# Road Network Vulnerability Analysis: Conceptualization, Implementation and Application

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**Abstract**

The paper describes a process for road network vulnerability analysis, from (i) the conceptual definition of vulnerability measures, through (ii) the derivation of practical indicators and models adapted to available data and their implementation in computational procedures, to (iii) the application of the methodology in case studies. In the first step, the vulnerability concept is defined and quantified formally, and distinct user and technological perspectives are highlighted. In the second step, the conceptual measures are adapted and calculated according to the conditions, requirements and goals of a particular analysis. The paper describes practical indicators and algorithms developed for large-scale vulnerability analyses. For the third step, the paper analyzes both single link closures and area-covering disruptions and the distribution of impacts among different regions in a case study on the Swedish road transport system. The spatial patterns are put in connection with the regional variations in location and travel patterns and network density. Finally, the implications for policy and possible approaches to vulnerability management are discussed.

**Keywords**: vulnerability, robustness, road network, transport, disruption, GIS

## 1. Introduction

Modern society relies upon the collection of systems and institutions known as the infrastructure to support the welfare and living standard of people. A downside of this dependency is that sudden failures and disruptions in the systems may cause severe strains on the society. Road network disruptions can threaten the possibility for people to receive medical care and other critical services. More generally, they impair people's accessibility to daily activities such as commuting to work, doing the shopping, etc. Furthermore, there may be large costs associated with remedies and restoration of the transport system to a fully operational state. It is thus of interest to study the magnitude and distribution of impacts due to disruptions in different parts of the network, so that resources for prevention, mitigation and restoration can be suitably allocated.

Disruptions can be caused by a wide range of events, some of which originate within the transport system, including traffic accidents and technical failures. Other events are external strains imposed on the system, often caused by nature, as with floods, landslides, heavy snowfall, storms, wildfires, earthquakes etc. While accidents and technical failures may have limited extents, disruptions caused by nature may cover large areas in the road network.

Road network vulnerability analysis can be defined as the study of potential degradations of the road transport system and their impacts on society, modeling the road infrastructure as a network with links (road segments) and nodes (intersections). Research interest in the topic grew in the early 2000s as part of a broader focus on critical infrastructure protection. Several recent natural disasters and terrorist attacks raised awareness that society is vulnerable to disruptions in these infrastructure systems. It was recognized by some researchers that new quantitative methods for assessing the consequences of severe, albeit seemingly unlikely, disruptions of the road transport system were needed (Berdica, 2002; D'Este and Taylor, 2003).

The subsequent vulnerability research has embraced a rich exploration of perspectives, metrics and methods. A number of papers have proposed frameworks or metrics for evaluating road network vulnerability (Chen et al., 2007; Jenelius et al., 2006; Qiang and Nagurney, 2008; Sullivan et al., 2010; Taylor and Susilawati, 2012). Other studies focus on the modeling and computational aspects of the analysis (Erath et al., 2009; Knoop et al., 2008; Luathep et al., 2011). A third line of research develops mathematical modeling and optimization techniques to identify worst-case scenarios, or best responses to such scenarios (Bell et al., 2008; Matisziw and Murray, 2009). Finally, many papers put most emphasis on the vulnerability evaluation itself (Bono and Gutiérrez, 2011; Dalziell and Nicholson, 2001; Tatano and Tsuchiya, 2008).

As the literature becomes more diverse and specialized, there is a growing need also for synthesis of various proposed methodologies into integrated analysis frameworks. The aim of this paper is thus to describe a process for large-scale road network vulnerability analysis: from (i) the conceptual definition of vulnerability measures, through (ii) the derivation of practical indicators and models adapted to available data and their implementation in computational procedures, to (iii) the application of the methodology in case studies. The intention is that such a comprehensive description will help researchers to identify parts of the process where additions and improvements can be made, and to see how contributions in one area can be connected to work in other parts of the process.

The task in the first step of the process is to formally define and quantify the concept of vulnerability, and to highlight different perspectives from which vulnerability can be viewed. This paper proposes that road network vulnerability is the societal risk of road infrastructure disruptions. The impacts of disruption scenarios for individuals are evaluated in economic terms. Two perspectives of vulnerability are distinguished: the first perspective focuses on the users and considers how different user groups are affected under various disruption scenarios. The second perspective focuses on the road network and considers how disruptions of different network elements affect the users and society overall.

In the second step, the conceptual measures are adapted and calculated according to the data, computational requirements and desired output of a particular analysis. This paper describes the derivation of one such set of practical indicators developed for large-scale vulnerability analyses using data from a travel demand forecasting model. A GIS-based approach and algorithms for computing the vulnerability indices for very large networks are presented.

In the third step the implemented measures are applied to generate useful information about the specific study area, or to draw more general conclusions regarding the factors contributing to vulnerability. This paper expands upon a series of studies of the Swedish road transport system (Jenelius 2009, 2010; Jenelius and Mattsson, 2012). Both single link closures and area-covering disruptions are considered and the distribution of impacts among users in different regions is investigated. The spatial patterns that are found are explained in terms of the properties of the vulnerability metrics and models, and are put in connection with the regional variations in location and travel patterns and network density.

## 2. Conceptualization: Perspectives and formal vulnerability measures

Transport system vulnerability is here seen as society's risk of transport system disruptions and degradations. Road network vulnerability analysis, in particular, focuses on the road transport system and models the physical inftrastructure as a network of links (road segments) and nodes (intersections). The notion of risk is adopted from Kaplan and Garrick (1981), who propose that the results of a risk analysis can be represented as a list of “triplets”, each consisting of a description of a particular scenario, the probability of that scenario occurring, and the impact of the scenario. The risk is then the set of all triplets.This definition of vulnerability is more general than the simplified notion that risk is the *product* of probability and consequence. Thus, a vulnerability analysis may well put particular focus on rare, extreme events. Furthermore, the focus is on the users of the road network, that is, people, businesses and services, rather than the network itself.

Consider an individual, denoted $n$, and a road network disruption scenario, denoted $\sigma$, from a scenario space $\Omega$. Each dimension of $\Omega$ represents a relevant aspect of the disruption, such as the element involved (the set of road network links and nodes), the duration, the time of occurrence, the levels of capacity reductions, etc. With each scenario $\sigma$ is further associated a “null” scenario $\sigma_0(\sigma) \in \Omega$ that represents the baseline, normal level of operations during the time of the disruption had it not occurred, and against which the impact of the disruption is assessed.

Vulnerability analysis involves comparing and aggregating the various aspects of the disruption impacts for different users under different scenarios. The impacts must therefore be expressed in units such that interpersonal comparisons and aggregations are meaningful. For cost-benefit analyses of vulnerability-reducing investments, it is desirable to express the disruption impacts in economic terms. This allows prevention, repair and restoration costs to be added and compared to the disruption impacts, such as delayed goods deliveries and reduced accessibility to societal services.

With these aims it is reasonable to adopt a *micro-economic* approach and view users (i.e., individuals, businesses etc.) as economic agents interacting with each other and the infrastructure. The individual is thus seen as a consumer of goods, activities, services and travel. Network disruptions often lead to increased travel times for travellers. An increase in travel time means that an individual may lose income, may have to sacrifice time from other activities, and may get reduced accessibility to societal services. The micro-economic framework postulates that individuals make decisions in order to maximize their obtained utility, while businesses or firms seek to maximize their profits, under the prevailing circumstances. The *compensating variation*, or CV for short, represents the smallest amount that the individual should be willing to accept as compensation for the disruption (or in the case of an improvement, the largest amount that the user should be willing to pay for it) (Mas-Colell et al., 1995). The compensating variation is used here as a formal measure of the impact of a disruption for individuals. For individual $n$ and disruption scenario $\sigma$ this quantity is denoted $\Delta C_n(\sigma)$. The framework is illustrated in Figure 1.

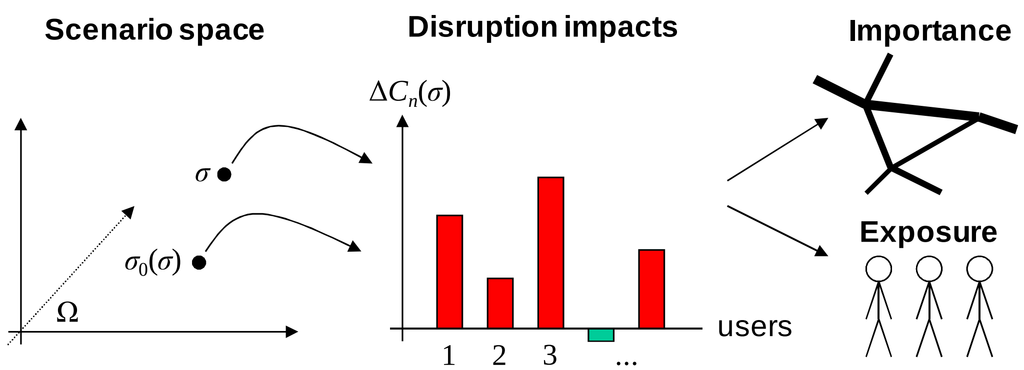


**Figure 1: Illustration of the framework for vulnerability analysis.**

### 2.1 Vulnerability and exposure

Vulnerability may be viewed from two different perspectives. The first perspective is to focus on the *societal* side of the system. For a particular individual one may ask: Under various conceivable disruption scenarios, how would the individual be affected, and what is the probability of each scenario occurring? One may also ask: What would be the impacts of the worst-case plausible scenario, and what are the long-run expected impacts of system disruptions?

Following Jenelius et al. (2006), the impact for a single user under a certain disruption scenario is referred to as the *exposure* of the user to that scenario (Taylor and Susilawati (2012) use the term "vulnerability" for essentially the same concept as exposure). Combining the exposure with the probability of the scenario gives the *vulnerability* of the user to that scenario. Exposure can thus be seen as conditional vulnerability from a societal perspective. The idea behind the exposure concept is to study and compare the situation for different individuals depending on the socioeconomic, demographic and geographic variables of interest, such as gender, age, income or residential location. This makes it possible to study the distribution of impacts among users, and identify groups of individuals that would be particularly severely affected by a certain scenario.

Within this formal framework the exposure of user $n$ to scenario $\sigma$ is simply

$$E(n \mid \sigma) = \Delta C_n(\sigma). \tag{1}$$

Rather than focusing on single individuals, one may more often be interested in the exposure of aggregate *groups* of individuals. The (mean) *user exposure* of a group $g = \{n_1, \dots, n_{N_g}\}$, where $N_g$ is the number of individuals in the group, to scenario $\sigma$ is then

$$UE(g \mid \sigma) = \frac{1}{N_g} \sum_{n \in g} \Delta C_n(\sigma). \tag{2}$$

It is often of interest to consider some aggregate measures of user exposure across a wide range of scenarios. One approach is to consider the worst case along one or several dimensions of the scenarios. The *worst-case exposure* thus captures the most severe impact for the user that a disruption of particular kind can have, regardless of the probability that this scenario will occur. Such an analysis can be useful for emergency preparedness and when scenario probabilities are highly uncertain, which is often the case.

To formalize the worst-case exposure of group $g$, the dimensions of the scenario space $\Omega$ are partitioned into two subspaces, denoted $\Omega_1$ and $\Omega_2$, such that a scenario $\sigma \in \Omega$ can be written as $\sigma = (\sigma_1, \sigma_2)$, where $\sigma_1 \in \Omega_1$ and $\sigma_2 \in \Omega_2$. Without loss of generality, consider the worst possible impacts along the dimensions in $\Omega_2$, while the dimensions in $\Omega_1$ are kept fixed at a certain point $\sigma_1$. Assuming that a maximum exists, the worst-case user exposure of $g$ with respect to $\Omega_2$ is then

$$UE^{\mathrm{wc}}(g \mid \sigma_1, \Omega_2) = \max_{\sigma_2 \in \Omega_2} UE(g \mid \sigma_1, \sigma_2), \tag{3}$$

As an example, if the worst-case impact of a single link closure in a road network is considered, $\Omega_2$ may represent the different links in the road network while $\Omega_1$ may represent different possible closure durations, times of occurrence etc., of which $\sigma_1$ is a particular case.

If each conceived disruption scenario is associated with a probability, another possible aggregation approach is to multiply probability and impact and consider the statistically expected vulnerability along one or several dimensions of the scenarios. Determining these probabilities, however, is an inherently difficult problem. A somewhat more manageable task, perhaps, is to assess the *relative* probabilities of different scenarios, so that probabilities can be normalized across all considered scenarios to add up to 1. This expected conditional vulnerability is referred to as *expected exposure* and makes it possible to study how vulnerability will tend to be distributed among individuals in the long run.

Formally, every considered disruption scenario $\sigma$ is associated with a probability normalized to 1 across all scenarios. Some dimensions of the scenario space $\Omega$ may be infinite (such as all possible closure durations), whereas others may be finite (such as all links in the network), which means that probabilities should be represented by a multivariate discrete-continuous distribution function $F_{\Omega}(x) = P(\sigma \leq x)$ where $\sigma \leq x$ is to be interpreted element-wise. Given a particular value $x_1$ for the dimensions $\Omega_1$ one can derive the conditional distribution function $F_{\Omega_2}(x_2 \mid x_1) = P(\sigma_2 \leq x_2 \mid \sigma_1 = x_1)$. The expected user exposure given $\sigma_1$ is then

$$UE^{\exp}(g \mid \sigma_1, \Omega_2) = \int_{\Omega_2} UE(g \mid \sigma_1, \sigma_2) dF_{\Omega_2}(\sigma_2 \mid \sigma_1). \quad (4)$$

## 2.2 Importance and criticality

The second perspective on vulnerability focuses on the *technological* side of the system. For a given component or group of components, here collectively called an *element*, one may ask: What is the probability that the element is disrupted (in a certain way, under certain conditions), and what would be the welfare impacts for society? Following Nicholson and Du (1994), the impact of a disruption of a given element is called the *importance* of the element. Many other terms have been used in different fields for the same concept, including “criticality” (Taylor and Susiwalati, 2012) and “vulnerability” (Knoop et al., 2008).

The main purpose behind the importance measure is to compare and rank different elements. This allows, for example, the identification of parts of the transport system where disruptions would be particularly severe. Disruptions of such elements represent worst-case scenarios and the elements can also be considered potential targets for antagonistic attacks on the system. Identifying important elements means that targeted measures can be taken to reduce the risk (i.e., the probability and/or consequences) of disruptions in those locations. Following Nicholson and Du (1994), the combination of importance and disruption probability is called the *criticality* of the element. Importance can thus be expressed as conditional criticality.

The concept of element importance entails that explicit or implicit assumptions are made about other scenario dimensions (time of occurrence, duration, degree of performance reduction, etc.), in a way that makes comparisons among elements meaningful. Thus, the importance of the element is calculated as the total impact of a certain disruption scenario involving the element, conditional on certain values for the other dimensions. Separating the element, denoted $e$, from the values for the other dimensions, jointly denoted $y$, the scenario can be written as $\sigma = (y, e)$. The importance of element $e$ can be defined with respect to a particular group of users $g$ as

$$I(e \mid y, g) = \sum_{n \in g} \Delta C_n(y, e). \quad (5)$$

In this paper only the overall importance of elements for all users is considered, and the group index is omitted.

# 3. Implementation: Derivation and computation of practical measures

## 3.1 From formal to practical measures

Depending on available data and the focus of the subsequent analysis, many different forms of practical vulnerability measures can be derived from the same formal framework. In this section one such set of practical indicators and algorithms is considered, adapted for large-scale vulnerability analyses of the Swedish road network (Jenelius 2009, 2010; Jenelius and Mattsson, 2012). In the data to which the vulnerability measures are adapted, the road network is modelled as a directed graph. Each network link $k$ has a fixed length and a travel time $t_k$. To the network is connected a set $N_{od}$ of special origin/destination (OD) nodes. All OD nodes have associated coordinates that allow them to be partitioned into geographical regions. The average number of trips of a certain type (e.g., work trips) being made during a certain time period (such as the annual average daily travel demand) between each OD node pair is available in demand matrices.

It should be noted that the data concern *trips*, while the formal measures concern *individuals*. Moving from users to trips could influence the analysis if a single user makes multiple trips and the impacts of a disruption are not additive across trips, as argued by Jenelius et al. (2011). It could also affect exposure comparisons between groups if the number of trips made per user varies between groups. Here, however, the term user is used even though the units of analysis will be trips.

Furthermore, the impact model is adapted to the level of detail in the analysis that the available data allows, which is relatively coarse. It is assumed that disruption scenarios consist of *complete closures* of one or several links for a certain duration $\tau$, which is typically assumed to be a few days at most. During this time the travel demand is inelastic to the disruption, so that all trips between each OD pair that would be made normally will also be made between the same OD pair given the disruption, although possibly postponed until the normal situation is restored. The travel demand per unit time between each OD pair $(i, j)$, denoted $x_{ij}$, is constant during the disruption.

Finally, the compensating variation for trip related to the disruption is assumed to be proportional to the increase in travel time or duration of postponement of the trip, i.e., the *delay* of the trip. People choose routes and departure times in order to minimize travel time. Moreover, the proportionality constant, i.e., the value of time, is assumed to be the same for all trips and individuals, so that a delay of a certain length is considered equally severe regardless of who is affected. The value of time is then a common proportionality constant for all trips and can be omitted in relative analyses.

Thus, a disruption scenario can be described with only two parameters: the element (the link or group of links) being closed, $e$, and the closure duration, $\tau$. A trip is also characterized by two factors: the OD pair $(i, j)$ and the departure time relative to the start and end of the disruption.

The total delay for all trips between $i$ and $j$ during the disruption given scenario $\sigma = (\tau, e)$ compared to the null scenario with all links fully operational is denoted $\Delta T_{ij}^{e}(\tau)$. In order to analyze spatial variations in exposure, trips can be grouped based on the regions where they start. Let $R$ denote the set of regions, let $r(i)$ be the region where OD node $i$ is located, and let $i \in r$ mean that OD node $i$ is located within region $r$. The total travel demand between $i$ and $j$ during the duration of the disruption is $x_{ij}\tau$, and the user exposure of the region to scenario $(\tau, e)$ is (compare with (2))

$$UE(r \mid \tau, e) = \frac{\sum_{i\in r}\sum_{j\in N_{od}} \Delta T_{ij}^{e}(\tau)}{\sum_{i\in r}\sum_{j\in N_{od}} x_{ij}\tau}. \tag{6}$$

The *worst-case* user exposure for a given closure duration $\tau$ is found by taking the maximum of $UE(r|\tau, e)$ across the set of considered elements $\mathcal{E}$, which corresponds to the general set $\Omega_2$ in the formal framework. Similarly, the *expected* user exposure is found by associating each element with a normalized closure probability $p(e|\tau)$ and calculating the expected impact across all considered elements as a weighted sum, corresponding to the general integrals in the formal framework. Thus, the worst-case and expected user exposure of region $r$ are (compare with (3) and (4))

$$UE^{\mathrm{wc}}(r|\tau, \mathcal{E}) = \max_{e\in\mathcal{E}} UE(r|e, \tau), \tag{7}$$

$$UE^{\mathrm{exp}}(r|\tau, \mathcal{E}) = \sum_{e\in\mathcal{E}} p(e|\tau) UE(r|e, \tau). \tag{8}$$

The overall importance of element $e$ is obtained by summing $\Delta T_{ij}^{e}(\tau)$ across all OD pairs (compare with (5)),

$$I(e|\tau) = \sum_{i \in N_{od}} \sum_{j \in N_{od}} \Delta T_{ij}^{e}(\tau). \tag{9}$$

**3.2 Representation of disruption scenarios**

Within the presented framework, different kinds of network disruption scenarios defined by closing one or more links and/or nodes are conceivable. Here, two kinds of scenarios are studied: single-link disruptions and area-covering network disruptions. The latter is a way of representing the form of spatial correlation among adjacent links/nodes that is typical for many external strains caused by nature. In principle, it would be straightforward to study any combination of disrupted links/nodes. From a computational point of view this is not computationally feasible for a large-scale network. Already considering all combinations of two disrupted links is beyond feasibility in the large-scale Swedish case study; there are more than 6 millions such different combinations (with its 34,704 links that may be disrupted, see Section 4.1).

For single-link disruptions, simultaneous closures of both directions of two-way road segments are considered. Since a directed graph model of the road network is used, a two-way road segment is represented by two links in opposite directions. An element $e$ thus consists of a pair of directed links for two-way streets, and a single link for one-way streets. In a full-range analysis, the set $\mathcal{E}$ consists of all such elements $e$, with some exceptions. Parts of the network outside of the study area may have been kept to provide alternative routes and reduce border effects, but should not be closed in the analysis. Other links may not represent physical road segments and should also be excluded, such as connectors to/from OD nodes, ferry lines, etc.

For area-covering disruptions, the methodology presented by Jenelius and Mattsson (2012) is considered here. In this approach, a complete coverage of the study area is made using evenly displaced grids of uniformly shaped and sized *cells* (see Figure 2). Each cell represents the precise spatial location and extent of a disrupting event. To simulate the event, any road links (including OD connectors to prevent intra-zonal trips, but excluding ferry links and external links) intersecting the cell (fully or partially) are completely closed for the duration of the disruption, while all links outside the cell are completely unaffected. An element $e$ thus contains all links that intersect a particular cell, and the set $\mathcal{E}$ contains the elements corresponding to every grid cell that intersects the study area. The size of the grid cells is chosen to capture the characteristics of the kind of events that are relevant for the analysis.

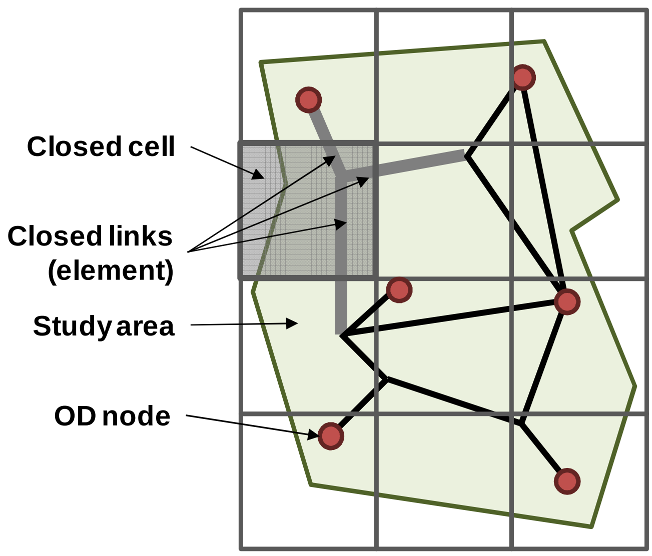

**Figure 2: Illustration of the grid-based approach to full-range area-covering disruption analysis. The geographical study area is represented by the irregular polygon.**

### 3.3 Computation of vulnerability measures

The spatial nature of transport systems and disruption hazards makes GIS a natural tool for performing vulnerability analyses. If GIS layers of the regions and the OD nodes are available, GIS methods can be used to identify the region in which each OD node is located. This information is used to calculate measures of regional exposure. For the grid-based approach to analyzing area-covering disruptions, GIS methods can be further used to create the grids, each represented as a layer of cells. All cells not covering the study area are easily identified and trimmed away. With a layer containing the road network links, it is then straightforward to identify all links intersecting each cell in order to define the network element associated with the cell. Once the disruption impact have been performed, the GIS environment is also useful for visualizing the results of the calculations.

To calculate the disruption impacts the model described in Jenelius (2010) is considered here. Users know the shortest path and its travel time for both the null scenario and the disruption scenario, as well as the closure duration, denoted $\tau$, once the closure occurs. If there are available routes, a user thus chooses to travel along the new shortest route or to wait until the links in element $e$ are reopened if this means reaching the destination faster. The difference in travel time between the new and the original shortest route is denoted $\Delta t_{ij}^{e}$. If there are no alternative routes, expressed as $\Delta t_{ij}^{e} = \infty$, the user waits until the closure is lifted. Assuming that travel demand is evenly or randomly distributed over time, the total delay $\Delta T_{ij}^{e}$ during the closure is (for a derivation, see Jenelius, 2010)

$$\Delta T_{ij}^{e}(\tau) = \begin{cases} x_{ij}\Delta t_{ij}^{e}\left(\tau - \frac{\Delta t_{ij}^{e}}{2}\right) & \text{if } \Delta t_{ij}^{e} < \tau, \\ \frac{x_{ij}\tau^2}{2} & \text{otherwise.} \end{cases} \tag{10}$$

The model is adapted to very large, mainly uncongested road networks, where computation time and memory consumption are important issues. In particular, it is assumed that the closure of a link does not affect the travel time on any other link. This approximation is considered reasonable for most of the Swedish road network used in the case study. The model also makes strong assumptions on the information available to the users and on their capability to react in an optimal way to this information. For comparisons between different disruption scenarios, the model is arguably good enough. Should a more sophisticated and computationally efficient delay model be developed, it would be straightforward to replace the present one.

The most straightforward way of computing the impacts $\Delta T_{ij}^{e}(\tau)$ for all OD pairs and scenarios is to calculate the travel times $t_{ij}^{0}$ between all OD pairs in the null scenario, close element $e$ and compute the disruption travel times $t_{ij}^{e}$ for all OD pairs. The delays are then found from the difference in travel times $\Delta t_{ij}^{e}$. The process is repeated for each element $e \in \mathcal{E}$.

Large-scale vulnerability analysis requires significant amounts of computations to calculate the impacts of all disruption scenarios for all users. The computational load can be decreased significantly, however, by making use of the property that OD travel times are independent in the absence of congestion. An alternative approach is thus to start from a specific OD node $i$ and calculate the shortest path tree to all other nodes, for example using Dijkstra's algorithm. Then only such elements $e \in \mathcal{E}$ where at least one link $k \in e$ is in the shortest path tree from $i$ need to be considered; elements with no link in the shortest path tree will not lead to delays for origin $i$ when closed. This reduces the number of shortest path calculations that must be performed.

If element $e$ has at least one link in the shortest path tree, the element is closed and the shortest path tree is updated. The update can be performed by running Dijkstra's algorithm from scratch. However, it is possible to increase computational performance further by using a *reoptimization* algorithm on the initial shortest path tree. When all elements $e \in \mathcal{E}$ have been considered, the process is repeated for each origin node $i \in N_{od}$.

In the implementation considered here, Dijkstra's algorithm is performed with approximate buckets (Cherkassky et al., 1993), which has been found to be particularly fast for road networks (Zhan and Noon, 1998). The reoptimization algorithm is adapted from Buriol et al. (2004) and uses the same approximate buckets structure. The reoptimization algorithm requires that the travel time on a single link is increased. An element closure is therefore implemented by closing the element links progressively and updating the shortest path tree after each additional link is closed, and a link is closed by setting the link travel time to a sufficiently large constant. The reoptimization method is therefore particularly efficient if the elements contain few links, such as in single-link closure analysis.

The procedure for calculating the worst-case and expected exposure measures Eqs. (7) and (8) and the importance measure (9) for a set of elements $\mathcal{E}$ is shown in Algorithm 1. For each origin node $i$, the algorithm calls a sub-procedure that performs the shortest path calculations and computes the delay (10), shown in Algorithm 2.

**Algorithm 1:** ImportanceExposure

*Input*: Closure duration $\tau$, elements set $\mathcal{E}$
*Output*: Importance $I(e|\tau)$ for all elements $e \in \mathcal{E}$, worst-case user exposure $UE^{\mathrm{wc}}(r|\tau,\mathcal{E})$ and expected user exposure $UE^{\mathrm{exp}}(r|\tau,\mathcal{E})$ for all regions $r \in R$

```
I(e|τ) ← 0 for all e ∈ ℰ
TE(r|τ,e) ← 0 for all r ∈ R
For each origin node i ∈ N_od
        (ΔT_i^e)_{e∈ℰ} ← SingleSourceCloseAllElements(i,τ,ℰ)

        For each element e ∈ ℰ
                I(e|τ) ← I(e|τ) + ΔT_i^e
                TE(r(i)|τ,e) ← TE(r(i)|τ,e) + ΔT_i^e
        End for
End for

For each region r ∈ R
        UE(r|τ,e) = TE(r|τ,e)/(∑_{i∈r} ∑_j x_ij τ)
        UE^exp(r|τ,ℰ) ← ∑_{e∈ℰ} p(e|τ)UE(r|τ,e)
        UE^wc(r|τ,ℰ) ← max_{e∈ℰ} UE(r|τ,e)
End for
```

**Algorithm 2:** SingleSourceCloseAllElements

*Input:* Origin $i \in N_{od}$, closure duration $\tau$, elements set $\mathcal{E}$
*Output:* Total origin disruption impact $\Delta T_i^e$ for every element $e \in \mathcal{E}$

*Calculate initial OD travel times and shortest path tree*
$\left[\left(t_{ij}^0\right)_{j \in N_{od}}, SPT_i^0\right] \leftarrow \text{InitialShortestPaths}(i)$

For each element $e \in \mathcal{E}$
    $\left(t_{ij}^e\right)_{j \in N_{od}} \leftarrow \left(t_{ij}^0\right)_{j \in N_{od}}$
    $SPT_i \leftarrow SPT_i^0$
    $\Delta T_i^e \leftarrow 0$
    For each link $k \in e$
        *Save initial link travel time, close link by adding sufficiently large penalty M, update shortest path tree*
        $t_k^0 \leftarrow t_k$
        $t_k \leftarrow t_k + M$
        $\left[\left(t_{ij}^e\right)_{j \in N_{od}}, SPT_i\right] \leftarrow \text{ReoptimizeShortestPaths}(i, k, M, SPT_i)$
    End For

    For each destination node $j \in N_{od}$
        *Calculate closure impact for destination*
        $\Delta t_{ij}^e = t_{ij}^e - t_{ij}^0$
        If $\Delta t_{ij}^e < \tau$
            $\Delta T_i^e \leftarrow \Delta T_i^e + x_{ij} \cdot \Delta t_{ij}^e \left(\tau - \frac{\Delta t_{ij}^e}{2}\right)$
        Else
            $\Delta T_i^e \leftarrow \Delta T_i^e + x_{ij} \cdot \frac{\tau^2}{2}$
        End If
    End For

    For each link $k \in e$
        *Restore initial link travel time*
        $t_k \leftarrow t_k^0$
    End For
End For

## 4. Application: Spatial vulnerability analysis of Sweden

For the third step of the analysis process, the methodology developed in Section 3 is here applied to the Swedish road network. In the grid-based analysis of area-covering disruptions, 12.5 × 12.5 km$^2$ square cells are used to represent the disrupting events. Four grids are used, symmetrically displaced in two longitudinal and two latitudinal steps, so that four different cells cover every point in the study area. Results are illustrated with a 12-hour closure duration for both single-link and area-covering closures. For some variations of the scenario assumptions (cell size and closure duration), see Jenelius (2009, 2010) and Jenelius and Mattsson (2012).

To calculate expected regional user exposure, each disruption scenario must be associated with a normalized probability of occurrence. Here the approach sometimes known as Laplace's Principle of Indifference (e.g., Keynes, 1921) is used, which says that all scenarios should be regarded as equally probable if there is no evidence to the contrary. Although there is good reason to believe that disruption probabilities vary geographically, empirical basis for a more refined model is currently lacking. Thus, for single link closures it is assumed that every road segment of unit length has the same closure probability. Hence, the closure probability is proportional to the length of the link, which represents a first approximation of the relative probability that some external event disrupts each link. For area-covering disruptions, accordingly, it is assumed that each cell has the same closure probability.

### 4.1 Data

The network and travel demand data (including both car and truck trips) used for the analysis were obtained from the Swedish national travel demand model system Sampers (Beser and Algers, 2001). For more information about this source of data, see Jenelius and Mattsson (2012). The travel time of each link in the original undisrupted network is calculated in Sampers with user equilibrium traffic assignments, which means that initial congestion is considered in the study. After some pre-processing the road network model consists of 32,759 nodes, including 8764 OD nodes, and 86,940 directed links. For the single-link closure analysis there are 34,704 elements in $\mathcal{E}$ to consider, which is the total number of one- and two-way road segments excluding OD node connectors, ferry links and a few external links. For the area-covering disruption analysis there are 9510 elements in $\mathcal{E}$, which is the total number of cells in the four grids covering the study area.

Figure 3 displays some properties of the study area, Sweden, related to location and travel patterns. The left map shows the flow on each link in the road network based on all-or-nothing (shortest path) assignment in the undisrupted network. The map highlights that much of the traffic is concentrated to a few urban areas in the south (Stockholm, Gothenburg and Malmö/Helsingborg) and the highway corridors connecting them. The right map shows the locations of the OD nodes and the level of travel demand generated from each origin according to the OD matrix. It can be seen that travel demand tends to be concentrated to the east coast in the northern parts of the study area, while it is fairly evenly distributed in the southern parts.

### 4.2 Regional worst-case user exposure

For single-link elements, the worst-case regional user exposure (Eq. (7)) represents the largest possible impact of a single link closure of a particular duration on the users starting within the region. It can be seen that the worst-case exposure will be high if a large share of the regional trips normally use a link with particularly poor alternatives. It follows from the impact model (10) that the longer the closure duration, the more likely it is that the most important link for the region is a cut link, i.e., a link without alternatives. In the case of Sweden, the presence or absence of cut links in a region has little connection with the general density of the regional road network (Jenelius, 2009). Furthermore, adding a single new link that provides redundancy to a cut link could drastically improve the worst-case exposure of a region. This discussion also implies that the metric is quite sensitive to the details of the network model.

For cell elements, the impact of a closure is largely determined by the concentration of travel demand within the cell itself. As a consequence of this, the worst-case user exposure will be high if a large share of the total regional travel demand is concentrated to the area covered by the disruption, whereas the network density is of little influence (Jenelius and Mattsson, 2012). Thus, regions that have a central hub where a large share of the trips originate and terminate are particularly exposed to this kind of scenario. At the opposite end are regions with highly dispersed location and travel patterns.

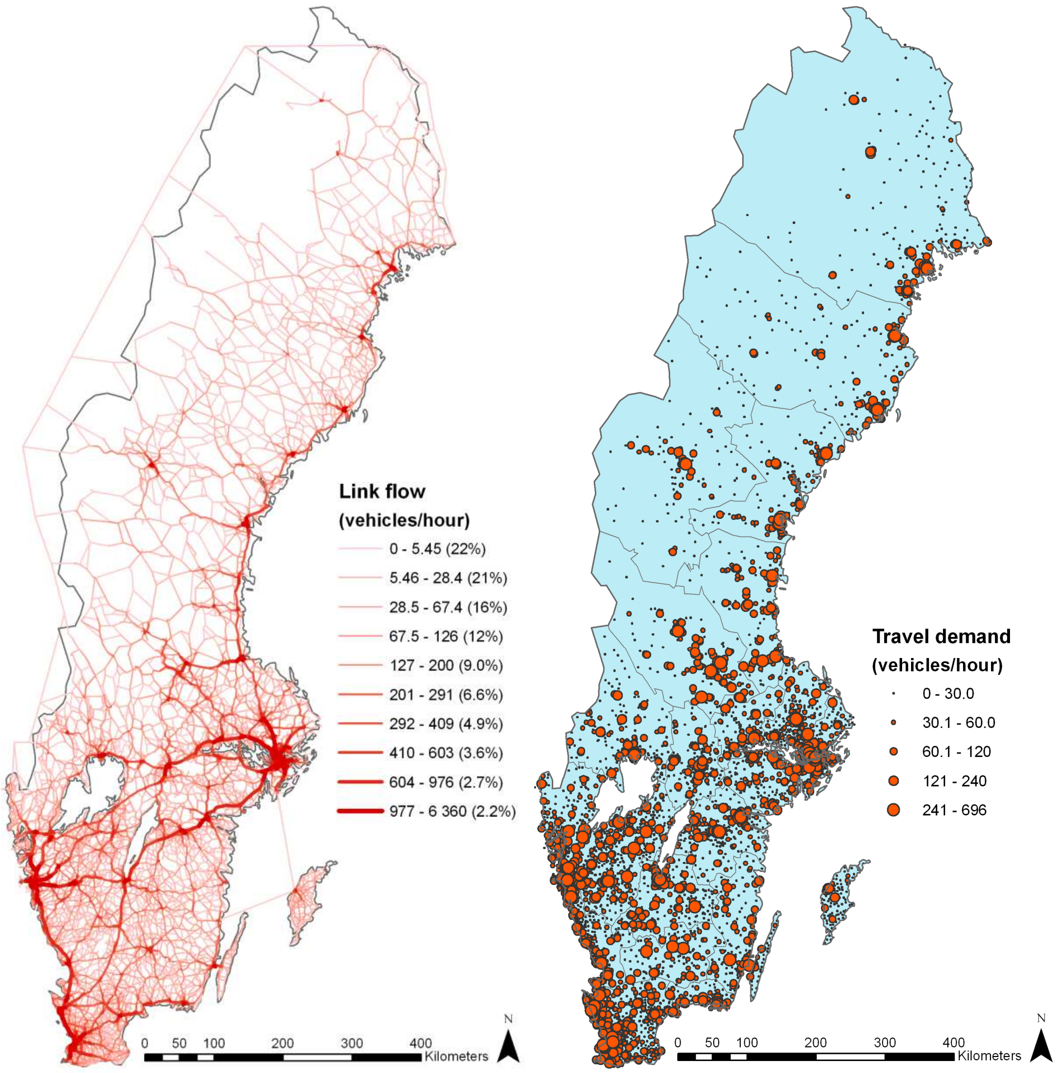


**Figure 3: Characteristics of the study area, Sweden. Left: Annual average daily link flow (vehicles/hour). Right: Annual average daily outbound travel demand of origin/destination nodes (vehicles/hour).**

Figure 4 shows the worst-case user exposure of every county in Sweden with respect to single link closures to the left and 12.5 km cell closures to the right. It can be noted that the two maps do not show great similarity. This is not unexpected since the worst-case exposure to single link closures is highly dependent on the more or less random locations of cut links. For cell closures the spatial pattern reflects the extent to which the travel is concentrated to a single hub in each county.

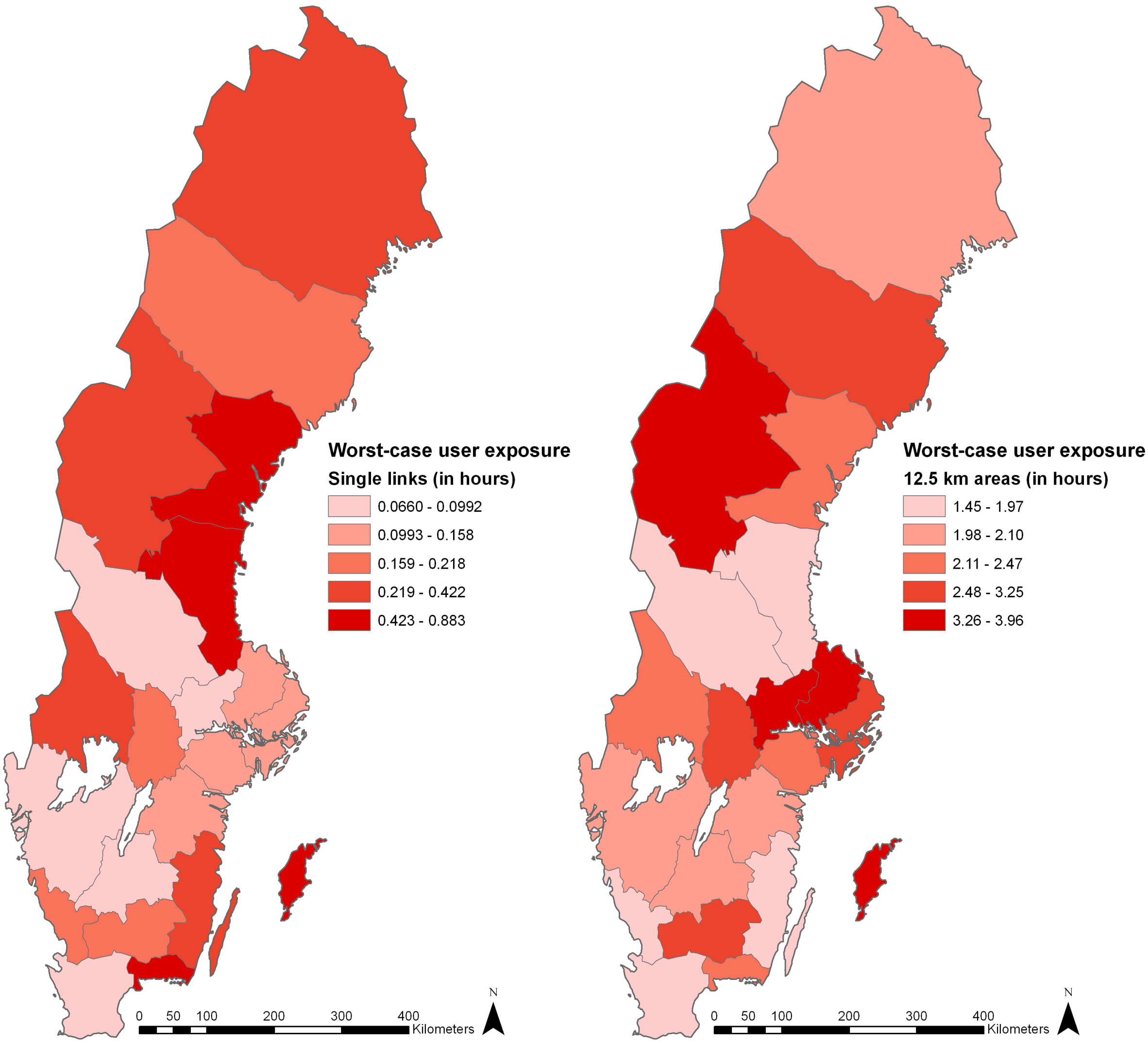


**Figure 4: Worst-case user exposure of Swedish counties, 12 hour closure duration. Left: Single link closures. Right: 12.5 × 12.5 km$^2$ cell closures.**

### 4.3 Regional expected user exposure

For single-link elements, the regional expected user exposure (Eq. (8)) is large if the trips are long on average, so that the users have a large chance of using the road segment that is closed, and if the regional network density is low, so that the alternative routes are considerably worse on average (Jenelius, 2009). For long closure durations, regions where a large share of the trips normally use cut links are particularly exposed. Thus, expected exposure is influenced by travel patterns as well as the development of the road network.

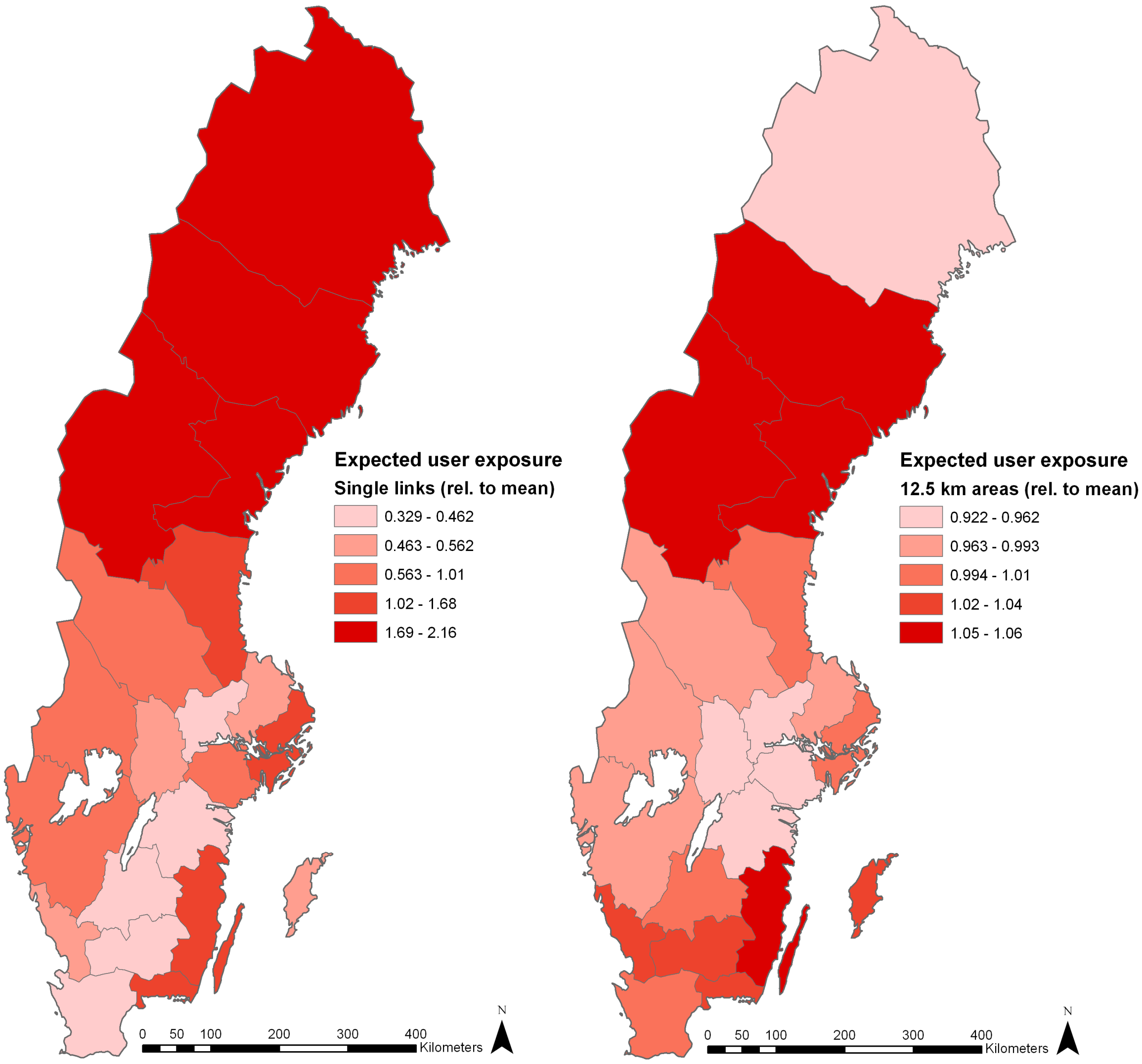


**Figure 5: Expected user exposure of Swedish counties, 12 hour closure duration. Left: Single link closures. Right: 12.5 × 12.5 km$^2$ cell closures.**

For cell elements, the determinants behind the expected user exposure are not as easy to characterize. For example, the concentration or dispersion of the population within the region, although critical for the worst-case scenario, should have only limited effect for the expected exposure. This is because a trip cannot be made if either its origin or its destination is located within the disrupted cell, and the mean impact is not dependent on whether each of a few cells disrupts a large share of the trips or whether each of many cells disrupts a small share of the trips. However, the factors that underlie the expected exposure to single link closures, trip length and network density, should also be influential under cell closures, in particular when the cells are small. Long trips run a larger risk of being affected by area-covering closures, which increases the expected user exposure of the region. Furthermore, poor redundancy in the network means that through trips will have worse or no alternative routes to take when a cell is closed. The longer the closure duration, the larger influence cells with no redundancy around them ("cut cells") should have.

Figure 5 presents the expected user exposure of the Swedish counties with respect to single link closures to the left and 12.5 km cell closures to the right. As expected, some correlation can be discerned between the two maps, suggesting that similar factors underlie both vulnerability metrics. There are also noticeable differences, however, for example that the northernmost county (Norrbottens län) is highly exposed to single link closures while relatively unexposed to area-covering closures compared to the other counties. This difference may be an effect of the sparse regional road network, which means that area-covering disruptions only have moderately worse impacts than single link closures, whereas the differences are much larger in other areas.

### 4.4 Link and cell importance

A single link is important if it is used by many, i.e., if the flow on the link is high, and if the alternatives for the affected users are poor on average (Jenelius, 2010). The quality of the alternatives, in turn, depends on the local redundancy in the network. As a result, we expect to find important links in densely populated areas, because of large flows on the links, as well as in sparse areas, because of poorly developed networks. The longer the closure duration, the more important are cut links considered relative to other links.

Disruption of a cell means that no trips can be made within, into or out of the area covered by the cell; hence, all such travel demand is unsatisfied (i.e., delayed until the links are reopened). In addition, some trips normally going through the cell may suffer delays or may not be possible to make during the closure. For small cells, representing very local disruptions, few links and OD nodes will be contained in each cell. Hence, cell importance will correspond closely to link importance in this case.

For large cells, on the other hand, the number of internal, inbound and outbound trips will dominate over through trips, and the importance of a cell will mainly be determined by the travel demand generated within the cell itself. In other words, the impacts will be largest where the most people are localized. Therefore, location patterns rather than network structure or travel patterns play the most significant role for the importance of large cells (Jenelius and Mattsson, 2012). As for single links, the longer the closure duration, the larger influence unsatisfied demand has relative to through trips that suffer delays.

Figure 6 shows the importance of every link in the Swedish road network model to the left and every $12.5 \times 12.5$ km$^2$ cell in the four grids covering the study area to the right, assuming a 12-hour closure in both cases. The mean importance of the four cells covering each point in the study area is shown. The map thus has a resolution of half the cell size, i.e., $6.25 \times 6.25$ km$^2$ squares. The left map shows that many important links can be found around the two main urban areas Stockholm and Gothenburg on the east and west coasts, respectively. Comparison with Figure 3 shows that these links are mainly important because of the large number of travellers using them (since congestion effects are not considered in the calculations, these links are likely even more important in reality). There is also a significant number of important links in the sparse northern regions. These are important mainly because of the poor local redundancy; in some cases there are no alternative routes at all.

The right map showing cell importance bears some similarity to the left map in that some influence of the network structure can be seen, particularly in the north; this is an effect of the relatively small cell size. However, there is an even clearer influence from the concentration of travel demand (Figure 3). This confirms the general observation that the impacts of area-covering disruptions are most severe in regions with highly concentrated travel demand. Hence, the southernmost county (Skåne län), where both the population and the road network are dense, is typically affected much worse by area-covering closures than single link closures in terms of overall impact. This is in compliance with anecdotic observations that e.g. snow storms regularly lead to severe disruptions in this county despite its very dense road network.

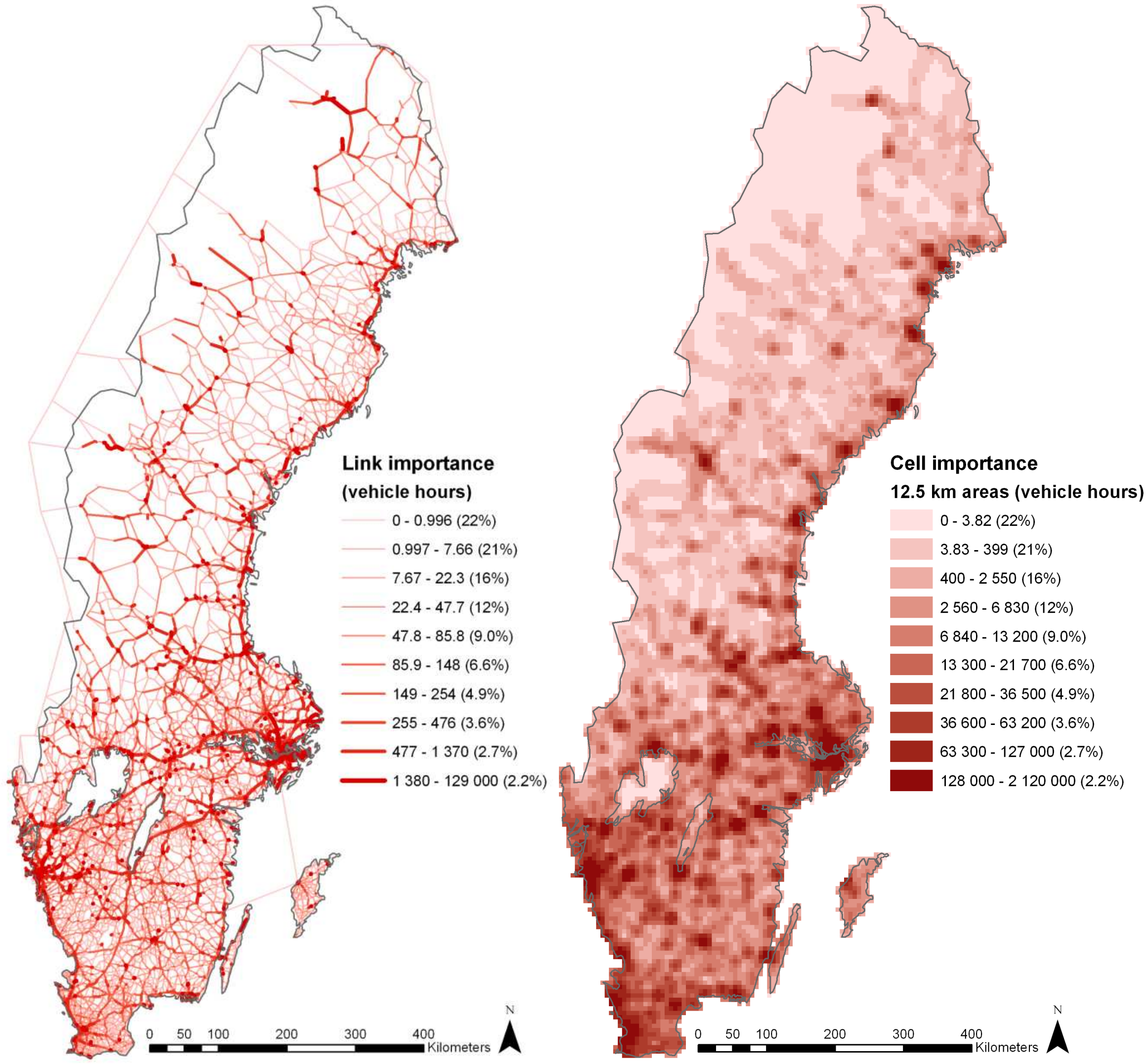


**Figure 6: Element importance, 12 hour closure duration. Left: Single links. Right: 12.5 × 12.5 km$^2$ cells (Jenelius and Mattsson, 2012). The percentages indicate the share of elements in each category.**

## 5. Conclusion and discussion

The paper described a process for road network vulnerability analysis, from the conceptual definition of vulnerability measures, through the derivation of practical indicators and models adapted to available data and their implementation in computational procedures and algorithms, to the application of the methodology in case studies. The paper proposed that road network vulnerability is the societal risk of road infrastructure disruptions, and that the impacts of disruption scenarios for individuals be evaluated in economic terms. From this standpoint, two perspectives of vulnerability were distinguished, focusing on (i) the users, and (ii) the road network. The paper described the derivation of practical indicators, a GIS-based approach and algorithms developed for large-scale vulnerability analyses. Finally, the paper presented a case study of the Swedish road network considering both single-link closures and area-covering disruptions.

A vulnerability analysis process such as outlined here provides the background and starting point for an evaluation of various measures to reduce vulnerability, if needed. Still not enough is known about how to best manage the vulnerability with emergency preparedness, infrastructural reinforcements and expansions, operations and maintenance procedures, etc. This suggests that there is a need for more *normative* approaches in model-based vulnerability analysis. That is, given the society's current state of vulnerability to disruptions in the road transport system, what actions should be taken? In the planning stage, a vulnerability analysis can for example guide the alignment and standard of a new road, or support the building of a new road that among other benefits provides some redundancy to the existing roads. To promote equity and regional development, it may be desirable for authorities to direct investments and actions aimed at reducing exposure so as to particularly benefit certain disadvantaged groups.

Some general observations regarding the impact of new infrastructure projects may be made from the study presented in this paper. Given certain spatial extents, durations and relative probabilities, the study found significant influences on regional variations in vulnerability from travel patterns, location patterns and the development of the road network. In practice, road investments will typically have little influence on these fundamental properties of the transport system and the population distribution within a reasonable planning horizon. Therefore we believe that resource allocation for reducing vulnerability is more an issue of prevention and preparedness for quick mitigation and restoration than redundancy-providing but expensive infrastructure investments.

During the operations stage, different actions can be taken to reduce the vulnerability depending on the type of identified hazard or threat. As examples of how to reduce the likelihood of incidents, traffic accidents may be avoided by straightening or widening the road or reducing the speed limit, technical failures may be avoided with more thorough inspection and maintenance, and natural hazards may be avoided by upgrading the road structure, such as switching to larger drain pipes to handle floods. To reduce the consequences of a disruption once it has occurred the main issue is to restore the performance of the network as rapidly as possible, for example by increasing the resources for stand-by maintenance preparedness, while information provision and management through Intelligent Transport Systems (ITS) and other channels is important for limiting the effects of the degradation while it lasts.

In order to allocate resources for reducing vulnerability efficiently, it is necessary to combine the impact calculations with estimates of the frequencies with which different kinds of disruptive events will occur in different parts of the study area. Given the lack of observations of rare events and the influence of local geographical features, this is a difficult task. Development of the methodology for probability assessments in vulnerability analysis should thus be an important area for further work.

## Acknowledgements

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