The traveler costs of unplanned transport network disruptions: An activity-based modeling approach

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Abstract

In this paper we introduce an activity-based modeling approach for evaluating the traveler costs of transport network disruptions. The model handles several important aspects of such events: increases in travel time may be very long in relation to the normal day-to-day fluctuations; the impact of delay may depend on the flexibility to reschedule activities; lack of information and uncertainty about travel conditions may lead to under- or over-adjustment of the daily schedule in response to the delay; delays on more than one trip may restrict the gain from rescheduling activities. We derive properties such as the value of time and schedule costs analytically. Numerical calculations show that the average cost per hour delay increases with the delay duration, so that every additional minute of delay comes with a higher cost. The cost varies depending on adjustment behavior (less adjustment, loosely speaking, giving higher cost) and scheduling flexibility (greater flexibility giving lower cost). The results indicate that existing evaluations of real network disruptions have underestimated the societal costs of the events.

KEYWORDS: transport network disruption, delay cost, schedule adjustment, activity-based model, information

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1 Introduction

Disruptions in the road transport system, caused by for example extreme weather, infrastructure failures or severe car crashes, can have significant societal consequences. A partial or complete loss of capacity on a road link or across a larger area may lead to travel time increases that spread to the surrounding network through congestion and queues. For individuals, sudden travel time increases can impair the ability to commute to work and take part in other daily activities such as dropping off and picking up children from daycare, shopping, or meeting friends. For businesses, negative impacts arise from delayed deliveries and supplies, loss of manpower and customers, increased freight costs, etc.

For many reasons it is desirable to express the impacts of network disruptions in monetary terms. Immediately after an event has occurred, the costs of different restoration schemes can then be compared with their estimated benefits to determine which scheme would be the most efficient. For example, contractors may be given a bonus for every day ahead of normal schedule functionality is restored, as was done after the Northridge earthquake in California in 1994 (Wesemann et al., 1996) and the collapse of the I-35W Mississippi River Bridge in Minneapolis in 2007 (Xie and Levinson, 2009). The size of the bonus should be proportional to economic losses avoided by the early restoration.

In the planning process, transport models can be used to identify critical links and areas where the impacts of disruptions would be particularly severe (Jenelius et al., 2006; Jenelius and Mattsson, 2008; Taylor et al., 2006; Matisziw and Murray, 2009). With impacts valued as economic losses, cost-benefit analysis can be applied to determine if, how much, and where resources should be allocated to reduce the likelihood or consequences of potential future disruptions.

Estimates of the costs due to travel time increases have been performed in connection with several major real-world disruptions, e.g., the 1994 Northridge earthquake (Wesemann et al., 1996), the 2007 bridge collapse in Minneapolis (Xie and Levinson, 2009) and the 2006 landslide in Småröd, Sweden (MSB, 2009).[1] In these studies, the approach was to calculate the delays caused by the disruption using a transportation model system, typically based on deterministic or stochastic user equilibrium traffic assignment.[2] The calculated delays were then multiplied with a standard value of time to obtain a monetary cost. Using this approach, the delay costs of freeway closures due to the 1994 Northridge earthquake were estimated to

[1] In the landslide in Småröd, Sweden in December 2006, 500 meters of the European highway E6 and 200 meters of a nearby railroad were carried away. The highway and railway were restored in February 2007 (MSB, 2009).

[2] Unless otherwise stated, the term *delay* refers throughout the paper to journey delay, i.e., travel time in excess of the normal travel time. Depending on how the traveler adjusts her schedule, journey delay may result in schedule delay, i.e., late or early arrival.

have exceeded 1.6 million USD per day (Wesemann et al., 1996), the delay costs of the 2007 Minneapolis bridge collapse were estimated to between 71 thousand USD and 220 thousand USD per day (Xie and Levinson, 2009) depending on assumptions about traveler response, and the delay costs of the 2006 Småröd landslide to about 5.5 million SEK (ca. 750 thousand USD) per day (MSB, 2009).

This approach is adopted also in many model-based studies of road network vulnerability (e.g., Jenelius et al., 2006; Jenelius, 2009; Taylor et al., 2006; Erath et al., 2009). Since relative comparisons between different disruption scenarios are independent of the value of time (assuming a single value for all users), some studies report the delays directly.

However, there are several reasons to believe that the cost per hour delay due to a significant unplanned transport network disruption is higher than the ordinary value of time. First, it is well known that delay time is valued higher by travelers than typical travel time (e.g., Wardman, 2001; Bates et al., 2001). Empirical estimates of linear schedule delay models, in which costs rise proportionally to the lateness (or earliness) of arrival in relation to some preferred arrival time, have suggested that schedule delay is valued approximately three times as high as ordinary travel time (Small, 1982; Tseng and Verhoef, 2008). In fact, there is evidence that the cost increases faster than linearly with delay as the time lost from all other possible activities accumulates (Börjesson and Eliasson, 2009).

Second, the unexpected nature of unplanned disruptions means that travelers are unable to adjust their schedules adequately beforehand to compensate for the travel time increases. Also, the disturbance of normal travel conditions and associated uncertainty during the first few days following an event should lead to high costs, as travelers learn and adapt to new routes, departure times, modes and more complex adjustments (Hunt et al., 2002; Cairns et al., 2002; Zhu et al., 2008; He et al., 2009).

Third, it is likely that a disruption induces travel time increases on more than one trip during the day, such as both the morning and the evening commute. If delay occurs only on the morning trip, a flexible work schedule makes it possible to compensate for late arrival to work by working longer in the evening. If delay affects both commute trips, however, this restricts the possibility to make up for late arrival by working later, which should amount to higher costs.

This paper aims to develop an approach to assessing the traveler costs of unplanned road network disruptions that incorporates the aspects mentioned above. Thus, we do not assume that the cost is necessarily linear in the delay; rather, we formulate a simple activity-based model of the daily travel decisions as the foundation for the analysis (Axhausen and Gärling, 1992; Bowman and Ben-Akiva, 2001; Ashiru et al., 2004; Ettema et al., 2007). Within this framework, delay costs arise explicitly from the time that is lost from activities that would be more beneficial than the extra time spent traveling.

Our setting differs from that which is considered in studies of the value of travel time *reliability* or, equivalently, the cost of travel time uncertainty (e.g., Noland and Small, 1995; Bates et al., 2001; Fosgerau and Karlström, 2010). While the latter literature considers the ex ante impacts of stochastic fluctuations in travel time in terms of early departures (safety margins) and late arrivals, our paper is concerned with the ex post impacts of sudden large increases in travel time.

The basic model formulation is adopted from Ettema and Timmermans (2003). In their paper the model was mainly used for numerical estimation, and the authors did not fully work out its analytical properties. In particular, the value of travel time savings within their model, did not take into account that saved time in the long run can be allocated to any activity during the day in a way that maximizes utility. Also, although they included flexibility of work hours as a parameter in the model estimation, they did not consider the effect of scheduling flexibility on the optimality conditions, the value of travel time and schedule costs theoretically. In this paper, we derive these properties analytically for any level of scheduling flexibility, and we demonstrate the relationship between this approach and linear schedule delay models.

With the model we study the impact of exogenous increases in travel time under different types of adjustment behavior, reflecting the different levels of information and certainty that travelers may have about the post-disruption travel conditions (these adjustment profiles are based on empirical evidence from the 2007 Minneapolis bridge collapse). With model parameters calibrated against the empirical results of Tseng and Verhoef (2008), we investigate how journey delay costs vary with delay duration. We also compare the impacts of a disruption affecting the morning and the evening commute symmetrically with disruptions affecting only one of the trips, to determine the value of being able to reschedule the remaining day following a long delay.

The paper is organized as follows. In Section 2 the model is formulated and optimality conditions for the daily schedule, as well as analytical expressions for the value of travel time and schedule delay costs, are derived. In Section 3 the characteristics of unplanned network disruptions are discussed, adjustment profiles are defined and journey delay costs under different adjustment profiles are identified. Section 4 describes the numerical utility specifications and parameter estimates and Section 5 presents the results from the analysis. The modeling approach, results and possible extensions are discussed and conclusions are drawn in Section 6.

2 Theoretical model

The modeling framework postulates that individuals spend the day taking part in activities and traveling between activities. A daily schedule is a sequence of activities and trips with a specified start time and duration for each activity and trip. The individuals have preferences among the set of feasible schedules which are expressed with a utility function U.

The utility derived from taking part in an activity is assumed to be independent of other activities but to depend in general on both the time of day and on the duration of the participation. The utility gained from spending another unit of time on activity i at time t is expressed in the form of a marginal utility function $u_i(t; t_{si})$, where t_{si} is the start time of the activity. Specifically and in accordance with Ettema and Timmermans (2003), we assume that the marginal utility depends on a linear combination of the time of day t and the duration $t - t_{si}$, i.e., $u_i(t - \xi_i t_{si})$, where $\xi_i \in [0, 1]$ is a parameter expressing the scheduling flexibility of the activity. Note that $\xi = 0$ means that marginal utility depends only on time of day, while $\xi = 1$ means that marginal utility depends only on time since arrival, i.e., activity duration. Flexibility here thus refers to the degree to which the utility of taking part in an activity is independent of the time of day and is not associated with any particular assumption about the shape of the marginal utility function. In general, time-of-day dependencies arise from, e.g., fixed activity start and end hours and benefits of coordination with others. Duration dependencies arise from, e.g., start-up and fatigue effects.

The marginal utility derived from traveling, denoted ν, is assumed to be constant.[3] The time required to travel from activity i to $i + 1$ is deterministic and depends in general on the departure time t_{di}, $T_i(t_{di})$. Similar to Noland and Small (1995) we may write the travel time on trip i as the sum of a component T_i that is independent of departure time and a congestion profile $T_{xi}(t_{di})$, i.e., $T_i(t_{di}) = T_i + T_{xi}(t_{di})$. To ensure that departing later cannot make a traveler arrive earlier (i.e., the FIFO principle) we require that $T_i'(t) = T_{xi}'(t) > -1$.

In this paper we consider a daily schedule that consists of three activities and two intermediate trips. Although the model is quite general, we will typically interpret activity 1 as being at home in the morning, activity 2 as being at work during the day and activity 3 as being at home in the evening. Correspondingly, trips 1 and 2 represent the commute from home to work in the morning and back in the evening, respectively.

We assume that the schedule of a day is independent of preceding and subsequent days. This means that we can fix two times $t = 0$ and $t = 1$ that mark the start and end of the day, respectively. There is thus no need to distinguish between time-of-day and duration-dependent utility for activities 1 and 3. For activity 2 we assume that the marginal utility at time t depends on the duration $t - t_{s2}$ as well as the time of day t, where a parameter ξ expresses the scheduling flexibility.

[3]This assumption can be generalized so that utility depends nonlinearly on travel duration, as some empirical evidence suggests (e.g., Redmond and Mokhtarian, 2001). For the present analysis we lack the necessary data to employ such functions.

For work trips, late arrival or early departure can have both short-run (e.g., penalties) and long-run negative impacts on the wage, which in turn would affect the available budget for consumption. Thus, it is reasonable to interpret U as a partially indirect utility function, implicitly incorporating that goods are consumed optimally for any income. The marginal utility of the work activity $u_2(t - \xi t_{s2})$ then arises both from the direct utility derived from working and the effect on the direct utility from goods consumption through the budget constraint (c.f. Small, 1982).

In summary, we have the following notation:

t_{d1} departure time of trip 1 (end time of activity 1).

t_{d2} departure time of trip 2 (end time of activity 2).

$T_1(t_{d1}) = T_1 + T_{x1}(t_{d1})$ duration of trip 1 as a function of departure time.

$T_2(t_{d2}) = T_2 + T_{x1}(t_{d2})$ duration of trip 2 as a function of departure time.

$t_{s2} = t_{d1} + T_1(t_{d1})$ start time of activity 2 (arrival time of trip 1).

$t_{s3} = t_{d2} + T_2(t_{d2})$ start time of activity 3 (arrival time of trip 2).

$u_1(t)$ marginal utility of activity 1 at time t.

$u_2(t - \xi t_{s2})$ marginal utility of activity 2 at time t, given start time t_{s2} and scheduling flexibility ξ.

$u_3(t)$ marginal utility of activity 3 at time t.

ν marginal utility of travel.

Given that the number and sequence of activities have been fixed, the remaining decision variables for the individual are the durations of the activities (where two durations determine the third) or, equivalently, the departure times of the two trips. The utility associated with the schedule (t_{d1}, t_{d2}) is then (c.f. Ettema and Timmermans, 2003; Zhang et al., 2005)

$$\begin{aligned} U(t_{d1}, t_{d2}) = &\int_0^{t_{d1}} u_1(t)dt + \int_{t_{d1}+T_1(t_{d1})}^{t_{d2}} u_2\big(t - \xi[t_{d1} + T_1(t_{d1})]\big)dt \\ &+ \int_{t_{d2}+T_2(t_{d2})}^{1} u_3(t)dt + \nu[T_1(t_{d1}) + T_2(t_{d2})]. \end{aligned} \tag{1}$$

2.1 Optimal schedule

Under normal pre-disruption travel conditions the individual chooses morning and evening departure times in order to maximize her utility. Thus, the individual solves

$$\max_{t_{d1},t_{d2}} \; U(t_{d1}, t_{d2}) \tag{2}$$

$$\text{s.t.} \quad 0 \leq t_{di} \leq 1, \qquad i = 1, 2, \tag{3}$$

$$t_{d1} + T_1(t_{d1}) \leq t_{d2}, \tag{4}$$

$$t_{d2} + T_2(t_{d2}) \leq 1. \tag{5}$$

Constraint (3) ensures that both trips are made during the day, constraint (4) ensures that the traveler arrives at activity 2 before departing again, while constraint (5) ensures that the traveler arrives at activity 3 before the end of the day.

The marginal utility functions u_i are assumed to be continuously differentiable, initially increasing (representing a warm-up period) but ultimately decreasing (representing a cool-down period), regardless of arrival time. With realistically formulated utility functions the constraints will be non-binding, so that optimal departure times satisfy $\partial U/\partial t_{d1} = \partial U/\partial t_{d2} = 0$.

To make the analysis more transparent, let us in the following consider the special case where travel time is independent of departure time, i.e, $T_i(t_{di}) = T_i$ for $i = 1, 2$; we return to the general case in Section 2.4. If, in this special case, we assume that travel times are such that the individual can always depart during the cool-down period and arrive during the warm-up period of each activity, i.e., that $u_1'(t_{d1}) < 0$, $u_2'([1-\xi]t_{s2}) > 0$, $u_2'(t_{d2} - \xi t_{s2}) < 0$ and $u_3'(t_{s3}) > 0$, then the utility function is concave and there is a unique maximum.

For any scheduling flexibility ξ the optimality conditions require that the departure times simultaneously satisfy

$$u_1(t_{d1}^*) = [1-\xi]u_2\big([1-\xi]t_{s2}^*\big) + \xi u_2(t_{d2}^* - \xi t_{s2}^*), \tag{6}$$

$$u_2(t_{d2}^* - \xi t_{s2}^*) = u_3(t_{s3}^*), \tag{7}$$

where $t_{s2}^* = t_{d1}^* + T_1$ and $t_{s3}^* = t_{d2}^* + T_2$ are the arrival times associated with the optimal departure times t_{d1}^* and t_{d2}^*. Note that in general the optimal timing of trip 1 depends on the departure time of trip 2, while the optimal timing of trip 2 depends on the arrival time of trip 1.

If activity 2 is completely fixed ($\xi = 0$) the optimality conditions give that $u_1(t_{d1}^*) = u_2(t_{s2}^*)$ and $u_2(t_{d2}^*) = u_3(t_{s3}^*)$, i.e., the marginal utilities at the start and the finish must be equal (c.f. Ettema and Timmermans, 2003; Tseng and Verhoef, 2008). In this special case the optimal timing of one trip is independent of the travel time and timing of the other trip.

In the other extreme, if the schedule is completely flexible ($\xi = 1$) optimal departure times will satisfy $u_1(t^*_{d1}) = u_2(t^*_{d2} - t^*_{s2}) = u_3(t^*_{s3})$. That is, since the departure time from activity 1 and the arrival time to activity 3 are in one-to-one correspondence with the duration of the respective activity, the marginal utility of the duration of each activity must be equal (compare with pure time allocation models, e.g., Jara-Diaz et al. (2008)).

Given that the traveler arrives to activity 2 at t_{s2} (not necessarily the optimal arrival time t^*_{s2}), there is an optimal departure time from that activity, $t^*_{d2}(t_{s2})$, and an associated arrival time to activity 3, $t^*_{s3}(t_{s2})$. It will be useful in the following to calculate the marginal effect on utility of a change in the arrival time t_{s2}, assuming that trip 2 is optimally timed in response to the change. Thus, let $\tilde{U}_2(t_{s2}, t_{d2})$ be the utility derived from time t_{s2} to the end of the day. We then introduce the "backward optimal" marginal utility function $\tilde{u}_2(t_{s2}) = -d\tilde{U}_2(t_{s2}, t^*_{d2}(t_{s2}))/dt_{s2}$, which is the marginal change in subsequent utility due to an earlier arrival time. From the envelope theorem (e.g., Mas-Colell et al., 1995) it follows that $\tilde{u}_2(t_{s2})$ is given by

$$\tilde{u}_2(t_{s2}) = [1-\xi]u_2\big([1-\xi]t_{s2}\big) + \xi u_2\big(t^*_{d2}(t_{s2}) - \xi t_{s2}\big). \tag{8}$$

The optimality conditions (6)–(7) can thus be summarized as $u_1(t^*_{d1}) = \tilde{u}_2(t^*_{s2})$.[4] Note that $\tilde{u}_2(t_{s2}) = u_2(t_{s2})$ with $\xi = 0$ and $\tilde{u}_2(t_{s2}) = u_2(t^*_{d2}(t_{s2}) - t_{s2})$ with $\xi = 1$.

2.2 Travel costs

By definition, the utility derived from a daily schedule depends not only on the amount of time spent traveling and in activities but also on the timing of the activities and trips. In the following we will assume that the marginal utility of income within our framework is constant. This means that we can choose the scale of utility such that U is a money metric utility function. Thus, at any time t, differences among marginal utilities $u_i(t; t_{si})$ and ν represent the willingness to pay for (or accept) spending an additional unit time in one activity rather than the other (Tseng and Verhoef, 2008). In particular, $\alpha_i(t; t_{si}) \equiv u_i(t; t_{si}) - \nu$ then represents the willingness to pay to spend one additional unit of time in activity i rather than traveling, given arrival to activity i at t_{si}. Note that when activity marginal utilities depend on arrival times, this is in general not equal to minus the willingness to pay for the opposite transition, i.e., to spend one additional unit of time traveling rather than starting activity i, which is $u_i(t; t) - \nu$.

[4] This can be seen as a dynamic programming approach to optimizing the daily activity schedule, which is generalizable to more complex schedules (see, e.g., Karlström, 2005).

Following Tseng and Verhoef (2008), we define the cost associated with a trip of a certain travel time and timing as the willingness to accept the trip in relation to an optimally timed instantaneous transition from origin to destination. Within our two trips model, it is reasonable to calculate the travel cost of trip 1 under the condition that trip 2 is timed optimally given the arrival time of trip 1. If travel were instantaneous, the optimal time to travel, denoted t^*, would be the solution to $u_1(t^*) = \tilde{u}_2(t^*)$, i.e., the point in time when the marginal utility at activity 1 equals the backward optimal marginal utility of activity 2.

With a positive travel time, the marginal cost of travel at any time t arises from the difference in the marginal utility of travel ν and the largest of $u_1(t)$ and $\tilde{u}_2(t)$, where $u_1(t) > \tilde{u}_2(t)$ for $t < t^*$ and $u_1(t) < \tilde{u}_2(t)$ for $t > t^*$. The total cost of a trip starting at t_{d1} and ending at t_{s2} is

$$C_1(t_{d1}, t_{s2}) = \int_{t_{d1}}^{t^*} [u_1(t) - \nu]dt + \int_{t^*}^{t_{s2}} [\tilde{u}_2(t) - \nu]dt, \tag{9}$$

or equivalently, with the marginal cost functions $\alpha_1(t) = u_1(t) - \nu$, $\beta_1(t) = u_1(t) - \tilde{u}_2(t)$ and $\gamma_1(t) = \tilde{u}_2(t) - u_1(t)$,

$$C_1(t_{d1}, t_{s2}) = \int_{t_{d1}}^{t_{s2}} \alpha_1(t)dt + \begin{cases} \int_{t_{s2}}^{t^*} \beta_1(t)dt & \text{if } t_{s2} < t^*, \\ \int_{t^*}^{t_{s2}} \gamma_1(t)dt & \text{if } t_{s2} \geq t^*. \end{cases} \tag{10}$$

Tseng and Verhoef (2008) derived the corresponding formula within their model for the timing of the morning commute, which is equivalent to the special case $\xi = 0$ of our model. They thus considered the work activity to be completely fixed so that subsequent trips are irrelevant for the timing problem, which means that $\tilde{u}_2(t)$ simplifies to $u_2(t)$. In Section 4.1 we make use of the relationship with the model of Tseng and Verhoef (2008) to calibrate the travel costs of the morning commute.

Travel costs are also captured in the schedule delay model of for example Vickrey (1969) and Small (1982), where costs arise if an individual arrives before or after a preferred arrival time t^*. Specifically, the cost of a trip starting at time t_d and ending at time $t_s = t_d + T$ in that model is, for some parameters α, β and γ,[5]

$$C_1(t_d, t_s) = \alpha T + \begin{cases} \beta[t^* - t_s] & \text{if } t_s < t^*, \\ \gamma[t_s - t^*] & \text{if } t_s \geq t^*. \end{cases} \tag{11}$$

It can be seen that the linear model (11) is obtained as a special case of (10) when the three marginal cost functions $\alpha_1(t)$, $\beta_1(t)$ and $\gamma_1(t)$ are constant over time. It

[5]This linear specification, sometimes with an additional discrete lateness penalty, is frequently used to evaluate the value of travel time reliability, whereby the travel time T is treated as stochastic with to the traveler known distribution (e.g., Noland and Small, 1995; Fosgerau and Karlström, 2010).

can also serve as an approximation of (10) when the deviations from the normal travel time and trip timing are small. In the context of large unplanned network disruptions, however, deviations may be very large and approximation errors may be significant if $\alpha_1(t)$, $\beta_1(t)$ and $\gamma_1(t)$ vary over time as empirical evidence suggests (Tseng and Verhoef, 2008).

More generally, we can define the total travel cost of a given daily schedule as the willingness to accept the schedule instead of instantaneous, optimally timed transitions between activities. If we make the travel times explicit in the utility function, i.e., $U(t_{d1}, t_{d2}; T_1, T_2)$, we can write the travel cost as

$$C(t_{d1}, t_{d2}; T_1, T_2) = U(t^*_{d1}, t^*_{d2}; 0, 0) - U(t_{d1}, t_{d2}; T_1, T_2). \tag{12}$$

2.3 Value of travel time savings

An increase in travel time affects utility not only through the marginal utility of travel but also through the reduction in activity participation that must occur due to the limited time available in a day (e.g., Jara-Diaz, 2000). To capture the full, long-run impact of an exogenous change in travel time, it is reasonable to calculate its value given that individuals adjust their schedule optimally to the change. The full effect on utility of a change in travel time is then dU^*/dT, where U^* is the utility under the optimal schedule and T is a generic travel time.[6] If, as before, we assume that U is a money metric utility function, the value of time is simply $-dU^*/dT$, or equivalently, dC^*/dT.

In general, the impact on utility depends on which trip travel time is changed on, since this determines which activities are most affected. In other words, the value of travel time savings varies between trips. On the other hand, the value of time on one trip generally depends on the baseline travel times on both trips from which the change occurs. From the envelope theorem, the value of saving time on each trip is

$$\frac{dC^*}{dT_1} = \tilde{u}_2(t^*_{s2}) - \nu, \tag{13}$$

$$\frac{dC^*}{dT_2} = u_3(t^*_{s3}) - \nu. \tag{14}$$

If activity 2 is fixed ($\xi = 0$) we have $\tilde{u}_2(t^*_{s2}) = u_2(t^*_{s2})$, and from the optimality conditions it follows that the value of saving travel time on one trip is independent of the travel time and timing of the other trip. If activity 2 is completely flexible

[6]Ettema and Timmermans (2003) define the value of time on trip 1 based on the partial derivative of U with respect to T_1 holding both departure times fixed. Instead, our model allows gained time in the long run to be distributed across all activities. Furthermore, we also account for the possibility of flexible work hours, and consider the value of travel time for trip 2.

($\xi = 1$) we have $\tilde{u}_2(t^*_{s2}) = u_2(t^*_{d2} - t^*_{s2})$, and since the optimality conditions require that $u_2(t^*_{d2} - t^*_{s2}) = u_3(t^*_{s3})$, the value of saving travel time will be the same regardless of which trip travel time is saved on. In this special case there is thus a single value of time for both trips.

It is also of interest to know the optimal schedule adjustments dt^*_{di}/dT_j and dt^*_{si+1}/dT_j, $i, j = 1, 2$, in response to a change in travel time. Expressions for these are derived in the Appendix for the general case with deparure time-dependent travel times. With our assumptions about the shape of the marginal utility functions, fixed travel times mean that an increase in travel time moves the optimal departure and arrival times of the trip earlier and later, respectively; that is, $dt^*_{di}/dT_i < 0$ and $dt^*_{si+1}/dT_i > 0$ for $i = 1, 2$.

With knowledge about the optimal schedule adjustments, the value of time on each trip can be further decomposed in terms of the fractions of an increase in travel time that are taken from each of the three activities. Since the available time in a day is fixed, these fractions must sum to one. With $\xi = 0$ the optimal departure time adjustment for each trip depends on the relative steepness of the marginal utilities of the origin and destination activities. That is, if the marginal utility decreases more steeply at the origin than it increases at the destination, more time will be taken from the latter activity, and vice versa,

$$\frac{dC^*}{dT_i}_{\xi=0} = \frac{-u'_{i+1}(t^*_{si+1})}{u'_i(t^*_{di}) - u'_{i+1}(t^*_{si+1})} u_i(t^*_{di}) + \frac{u'_i(t^*_{di})}{u'_i(t^*_{di}) - u'_{i+1}(t^*_{si+1})} u_{i+1}(t^*_{si+1}) - \nu, \quad i = 1, 2. \tag{15}$$

With $\xi = 1$ time is taken from all three activities in proportions that are independent of which trip travel time is changed on and, again, determined by the relative steepness of the marginal utility functions,

$$\begin{aligned}\frac{dC^*}{dT}_{\xi=1} = &\frac{-u'_2(t^*_{d2} - t^*_{s2})u'_3(t^*_{s3})}{u'_1(t^*_{d1})u'_2(t^*_{d2} - t^*_{s2}) - u'_1(t^*_{d1})u'_3(t^*_{s3}) - u'_2(t^*_{d2} - t^*_{s2})u'_3(t^*_{s3})} u_1(t^*_{d1}) \\ &+ \frac{-u'_1(t^*_{d1})u'_3(t^*_{s3})}{u'_1(t^*_{d1})u'_2(t^*_{d2} - t^*_{s2}) - u'_1(t^*_{d1})u'_3(t^*_{s3}) - u'_2(t^*_{d2} - t^*_{s2})u'_3(t^*_{s3})} u_2(t^*_{d2} - t^*_{s2}) \\ &+ \frac{u'_1(t^*_{d1})u'_2(t^*_{d2} - t^*_{s2})}{u'_1(t^*_{d1})u'_2(t^*_{d2} - t^*_{s2}) - u'_1(t^*_{d1})u'_3(t^*_{s3}) - u'_2(t^*_{d2} - t^*_{s2})u'_3(t^*_{s3})} u_3(t^*_{s3}) - \nu.\end{aligned} \tag{16}$$

2.4 Departure time-dependent travel times

Compared to the special case above, results are less clear-cut when travel times depend on departure times. For example, without any further specification of the travel time profiles $T_{xi}(t)$, the utility maximization problem is in general not concave and optimal schedules need not be unique.

For the study of the value of travel time, it is necessary to distinguish between the effect of an exogenous change in travel time and the endogenous dependency of travel time on departure time. In our analysis we assume that the exogenous change in travel time on a trip is uniform and independent of departure time. The direct effect of an exogenous change in travel time dT_i is an equal change in arrival time $dt_{si} = dT_i$. Meanwhile, the direct effect of a change in departure time dt_{di} is a change in travel time $T'_{xi}(t_{di})dt_{di}$ and arrival time $dt_{si} = [1 + T'_{xi}(t_{di})]dt_{di}$. At optimum, an exogenous change in travel time can affect the departure time and thus induce an endogenous change in travel time as well.

Formulas (9), (10) and (12) for the travel costs remain valid in the general case, as do formulas (13) and (14) for the marginal value of time. However, it is no longer necessarily true that an increase in travel time moves the optimal departure and arrival times on the trip earlier and later, respectively, nor that a completely flexible schedule ($\xi = 1$) implies a single value of time for both trips. All derivations for the general case can be found in the Appendix.

3 Unplanned disruptions

Empirical evidence tells us that major unplanned transport network disruptions are generally followed by a time—on the order of days or weeks—of uncertainty, learning and adaptation for the travelers. If the network disruption is long-lasting, the traffic eventually approaches a new equilibrium-like state, where travelers have received sufficient information about the new travel conditions and adjusted their travel decisions accordingly. Observations are fairly consistent in that the most common responses by individuals are changes in departure time and route choice. To a lesser extent people cancel or consolidate (mainly non-work) trips, whereas people are relatively reluctant to change travel mode (Wesemann et al., 1996; Giuliano and Golob, 1998; Hunt et al., 2002; Cairns et al., 2002; Clegg, 2007; Zhu et al., 2008).[7]

The delay costs during the first few days of a disruption can be expected to be particularly high, both because the travel times themselves are longer (due to, for example, suboptimal route choices) and because the travelers are less able to adapt

[7]For example, in a survey following the 2007 Minneapolis bridge collapse (used further below), people who stated that they were affected by the bridge collapse responded that they adjusted in the following ways: changed departure time (75.3%), changed route (72.3%), avoided destinations (61.0%), cancelled trips (14.3%), worked from home (9.1%), and changed mode (6.5%) (Zhu et al., 2008).

their schedules to the journey delays. Immediately following the event, some individuals may be completely uninformed of the disruption when scheduling a trip. Such individuals will depart as normal and, by incurring much longer than expected travel times, arrive late to their destinations.

Soon, however, information about the event will start to spread among travelers through the media, family, friends and other channels (Zhu et al., 2008). Still, even when most travelers have received information about the occurrence of the disruption, there likely remains unusually large uncertainty about the travel conditions for a significant time period. During this period, travelers will plan their travel as best as they can given their predictions of the travel conditions they will face. Some travelers may underestimate the induced changes in travel conditions and arrive late to their destinations. Others may overestimate the changes or, given the uncertainty, add a safety margin to the estimated delay in order to reduce the risk of arriving late to work (Noland and Small, 1995; Fosgerau and Karlström, 2010). If travel conditions turn out to be better than feared, these individuals will arrive early at their destinations.

To investigate whether empirical data supports these hypothesized schedule adjustment types, we use observations from the collapse of the I-35W bridge across the Mississippi River in Minneapolis, Minnesota on 1 August 2007. The collapse resulted in tragic causalties as well as a significant, long-lasting disturbance to the transportation system of the city. A hand-out, mail-in survey was conducted in September 2007 to investigate how travelers were affected by and adjusted to the disruption (Zhu et al., 2008).

Respondents were asked (among other things) about the departure time, travel time, travel mode and route choice in their morning commute during each of four periods: before the bridge collapse, the day after the collapse, the following weeks, and at the time of the survey. Unfortunately for the present analysis, they were not asked about their evening commute. They were also asked whether they have a flexible work schedule or not. Out of a total of 1000 handed out surveys, 141 usable responses were obtained. Here we consider the 79 respondents who stated that they commuted by car before the collapse. Of these, 56 respondents had their pre-collapse schedule affected by more than 5 minutes through changed travel time and/or departure time in at least one post-collapse period; we will refer to this group as “affected car users”.

According to the theoretical model with travel times independent of departure times, an increase in travel time on the morning trip will move the optimal departure time of that trip earlier and the optimal arrival time later (whether the departure time or the arrival time will change the most depends on the relative steepness of the marginal utility functions). The survey data, however, contains many responses where this principle is violated, in particular the morning after the collapse.

We do not have information about the time-of-day variation of travel times for each respondent before and after the disruption, and hence we cannot in principle rule out that the seemingly suboptimal responses noted above are actually optimal due to changes in the congestion profile. However, we think that a more important explanation for the observed behavior is the lack of reliable information that characterizes the time immediately following a major disruption such as this.

On the first day, 13 (23%) of the affected car users report departing at the same time as before the collapse. Of these, eight (14% of affected car users) report increased travel times, ranging between 17% and 50% of the pre-collapse travel times (two report unchanged and three report reduced travel times). Hence, there is a tendency for under-adjustment of the schedule as compared to the theory. At the same time, 26 (46%) respondents report departing earlier than before the collapse the first day. Of these, nine (16% of affected car users) arrive at work earlier than before the collapse (six report unchanged and eleven report later arrival times). Hence, there is also a tendency for over-adjustment of the schedule.[8] Both these effects, in particular the under-adjustment, seem to decline the following weeks after the collapse.

3.1 Adjustment profiles and delay costs

Based on the empirical observations, we formulate a number of stylized adjustment profiles for which we then compare journey delay costs within our model.

No adjustment. The first type of response that we consider is to not adjust the departure time on any of the two trips when faced with delays. This is often realistic for trip 1 when travelers are completely uninformed of the disruption, but is probably unrealistic for trip 2 when travelers have been exposed to the event. This adjustment profile may therefore represent the upper extreme of an initially uninformed traveler.

No + optimal adjustment. Alternatively, we may assume that the traveler departs as normal in the morning but, being exposed to the disruption on trip 1, is able to gather information and schedule trip 2 optimally given the delay and the actual arrival time of trip 1. This adjustment profile may represent the lower extreme of an initially uninformed traveler and provides, in combination with the no adjustment profile, a span in which the impact for any initially uninformed traveler should lie.

[8] We found that the share of people who over-adjusted their departure time was significantly (at the $\alpha = 0.05$ level) higher among those with stated fixed work schedule than among those with stated flexible schedule, which suggests that people with fixed schedules are more inclined to add safety margins to travel times in order not to arrive late. We also tested whether household conditions such as the number of small and large children restricted the possibility to depart earlier in the morning, but found no significant effects.

Overadjustment. The third type of response is to overestimate the actual delays on both trips, either erroneously or deliberately as a safety margin. The departure times are then optimized with respect to the overestimated travel times and, for trip 2, the actual arrival time of trip 1. In the analysis we assume, somewhat arbitrarily, that journey delays are systematically overestimated by 50%. In that case, travelers with a baseline travel time of 0.5 h and an actual delay of 1.0 h would reschedule their trip assuming a 2.0 h travel time.

Over + optimal adjustment. In correspondence with the no + optimal response profile, we also consider the case where a traveler overestimates the journey delay by 50% before making trip 1 but is able to optimally reschedule trip 2 given the true delay on trip 2 and the actual arrival time of trip 1.

Optimal adjustment. Finally, as a baseline we consider the case where travelers are able to perfectly predict the travel conditions and adjust their schedules optimally in response to the delay. If the disruption is long-lasting, this should be the behavior that emerges over time, as travelers learn and adapt to the changed travel conditions.

The impact of journey delay for a traveler depends not only on her adjustment profile but on the flexibility of her schedule for adjustments. With a highly fixed work schedule, arriving late to work means that much of this time will be lost without possibility of recovery. With a relatively flexible schedule, however, time lost during the morning warm-up period can be somewhat compensated for by a more productive period in the evening and possibly by sacrificing some time at home in the evening. Similarly, with a highly fixed schedule, arriving early to work means that time will be spent unproductively at work compared to the time lost in the morning. With a flexible schedule, work can be carried out about as productively as normally, but productivity will decrease earlier in the evening.

Figure 1 shows how utility losses due to delays occur during the day with the no adjustment, the overadjustment and the optimal adjustment profiles for a generic set of marginal utility curves—in particular, activity 2 is assumed to be semi-flexible ($\xi = 0.5$). At any time, a cost (shown in red) or gain (shown in green) arises proportionally to the difference between the current marginal utility and the marginal utility at the same time before the disruption.

3.2 Trip cancellation

Instead of adjusting trip departure times, a possible way of responding to expected long journey delays is to cancel the trip altogether. Some individuals, for example, may gain more by working from home one day than traveling to work with long

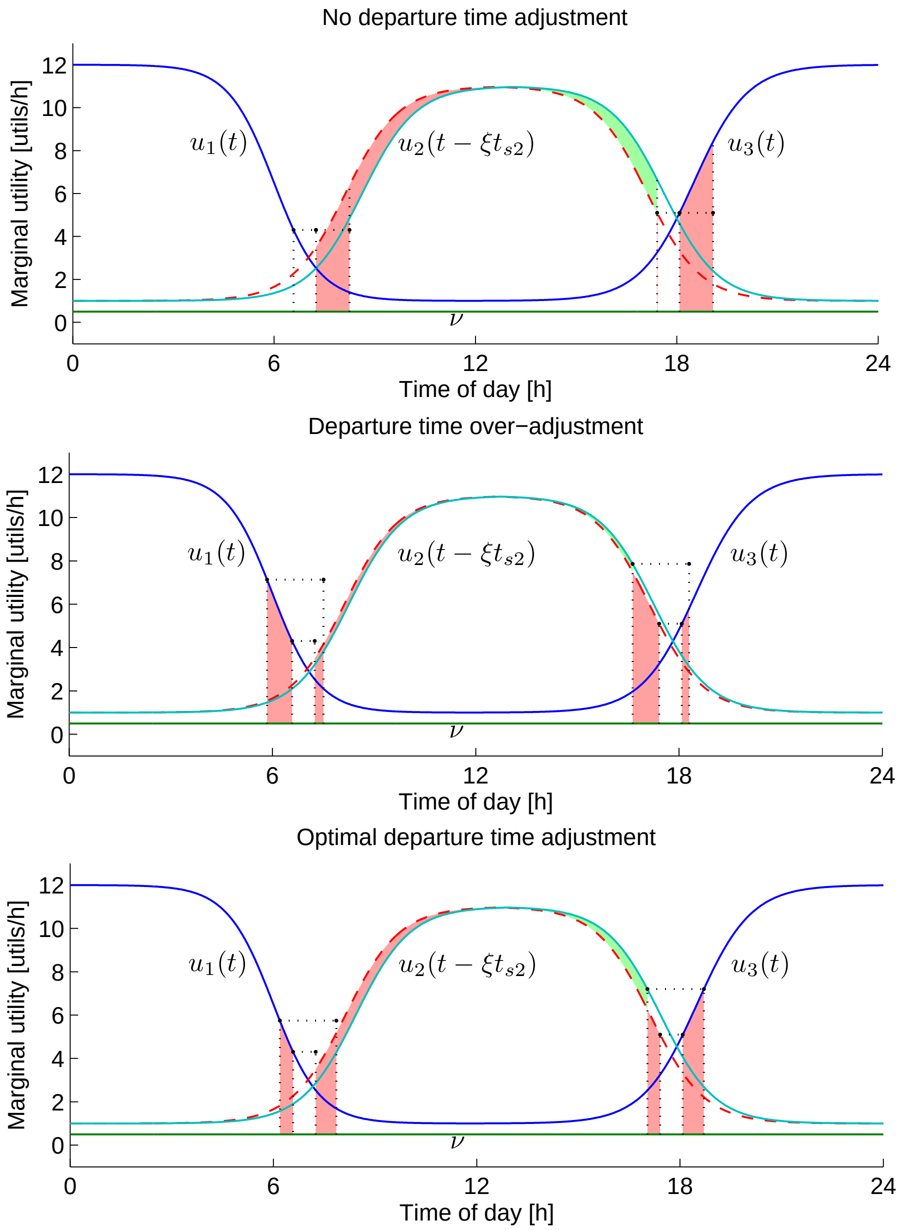

Figure 1: Utility losses due to journey delay under different schedule adjustment profiles and a semi-flexible work schedule. Solid curves show generic marginal activity utilities, dashed curves show pre-disruption optimal marginal utilities, dotted lines mark the intervals spent traveling, red areas show utility losses, green areas show utility gains. Symmetric baseline travel times (2 × 40 minutes) and journey delays (2 × 60 minutes) independent of departure times. Top: No departure time adjustment. Middle: Departure time over-adjustment. Bottom: Optimal departure time adjustment.

delays. In our setting, assuming that activities 1 and 3 represent being at home in the morning and the evening, respectively, this schedule would yield the utility

$$U_h = \int_0^1 \max\{u_1(t), u_3(t)\} dt. \tag{17}$$

Cancelling the trip is optimal if the utility lost from spending time traveling is greater than the utility gained from taking part in the activity itself, so that $U < U_h$.[9] In an uncertain environment, people may also cancel trips if they *expect* the travel times to be longer than they actually would be. In extreme situations some people may have no possible route to reach their destinations during the disruption, in which cases they will be forced to cancel or postpone trips.

The survey data from the Minneapolis bridge collapse shows that only one of the 56 affected car users cancelled the morning trip to work the day following the event. This suggests that trip cancellation is a relevant adjustment strategy for some individuals, but that other strategies such as route and departure time changes are much more dominant, which also previous investigations have shown (Giuliano and Golob, 1998; Hunt et al., 2002; Cairns et al., 2002; Zhu et al., 2008).

4 Cost specifications

To obtain numerical values for the delay costs we need to specify functional forms and parameter values for the the willingness to pay for spending time at one location rather than another.[10] We use logistic functions to represent the time-varying marginal utility, capturing the warm-up and cool-down periods of each activity. We interpret activities 1 and 3 as being at home in the morning and the evening, respectively, and specify the marginal cost functions $\alpha_1(t) = u_1(t) - \nu$ and $\alpha_3(t) = u_3(t) - \nu$ as

$$\alpha_1(t) = \alpha_1^{\max} - \frac{\alpha_1^{\max} - \alpha_1^{\min}}{1 + \exp(-\zeta_1[t - \omega_1])} \quad 0 \le t \le 1, \tag{18}$$

$$\alpha_3(t) = \alpha_3^{\min} + \frac{\alpha_3^{\max} - \alpha_3^{\min}}{1 + \exp(-\zeta_3[t - \omega_3])} \quad 0 \le t \le 1, \tag{19}$$

where $\alpha_i^{\max}$ and $\alpha_i^{\min}$, $i = 1, 3$, are upper and lower limits for the willingness to pay for being at home in the morning and the evening, respectively, rather than

[9] The option of trip cancellation can be incorporated explicitly in an extended version of our model as a discrete decision variable that precedes the choices of departure times.

[10] As we adopt the model formulation of Ettema and Timmermans (2003), it would seem natural to make use of the marginal utility functions proposed and estimated by the authors. However, their utility functions were estimated so as to reproduce observed departure and arrival times at optimum, without any consideration of money trade-offs. Hence, there is no reason to assume that the chosen scale of utility in that paper represents the travelers' willingness to pay for different schedules.

traveling. Since being at home is essentially one single activity, we restrict the parameters so that $\alpha_1^{\max} = \alpha_3^{\max}$ and $\alpha_1^{\min} = \alpha_3^{\min}$. The parameters ω_i control the timing of the cool-down and warm-up periods, while ζ_i control the durations of the same periods (larger values meaning swifter changes).

Similarly, the marginal cost of the work activity $\alpha_2(t - \xi t_{s2}) = u_2(t - \xi t_{s2}) - \nu$ is specified as

$$\alpha_2(t - \xi t_{s2}) = \begin{cases} \alpha_2^{\min} + \dfrac{\alpha_2^{\max} - \alpha_2^{\min}}{1 + \exp(-\zeta_2^1[t - \xi t_{s2} - \omega_2^1])} & 0 \le t \le t_2^{\text{shift}}, \\ \alpha_2^{\max} - \dfrac{\alpha_2^{\max} - \alpha_2^{\min}}{1 + \exp(-\zeta_2^2[t - \xi t_{s2} - \omega_2^2])} & t_2^{\text{shift}} < t \le 1. \end{cases} \tag{20}$$

Here, parameters ω_2^1 and ω_2^2, together with the flexibility parameter ξ and arrival time t_{s2}, control the timing of the warm-up and cool-down periods, while ζ_2^1 and ζ_2^2 control the duration of each period. The transition between the two phases occurs when the marginal utilities intersect, which can be shown to be at $t_2^{\text{shift}} = [\zeta_2^1\omega_2^1 + \zeta_2^2\omega_2^2]/[\zeta_2^1 + \zeta_2^2] + \xi t_{s2}$. Again, $\alpha_2^{\max}$ and $\alpha_2^{\min}$ are upper and lower limits, respectively, for the willingness to pay for being at work rather than traveling.

4.1 Parameter calibration

Tseng and Verhoef (2008) estimated the time-varying willingness to pay for spending time at home and at work in the morning relative to traveling. Considering that work hours may be at least partially flexible, this corresponds to the marginal cost functions $\alpha_1(t) = u_1(t) - \nu$ and $\tilde{\alpha}_2(t) = \tilde{u}_2(t) - \nu$, respectively (see Section 2.2). Their framework was based on a stated preference survey of trip timing choices among discrete predefined time intervals, and they specified a mixed logit model to estimate average willingness to pay values as well as standard deviations between individuals in each time interval. It is notable that the empirical study found the marginal travel cost to be quite low and even negative around t^*, which suggests that individuals sometimes prefer traveling rather than, e.g., arriving early to work.

We use the empirical results of Tseng and Verhoef (2008) as reference for calibrating the parameters in our marginal utility functions. Parameter values are assumed to vary among individuals, and we calibrate both mean values and standard deviations to replicate the mean willingness to pay and the variability between individuals estimated in the empirical study. Of course, since we model the entire day and the empirical study only considered the morning commute, there are many degrees of freedom in chosing parameter distributions, in particular regarding the evening commute and scheduling flexibility. Some further empirical hints can be obtained from Hess et al. (2007), who estimated a joint schedule delay model for the morning and evening commutes and found, loosely speaking, that the costs associated with early and late departure from work are typically smaller than the costs of early and late arrival to work. Still, the results presented in this paper should be seen mainly as an illustration of the approach, to be followed by more rigorous estimation and more reliable results in future work.

The process of calibrating the parameters was as follows. From a given specification of parameter means and standard deviations, a collection of n synthetic "individuals" was created, where each individual is represented by a set of parameter values randomly drawn from independent normal distributions.[11] To ensure that the marginal utility of work was properly located within the day for any level of scheduling flexibility, the process of determining ω_2^1 and ω_2^2 involved an additional step: After the initial draw of these parameter values, the optimal arrival time t_{s2}^* for the inflexible case $\xi = 0$ was calculated. The values for ω_2^1 and ω_2^2 were then updated by subtracting ξt_{s2}^* from the initially drawn values, with ξ being the actual drawn value of the flexibility parameter.

Then, for each individual, $\alpha_1(t)$ and the backward optimal marginal cost function $\tilde{\alpha}_2(t)$ were calculated and the ideal transition time t^* when the two curves intersect was identified; $\tilde{\alpha}_2(t)$ was calculated assuming a travel time of 40 minutes for trip 2 (see Section 2.2). The marginal cost curves were then expressed as functions of $t' = t - t^*$, i.e., time away from t^*, producing n realizations of $\alpha_1(t')$ and $\tilde{\alpha}_2(t')$. Finally, the mean value and standard deviation of $\alpha_1(t')$ and $\tilde{\alpha}_2(t')$ across all individuals were calculated for every t'. The resulting time-varying curves were compared with the empirical results of Tseng and Verhoef (2008). Parameter means and standard deviations were adjusted heuristically and the calculations above repeated until a satisfactory correspondence was obtained. A sample size of $n = 20$ was deemed sufficient to produce stable results.

Table 1 shows the calibrated parameter values. Figure 2 shows the average marginal cost curves $\alpha_1(t)$, $\alpha_2(t - \xi t_{s2}^*)$ and $\alpha_3(t)$ with the calibrated parameter values, including confidence intervals. Figure 3 shows the average marginal cost curves $\alpha_1(t')$ and $\tilde{\alpha}_2(t')$ for the morning trip in comparison with the corresponding empirical results reported by Tseng and Verhoef (2008), including confidence intervals.

5 Results: Traveler costs of disruptions

For a given traveler the effect of a journey delay ΔT is a change in travel cost ΔC that depends on the size of ΔT, the allocation of ΔT on trip 1 and 2, the baseline

[11] The independence assumption means that calibrated standard deviations need to be quite small to reproduce the variation found by Tseng and Verhoef (2008). In a more realistic representation parameters would likely be correlated, which would allow for larger variation in each parameter.

Activity 1 (morning home)	Activity 2 (work)	Activity 3 (evening home)
$\alpha_1^{\max} = 19\ (1.0)$	$\alpha_2^{\max} = 25\ (1.0)$	$\alpha_3^{\max} = \alpha_1^{\max}$
$\alpha_1^{\min} = -10\ (0.5)$	$\alpha_2^{\min} = -35\ (0.5)$	$\alpha_3^{\min} = \alpha_1^{\min}$
$\zeta_1 = 60\ (0.5)$	$\zeta_2^1 = 80\ (0.5)$	$\zeta_3 = 65\ (0.5)$
$\omega_1 = 0.271\ (0.001)$	$\zeta_2^2 = 40\ (0.5)$	$\omega_3 = 0.708\ (0.001)$
	$\omega_2^1 = 0.292\ (0.002)$	
	$\omega_2^2 = 0.688\ (0.002)$	
	$\xi = 0.20\ (0.05)$	

Table 1: Mean parameter values for the activity marginal utility functions (standard deviations in parentheses). The values for ω_2^1 and ω_2^2 represent the case $\xi = 0$ (see the text).

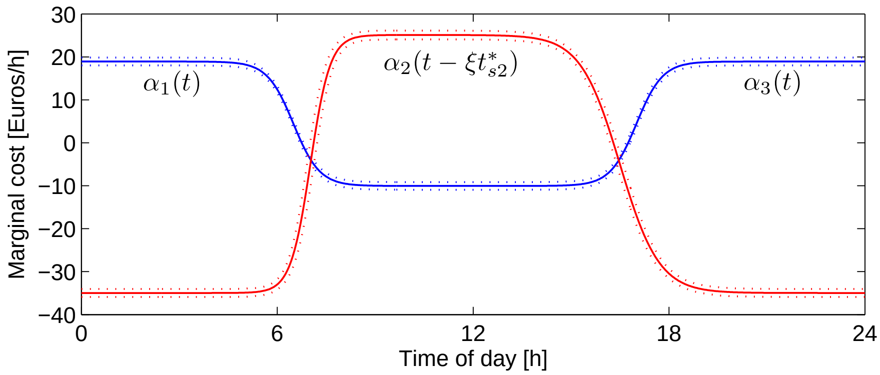

Figure 2: Average marginal cost curves based on random draws from normal distributions defined by the mean and standard deviation of the parameter estimates shown in Table 1. The dotted lines show confidence intervals defined by two times the standard deviation at each point in time.

pre-disruption travel time on both trips, and the response in terms of schedule adjustments. The delay costs vary between individuals due to different preferences, as captured by the parameter values in the marginal cost functions. As in the calibration process, we have calculated the delay costs across a sample of synthetic individuals, each represented by a set of parameter values randomly drawn from normal distributions defined by the means and standard deviations of the parameters shown in Table 1. To explicitly study the influence of scheduling flexibility we set ξ to precise values, 0 or 1, and the location parameters ω_2^1, ω_2^2 are adjusted accordingly as described in Section 4.1.

Figure 4 shows the average cost per hour delay, $\Delta C/\Delta T$ for both a completely fixed ($\xi = 0$) and a completely flexible ($\xi = 1$) work activity. The cost is calculated for delays ranging from 0 to 5 hours symmetrically distributed on the two trips, for the five schedule adjustment profiles described in Section 3.1. The value of time is

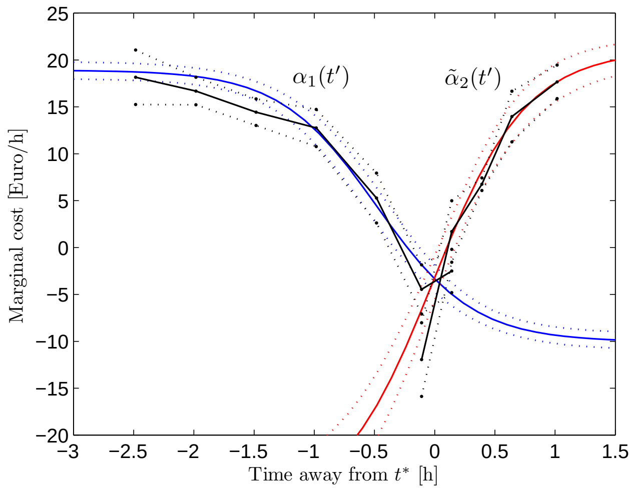

Figure 3: Marginal cost curves relative to the ideal transition time t^* based on random draws from normal distributions defined by the mean and standard deviation of the parameter estimates shown in Table 1. The dotted lines show confidence intervals defined by two times the standard deviation at each point in time. The black lines show the values estimated by Tseng and Verhoef (2008).

calculated for a marginal change in travel time also symmetrical in the two trips, and the baseline travel time is set to 40 minutes on both trips. Baseline travel times and delays are independent of departure times, i.e., there is no time-varying congestion. Table 2 reports results corresponding to Figure 4 for baseline travel times 30 minutes and 60 minutes on each trip.

As can be seen from Figure 4, the delay costs increase rapidly for all adjustment types, although at diminishing rates, reflecting the shapes of the marginal cost functions. By construction the delay cost is lowest for the optimal adjustment profile, since optimizing the schedule against the delays is equivalent to minimizing the travel costs. The costs under the optimal adjustment profile are almost identical for a fixed and a flexible work schedule, which means that we can make relevant comparisons for the other adjustment profiles.

For the two profiles involving no adjustment of the morning departure time, the costs are slightly higher with a fixed work schedule, which reflects that time lost from work is more costly in the morning than in the evening. Particularly with a flexible schedule, the cost for the no + optimal adjustment type—i.e., travelers who retain their pre-disruption departure time on trip 1 but optimally reschedule trip 2—is only slightly lower than the pure no adjustment profile. In this case there is thus little to be gained from rescheduling the evening trip since it will be optimal to take most of the journey delay in the form of late arrival at home.

The costs for the two over-adjustment profiles are restricted to even smaller inter-

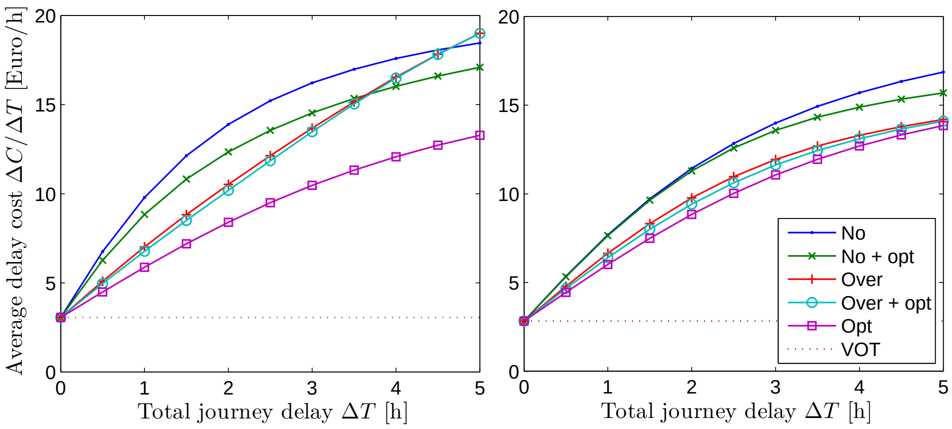

Figure 4: Average delay cost per hour delay, $\Delta C/\Delta T$, with delay symmetrical on the two trips. Baseline travel time is 2 × 40 minutes. Left: Fixed work schedule ($\xi = 0$). Right: Flexible work schedule ($\xi = 1$).

vals, which shows that the early departure and arrival in the morning are the main contributors to the delay costs. These intervals should not be interpreted as upper and lower bounds for travelers who over-estimate delays, since it is of course possible to overestimate delays by more or less than 50% which was assumed here. With a fixed schedule over-adjustment leads to lower costs than no adjustment for moderate delays, but this advantage diminishes and is even reversed for long delays. The high costs for long delays are more due to the inconvenience of arriving early to work than to that of departing early from home. With a flexible schedule the costs of over-adjustment are considerably lower, since there is no cost associated with arriving early to work; on the contrary, early arrival gives more time to spend at work or at home in the evening.

Table 2 shows that delay costs become less dependent on the baseline travel time as delays increase. However, the marginal value of time is significantly higher for longer baseline travel times, which means that delay costs *relative* to the value of time decrease with the baseline travel time. Note that since travel time is treated as exogenous, these are effects of the shape of the marginal utility functions only—we have not taken into account that individuals with different utility functions may choose different baseline travel times according to their preferences (self-selection).

It may be noted that the calculated values of time appear to be significantly lower than those found in most empirical studies, although they are in line with the results of Tseng and Verhoef (2008). Given the many degrees of freedom in the calibration process, we should not put too much significance on the precise numbers. Still, it should be remembered that these values refer to optimally timed trips in very simple daily schedules when there is no time-varying congestion; trips that are not

Delay ΔT	Adj. trip1+2 No	No+opt.	Over	Over+opt.	Optimal
$\xi = 0$					
Base. $T = 1.0$ h	VOT = 1.15 (0.30)				
1.0 h	8.20 (0.33)	7.26 (0.32)	5.20 (0.31)	4.95 (0.31)	4.00 (0.30)
3.0 h	15.3 (0.34)	13.5 (0.37)	12.2 (0.33)	12.0 (0.34)	8.97 (0.30)
5.0 h	17.9 (0.34)	16.3 (0.37)	17.9 (0.35)	17.9 (0.35)	12.2 (0.32)
Base. $T = 2.0$ h	VOT = 6.82 (0.31)				
1.0 h	12.7 (0.33)	11.8 (0.33)	10.4 (0.32)	10.2 (0.32)	9.37 (0.31)
3.0 h	17.7 (0.34)	16.4 (0.37)	16.2 (0.34)	16.1 (0.35)	13.1 (0.33)
5.0 h	19.4 (0.36)	18.4 (0.36)	20.8 (0.37)	20.8 (0.37)	15.1 (0.35)
$\xi = 1$					
Base. $T = 1.0$ h	VOT = 0.99 (0.38)				
1.0 h	5.98 (0.38)	5.93 (0.38)	4.86 (0.39)	4.61 (0.39)	4.20 (0.39)
3.0 h	12.9 (0.33)	12.6 (0.34)	10.7 (0.36)	10.3 (0.35)	9.66 (0.36)
5.0 h	16.2 (0.33)	15.1 (0.36)	13.4 (0.36)	13.2 (0.35)	12.9 (0.35)
Base. $T = 2.0$ h	VOT = 6.49 (0.40)				
1.0 h	10.8 (0.37)	10.8 (0.37)	9.94 (0.38)	9.74 (0.38)	9.41 (0.38)
3.0 h	15.9 (0.34)	15.3 (0.36)	14.0 (0.36)	13.9 (0.36)	13.5 (0.36)
5.0 h	18.1 (0.34)	16.7 (0.39)	15.6 (0.37)	15.6 (0.37)	15.5 (0.37)

Table 2: Average delay cost per hour delay, $\Delta C/\Delta T$ (Euro/h). Travel times and delays are symmetrical on the two trips. Standard deviations are shown in parentheses.

optimally timed within more complex schedules and subject to varying congestion may be associated with much higher values of time.

5.1 Influence of delay distribution

We have compared the delay costs for the scenario above in which delay is symmetrical on the two trips with those under two other possible disruption scenarios. The first is a significant but brief disruption (e.g., a severe car crash) that only affects the travel time on trip 1; the second scenario is a disruption that occurs in the middle of the day and only affects the travel time on trip 2. The purpose of the comparison is to investigate the interplay between the increasing marginal delay cost that was found above (which should mean that delay concentrated to one trip has a higher cost) and the possibility to make up for time lost in the morning by rescheduling the remaining day (which should mean that delay has a higher cost if concentrated to trip 2 than to trip 1).

For the first scenario with delay concentrated to trip 1 we consider the three basic adjustment profiles—no adjustment, over-adjustment and optimal adjustment—on trip 1; for trip 2 we assume that all travelers are informed that the disruption is over and are able to adjust the departure time optimally given the arrival time to

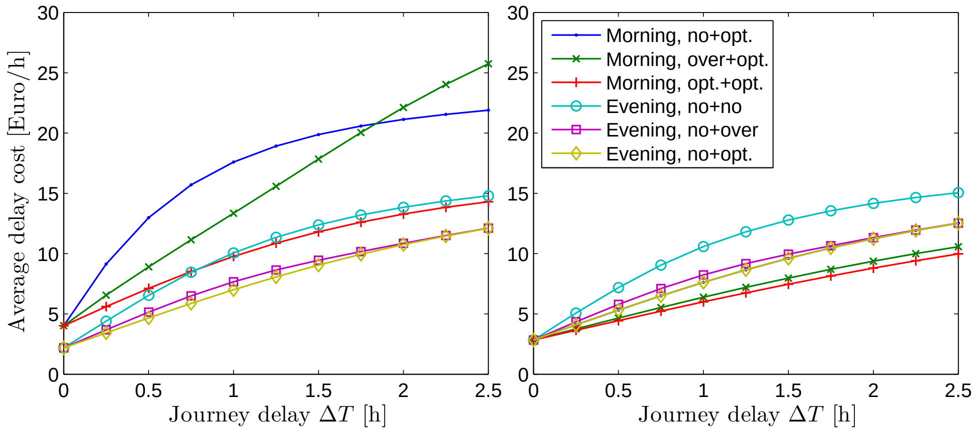

Figure 5: Average delay cost per hour delay, $\Delta C/\Delta T$, with delay only on the morning trip or only on the evening trip. Baseline travel time is 2×40 minutes. Left: Fixed work schedule ($\xi = 0$). Right: Flexible work schedule ($\xi = 1$).

work. Correspondingly, for the second scenario with delay concentrated to trip 2 we consider the same type of adjustment on trip 2 (no, over- and optimal adjustment given delay and arrival time to work); we assume that all travelers are uninformed of the disruption to occur later in the day when they depart in the morning and make no adjustment of the departure time for trip 1.

Figure 5 shows the delay cost in each of the two alternative scenarios for a fixed and a flexible work schedule. Again, the baseline travel time is 40 minutes on each trip, so that results can be compared with Figure 4 above. Considering first the scenario with delay only on the evening trip, we see that the delay costs under each of the three adjustment profiles are very similar with a fixed and a flexible work schedule. In other words, there is in this case no benefit of having a flexible schedule, since there is no more slack in the schedule to make up for the delay that day. For both levels of flexibility, some reduction in cost can be gained from rescheduling the evening trip.

With a fixed work schedule the scenario with delay only on the morning trip leads to higher costs than the scenario with delay only in the evening. This is because the marginal costs increase more steeply in the morning and there is no possibility to compensate for time lost in the morning later in the day. The costs vary greatly depending on how well the traveler is able to anticipate the delay and adjust her departure time in the morning. Note that the cost curves for the two scenarios converge to two different values as the delay tends to zero, representing the different values of time on the two trips.

With a fully flexible work schedule the costs for the scenario with delay only on the morning trip are drastically lower than with a fixed schedule. Furthermore, the costs are lower than with delays only in the evening. This clearly illustrates that flexibility in the timing of activities can have a significant influence on the cost. Note that all cost curves converge to the single value of time as the delay tends to zero. Another effect of a completely flexible work activity is that the costs of the no+optimal adjustment profile for the morning disruption scenario and the evening disruption scenario are identical.

6 Discussion and conclusion

The main point of this paper has been to introduce activity-based modeling as a viable approach to evaluating the traveler costs of transport network disruptions. The modeling framework is able to handle several important aspects of such events: journey delays may be very long in relation to normal day-to-day travel time fluctuations; the impact of delay may depend on the flexibility to reschedule activities; lack of information and uncertainty about travel conditions may lead to under- or over-adjustment of the daily schedule in response to the delay; delays on more than one trip may restrict the gain from rescheduling activities.

The numerical calculations show that the average cost per hour delay increase with the delay duration, so that every additional minute of journey delay comes with a higher cost. The cost varies depending on adjustment behavior (less adjustment, loosely speaking, giving higher cost) and scheduling flexibility (greater flexibility giving lower cost). The results also show that with a fixed work schedule, a disruption affecting only the morning commute is more costly than if the same total delay is equally distributed on both trips, which in turn is more costly than a disruption affecting only the evening commute. With a flexible work schedule on the other hand, a disruption in the morning only is less costly than a disruption in the evening, reflecting the benefit of being able to adjust the remaining schedule in response to earlier events.

The delay costs are typically significantly higher than the ordinary value of time, which suggests that existing evaluations of real network disruptions such as the 1994 Northridge earthquake (Wesemann et al., 1996), the 2006 landslide in Småröd, Sweden (MSB, 2009) and the 2007 bridge collapse in Minneapolis (Xie and Levinson, 2009), have underestimated the societal costs of the disruptions. It also indicates that actions taken to reduce the impacts of a disruption, such as reducing the time until the network capacity is restored or providing up-to-date and relevant information, could have greater economic benefits than previously estimated. By comparing the delay costs of an uninformed versus an informed traveler we get a measure with which the value of information can be assessed.

The model employed here is quite simple, involving only three activities, two trips and two departure times as decision variables. It is in principle straightforward to extend the model to include more activities and trips as well as day-to-day dependencies. This could also include other choice dimensions such as route, destination and mode choice. Although the rapidly increasing number of decision variables would make the analysis (and estimation) increasingly challenging, the essential features would remain the same: delays will give less time to take part in activities, which leads to costs that depend on scheduling flexibility, adjustment response, etc. The concept of the backward optimal marginal utility function (and its counterpart the forward optimal marginal utility function introduced in the Appendix), properly generalized, is still relevant for calculating the value of time and delay costs on different trips. It is reasonable to assume that more complex schedules, involving more scheduling constraints, will lead to higher costs in the event of large delays. On the other hand, some delays may be avoided by rearranging the schedule and traveling to other locations, in particular if they occur early in the day.

We have assumed in this paper that the marginal utility of travel is constant and independent of the duration of the trip. This means that delay costs arise only due to the time lost from activities. In reality it is reasonable that, beyond the normal travel time, the marginal utility of travel decreases with the delay due to fatigue, discomfort, irritation, etc. If this effect is found empirically and is incorporated in the model, the cost of long delays could well be greater than what has been indicated here.

For future applications of models of this kind there is a need to develop the estimation procedure of the utility functions. For our purposes it is necessary to estimate both individuals' preferences among possible schedules and the substitution rates between money compensations and deviations from the optimal schedules. Estimation of activity-based models is currently an active research topic (e.g., Timmermans, 2005). It is also possible that approaches may be adopted from the schedule delay and travel time reliability literature reliability (e.g., Hess et al., 2007).

An interesting topic for future work is to integrate our cost model with a model of the dynamic traffic evolution following a disruption, that would provide the travel times as input to the cost model. In a fully integrated model these costs would, in turn, affect traveler's decisions the following days, which give rise to new costs, etc. Indeed, a cost model such as the present provides an economic foundation for determining the day-to-day evolution of traffic which is often modeled using quite ad hoc iterative schemes. Another direction for further work is to treat travel times as stochastic and integrate the cost of vulnerability and the value of reliability in the same framework and analysis.

Besides travelers' delay costs, there may be many other impacts that contribute to the total societal costs of transport network disruptions. For firms, employees arriving late, delayed deliveries, cancelled business meetings, etc. lead to productivity losses. A large disruption may cause a substantial part of the work force of a firm to arrive late, which might reduce the impact for the individual but increase the cost for the firm. There are also welfare losses associated with not being able to reach societal services such as emergency health care as fast as normally. Analyzing and synthesizing these other costs are topics for further research.

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A Appendix: Mathematical derivations

Here we derive expressions for the optimal schedule adjustments following marginal changes in travel time in the general case when travel times are functions of departure times, $T_i(t_{di}) = T_i + T_{xi}(t_{di})$, $i = 1, 2$. For brevity, not all expressions are written out explicitly but can be obtained by merging the constituting components.

Restating equation (1), the utility of the daily schedule (t_{d1}, t_{d2}) given fixed travel time components T_1, T_2 is

$$U(t_{d1}, t_{d2}; T_1, T_2) \equiv \int_0^{t_{d1}} u_1(t)dt + \int_{t_{d1}+T_1+T_{x1}(t_{d1})}^{t_{d2}} u_2\big(t - \xi[t_{d1} + T_1 + T_{x1}(t_{d1})]\big)dt \\ + \int_{t_{d2}+T_2+T_{x2}(t_{d2})}^{1} u_3(t)dt + \nu[T_1 + T_{x1}(t_{d1}) + T_2 + T_{x2}(t_{d2})]. \tag{21}$$

First-order partial derivatives are given by

$$\frac{\partial U}{\partial t_{d1}} = u_1(t_{d1}) - \nu \\ - [1 + T'_{x1}(t_{d1})]\big[[1 - \xi]u_2([1 - \xi]t_{s2}) + \xi u_2(t_{d2} - \xi t_{s2}) - \nu\big], \tag{22}$$

$$\frac{\partial U}{\partial t_{d2}} = u_2(t_{d2} - \xi t_{s2}) - \nu - [1 + T'_{x2}(t_{d2})][u_3(t_{s3}) - \nu]. \tag{23}$$

First-order necessary optimality conditions require that $\partial U/\partial t_{d1} = \partial U/\partial t_{d2} = 0$ at (t^*_{d1}, t^*_{d2}).

Total differentiation of (21) at optimum gives

$$\frac{dU^*}{dT_i} = \frac{dt^*_{d1}}{dT_i}[u_1(t^*_{d1}) - \nu] - \frac{dt^*_{s2}}{dT_i}\big[[1-\xi]u_2([1-\xi]t^*_{s2}) + \xi u_2(t^*_{d2} - \xi t^*_{s2}) - \nu\big]$$
$$+ \frac{dt^*_{d2}}{dT_i}[u_2(t^*_{d2} - \xi t^*_{s2}) - \nu] - \frac{dt^*_{s3}}{dT_i}[u_3(t^*_{s3}) - \nu], \quad i = 1, 2. \tag{24}$$

A.1 The morning trip

The utility from the arrival to activity 2 at time t_{s2} and onward given departure time t_{d2} is

$$\tilde{U}_2(t_{s2}, t_{d2}) \equiv \int_{t_{s2}}^{t_{d2}} u_2(t - \xi t_{s2})dt + \int_{t_{d2}+T_2(t_{d2})}^{1} u_3(t)dt + \nu T_2(t_{d2}). \tag{25}$$

To satisfy optimality condition (23), a change in arrival time t_{s2} incurs an adjustment of the optimal departure time $t^*_{d2}(t_{s2}) \equiv \arg\max_{t_{d2}} \tilde{U}_2(t_{s2}, t_{d2})$ and associated arrival time $t^*_{s3}(t_{s2})$ given by

$$\frac{dt^*_{d2}}{dt_{s2}} = -\frac{\partial^2 \tilde{U}_2/\partial t_{s2}\partial t_{d2}}{\partial^2 \tilde{U}_2/\partial t^2_{d2}}, \tag{26}$$

$$\frac{dt^*_{s3}}{dt_{s2}} = \big[1 + T'_{x2}\big(t^*_{d2}(t_{s2})\big)\big]\frac{dt^*_{d2}}{dt_{s2}}, \tag{27}$$

evaluated at $(t_{s2}, t^*_{d2}(t_{s2}))$, where

$$\frac{\partial^2 \tilde{U}_2}{\partial t_{s2}\partial t_{d2}} = -\xi u'_2(t_{d2} - \xi t_{s2}), \tag{28}$$

$$\frac{\partial^2 \tilde{U}_2}{\partial t^2_{d2}} = u'_2(t_{d2} - \xi t_{s2}) - [1 + T'_{x2}(t_{d2})]^2 u'_3(t_{s3}) - T''_{x2}(t_{d2})[u_3(t_{s3}) - \nu]. \tag{29}$$

Note: $\xi = 0$ gives $dt^*_{d2}/dt_{s2} = 0$, while $\xi = 1$ with constant travel times gives $dt^*_{d2}/dt_{s2} = u'_2(t^*_{d2}(t_{s2}) - t_{s2})/[u'_2(t^*_{d2}(t_{s2}) - t_{s2}) - u'_3(t^*_{s3}(t_{s2}))]$. Furthermore, our assumptions that $u'_2(t_{d2} - \xi t_{s2}) < 0$ and $u'_3(t_{s3}) > 0$ and fixed travel times give $dt^*_{d2}/dt_{s2} \in [0, \xi]$ for any ξ.

The "backward optimal" marginal utility function is obtained as

$$\tilde{u}_2(t_{s2}) \equiv -\frac{d\tilde{U}_2(t_{s2}, t^*_{d2}(t_{s2}))}{dt_{s2}} = [1-\xi]u_2([1-\xi]t_{s2}) + \xi u_2\big(t^*_{d2}(t_{s2}) - \xi t_{s2}\big). \tag{30}$$

Note: $\xi = 0$ gives $\tilde{u}_2(t_{s2}) = u_2(t_{s2})$ and $\xi = 1$ gives $\tilde{u}_2(t_{s2}) = u_2(t^*_{d2}(t_{s2}) - t_{s2})$.

The derivative of $\tilde{u}_2$ with respect to the arrival time t_{s2} is

$$\tilde{u}'_2(t_{s2}) = [1-\xi]^2 u'_2([1-\xi]t_{s2}) + \xi\left[\frac{dt^*_{d2}(t_{s2})}{dt_{s2}} - \xi\right] u'_2\big(t^*_{d2}(t_{s2}) - \xi t_{s2}\big). \tag{31}$$

The total daily utility given departure times t_{d1}, $t^*_{d2}(t_{s2})$ and fixed travel time components T_1, T_2 is

$$U_1(t_{d1}; T_1, T_2) \equiv U\big(t_{d1}, t^*_{d2}\big(t_{d1} + T_1 + T_{x1}(t_{d1})\big); T_1, T_2\big) \tag{32}$$

To satisfy optimality condition (22), an increase in travel time T_1 incurs an adjustment of the optimal departure time t^*_{d1} and associated arrival time t^*_{s2} given by

$$\frac{dt^*_{d1}}{dT_1} = -\frac{\partial^2 U_1/\partial t_{d1}\partial T_1}{\partial^2 U_1/\partial t^2_{d1}}, \tag{33}$$

$$\frac{dt^*_{s2}}{dT_1} = 1 + [1 + T'_{x1}(t^*_{d1})]\frac{dt^*_{d1}}{dT_1}, \tag{34}$$

evaluated at t^*_{d1}, where

$$\frac{\partial^2 U_1}{\partial t_{d1}\partial T_1} = -[1 + T'_{x1}(t_{d1})]\tilde{u}'_2(t_{s2}), \tag{35}$$

$$\frac{\partial^2 U_1}{\partial t^2_{d1}} = u'_1(t_{d1}) - [1 + T'_{x1}(t_{d1})]^2\tilde{u}'_2(t_{s2}) - T''_{x1}(t_{d1})[\tilde{u}_2(t_{s2}) - \nu]. \tag{36}$$

Note: With constant travel times, $\xi = 0$ gives

$$\frac{dt^*_{d1}}{dT_1}_{\xi=0} = \frac{u'_2(t^*_{s2})}{u'_1(t^*_{d1}) - u'_2(t^*_{s2})}, \tag{37}$$

while $\xi = 1$ gives

$$\frac{dt^*_{d1}}{dT_1}_{\xi=1} = \frac{u'_2(t^*_{d2} - t^*_{s2})u'_3(t^*_{s3})}{u'_1(t^*_{d1})u'_2(t^*_{d2} - t^*_{s2}) - u'_1(t^*_{d1})u'_3(t^*_{s3}) - u'_2(t^*_{d2} - t^*_{s2})u'_3(t^*_{s3})}. \tag{38}$$

Furthermore, our assumptions about the shape of the marginal utility functions and constant travel times give $dt^*_{d1}/dT_1 \in [-1, 0]$ and $dt^*_{s2}/dT_1 \in [0, 1]$ for any ξ.

Finally, we have $dt^*_{d2}/dT_1 = dt^*_{d2}/dt_{s2} \cdot dt^*_{s2}/dT_1$ and $dt^*_{s3}/dT_1 = dt^*_{s3}/dt_{s2} \cdot dt^*_{s2}/dT_1$.

A.2 The evening trip

The derivations for the evening trip are analogous to those for the morning trip but are carried out here for completeness. The utility up to the departure from activity 2 at time t_{d2} given departure time t_{d1} from activity 1 is

$$\hat{U}_2(t_{d1}, t_{d2}) \equiv \int_0^{t_{d1}} u_1(t)dt + \int_{t_{d1}+T_1(t_{d1})}^{t_{d2}} u_2\big(t - \xi[t_{d1} + T_1(t_{d1})]\big)dt + \nu T_1(t_{d1}). \tag{39}$$

To satisfy optimality condition (22), a change in departure time t_{d2} incurs an adjustment of the optimal departure time $t^*_{d1}(t_{d2}) \equiv \arg\max_{t_{d1}} \hat{U}_2(t_{d1}, t_{d2})$ and associated arrival time $t^*_{s2}(t_{d2})$ given by

$$\frac{dt^*_{d1}}{dt_{d2}} = -\frac{\partial^2 \hat{U}_2/\partial t_{d1}\partial t_{d2}}{\partial^2 \hat{U}_2/\partial t^2_{d1}}, \tag{40}$$

$$\frac{dt^*_{s2}}{dt_{d2}} = \big[1 + T'_{x1}\big(t^*_{d1}(t_{d2})\big)\big]\frac{dt^*_{d1}}{dt_{d2}}, \tag{41}$$

evaluated at $(t^*_{d1}(t_{d2}), t_{d2})$, where

$$\frac{\partial^2 \hat{U}_2}{\partial t_{d1}\partial t_{d2}} = -\xi[1 + T'_{x1}(t_{d1})]u'_2(t_{d2} - \xi t_{s2}), \tag{42}$$

$$\begin{aligned}\frac{\partial^2 \hat{U}_2}{\partial t^2_{d1}} &= u'_1(t_{d1}) - [1 + T'_{x1}(t_{d1})]^2\big[[1-\xi]^2 u'_2([1-\xi]t_{s2}) - \xi^2 u'_2(t_{d2} - \xi t_{s2})\big] \\ &\quad - T''_{x1}(t_{d1})\big[[1-\xi]u_2([1-\xi]t_{s2}) + \xi u_2(t_{d2} - \xi t_{s2}) - \nu\big].\end{aligned} \tag{43}$$

Note: $\xi = 0$ gives $dt^*_{d1}/dt_{d2} = 0$, while $\xi = 1$ with constant travel times gives $dt^*_{d1}/dt_{d2} = u'_2(t_{d2} - t^*_{s2}(t_{d2}))/[u'_1(t^*_{d1}(t_{d2})) + u'_2(t_{d2} - t^*_{s2}(t_{d2}))]$. Furthermore, our assumptions that $u'_1(t_{d1}) < 0$, $u'_2([1-\xi]t_{s2}) > 0$ and $u'_2(t_{d2} - \xi t_{s2}) < 0$ and constant travel times give $dt^*_{d1}/dt_{d2} \in [0, 1/\xi]$ for any $\xi > 0$.

The "forward optimal" marginal utility function is obtained as

$$\hat{u}_2(t_{d2}) \equiv \frac{d\hat{U}_2(t^*_{d1}(t_{d2}), t_{d2})}{dt_{d2}} = u_2\big(t_{d2} - \xi\big[t^*_{d1}(t_{d2}) + T_1\big(t^*_{d1}(t_{d2})\big)\big]\big). \tag{44}$$

The derivative of $\hat{u}_2$ with respect to the departure time t_{d2} is

$$\hat{u}'_2(t_{d2}) = \left[1 - \xi\frac{dt^*_{d1}(t_{d2})}{dt_{d2}}\big[1 + T'_{x1}\big(t^*_{d1}(t_{d2})\big)\big]\right]u'_2\big(t_{d2} - \xi t^*_{s2}(t_{d2})\big). \tag{45}$$

The total daily utility given departure times $t^*_{d1}(t_{d2})$, t_{d2} and fixed travel time components T_1, T_2 is

$$U_2(t_{d2}; T_1, T_2) \equiv U\big(t^*_{d1}(t_{d2}), t_{d2}; T_1, T_2\big) \tag{46}$$

To satisfy optimality condition (23), an increase in travel time T_2 incurs an adjustment of the optimal departure time t^*_{d2} and associated arrival time t^*_{s3} given by

$$\frac{dt^*_{d2}}{dT_2} = -\frac{\partial^2 U_2/\partial t_{d2}\partial T_2}{\partial^2 U_2/\partial t^2_{d2}}, \tag{47}$$

$$\frac{dt^*_{s3}}{dT_2} = 1 + [1 + T'_{x2}(t_{d2})]\frac{dt^*_{d2}}{dT_2}. \tag{48}$$

evaluated at t_{d2}^*, where

$$\frac{\partial^2 U_2}{\partial t_{d2} \partial T_2} = -[1 + T'_{x2}(t_{d2})] u'_3(t_{s3}), \tag{49}$$

$$\frac{\partial^2 U_2}{\partial t_{d2}^2} = \hat{u}'_2(t_{d2}) - [1 + T'_{x2}(t_{d2})]^2 u'_3(t_{s3}) - T''_{x2}(t_{d2})[u_3(t_{s3}) - \nu]. \tag{50}$$

Note: With constant travel times, $\xi = 0$ gives

$$\frac{dt_{d2}^*}{dT_2}_{\xi=0} = \frac{u'_3(t_{s3}^*)}{u'_2(t_{d2}^*) - u'_3(t_{s3}^*)}, \tag{51}$$

while $\xi = 1$ gives

$$\frac{dt_{d2}^*}{dT_2}_{\xi=1} = \frac{u'_1(t_{d1}^*) u_3(t_{s3}^*) + u'_2(t_{d2}^* - t_{s2}^*) u'_3(t_{s3}^*)}{u'_1(t_{d1}^*) u'_2(t_{d2}^* - t_{s2}^*) - u'_1(t_{d1}^*) u'_3(t_{s3}^*) - u'_2(t_{d2}^* - t_{s2}^*) u'_3(t_{s3}^*)}. \tag{52}$$

Furthermore, our assumptions about the shape of the marginal utility functions and constant travel times give $dt_{d2}^*/dT_2 \in [-1, 0]$ and $dt_{s3}^*/dT_2 \in [0, 1]$ for any ξ.

Finally, we have $dt_{d1}^*/dT_2 = dt_{d1}^*/dt_{d2} \cdot dt_{d2}^*/dT_2$ and $dt_{s2}^*/dT_2 = dt_{s2}^*/dt_{d2} \cdot dt_{d2}^*/dT_2$.

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