \) updates. The computation of removing and replacing all 6,362 links and storing the results takes about 15–20 minutes with this algorithm on a Xeon 1.8 GHz processor. The programs are implemented in C++ and are described in Jenelius (2004).

4.4 Topological properties

In recent years, the properties of complex networks have become a topic for a great deal of research. One interesting question has been whether networks from different fields have any characteristics in common. In the literature, there are particularly three measures that have gained a lot of attention: the average geodesic distance, the clustering coefficient and the degree distribution (Albert and Barabási, 2002; Newman, 2003). A study of these measures for road networks is called for by Waters (2004).

The geodesic distance between two nodes is the minimum number of links that have to be traversed in order to get from one node to the other. The average geodesic distance between every pair of nodes, here denoted by \( l \), is a measure of how compact or scattered the network structure is.
The clustering coefficient $C$ is used to measure to what extent the nodes are forming small, tightly connected groups, which is common in, e.g., social networks. The clustering coefficient $C_i$ of a node $i$ is defined as

$$C_i = \begin{cases} \frac{L_i}{k_i (k_i - 1)/2} & \text{if } k_i > 1, \\ 0 & \text{else,} \end{cases}$$

where $k_i$ is the degree, i.e., the number of neighbours, of $i$ and $L_i$ is the number of links between the neighbours of $i$. The denominator is the maximum possible number of links between the neighbours of $i$. $C$ is the average of $C_i$ over all nodes. Figure 3 gives a simple example of how to calculate the clustering coefficient of a node.

The degree distribution $P(k)$ is the fraction of nodes with degree $k$ for $k = 0, 1, 2, \ldots$. The degree distributions of many studied networks have been found to follow power-laws, $P(k) \propto k^{-\gamma}$, for some positive parameter $\gamma$ usually ranging between 2 and 3. The functional form of $P(k)$ has been found to have a great impact on the robustness of the network structure against random node failures as well as hostile node attacks (Albert and Barabási, 2002).

In Table 2 are shown the clustering coefficient, the average geodesic distance and some other topological measures of the initial network in its undirected form, with and without the centroids, as well as the final simplified network. $N$ and $L$ denote the number of nodes and links, respectively. The clustering coefficient is shown relative to what it would be on average if the links were drawn completely at random, which is $C_{\text{rand}} = 2L/(N(N - 1))$. $\bar{k}$ denotes the average degree of the nodes, while $\bar{d}$ is the average travel distance (in kilometres) and $\bar{t}$ is the average travel time (in minutes) between every pair of nodes.

For comparison, Table 2 also includes some other infrastructure networks of similar size. The first is the high voltage power transmission network of the Nordic countries (Sweden, Norway, Finland and parts of Denmark), which has been studied in Holmgren, Å. (2004). The second is the high voltage power transmission network of the western United States (Watts and Strogatz, 1998). In both these networks, the links represent power lines and the nodes represent generators and substations. The third network is the physical structure of the Internet (Newman, 2003), where the nodes represent domains and the links are connections between the domains. The final section of the table contains a few road networks that have been used to test algorithms for shortest paths and...
traffic assignment. The first and third of these are used in Zhan and Noon (1998) and the second in Holmgren, J. (2004).

### Table 2: Topological measures of the initial and the simplified road network, as well as other undirected networks.

<table>
<thead>
<tr>
<th>Road network version</th>
<th>N</th>
<th>L</th>
<th>$\bar{k}$</th>
<th>$C/C_{\text{rand}}$</th>
<th>$l$ [km]</th>
<th>$\bar{l}$ [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial w centroids</td>
<td>26,989</td>
<td>30,376</td>
<td>2.25</td>
<td>149.9</td>
<td>206.8</td>
<td>516.7</td>
</tr>
<tr>
<td>Initial w/o centroids</td>
<td>25,590</td>
<td>28,977</td>
<td>2.26</td>
<td>152.5</td>
<td>206.7</td>
<td>518.6</td>
</tr>
<tr>
<td>Simplified</td>
<td>4,470</td>
<td>6,362</td>
<td>2.85</td>
<td>131.9</td>
<td>55.98</td>
<td>395.3</td>
</tr>
<tr>
<td>Infrastructure network</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nordic power grid(^a)</td>
<td>4,789</td>
<td>5,572</td>
<td>2.33</td>
<td>34.16</td>
<td>21.75</td>
<td></td>
</tr>
<tr>
<td>W. USA power grid(^b)</td>
<td>4,941</td>
<td>6,594</td>
<td>2.67</td>
<td>148.3</td>
<td>18.99</td>
<td></td>
</tr>
<tr>
<td>Internet(^c)</td>
<td>10,697</td>
<td>31,992</td>
<td>5.98</td>
<td>697.4</td>
<td>3.31</td>
<td></td>
</tr>
<tr>
<td>Other road network</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Georgia, USA (coarse)(^d)</td>
<td>2,878</td>
<td>4,214</td>
<td>2.92</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chicago Regional, USA(^e)</td>
<td>12,982</td>
<td>19,509</td>
<td>3.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Louisiana, USA (fine)(^d)</td>
<td>35,793</td>
<td>49,440</td>
<td>2.76</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


It can be seen that the average topological distance $l$ is far bigger in the road network than in the power grids and the Internet. This indicates that the constraint that geographical distance puts on which nodes are connected is the strongest for the road network. It is simply not economically profitable to build direct links between distant locations, and such links are more likely to be established by, e.g., flight or train. For the Internet on the other hand, the cost of distance is much lower and many nodes have direct connections between each other. Meanwhile, the clustering coefficient $C$ of the simplified road network is similar to that of the power grid of Western USA but significantly lower than for the Internet. Thus, given that we can reach two locations (nodes) directly from where we are, it is less likely in the road network than in the Internet that there exists a direct link between these two locations. It may also be noted that the average degree is similar in all road networks, around or slightly below three.

Table 3 shows the degree distribution of the road network before and after the simplification process in absolute numbers. Without the centroids, the maximum degree is six, which is a more restrictive bound than in most other networks due to the limited space available at road intersections. Note that in the simplified form, all nodes with degree one and two are demand nodes. For the two power grids the degree distributions decay exponentially, with maximum degrees of 14 and 19, respectively (Holmgren, A., 2004). The degree distribution of the Internet decays according to a power-law, which means that some nodes have very high degrees (Newman, 2003).

### Table 3: The degree distribution of the initial and the simplified road network. For each value $k = 0, 1, 2, \ldots$, the number of nodes with degree $k$ is shown.

<table>
<thead>
<tr>
<th>Road network version</th>
<th>$k = 0$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial w centroids</td>
<td>1</td>
<td>2,816</td>
<td>15,870</td>
<td>7,126</td>
<td>1,081</td>
<td>82</td>
<td>9</td>
<td>2</td>
<td>2</td>
<td>26,989</td>
</tr>
</tbody>
</table>
4.5 Results

We now present some of the results from the case study. The importance of each individual link with respect to the whole network is calculated using the three variants introduced in section 3.1: global, demand-weighted and unsatisfied demand-related importance. For each municipality, the global, demand-weighted and unsatisfied demand-related exposure is calculated with respect to the two scenarios described in section 3.2: First, the average-case scenario, in which we calculate the expected effects of closing a randomly selected link; second, the worst-case scenario, in which the most important link is closed. All formulas can be found in the Appendix.

4.5.1 Link importance for the whole network

The global importance of each link for the whole network is measured by the increase in travel time per OD pair when the link is closed (see Formula 1 in the Appendix). It can be seen from Figure 4 that the most important links are sections of the E 4 European highway that stretches along the entire coast. It is clear that this road constitutes an important backbone for fast access across the network. This is to be expected since the road allows travel at high speed and since many demand nodes are located close to the coast. We can also see that the most important link, located in the middle part of the E 4 in the rocky High Coast region, causes an average increase in travel time by more than ten minutes per OD pair when closed.

<table>
<thead>
<tr>
<th>Road network version</th>
<th>k = 0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial w/o centroids</td>
<td>1</td>
<td>1,484</td>
<td>16,627</td>
<td>6,730</td>
<td>716</td>
<td>30</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>25,590</td>
</tr>
<tr>
<td>Simplified</td>
<td>0</td>
<td>141</td>
<td>748</td>
<td>3,248</td>
<td>323</td>
<td>9</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4,470</td>
</tr>
</tbody>
</table>

**Figure 4**: Global link importance for the whole network (Formula 1).

**Figure 5**: Demand-weighted link importance for the whole network (Formula 2).
The demand-weighted importance of each link is the increase in travel time per trip when the link is closed (Formula 2). The most important links are generally short roads within cities and do not show well on the map in Figure 5. In particular, many of the links are parts of the E 4 going through Gävle and Umeå, two of the three largest municipalities in the area. These road sections therefore accommodate both regional and local traffic. The most important link, located in the city of Gävle to the south, causes an average increase in travel time per trip by more than one minute when closed. However, these results should be treated with some caution since the network data does not include all the small streets in the cities. The effects of closing a city street may therefore appear more severe than they would be in reality.

Similarly, the links that cause the largest relative amount of unsatisfied demand when closed (Formula 3) are generally short roads in or near cities. In particular, Figure 6 shows that many of the links are located east of the E 4, close to the coast where there is little room for alternative links. Others are located in the sparsely populated mountain areas and are often sections of roads that continue into Norway. Still others lie on the southern boundary and can be considered a boundary effect, since they would not divide the network if the study area were larger. When the most important link, again located in Gävle, is closed, nearly one in every forty vehicles is unable to reach its destination. Since this measure also is sensitive to inaccuracies in the network data, the results should not be taken too definitely.

4.5.2 Municipality exposure

Average-case scenario

Global municipality exposure is measured by the average increase in travel time per origin in the municipality and destination in the whole network. In the average-case scenario a randomly chosen non-cut link is closed (Formula 4). As shown in Figure 7, the most exposed municipalities are located in the northernmost part of the region. Here the road network and the population are particularly sparse. In any network, however, peripheral nodes will suffer the most when a random link is removed, since they rely on a smaller number of links in their proximity for their connection to the rest of the network. As a result of this the southernmost region is affected as well, even though the network is denser here. This is an example of a border effect that should be carefully considered, and because of such effects it might be necessary to include a larger network than the one under study.
The demand-weighted exposure of a municipality is the increase in travel time per trip beginning in the municipality and ending anywhere in the network (Formula 5). Since most demand is to nearby destinations, the effects of closing a link should be more local here than in the global perspective, and a few links should be responsible for a large part of the exposure of each municipality. Figure 8 shows that there appear to be some inland regions where the exposure of the municipalities is higher than elsewhere. One reason for this is that the local road networks in these areas are sparser, with fewer good alternatives, than elsewhere. The most exposed municipalities are also among the most sparsely populated. One thing to note is that in the northernmost municipalities the trips appear to be very little affected by a random link closure. This is not the full picture, however, as we will see when we turn to the measure of relative unsatisfied demand.

Figure 9 shows that the northernmost municipality of Kiruna is among the most exposed ones regarding the relative amount of unsatisfied demand (Formula 6). Indeed, several links in this region will cut off demand nodes from the rest of the network when closed. For the non-cut links, however, the finite increases in travel time per trip are small, as seen above. The conclusion is that unsatisfied demand should be the first measure to study, since being unable to reach your destination is a more serious scenario. From the figure we also notice that several of the most densely populated coastal municipalities are highly exposed, while the least exposed ones are located in the centre of the region, away from the boundaries. This could also be expected, if the topography does not place restrictions on road construction: in general, internal nodes have a higher connectivity (higher degree).
Worst-case scenario

Figure 10 shows that the municipalities in the northern parts of the area are globally exposed to the closure of the most important link (Formula 7). Somewhat surprisingly, however, the south-east corner, where both the population and the road network is relatively dense, is the most exposed part of the area. It is in fact a short section of the E 4 European highway with no nearby alternative links that is the most important one for all the municipalities in this region. This is another indication of the great importance of this road.

The qualitative situation for the demand-weighted municipality exposure (Formula 8) shown in Figure 11 is similar to that in the average-case scenario (Figure 8), which strengthens the notion that a few important links are responsible for most of the travel time increase for each municipality. The most important link is always located within the same municipality. Again, the view that the northernmost region and a few other municipalities appear to withstand the event well must be revised when unsatisfied demand is considered. The most exposed municipality is Arjeplog to the northwest, where travellers will experience an average increase in travel time by almost one and a half hour when a certain link is closed.

Finally, for the unsatisfied demand-related municipality exposure (Formula 9), the general picture shown in Figure 12 is also similar to the average-case scenario (Figure 9). This confirms that only one or a few links account for the majority of unsatisfied demand in each municipality. In the north-western municipalities of Dorotea and Sorsele, over 80 per cent of the trips will not reach their destinations if a certain link in the same municipality is closed.
5. Conclusions

We have defined the concepts of importance and exposure as criticality and vulnerability, respectively, but cleared from the component of “probability of adverse events” inherent in these concepts. Importance and exposure are therefore conditional concepts which tell us “what is the damage, given that something happens”. Further, we have defined a collection of operational measures of these concepts, based on the increase in generalised travel cost, weighted and averaged in different ways.

We have defined the importance of a link or group of links in a road network to be the increase in the generalised cost of travel when that link or group of links is closed. Link importance can been calculated with respect to single nodes, to groups of nodes (e.g., municipalities) and to the whole network by properly aggregating the increase in travel cost. The most important links from the point of view of the whole network has been presented.

We have used the unweighted increases in travel costs to render an “equal opportunities” perspective on importance and exposure, whereas by weighting them with travel demand, we have obtained measures from the perspective of “social efficiency”. For the case where the network is divided in two disconnected parts, we have defined and calculated the unsatisfied demand, i.e., the travel demand between the two parts that cannot come through.

Furthermore, the exposure to links failure has been defined for one node, a group of nodes or for the whole network. The exposure then is the increase in aggregate travel cost, given that the link has failed. Exposure has been calculated for the case of a random failure (average time increase over all removed links) and for the case of a di-
rected, informed attack (worst-case time increase, if the most important link was removed), and the results have been presented here for municipalities.

We have then applied these measures to the road network of northern Sweden, and found them to be useful additional planning tools. With all measures imported into a GIS with an appropriate network, the analyst can choose the level of detail at his or her own discretion. For example, it is possible to see which are the most important links for a certain municipality, or which individual nodes are most exposed to road disruptions.

The links that disrupt the network completely should be considered most carefully (the measure of relative unsatisfied demand). Questions that need attention are what real alternatives there are for those people to get to work, or in the case of an emergency situation, how rescue and repair vehicles can get through. In the northern and more sparsely populated municipalities, it is not uncommon with cross-country vehicles and snowmobiles for private trips, but what about heavier trucks?

6. Discussion

In order to obtain reasonable run times, we have chosen a comparatively sparse network with no indicated congestion. Because of the extent of the network, the city networks are also somewhat sparser than in reality, in the sense that smaller streets, which might be used for alternative routes in case of disruptions, are not represented. We have simplified this network further by excluding centroids, centroid connectors, and intermediate nodes, where only some characteristics of minor importance, like direction or speed, changes. Our simplifications should not have any negative effect on the accuracy, which is rather due to the resolution in the original network, the up-splitting of demand into centroids etc.

These are the simplifications on the supply side. They might overestimate our results in the cities—in reality one might not have to go so far for a bypassing route. On the demand side, on the other hand, the simplification of constant flow functions (volume-delay-functions) works in the opposite direction: if there is any congestion in the normal case, then all of the alternative routes would be all the more congested in the case of the disruption of a link. In any circumstance, the cities included in the study are comparatively small, and congestion is therefore not a matter of concern.

In the absence of congestion, the simplification of inelastic demand will only affect the demand-weighted measures and measures of unsatisfied demand, which will both be
overestimated—by how much is a matter of the elasticities of trip frequency (which depend on the trip purpose), and the accessibility to other modes of transport (cross-elasticities of mode choice). The unweighted (“equality”) measures, which are only dealing with delay, remain unaffected.

An inelastic demand function approximates the situation of, e.g., a work trip with few alternative modes than car available (which is not unlikely in large parts of the study area). An alternative interpretation is that the drivers do not get the interruption information until they enter the link in question—i.e., there is no possibility to change the decision on making the trip. If the drivers had had that information before starting from home, there is a chance that the trip would have been cancelled.

It is obvious that network vulnerability or lack of reliability is an additional cost for the users: individuals, companies and the society as a whole. Therefore, vulnerability and reliability issues should be included in the assessment of road projects. Road administrations use cost-benefit-analysis for the assessment of different alternatives in road planning. Our measures could be included in such cost-benefit-analyses for comparisons between different alternatives at all levels of the planning process: in the investment stage—are complementary roads needed, rather than increasing the capacity of existing ones, and which roads are the most urgent to secure from the point of view of thaw weakening?—, in the maintenance stage—where to first repair pot-holes and traffic lights, clear ditches and culverts?—, as well as in the operational stage—in case of a snow storm, which roads are the first ones that need to be ploughed?

Partly for economic reasons, the focus now is on how the road network performs under normal circumstances. Hazardous events that cause road closures will however inevitably occur, and considering the interdependence between the road infrastructure and rescue and repair vehicles, the situation can stall. Identifying and augmenting critical links can help prevent this from happening.

7. Further research

Depending on interest, this research could be led in several different directions. We have analysed the situation that the closure of road links did not have any effect on the users’ decisions to travel. A more realistic scenario for more prolonged breakdowns is that travel demand will go down when the cost (in time etc.) rises. Therefore, and in order to be able to use our measures for accessibility and cost-benefit analysis, it would be an improvement to introduce some elastic demand function.

Furthermore, for the analysis of denser cities and metropolitan areas, congestion would need to be taken account of. Unfortunately, the methods for finding the shortest routes in a congested network have until recently been very time demanding for large networks, because of the interactions between the link costs, route choice and elastic demand. As before, the same calculations have to be repeated after every link is removed. A study with a congested network was presented in Berdica (2002b) for the city of Stockholm, where some of the most important links (bridges) were removed, using commercial software for calculating network equilibrium. With this kind of software,
because of its time consumption it is only feasible to study a limited number of scenarios.

There might be a need for more realistic modelling of the failure caused by the adverse event, than just removing one link at a time. Scenario building can be employed to find out the nature of the hazardous events that can damage the network. For example, events such as a storm or a nuclear discharge could affect a wide area of the network rather than a single link. Other events to be modelled in this way could include natural phenomena like thaw and floods, or human-induced activities like industrial discharges, bridge collapses, big car and lorry accidents, or malign damage caused by sabotage and terrorist attacks. The calculation of the increased travel cost in such scenarios will be very short, since there is only one alternative scenario instead of \( L \) (the number of links) like in the present case. On the other hand, it will be more important to include demand elasticity and network congestion, since the network will be under a higher pressure.

This paper has focused on the consequences of hazardous events in the road network, as captured in the measures of importance and exposure. A natural continuation could therefore be to look at the probability of these events to occur. Some probabilities might be possible to estimate, as those for traffic accidents or some weather- or climate-related events, but for the really severe scenarios and accidents, there is the problem shared by all risk analysis to estimate probabilities of very rare events.

Acknowledgements

We would like to thank the Swedish Research Council for Environment, Agricultural Sciences and Spatial Planning (Formas) for their financial support. We also wish to thank Lars Johansson at the Swedish Road Administration (SRA) for providing the road network, travel demand and traffic data. Finally, we are indebted to Åke Holmgren and an anonymous referee for their comments on the text. All remaining errors and unclear points are however due to the authors themselves.

A preliminary version of this article was presented at the 8th Nectar Conference, June 2–4 2005, Las Palmas, Gran Canaria, Spain.

References


Berdica, K. (2002a) An introduction to road vulnerability: what has been done, is done and should be done. Transport Policy, 9, 117–127.


**APPENDIX: FORMULAS**

**Definitions**

Let $V$ and $E$ denote the set of nodes and the set of links of the network, respectively, while $L = |E|$ is the number of links. Let $V^d \subseteq V$ be the set of demand nodes (the nodes where all trips begin and end), and let $N^d = |V^d|$ be the number of demand nodes. Let $M$ denote the set of municipalities, let $V^d_m \subseteq V^d$ be the set of demand nodes in municipality $m \in M$ and let $N^d_m = |V^d_m|$ be the number of nodes in $V^d_m$.

Furthermore, let $c_{ij}^{(k)}$ be the travel cost between demand nodes $i$ and $j$ when link $k$ is closed. $c_{ij}^{(0)}$ signifies the travel cost in the initial, undamaged network. Let $x_{ij}$ be the demand from $i$ to $j$. The unsatisfied demand $u_{ij}^{(k)}$ is then defined as

$$u_{ij}^{(k)} = \begin{cases} x_{ij} & \text{if } c_{ij}^{(k)} = \infty, \\ 0 & \text{if } c_{ij}^{(k)} < \infty, \end{cases} \; i, j \in V^d, k \in E.$$

Finally, let $E^\text{nc} \subseteq E$ be the set of non-cut links, i.e., links such that when a link in $E^\text{nc}$ is closed, the network remains fully connected. $L^\text{nc} = |E^\text{nc}|$ is then the number of links in $E^\text{nc}$.

**Link importance for the whole network**

- **Global importance**: Increase in travel cost per OD pair.

  $$\text{Importance}_{\text{glob}}^{\text{net}}(k) = \frac{1}{N^d(N^d-1)} \sum_i \sum_{j \neq i} (c_{ij}^{(k)} - c_{ij}^{(0)}), \; k \in E^\text{nc} \quad (1)$$

- **Demand-weighted importance**: Increase in travel cost per trip.

  $$\text{Importance}_{\text{dem}}^{\text{net}}(k) = \frac{\sum_i \sum_{j \neq i} x_{ij}(c_{ij}^{(k)} - c_{ij}^{(0)})}{\sum_i \sum_{j \neq i} x_{ij}}, \; k \in E^\text{nc} \quad (2)$$

- **Relative unsatisfied demand**.

  $$\text{Importance}_{\text{uns}}^{\text{net}}(k) = \frac{\sum_i \sum_{j \neq i} u_{ij}^{(k)}}{\sum_i \sum_{j \neq i} x_{ij}}, \; k \in E \quad (3)$$

**Municipality exposure**

- **Average-case scenario**: Expected effects of closing a randomly chosen link.
– Global exposure: Increase in travel cost per OD pair.
\[
\text{Exposure}^{\text{glob}}_{\text{rand}}(m) = \frac{1}{L \cdot N^d_m(N^d_m-1)} \sum_{k \in E^c} \sum_{j \in V^d_m \setminus j \neq i} \sum_{i \in V^d_m} (c^{(k)}_{ij} - c^{(0)}_{ij}), \quad m \in M
\] (4)

– Demand-weighted exposure: Increase in travel cost per trip.
\[
\text{Exposure}^{\text{dem}}_{\text{rand}}(m) = \sum_{k \in E^c} \sum_{j \in V^d_m \setminus j \neq i} x_{ij} \left( c^{(k)}_{ij} - c^{(0)}_{ij} \right) \frac{1}{L \cdot \sum_{i \in V^d_m \setminus j \neq i} x_{ij}}, \quad m \in M
\] (5)

– Relative unsatisfied demand.
\[
\text{Exposure}^{\text{uns}}_{\text{rand}}(m) = \sum_{k \in E^c} \sum_{j \in V^d_m \setminus j \neq i} u^{(k)}_{ij} \frac{1}{L \cdot \sum_{i \in V^d_m \setminus j \neq i} x_{ij}}, \quad m \in M
\] (6)

– Worst-case scenario: Effects of closing the most important link.

– Global exposure: Increase in travel cost per OD pair.
\[
\text{Exposure}^{\text{glob}}_{\text{max}}(m) = \max_{k \in E^c} \frac{1}{N^d_m(N^d_m-1)} \sum_{i \in V^d_m \setminus j \neq i} \sum_{j \in V^d_m} (c^{(k)}_{ij} - c^{(0)}_{ij}), \quad m \in M
\] (7)

– Demand-weighted exposure: Increase in travel cost per trip.
\[
\text{Exposure}^{\text{dem}}_{\text{max}}(m) = \max_{k \in E^c} \sum_{j \in V^d_m \setminus j \neq i} x_{ij} \left( c^{(k)}_{ij} - c^{(0)}_{ij} \right) \frac{1}{\sum_{i \in V^d_m \setminus j \neq i} x_{ij}}, \quad m \in M
\] (8)

– Relative unsatisfied demand.
\[
\text{Exposure}^{\text{uns}}_{\text{max}}(m) = \max_{k \in E^c} \sum_{i \in V^d_m \setminus j \neq i} u^{(k)}_{ij} \frac{1}{\sum_{j \in V^d_m \setminus j \neq i} x_{ij}}, \quad m \in M
\] (9)