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The Value of New Public Transport Links for Network Robustness and Redundancy

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ABSTRACT

A common argument for introducing new links or services to transport networks is that they will contribute to greater capability to withstand system breakdowns. This paper presents a methodology for assessing the value of new links for public transport network robustness, considering disruptions of other lines and links as well as the new links themselves. The value is evaluated in terms of passenger welfare under disruptions and can be compared to traditional welfare benefits and investment costs. Distinction is made between the value of robustness, defined as the change in welfare during disruption compared to the baseline network, and the value of redundancy, defined as the change in welfare losses due to disruption. The paper introduces the total values of robustness and resilience by considering a full space of scenarios and their respective frequencies. Using a model that considers passengers’ dynamic travel choices, stochastic traffic conditions, timetables and capacity constraints, results are more nuanced than analyses based only on network topology and other static attributes. A new cross-radial light rail transit line in Stockholm, Sweden is evaluated. The new link increases welfare levels under all scenarios and has a positive value of robustness. However, disruption costs increase under some scenarios and the value of redundancy is negative. In general, the value of redundancy depends on the new link’s role as complement or substitute and passengers’ ability to utilize spare capacity during short-term unexpected disruptions.

1. INTRODUCTION

Public transport systems are sensitive to degradations of the technical or physical infrastructure, for example electrical failures and malfunctioning vehicles. Disruptions can also arise from degradations of the services, such as crew strikes and accidents. Due to rigid constraints in terms of line operations, timetables, vehicle and personnel stock etc., service disruptions in public transport networks are prone to having wide and sustained implications. In addition to the immediate effect on the network links directly concerned, the dynamic nature of public transport supply results in impacts on service availability and capacity further downstream and in some cases even upstream. In case of disruptions, the impact for travellers in terms of delays and inconvenience depends on the availability of alternative travel options, i.e., the amount of redundancy in the public transport network.
Given the significant role of urban public transport systems for accessibility and sustainability, an important goal in planning and management is to increase their robustness, i.e., their ability to maintain functionality under service disruptions. Although the main reason for investing in new network capacity is typically to reduce travel times during normal operating conditions, the argument that the new link or lines will add redundancy and increase the robustness of the transport system is also often heard (in Sweden, a current example is the planned freeway bypass of Stockholm which is intended to relieve the inner city from through traffic). However, while the effects on travel times and welfare under normal conditions are often assessed with planning models and cost-benefit analysis, the claims of increased robustness are in general not backed up by evidence from real evaluations or simulations.

The purpose of this paper is thus to develop a methodology for evaluating the value for robustness and redundancy of extending the public transport network. In particular, the aim is to assess whether a proposed new link or line indeed contributes to make the system more robust or not, to disentangle and compare this effect with the general increase in network efficiency under normal conditions, and to express the value of robustness in monetary welfare terms.

From a network topological viewpoint, the argument that new links increase robustness makes intuitive sense, and previous studies have suggested that network topology has effects on public transport robustness. It is well-known that the connectivity of radial networks is highly vulnerable to disruption, isolating one branch from the remaining network. Derrible and Kennedy (2010a,b) suggested that robustness of subway systems corresponds to the number of cyclic paths available in the network, representing the possibility to use alternative routes under disruption. According to this criterion, the subways of Tokyo and Seoul are particularly robust but also those of Madrid, Paris, Osaka, London and Moscow.

Ash and Newth (2007) developed an evolutionary algorithm for adding links that improve network robustness. Based on the algorithm results they concluded that the most robust networks were characterized by high clustering, modularity (i.e., a structure of highly connected sub-networks, loosely connected to other sub-networks) and long path length compared with random network evolution. Meanwhile, Roth et al. (2012) showed that the world’s largest subway networks share a very similar topology, with a central core from which branches radiate.

Recently, a number of studies of public transport robustness have extended the topological analysis to system-based approaches that consider further aspects of demand and supply. Rodriguez-Nunez and Garcia-Palomares (2014) assumed that link travel times and an OD travel demand matrix are known and that travelers choose the fastest route in the network to reach their destinations. They defined the importance of a link as the unsatisfied demand, i.e., the number of trips that cannot be carried out, or as the increase in average travel time assuming that affected travelers make the fastest available detour, and emphasized the importance of circular lines in providing travel alternatives in case of disruptions. De-Los-Santos et al. (2012) performed a similar network scan evaluation while also considering the case of a replacement service for the closed link. Jin et al. (2014) evaluated the lack of resilience of metro networks as the expected share of unsatisfied demand under a set of disruption scenarios. They considered a two-stage stochastic programming model to increase resilience by better integration with and higher capacity on parallel bus lines which was applied to real data from Singapore. For a more thorough discussion of recent studies of public transport robustness and resilience, see Mattsson and Jenelius (2015).

A limitation of previous studies is that specialized measures are used to evaluate disruption impacts, such as the amount of unsatisfied demand. While such analyses reveal many useful insights, they do not fit into the standard appraisal schemes used to assess the value of new links and lines. Hence, it is difficult to use the results to compare the magnitude
of robustness effects of new lines and links with the increase in welfare under normal operating conditions and the investment costs. As a result, the methods do not give guidance as to whether, and how much, robustness effects should influence the prioritization of alternative new investments.

Also, while several of the aforementioned studies have considered some aspects of demand and supply such as OD travel patterns and link travel times, none of the evaluations have taken into consideration the underlying system dynamics and their implications on cascading effects and adaptive rerouting travel decisions. In practice, passengers are likely to adjust their travel patterns and expectations according to the existence of the new links; however, the new links themselves may suffer disruptions as well. Furthermore, travellers’ ability to respond to an unexpected disruption in the short term may be far from optimal due to limited information and service constraints. The net impact on robustness is thus less than obvious.

The small example network in Figure 1 serves to illustrate the point. The baseline undirected network (Figure 1, left) consists of four nodes (A, B, C and D) and three links (AB, BD and DC). This tree network has no redundancy, so that travellers exercise no path choice and a disruption of any link will disconnect the network with no alternative travel options available for passengers. By adding a link between A and C (Figure 1, middle), trips between each pair of origin-destination nodes can go through two paths under normal operations. For example, trips between A and D can go through either node B or C, potentially enabling travellers to gain travel time savings. Moreover, the network provides a circuit which ensures that the removal of a single link will not result in a disconnected network. If the network is augmented with another link AD (Figure 1, right), this new link creates an additional route between each node pair, increasing network connectivity and redundancy.

![Figure 1. Simple network illustration. The baseline network (left) lacks redundancy, the circuit provides redundancy (middle) which could be enhanced by improved connectivity (right)](image)

The benefits of each additional link may hence consist of time savings under normal operations as well as in case of disruptions. The benefits induced by each additional link in case of a disruption depend on travellers’ ability to change routes in the short term. A static topological analysis considers an extended network with a disrupted link (e.g. Figure 1, right, with a disruption on link AD) as equivalent to a network without the disrupted link (e.g. Figure 1, middle). However, in reality these two situations may result in different passenger loads and travel times due to the dynamics of delay propagation and passenger rerouting decisions. This may even result in counter-productive results where the network is worse-off in case of disruption on the new link, giving rise to a paradox resembling the well-known Braess’s paradox (Braess et al. 2005).

In this paper, the value of robustness of the network extension is defined as the change in passenger welfare under network disruptions in the extended network compared to the baseline network. In general, a network extension increases service quality and welfare for some passengers under any conditions, as well as serves as an alternative travel option for other passengers during disruptions, and the value of robustness encapsulates both potential benefits. To isolate the benefit as an alternative travel option, the value of redundancy of the
network extension is defined as the value of robustness minus the difference in welfare under normal conditions in the extended network compared to the baseline network. Together, the two measures give a fuller representation of the contribution of new links to the ability of the network to withstand degradation.

The methodology builds on the dynamic, stochastic and multimodal notion of public transport network vulnerability, accounting for interactions between supply and demand and the accumulated effect of disruption on system performance, introduced by Cats and Jenelius (2014). A probabilistic path choice process is used to model passenger decisions, where the evaluation of alternative paths depends on passenger’s preferences and perceptions. The impact of link and line disruption is evaluated as the reduction in welfare (considering travel time, number of transfers, etc.) due to the disruption. The current paper expands upon existing methodology by considering a full space of disruption scenarios and associating each scenario with its frequency of occurrence. Unlike previous work, this framework can be used to assess the full long-term robustness and redundancy value of a new link and put this in relation to investment and maintenance costs as well as standard welfare effects.

The methodology is applied in a real-world case study for the high frequency public transport network of Stockholm, Sweden and the introduction of a cross-radial light rail line. A full-scan vulnerability analysis is carried out for both the baseline and the extended network, considering disruptions on every public transport line. To evaluate system performance under varying conditions, a dynamic public transport operations and assignment model, BusMezzo, is used.

The paper is organized as follows. Section 2 describes the methodology and the dynamic public transport model used in the application, Section 3 describes the design of the case study while results are presented and discussed in Section 4; Section 5 concludes the paper.

2. METHODOLOGY

2.1 Scenario welfare assessment

A public transport network expansion is an addition to the baseline network of one or several public transport lines, and/or a set of nodes and links representing new stops and connections. The focus of this paper is on two dimensions regarding the state of the network: (1) the network configuration that is available, i.e., the baseline network or the extended network, and (2) the network element that is disrupted, i.e., the set of disrupted links or lines. Other factors such as the start time and duration of the disruption are held fixed in all scenarios. Let $M_0$ denote the baseline network, and let $M = M_0 \cup \Delta M$ denote an extended network with new nodes, links and lines $\Delta M$. Further, let $\delta = 0$ denote a scenario with no disruption. A disruption scenario involving network configuration $M$ and disrupted network element $\delta$ can then be summarized as the pair $(M, \delta)$.

Travel demand is assumed to be connected to the network through a set of origin-destination (OD) nodes, $S_{OD}$. The set of travellers from origin $o \in S_{OD}$ to downstream destination $d \in S_{OD}$ on a given day is denoted $N_{od}$. The number of travellers may be stochastic to represent day-to-day variations. However, the demand for public transport is assumed here to be inelastic, that is, independent of changes in travel times etc. Passengers choose paths to maximize utility (e.g., a combination of in-vehicle time, walking time, waiting time, etc.) In general, which path between $o$ and $d$ a given traveller chooses a given day and time-of-day will depend on the properties of the different public transport lines and on prevailing service conditions, according to the preferences of the individual.
From the passengers’ perspective, the performance of a public transport system can be evaluated in terms of passenger welfare, the total utility or negative generalized travel cost of the passengers (Jenelius and Mattsson 2015). Since travel demand levels and line schedules are time-dependent, so are the impacts of network disruption. Day-to-day variations in demand and supply further imply that the impacts are stochastic a priori.

In this paper welfare is evaluated with a generalized cost function that is a combination of four factors: in-vehicle time, waiting time, walking time and number of transfers. With \( W_n(M, \delta) \) denoting the welfare of passenger \( n \) in scenario \((M, \delta)\), the expected total daily welfare in scenario \((M, \delta)\) across day-to-day supply and demand variations is

\[
W(M, \delta) = E\left[\sum_{o \in S_{cd}} \sum_{d \in S_{ad}} \sum_{n \in N_{cd}} W_n(M, \delta)\right].
\]

### 2.2 Values of robustness and redundancy

The value of a network expansion under normal operating conditions is evaluated as the difference in total expected welfare in the extended network compared to the baseline network,

\[
W(M+,0) - W(M_0,0).
\]

Thus, the value of network expansion is evaluated in terms of the overall impact on passenger in-vehicle time, waiting time, walking time and number of transfers.

The value of network extension \( \Delta M \) for robustness is evaluated as the difference in welfare during the disruption in extended network \( M+ \) compared to baseline network \( M_0 \). In other words, the value of robustness captures the change in the level of welfare that the transport system is able to maintain under a disruption. For a particular disruption scenario \( \delta \),

\[
VRob(\Delta M \mid M_0, \delta) = W(M+, \delta) - W(M_0, \delta).
\]

The robustness value can in general be both positive and negative depending on the element and disruption scenario, but except for Braess-like situations the value is expected to be positive in most cases.

A given network extension \( \Delta M \) may have beneficial effects under several different link disruptions. This paper introduces the total robustness value as the sum of the robustness values across a wide range of feasible scenarios, considering the occurrence frequency of each scenario. Further, it must be considered that the new nodes, links and/or lines forming extension \( \Delta M \) are themselves subject to disruptions. The impact of a disruption of element \( \delta \) in network configuration \( M \) is evaluated as the cost

\[
C(\delta \mid M) = W(M, \delta) - W(M,0).
\]

The larger overall negative impact of the disruption on passenger in-vehicle time, waiting time, etc., the more negative is \( C(\delta \mid M) \). The disruption cost is also a measure of the importance of the disrupted network element (Cats and Jenelius 2014).

Let \( F(\delta) \) denote the frequency (e.g., number of occurrences per year) of disruptions of element \( \delta \). It is assumed here that the network augmentation itself does not affect the disruption frequencies. The total value of robustness of network augmentation \( \Delta M \) (for example, per year) is then
Eq. (5) indicates a way to incorporate robustness effects of new infrastructure projects in economic appraisal schemes such as cost-benefit analysis. Different feasible projects can thus be evaluated based on their overall benefits in relation to their costs.

In general, a network extension increases welfare for some passengers under both normal and disturbed conditions, in addition to reducing impacts during disruptions for some passengers. For example, a new line that shortens travel times for many travellers under normal conditions typically does so also during disruptions of other lines, in particular if the disruption occurs far from the new line. The value of robustness encapsulates both these aspects of passenger benefits. However, it is of interest to isolate the value of the network extension specifically as a means to reduce the impacts of network disruptions from the general increase in the level of welfare. Thus, the value of redundancy of the network extension is defined as the value of robustness minus the difference in welfare under normal conditions in the extended network compared to the baseline network. For a particular disruption scenario $\delta$,

$$V_{\text{Red}}(\Delta M \mid M_0, \delta) = V_{\text{Rob}}(\Delta M \mid M_0, \delta) - [W(M_+, 0) - W(M_0, 0)]$$

(6)

redundancy of network extension $\Delta M$ is the difference between the disruption impact in the augmented network $M_+$ and the baseline network $M_0$,

$$V_{\text{Red}}(\Delta M \mid M_0, \delta) = C(\delta \mid M_+) - C(\delta \mid M_0).$$

(7)

The more the network extension reduces the cost of disruption scenario $\delta$, the larger is the value of redundancy. The value is negative, however, if the disruption cost is larger in the extended network compared to the baseline network. The total value of redundancy across a set of scenarios is

$$V_{\text{Red}}(\Delta M \mid M_0) = \sum_{\delta \neq 0} F(\delta) \cdot V_{\text{Red}}(\Delta M \mid M_0, \delta).$$

(8)

2.3 Dynamic public transport model

Public transport performance is assessed through the dynamic modelling of public transport supply and demand. BusMezzo, a stochastic and multimodal network model, is used for modelling the implications of alternative service disruptions on passenger assignment. The multi-agent public transport operations and assignment model was used in previous studies for modelling service reliability (Toledo et al. 2010), identifying critical links and the impacts of real-time information (Cats and Jenelius 2014) and the value of reserve capacity (Cats and Jenelius 2015).

The physical PTN is defined by a directed graph $G(S, E)$, where the node set $S$ represents stops and rail stations (all called stops here for simplicity), and the link set $E \subseteq S \times S$ represents direct connections between stops. Each link may be operated by one or several public transport lines. A line $l$ is defined by a sequence of stops $l = (s_{l1}, s_{l2}, \ldots, s_{ll})$, where $s_{l1}$ is the origin terminal and $s_{ll}$ is the destination terminal. Each link $e$ is associated with a riding time, which is the time from the departure from the upstream stop to the arrival at the subsequent downstream stop. The riding time may vary between trips and between days depending on the current traffic conditions. Similarly, each stop is associated with a dwell
time, which is the time required for a vehicle to stop for boarding and alighting. Like the riding times, the dwell times may vary between trips and days, depending on the current number of passengers, vehicle type, etc., and may be considered stochastic.

Each line \( l \) is operated with a set of vehicle trips according to a schedule. The departure time of a trip on line \( l \) from the origin terminal is in general a function of a scheduled departure time and the arrival time of the previous trip, which depends on the realized riding and dwell times of that trip.

Similarly to the vehicle lines, the physical path of a traveller is defined by a sequence of stops from the origin to the destination, that is, \( j = (s_{j,1}, s_{j,2}, \ldots, s_{j,|j|}) \), where \( o_j = s_{j,1} \) is the origin stop and \( d_j = s_{j,|j|} \) is the destination stop.

Each passenger and vehicle is represented as an individual agent in the simulation model. Each public transport vehicle is assigned a sequence of trips to carry out within the simulation period, accounting explicitly for terminal times and dispatching control. Each trip prescribes the physical network links that are to be traversed as well as the list of stops to be served along the line. Public transport vehicle progress consists of four time components: running, delay, dwell and terminal time. The public transport simulation model is integrated into an event-based mesoscopic traffic simulation model, where each link is divided dynamically into a running part and a queuing part. Running times are computed based on speed-density functions, while delay times are determined by stochastic queuing servers. Dwell times at stops are the outcome of a flow-dependent function. Vehicle occupancy is updated and capacity constraints are explicitly enforced.

Passengers are generated based on time-dependent origin-destination matrices. Passengers progress in the network by alternately choosing walking links, stops and public transport trips. For each passenger, the selected path is the outcome of a sequence of travel decisions – boarding, alighting and connection decisions. Each en-route travel decision is triggered by a simulation event (e.g., a train arrives at the stop) and involves the choice among a set of possible actions (e.g., whether to board or not). A probabilistic choice model is then deployed by calculating the expected utility of the each path alternative. The utility value depends on the anticipated properties of the different path elements and the preferences of the individual. Model outputs include measures of public transport performance as well as the experienced travel attributes per passenger. Total passenger welfare is calculated by summing over the experienced path utilities.

The dynamic passenger assignment model in BusMezzo enables the analysis of passenger redistribution due to changing service conditions. A short temporary, unplanned link disruption is simulated as a link closure incident, implying that no vehicle can enter the disrupted link during the disruption period. The disruption will have primary effects on passengers on-board vehicles queuing upstream of the disrupted link, and secondary effects on passengers boarding upstream vehicles or waiting at downstream stops. Furthermore, spillover effects can cause cascading impacts due to vehicle delays and passenger rerouting decisions.

3. APPLICATION

3.1 Stockholm rapid public transport network

The method for evaluating the impact of new links on network robustness is applied to the Stockholm rapid public transport network: services that operate with an average scheduled headway up to 5 minutes during the morning peak period. The city is built on fourteen islands and bridges hence constitute bottlenecks in the transport system. The rapid public transport
network consists of metro, trunk bus lines and a light rail train. The impact of the latter on network robustness is the subject of analysis in this case study.

The Stockholm public transport system is characterized by a highly radial structure (Figure 1). The metro network consists of seven lines which form three trunks (Blue line: 10-11; Red line: 13-14; Green line 17-19). It has a low degree of connectivity and mid-level of directness when compared with other metro systems in the world (Derrible and Kennedy 2010). The metro system is well-known for its transit-oriented development along satellite suburbs developed around far-reaching line branches. The three main metro trunks intersect only at one station (T-Centralen), and three additional stations allow transferring between two trunks (Fridhemsplan, Slussen and Gamla Stan). This network structure generates bottlenecks along the three metro trunks where branches merge and lines intersect. Bus trunk lines provide high coverage in the inner-city where the metro system has limited local accessibility.

3.2 A new cross-radial line

In 2002 a light rail transit (LRT) line, Line 22, was launched in order to provide better connections between the southern and western inner suburbs (Figure 2). The line functions as a cross-radial connection between major interchange stations (Gullmarsplan, Liljeholmen and Alvik) with the Green and Red lines. The orbital line provides a new connection between hubs that are strategically located along the southern and western edges of the inner city. Moreover, it allows passengers to travel between the southern and western parts of Stockholm without going through the oversaturated city centre line segments and transfer hubs. The LRT line was constructed with varying types of right-of-way along its corridor including mixed traffic, dedicated tramways and completely elevated tracks. The line is 11.5 km long and serves 17 stops. The LRT is undergoing several developments including an extension towards the northern suburbs providing connections to the Blue Line branches.

The development of the LRT is part of Stockholm’s strategic vision to extend the inner-city boundaries to its edges and stimulate the development of a more polycentric urban area (City of Stockholm 2011). In addition, the investment in the LRT was motivated by its ability to provide new travel alternatives that will relieve the congestion from the most critical metro segments. The availability of alternative connections could be particularly valuable in case of service disruptions. The purpose of this application is to assess the impact of the LRT on network robustness.
3.3 Scenario design

The scenario design consists of a combination of network configuration and service disruption scenarios.

3.3.1 Network configurations

The metro and trunk bus line network prior to the opening of the LRT is considered the baseline network, $M_0$, while the LRT service is added in the extended network, $M_+$. Figure 3 presents two representations of the network graph where nodes correspond to either stops (left) or transfer hubs (right) and links to line segments.
Public transport operations were simulated for the morning peak period (06:00-09:00), while passenger assignment was simulated only during the rush hour (07:00-08:00) in order to allow network warm-up and cleaning periods. Each public transport mode is simulated with distinct vehicle types, vehicle capacity, operating speed, traffic regime, dwell time function and control strategies. These sets of operational attributes yield different reliability and capacity levels depending on service design and right-of-way. Given the service frequency, travellers are assumed to depart randomly from the origins without consulting timetables. The case study network consists of 437 stops and has 700 public transport services which are assigned to 200 vehicles during the morning peak period.

An OD matrix was constructed by applying an iterative proportional fitting method based on a base OD matrix and passenger counts data that were available from the metropolitan public transport agency. A total of approximately 125,000 individual passenger trips are generated during the rush-hour. The overall travel demand is considered fixed under all scenarios, assuming that the temporary disruption does not lead to substantial changes in travel patterns.

A choice-set generation model was performed for each network configuration. The generation process performs a path search technique subject to filtering and dominancy rules. Paths are then merged to hyperpaths based on common stops and lines along the path. The procedure resulted in 321,511 alternative hyperpaths for the baseline network \((M_0)\) and alternative hyperpaths 615,111 for the extended network \((M+)\). The paths of the baseline network are a subset of the paths of the extended network; thus, all paths considered by passengers in the baseline network are also considered in the extended network. The path sets were then used as background choice sets in the dynamic passenger assignment model implemented in BusMezzo. The path utility function is defined as

\[
v_{a,n}(t) = \beta_{a}^{\text{wait}} i_{a,n}^{\text{wait}}(t) + \beta_{a}^{\text{int}} i_{a,n}^{\text{int}}(t) + \beta_{a}^{\text{walk}} i_{a,n}^{\text{walk}} + \beta_{a}^{\text{trans}} \text{trans}_{a},
\]

where \(i_{a,n}^{\text{wait}}(t)\) and \(i_{a,n}^{\text{int}}(t)\) are the time-dependent anticipated waiting time and in-vehicle time, respectively. \(i_{a,n}^{\text{walk}}\) is the expected walking time and \(\text{trans}_{a}\) is the number of transfers involved with the path alternative. Based on model estimation (Cats 2011), parameter values reflect a ratio of 1.75 between in-vehicle and waiting or walking times and a transfer penalty equivalent to approximately 8 in-vehicle minutes. Trip fare is fixed in the Stockholm network for a given OD pair and therefore does not affect passenger path decisions. Passengers are assumed to have prior-knowledge concerning planned headways and scheduled travel times. Real-time information concerning the next expected arrival time is provisioned at each stop across the network.

The value-of-time of in-vehicle time incorporates the impact of crowding based on the guidelines provided in the meta-analysis by Wardman and Whelan (2011). According to this meta-study, the value of in-vehicle time depends on whether a passenger has a seat available or not and the ratio between on-board occupancy and seat capacity where the in-vehicle time multiplier increases as a non-linear function of the load factor.

In order to obtain robust outputs, the stochastic simulation model requires the analysis of a number of simulation replications. The number of replications was calculated based on
the guidelines provided in Dowling et al. (2004) with an initial number of ten replications. All of the reported results were found to obtain in this case study a maximum allowed error of 1%, ensuring adequate statistical significance. The simulation generates a series of output files including the paths that were taken by each traveller and the corresponding disaggregated experienced travel time components. Execution time for a single run was less than 1 minute on a standard PC.

3.3.2 Service disruptions

The performance of both networks, $M_0$ and $M_+$, was analysed under normal operations and severe service disruptions. Note that normal operations include various sources of uncertainty inherent to the public transport operations environment.

Two types of unplanned service disruptions are considered in this study: line and segment breakdowns. A full network scan (i.e., independently considering every line in the network) of partial reductions in line operations was carried out, which enables the approximation of the total value of robustness and redundancy of the new network element. Line breakdowns were conceived as a 50% bi-directional reduction in service frequency. Such a disruption may be caused by for example construction or maintenance works, reduced traffic capacity (e.g., lane or track closure), crew absenteeism, limited strike or other causes of missed departures. The ordinary planned frequency of all metro and trunk bus lines is 12 departures per hour and was thus reduced to 6 departures per hour in the disruption scenarios, whereas the LRT line has normally 8 departures per hour and which decreased to 4 departures per hour in case of disruption.

In addition, system performance is analysed in case of a severe local disruption such as the closure of a critical segment (i.e., a set of consecutive links). Segment breakdowns were simulated as abrupt 30 minutes failures starting from 07:15. Such incidents may be caused in practice by technical or mechanical failures (of vehicles or signals) or infrastructure closure (e.g. traffic accident, terror threat or suicide attempt in the case of rail-bound services). Segments were identified through a generalized measure of passenger betweenness centrality. The procedure described in Cats and Jenelius (2014) found that the following two line segments are among the most critical segments in the case study network (Figure 3):

- Liljeholmen-Slussen (bi-directional), Red line - lines 13 and 14 ($\delta = L\leftrightarrow S$)
- Alvik-Fridhemsplan (bi-directional), Green line - lines 17,18 and 19 ($\delta = A\leftrightarrow F$)

Unexpected segment breakdown implies that vehicles cannot enter or move along the disrupted segment during the disruption period. When the link returns to normal operations at 07:45 traffic on the previously disrupted link resumes and the system recovers gradually. The dynamic public transport simulation model, BusMezzo, considers explicitly the disruption effects on vehicles queuing upstream of the closure, passengers restrained on-board, passengers waiting at downstream stops and secondary effects due to spill-over of rolling stock availability or passenger rerouting.

The model thus facilitates the analysis of upstream, downstream and horizontal cascading effects. In contrast, most previous studies of PTN robustness assumed that only the disrupted links are affected by the disruption while all other links continue to function regularly (e.g., von Ferber et al., 2012). The provision of real-time information enables passengers to reroute according to prevailing conditions. Depending on network configuration, passengers might be able to change paths. However, information is available only at the stops and hence does not alter passengers’ decisions concerning the access stop and alighting stop once on-board.
The scenario design results in 29 scenarios: normal operations of the base and extended networks; line breakdowns for each of the eleven lines included in the base network and for each of the twelve lines included in the extended networks; and two segment breakdown scenarios for each network. Each of the disruption scenarios is denoted by the network \((M_0 \text{ or } M_+)\) and the disrupted line or segment \((1, 2, 3, 4, 10, 11, 13, 14, 17, 18, 19\) or \(22; L\leftrightarrow S \text{ or } A\leftrightarrow F)\).

The construction of the LRT is expected to contribute to the public transport network robustness by providing alternative connections in case of service disruption. Notwithstanding, the LRT itself might also be subject to disruptions and hence impose additional risk for system performance in case of service failure. The scenario of normal operations without the LRT service, \((M_0, 0)\) is fundamentally different from the scenario with a breakdown of the LRT service, \((M_+, 22)\). Passengers travel decisions are based on a different set of network prior knowledge and real-time information provision. The dynamic representation of public transport network operations and passenger route choice makes it possible to distinguish between these two scenarios.

4. RESULTS

4.1 Passenger welfare analysis

As evident in Figure 4, either generalized in-vehicle time (32-41% of the generalized travel cost) or generalized waiting time (31-38% of the generalized travel cost) are the largest travel time component in all scenarios, although neither of them dominates the generalized travel time. Exceptionally long waiting times are observed in scenarios where knock-on effects result in denied boarding, such as disruptions on line 4 and on the critical segments. As expected, the highest on-board crowding conditions are observed in case of a disruption on one of the most saturated lines – metro lines 13, 14, 18 and 19 – during which large passenger flows condense on fewer vehicles.

Figure 4. Generalized travel time components for each scenario
Generalized walking time per passenger is highly rigid in the Stockholm network and remains approximately 6.5 minutes in all scenarios. This is expected since access and egress times, which dominate walking times, are not affected by service disruptions. Transfer penalties account for approximately six minutes (11-15% of the generalized travel cost), with the exception of scenario ($M_\star$, A↔F) where the new cross-radial LRT line, which may be used for bypassing the disrupted segment, induces additional transfers that amount to 9 minutes of generalized travel cost.

In general, average in-vehicle and waiting time components decrease in the presence of the LRT. A breakdown of the LRT leads to an increase of both in-vehicle time and waiting time (72 seconds and 28 seconds per passenger, respectively) compared to their baseline levels prior to the introduction of the LRT.

The total generalized travel time per passenger and the total passenger welfare $W(M, \delta)$ according to Eq. (1) for each scenario are summarized in Table 1. Based on the Swedish value of time (91.63 SEK per hour, approximately 10 € as of April 2015), the total passenger welfare under normal operations in the peak morning hour amounts to a loss of 11.4 million SEK. Compared to the baseline network, the LRT leads to a benefit of 273 thousand SEK during a single rush hour. Note that these gains refer only to demand originating and destined along the baseline network and hence do not account for demand generated elsewhere, including along the LRT corridor or induced demand due to the introduction of a new service.
Table 1. Passenger travel time and welfare consequences for each network disruption scenario

| Scenario (M, δ) | Generalized travel time per passenger [sec] | Total welfare W(M, δ) [SEK] | Cost of disruption C(δ | M) [SEK] | Value of robustness VRob(ΔM | M₀, δ) [SEK] | Value of redundancy VRed(ΔM | M₀, δ) [SEK] |
|-----------------|---------------------------------------------|-----------------------------|---------------------------------|----------------------------------|---------------------------------|
| Baseline network |                                             |                             |                                 |                                  |                                 |
| (M₀, 0)         | 2819                                        | -11,383,363                 |                                 |                                  |                                 |
| (M₀, 10)        | 2875                                        | -11,606,955                 | -223,092                        |                                  |                                 |
| (M₀, 11)        | 2873                                        | -11,599,440                 | -215,577                        |                                  |                                 |
| (M₀, 13)        | 2957                                        | -11,941,198                 | -557,334                        |                                  |                                 |
| (M₀, 14)        | 2985                                        | -12,052,195                 | -668,332                        |                                  |                                 |
| (M₀, 17)        | 2866                                        | -11,572,942                 | -189,079                        |                                  |                                 |
| (M₀, 18)        | 2945                                        | -11,891,949                 | -508,086                        |                                  |                                 |
| (M₀, 19)        | 2976                                        | -12,015,414                 | -631,551                        |                                  |                                 |
| (M₀, L↔S)       |                                             |                             |                                 |                                  |                                 |
| (M₀, 0)         | 2901                                        | -11,711,911                 | -328,048                        |                                  |                                 |
| (M₀, 10)        | 2883                                        | -11,641,797                 | -257,934                        |                                  |                                 |
| (M₀, 11)        | 2864                                        | -11,563,639                 | -179,776                        |                                  |                                 |
| (M₀, 13)        | 2938                                        | -11,863,481                 | -479,618                        |                                  |                                 |
| (M₀, 14)        | 2976                                        | -12,015,414                 | -631,551                        |                                  |                                 |
| (M₀, 17)        | 2900                                        | -11,711,911                 | -328,048                        |                                  |                                 |
| (M₀, 18)        | 2937                                        | -11,858,088                 | -474,225                        |                                  |                                 |
| (M₀, 19)        | 2945                                        | -11,891,949                 | -508,086                        |                                  |                                 |
| (M₀, L↔F)       | 2937                                        | -11,858,088                 | -474,225                        |                                  |                                 |
| (M₀, A↔F)       | 2893                                        | -11,681,852                 | -297,989                        |                                  |                                 |
| Extended network with LRT |                             |                             |                                 |                                  |                                 |
| (M₁, 0)         | 2752                                        | -11,111,071                 | 272,792                         |                                  |                                 |
| (M₁, 10)        | 2826                                        | -11,412,185                 | -301,114                        | 194,770                          | -78,022                        |
| (M₁, 11)        | 2855                                        | -11,529,454                 | -418,383                        | 69,986                           | -202,806                        |
| (M₁, 13)        | 2952                                        | -11,918,631                 | -807,560                        | 22,567                           | -250,225                        |
| (M₁, 14)        | 2912                                        | -11,755,874                 | -644,803                        | 296,321                          | 23,529                          |
| (M₁, 17)        | 2820                                        | -11,385,047                 | -273,976                        | 187,895                          | -84,897                        |
| (M₁, 18)        | 2809                                        | -11,746,548                 | -635,477                        | 145,401                          | -127,391                        |
| (M₁, 19)        | 2828                                        | -11,823,095                 | -712,024                        | 192,319                          | -80,473                        |
| (M₁, L↔S)       | 2933                                        | -11,842,802                 | -731,731                        | 15,286                           | -257,506                        |
| (M₁, A↔F)       | 2712                                        | -11,307,788                 | -196,717                        | 374,064                          | 101,272                        |
| (M₁, 22)        | 2923                                        | -11,803,229                 | -692,158                        |                                  |                                 |

4.3 Value of robustness

The value of the new LRT for network robustness is assessed by calculating the welfare savings under each disruption scenario, \(VRob(\Delta M \mid M₀, \delta)\), according to Eq. (3). Total passenger welfare \(W(M, \delta)\) under each operational scenario with and without the LRT line are contrasted in Figure 5. It is evident that the network extension does not only result in lower total generalized travel cost under normal operations but for all disruption scenarios. Thus, the results clearly suggest that the network extension increases network robustness.
Figure 5. Total welfare for each service disruption and network scenario

The fifth column of Table 1 further shows that the LRT is particularly effective in increasing network robustness to disruptions on bus trunk line 1 and the critical segment on the Green Line (A↔F). In these two cases, the new connections between the hubs of Alvik, Liljeholmen and Gullmarsplan (Figure 3) are especially instrumental in effectively maintaining welfare levels.

The LRT could itself be subject to service breakdowns. A disruption on the LRT results in a welfare loss of 690 thousand SEK in the peak hour, greater than any other line except for metro line 13. The welfare in the event of a LRT disruption is lower than in the undisrupted network without LRT ($M_0$, 0). In other words, passengers’ reliance on the LRT leads to a welfare loss compared to if it was non-existent in the first place. This phenomenon stems from the delay of on-board travellers as well as the additional travel costs induced by rerouting. Hence, in order to determine whether the overall contribution of a new line is positive, the consequences in the event of a disruption on the new line have to be taken into consideration.

The total value of robustness of the LRT is estimated across all line disruption scenarios according to Eq. (5). It is assumed here that the probability of a service disruption on a particular line is proportional to the route length of the line. In other words, longer lines are more likely to be disrupted than short lines. Assuming that a single line disruption occurs during the analysis period, the average robustness value across individual line disruptions, weighted by the respective share of route-km, amounts to 161 thousand SEK. However, the total robustness value has to account also for the impact of a disruption of the LRT itself (Eq. 5). Based on the same probability per route-km, LRT disruptions amount to a cost of 24 thousand SEK, which yields an overall robustness value of 128 thousand SEK.

Assuming that each route-km is disrupted once a year during the rush hour (equivalent to identical and independent failure probabilities of 0.00274), the total robustness value of the LRT is 25.7 million SEK when only rush hour operations are considered. This robustness value is greater than the total generalized travel cost associated with two days of rush hour operations, across the network. Furthermore, these benefits constitute 25% of the annual benefits attributed to the LRT due to travel time savings under normal operations.
4.4 Value of redundancy

The fourth column of Table 1 shows that both the baseline and the extended network are most vulnerable to disruptions on metro lines 13, 14, 18 and 19, which are the most heavily used and serve as the backbone of the public transport system (Figure 2). Disruption of bus line 4, the busiest trunk line, also results in a significant increase in total passenger travel time. The breakdown of critical segments of the Red and Green metro lines, which are traversed by several lines, has severe consequences that are comparable and in some cases even worse than the disruption of lines. These are considerable losses considering that the breakdown lasts only 30 minutes.

Note that line criticality changes due to the network extension. For example, line 1 becomes less critical than line 2 in the extended network whereas the reversed is true in the base case. While the extended network is less vulnerable to both disruptions than the baseline network, this change in criticality has implications for the prioritization of future robustness investments and resource allocation.

Figure 6 presents the total welfare loss $\mathcal{C}(\delta \mid M)$ in each disruption scenario according to Eq. (4) for the baseline and extended networks. As can be seen, disruption impacts are not consistently lower in the extended network than in the base network; in fact, the disruption cost is greater in the extended network for the majority of scenarios. This result may seem paradoxical in light of the result in Section 4.3 that the network extension increases welfare across all scenarios. However, the result stems from the fact that the LRT does not increase welfare to the same extent during disruptions compared to normal operating conditions. In other words, the new line cannot be fully utilized as an alternative travel route by passengers affected by the disruptions. As the rightmost column of Table 1 shows, this implies that the value of redundancy of the LRT (Eq. 7) is negative for those scenarios.

![Figure 6. Welfare loss per disruption scenario with and without the LRT line](image)

For example, when one of the Green metro lines is disrupted (line 17, 18 or 19), generalized travel costs are lower in the presence of the LRT (Figure 5), whereas the welfare loss compared with normal operations is greater (Figure 6). With the LRT, some passengers bypass the disrupted segment by switching from the metro to the lower capacity LRT line,
leading to knock-down effects. Similarly to the results reported by Cats and Jenelius (2014), having access to more information – in this case concerning alternative travel paths – can result in a greater welfare loss due to network effects.

The total value of redundancy across all line disruption scenarios is calculated according to Eq. (8). Based on the same probabilities as in Section 4.3, the total resilience value is -22.3 million SEK, assuming that each route-km is disrupted once a year during the rush hour. The fact that the value is negative means that the total cost of disruptions on other lines increase with the LRT when assessed against the reference value (i.e. higher total welfare under baseline scenario).

5. DISCUSSION AND CONCLUSION

The argument that new public transport links will add redundancy and increase the robustness of the transport system is sometimes made to motivate new investments, but sound methodologies to evaluate such benefits in relation to traditional increases in welfare (due to shorter travel times, fewer transfers, etc.) and investment costs have been lacking. The paper proposed such a method, where robustness effects are evaluated in terms of passenger welfare under disruptions. By considering a full space of disruption scenarios and the frequency of occurrence of each scenario, the methodology can be used to validate whether a new infrastructure project will add to the robustness of the system, and to help decision-makers prioritize among alternative projects with robustness effects in mind. By using a model that considers passengers’ dynamic travel choices, stochastic traffic conditions, timetables and capacity constraints, the results are more nuanced than analyses based only on network topology and other static attributes.

The approach was demonstrated by evaluating the robustness value of a new light rail transit line in Stockholm, Sweden. The results show that the LRT increases total welfare under normal operating conditions as well as under disruptions of every other line. Disruption of the LRT itself leads to considerable cost; in fact, passengers’ reliance on the LRT leads to a welfare loss compared to the baseline scenario without the LRT. However, the welfare increases under other disruptions outweigh this cost to yield a strongly positive value of robustness.

On the other hand, a majority of disruption scenarios yield larger losses of welfare (although from a higher level) in the extended network than the baseline network. This is a seemingly paradoxical result, similar to the famous Braess’ paradox, which arises from passengers switching to the low capacity LRT line and their inability to fully utilize the available network capacity in the short term following unexpected disruption. The outcome also indicates that the LRT serves as a complement to other public transport lines to a greater extent than as a substitute. As a result, the total value of redundancy of the LRT is negative.

Conclusions from the case study can be extended to other public transport networks with similar characteristics. In general, a new link or line is expected to have positive welfare effects under normal conditions as well as most disruption scenarios because of shorter travel times, fewer transfers, etc., for some passengers. However, the value of the network extension as an alternative during disruption depends on its role in relation to the disrupted link (complement or substitute) and the ability of passengers to adapt to the conditions (depending on real-time information provision etc.). As the case study shows, the cost of disruptions may be higher in the extended network. Thus, the answer to the question whether a new infrastructure investment will increase transport network robustness depends on the precise definition of robustness in mind: whether it is the general level of welfare during disruptions (defined here as the value of robustness) or the welfare losses compared to normal conditions.
during disruptions (the value of redundancy). In the authors’ opinion, both measures are needed to see the full picture.

The results highlight the benefit of using a dynamic public transport network model considering demand and supply interactions to evaluate network robustness to unexpected, temporary disruptions. A topological graph model, which considers only connectivity or shortest distances between stations, or a simple static model, which assumes that passengers reroute immediately and optimally, would likely systematically overestimate the robustness benefits of new infrastructure.

Although the case study used a full-scan approach and considered disruptions of every public transport line in the network, the set of scenarios was limited to one particular level of service frequency reduction. An important direction for further work is to study the impact of the severity of frequency reduction on passenger welfare impacts. This could give insights into whether priority should be put on mitigating rare but severe degradations, or frequent but moderate degradations. Cats et al. (2015) demonstrated the importance of accounting for exposure to disruptions and its determinants, defined as the expected share of time that a network element is disrupted depending on the disruption frequency and duration, on the identification of critical links and the expected welfare loss. Future research may account for differences between links and lines in their exposure to disruptions.

Another interesting area for further research is the incorporation of robustness aspects in network design. While this paper studied the value of a given new line and new set of links in an existing network, one could test alternative network structures. A potential approach could iteratively add and remove links as a method to test optimal network evolution (Ash and Newth, 2007).

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